Istituto Nazionale di Fisica Nucleare
Sezione di Napoli

Theory of rare kaon decays

Giancarlo D'Ambrosio
INFN Sezione di Napoli

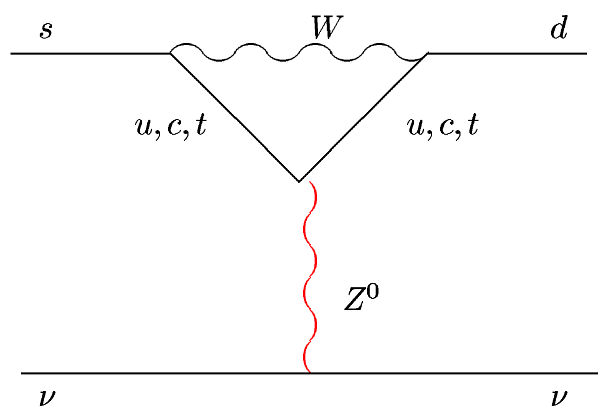
Outline

- Rare Kaon decays and MFV
- $K^+ \to \pi^+ l^+ l^- / K_S \to \pi^0 l^+ l^-$
- $K^+ \to \pi^+ \pi^0 l^+ l^-$ and other decays channels
- Conclusions

$K \to \pi\nu\overline{\nu}$

Why we need KOTO and NA62 experiments

$$A(s \to d\nu\overline{\nu})_{\rm SM} \sim \overline{s}_L\gamma_\mu d_L \quad \overline{\nu}_L\gamma^\mu\nu_L \times \left[\sum_{q=c,t} V_{qs}^* V_{qd}\, m_q^2\right]$$



$$\sim \quad \left[A^2\lambda^5\ (1-\rho-i\eta)m_t^2 + \lambda m_c^2\right]$$

SM $\quad \underbrace{V-A \otimes V-A}$ Littenberg

$\Downarrow$

$$\Gamma(K_L \to \pi^0\nu\overline{\nu}) \quad \begin{cases} \text{CP violating} \\ \Rightarrow\ J = A^2\lambda^6\eta \\ \text{Only } top \end{cases}$$

$K^+ \to \pi^+ \nu\bar{\nu}$

Misiak, Urban; Buras, Buchalla; Brod, Gorbhan, Stamou'11, Straub

$$B(K^+) \sim \kappa_+ \left[\left(\frac{\mathrm{Im}\lambda_t}{\lambda^5} X_t \right)^2 + \left(\frac{\mathrm{Re}\lambda_c}{\lambda} (P_c + \delta P_{c,u}) + \frac{\mathrm{Re}\lambda_t}{\lambda^5} X_t \right)^2 \right]$$

$\lambda_q = V_{qd}{}^* V_{qs}$

- κ_+ from K_{l3}
- P_c: SD charm quark contribution (30%±2.5% to BR)
  LD $\delta P_{c,u} \sim 4 \pm 2\%$
- $B(K^{\pm}) = (7.8 \pm 0.8 \pm 0.3) \times 10^{-11}$ first error parametric (V_{cb}), second non-pert. QCD
- E949 $B(K^{\pm}) = (1.73^{+1.15}_{-1.05}) \times 10^{-10}$

K_L

2.43+-0.39+-0.06

$$B(K_L) = (2.43 \pm 0.25 \pm 0.06) \times 10^{-11}$$ vs

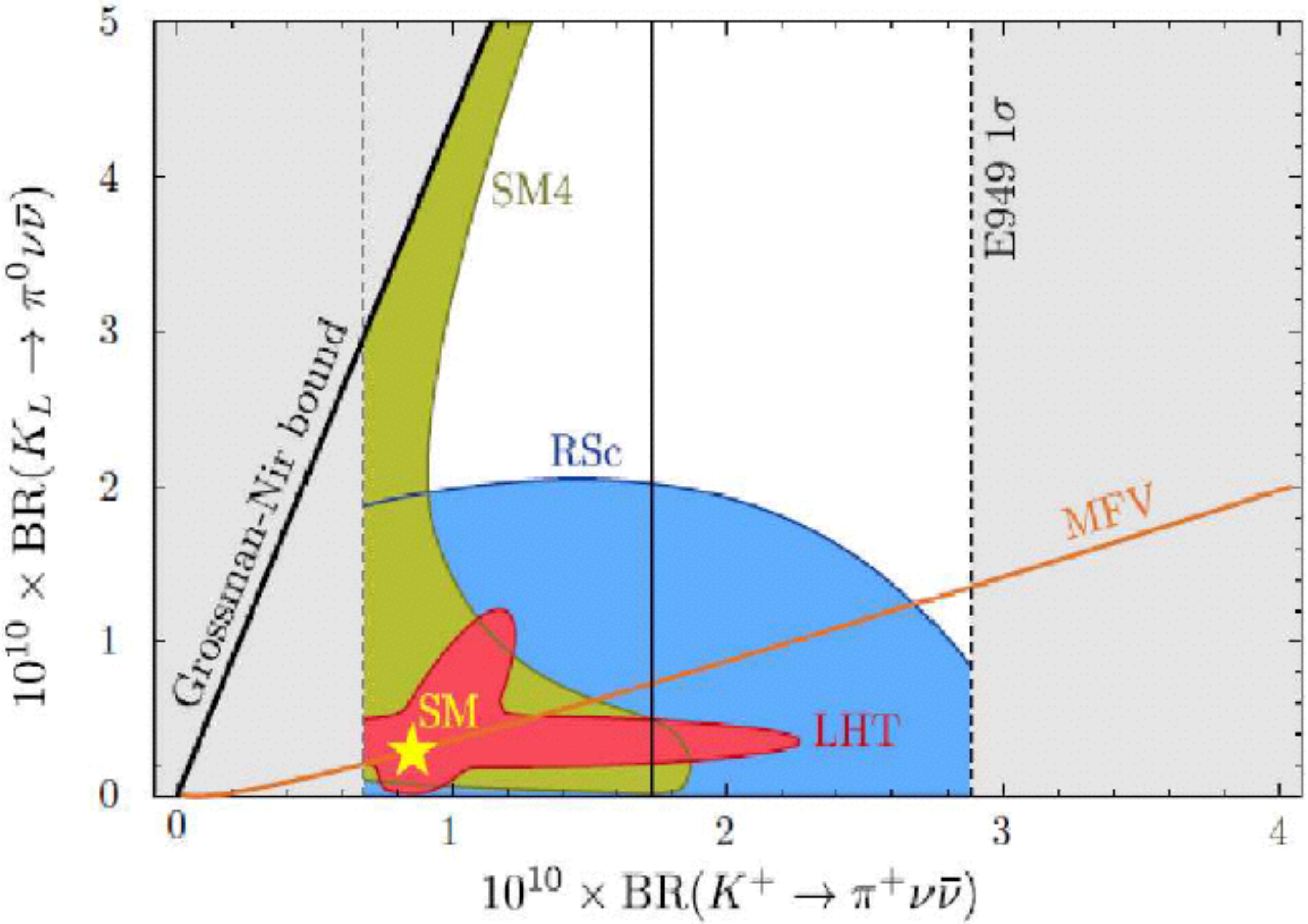
E391a $B(K_L) < 2.6 \times 10^{-8}$ at 90% C.L.

K_L Model-independent bound, based on $SU(2)$ properties dim-6 operators for $\bar{s}d\bar{\nu}\nu$ Grossman-Nir

$$B(K_L) \leq \frac{\tau_L}{\tau_+} \times B(K^{\pm})_{\rm E949} \leq 1.4 \times 10^{-9} \quad \text{at } 90\% C.L.$$

Loophole Fuyuto et al

NA62 , KOTO



Also Z' Buras et al,
Kneeglens Moriond 2015
Yamamoto et al 2015
and M.Blanke rev

Straub, CKM 2010 workshop (arXiv:1012.3893v2)

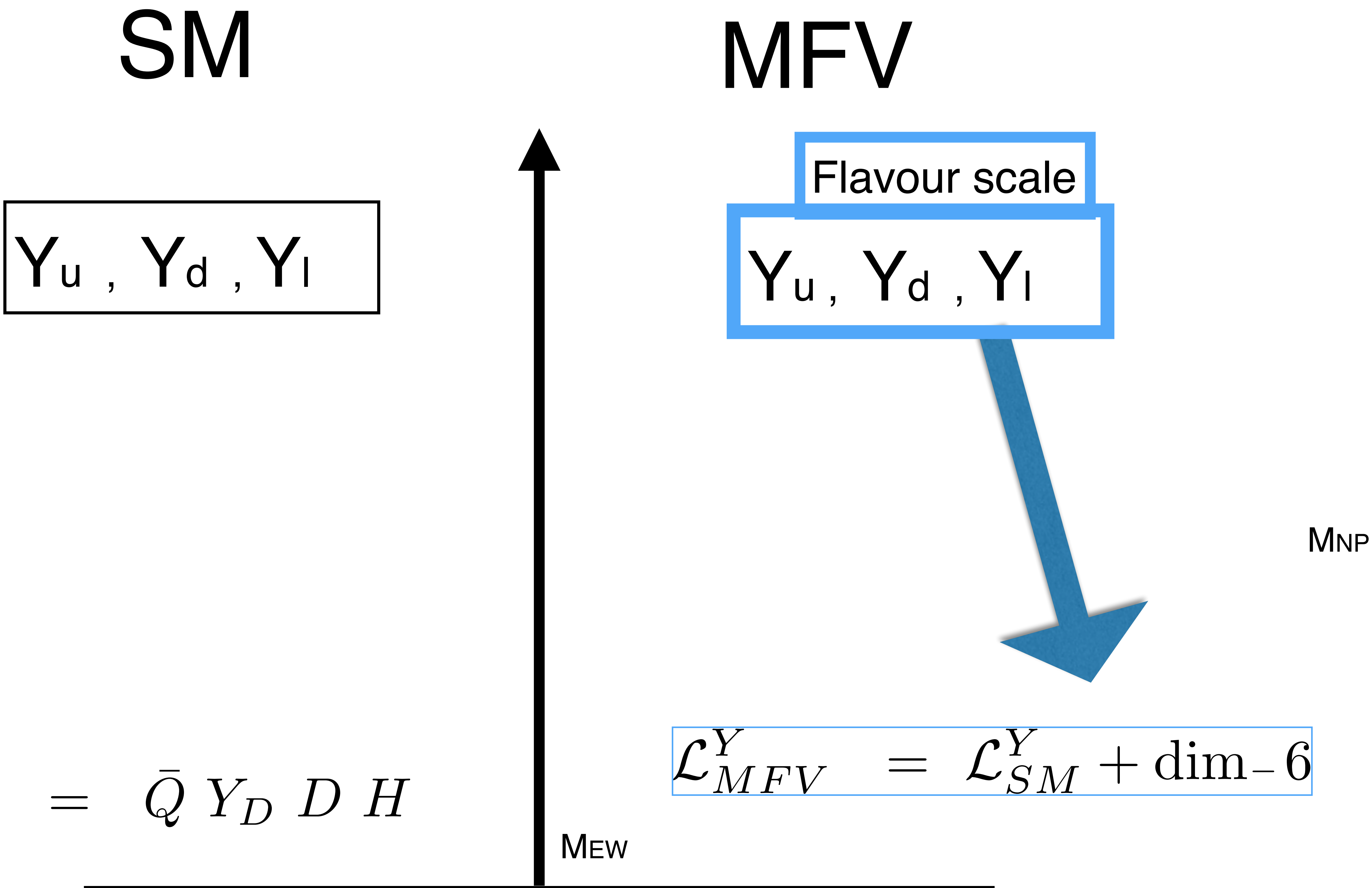
Generic Flavor structures strongly constrained

Operator	Bounds on Λ in TeV ($c_{NP}=1$) Re	Bounds on Λ in TeV ($c_{NP}=1$) Im	Bounds on c_{NP} ($\Lambda = 1$ TeV) Re	Bounds on c_{NP} ($\Lambda = 1$ TeV) Im	Observables
$(\bar{s}_L\gamma^\mu d_L)^2$	9.8×10^2	1.6×10^4	9.0×10^{-7}	3.4×10^{-9}	Δm_K; ϵ_K
$(\bar{s}_R d_L)(\bar{s}_L d_R)$	1.8×10^4	3.2×10^5	6.9×10^{-9}	2.6×10^{-11}	
$(\bar{c}_L\gamma^\mu u_L)^2$	1.2×10^3	2.9×10^3	5.6×10^{-7}	1.0×10^{-7}	Δm_D; $\lvert q/p\rvert_D$, ϕ_D
$(\bar{c}_R u_L)(\bar{c}_L u_R)$	6.2×10^3	1.5×10^4	5.7×10^{-8}	1.1×10^{-8}	
$(\bar{b}_L\gamma^\mu d_L)^2$	6.6×10^2	9.3×10^2	2.3×10^{-6}	1.1×10^{-6}	Δm_{B_d}; $\sin(2\beta)$ from $B_d\to\psi K$
$(\bar{b}_R d_L)(\bar{b}_L d_R)$	2.5×10^3	3.6×10^3	3.9×10^{-7}	1.9×10^{-7}	
$(\bar{b}_L\gamma^\mu s_L)^2$	1.4×10^2	2.5×10^2	5.0×10^{-5}	1.7×10^{-5}	Δm_{B_s}; $\sin(\phi_s)$ from $B_s\to\psi\phi$
$(\bar{b}_R s_L)(\bar{b}_L s_R)$	4.8×10^2	8.3×10^2	8.8×10^{-6}	2.9×10^{-6}	

Isidori Nir Perez 10

Problem already known since ’86 technicolour (Chivukula Georgi) susy (Hall Randall) extra dimensions (Rattazzi Zafferoni)

Maybe there is an energy gap between the theory of flavor and the EW scale , ameliorating also a clash from the scale of the bounds in the table above and the requirement of solving the hierarchy problem



$$\mathcal{L}_{SM}^{Y} = \bar{Q}\; Y_D\; D\; H$$

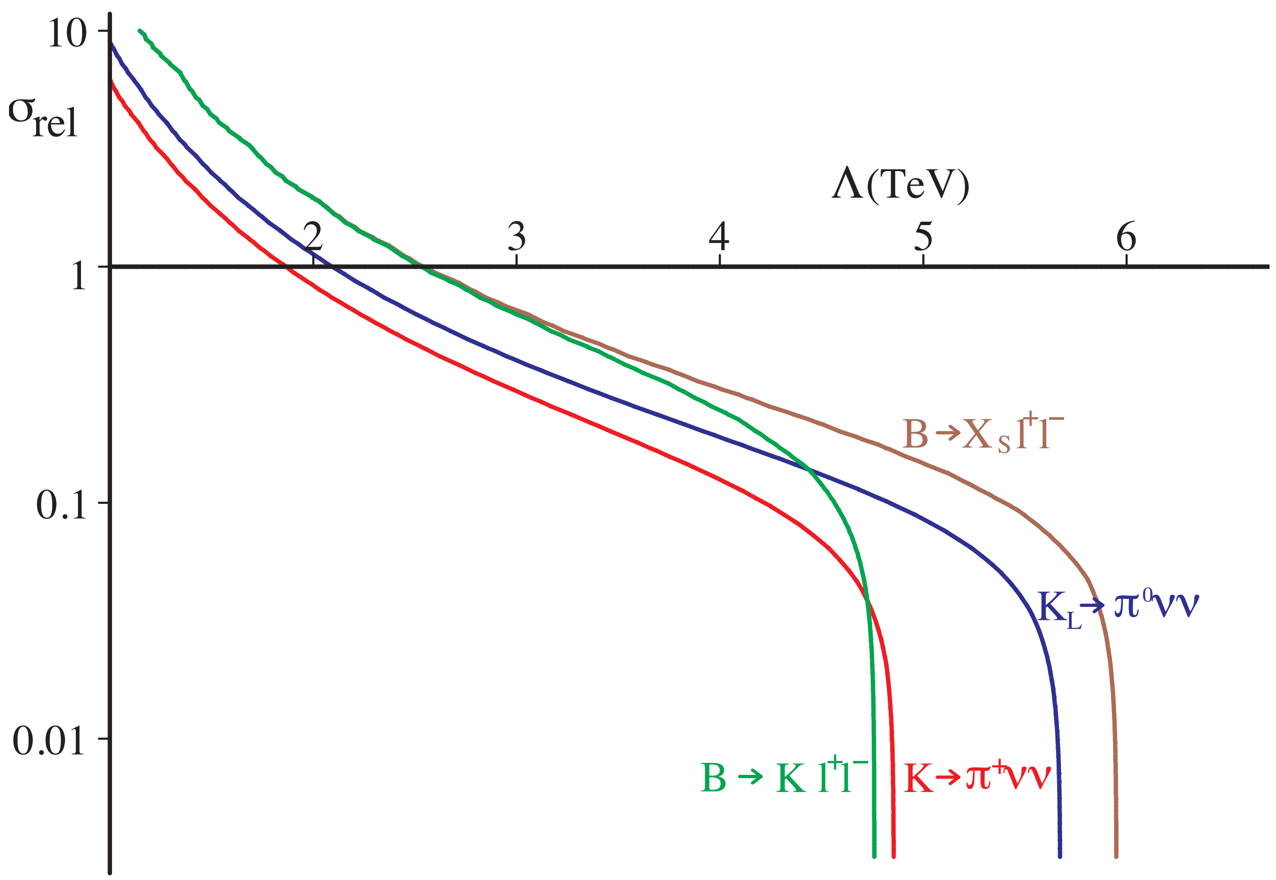
$$\mathcal{L}_{MFV}^{Y} = \mathcal{L}_{SM}^{Y} + \text{dim-}6$$

$$G_F = \overbrace{\mathrm{U}(3)_Q \otimes \mathrm{U}(3)_U \otimes \mathrm{U}(3)_D \otimes \mathrm{U}(3)_L \otimes \mathrm{U}(3)_E}^{\text{global symmetry}} + \overbrace{Y_{U,D,E}}^{\text{spurions}}$$

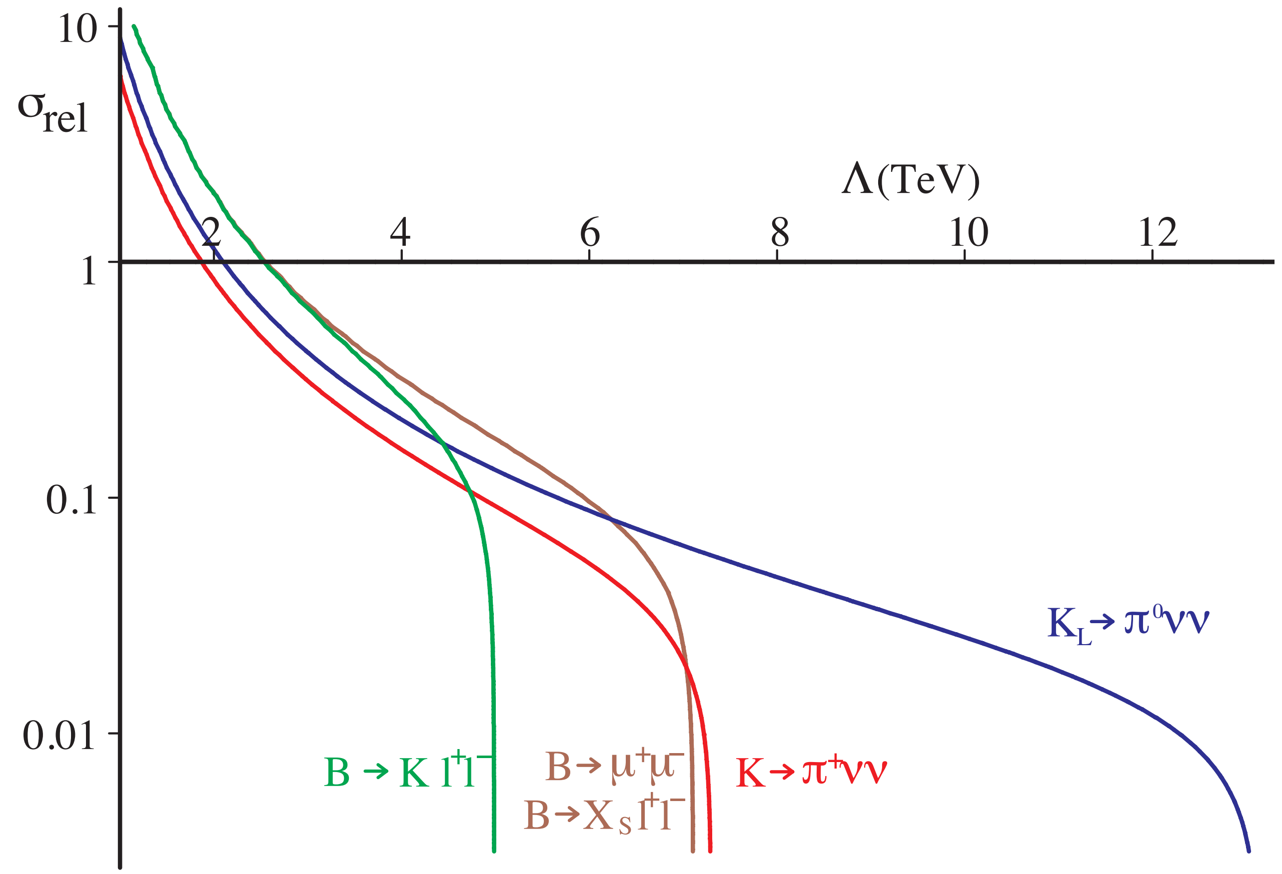
Bounds ameliorated

Minimally flavour violating dimension six operator	main observables	Λ [TeV] $-$	$+$	
$\mathcal{O}_0 = \frac{1}{2}(\bar{Q}_L\lambda_{\rm FC}\gamma_\mu Q_L)^2$	$\epsilon_K, \quad \Delta m_{B_d}$	6.4	5.0	
$\mathcal{O}_{F1} = H^\dagger\left(\bar{D}_R\lambda_d\lambda_{\rm FC}\sigma_{\mu\nu}Q_L\right)F_{\mu\nu}$	$B \to X_s\gamma$	8.3	13.4	
$\mathcal{O}_{G1} = H^\dagger\left(\bar{D}_R\lambda_d\lambda_{\rm FC}\sigma_{\mu\nu}T^a Q_L\right)G^a_{\mu\nu}$	$B \to X_s\gamma$	2.3	3.8	
$\mathcal{O}_{\ell 1} = (\bar{Q}_L\lambda_{\rm FC}\gamma_\mu Q_L)(\bar{L}_L\gamma_\mu L_L)$	$B \to (X)\ell\bar{\ell}, \quad K \to \pi\nu\bar{\nu}, (\pi)\ell\bar{\ell}$	3.1	2.7	*
$\mathcal{O}_{\ell 2} = (\bar{Q}_L\lambda_{\rm FC}\gamma_\mu\tau^a Q_L)(\bar{L}_L\gamma_\mu\tau^a L_L)$	$B \to (X)\ell\bar{\ell}, \quad K \to \pi\nu\bar{\nu}, (\pi)\ell\bar{\ell}$	3.4	3.0	*
$\mathcal{O}_{H1} = (\bar{Q}_L\lambda_{\rm FC}\gamma_\mu Q_L)(H^\dagger iD_\mu H)$	$B \to (X)\ell\bar{\ell}, \quad K \to \pi\nu\bar{\nu}, (\pi)\ell\bar{\ell}$	1.6	1.6	*
$\mathcal{O}_{q5} = (\bar{Q}_L\lambda_{\rm FC}\gamma_\mu Q_L)(\bar{D}_R\gamma_\mu D_R)$	$B \to K\pi, \quad \epsilon'/\epsilon, \ldots$	~ 1		

GD,Giudice,Isidori,Strumia



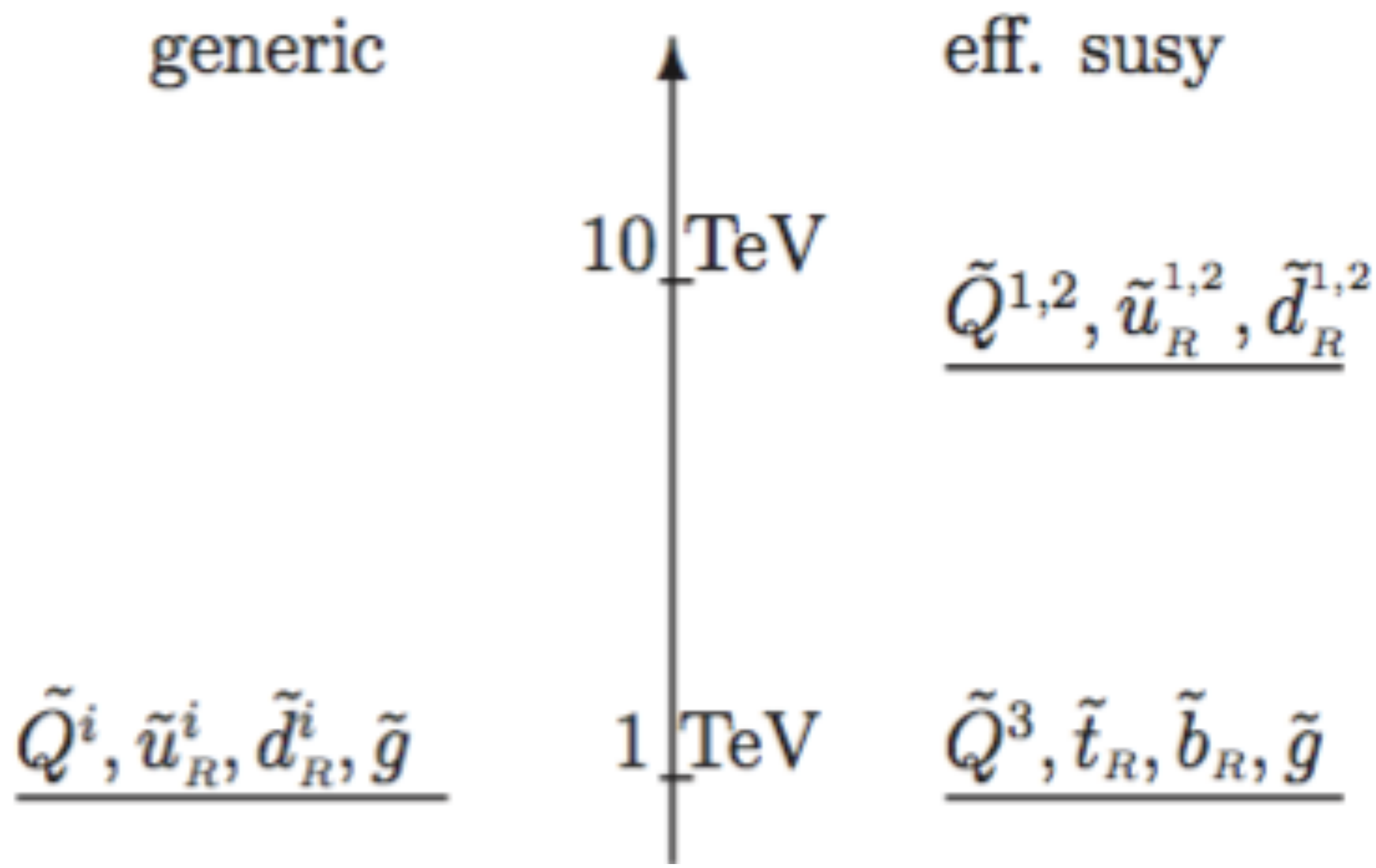
10% Overall CKM error



1% Overall CKM error

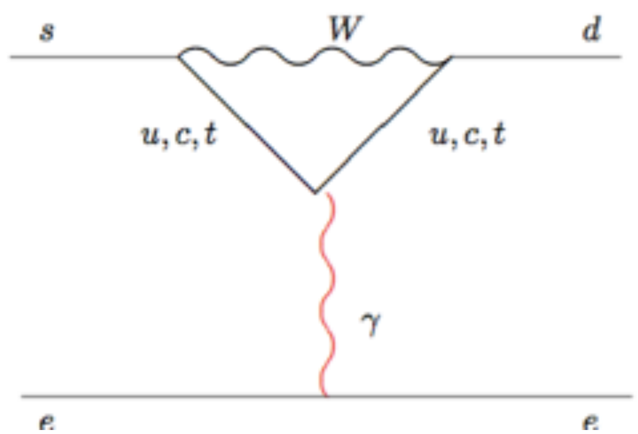
Kaplan et al, Dimopoulos Giudice, Barbieri et al , U(2)

susy spectrum

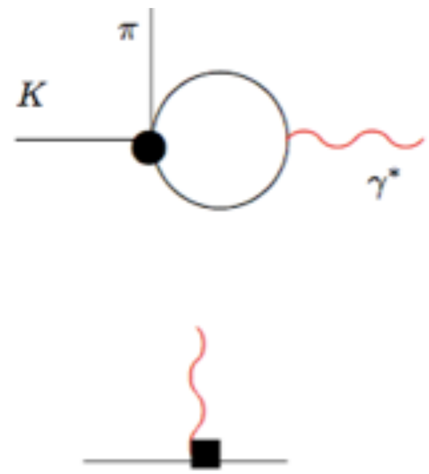


$K^+ \to \pi^+ e^+ e^-$ $K_S \to \pi^0 e^+ e^-$

SD



CHPT



CHPT

$$W^i = G_F m_K^2 (a_i + b_i z) + W^i_{\pi\pi}(z$$

$$i = \pm, S$$

$$a_i, b_i \sim O(1), \qquad z = \frac{q^2}{m_K^2}$$

- Observables $\Gamma(K^+ \to \pi^+ e^+ e^-)$, $\Gamma(K^+ \to \pi^+ \mu\overline{\mu})$, slopes
- a_i $O(p^4)$ $a_+ \sim N_{14} - N_{15}$, $a_S \sim 2N_{14} + N_{15}$ Ecker, Pich, de Rafae
- b_i $O(p^6)$ G.D., Ecker, Isidori, Portole
- a_+, b_+ in general not related to a_S, b_S

averaging flavour

$$a_+^{\text{exp.}} = -0.578 \pm 0.016$$
$$b_+^{\text{exp.}} = -0.779 \pm 0.066$$

Why testing Lepton Flavor Universality Violation (LFUV) in Kaon decays?

- Several anomalies in B-physics

[Bobeth, Hiller, Piranishvili (07)]

- LFUV from LHCb $$R(K) = \frac{\mathrm{Br}[B \to K\mu^+\mu^-]}{\mathrm{Br}[B \to K e^+ e^-]} = 0.745^{+0.090}_{-0.074} \pm 0.036$$

- P'_5 angular observable in $B \to K^* \mu^+ \mu^-$

[Descotes-Genon et al. (13 & 14); Altmannshofer & Straub (15); Jäger & Martin Camalich (16)]

- Also in semi-leptonic B-decays $B \to D(D^*)\tau\nu$ see FLAG

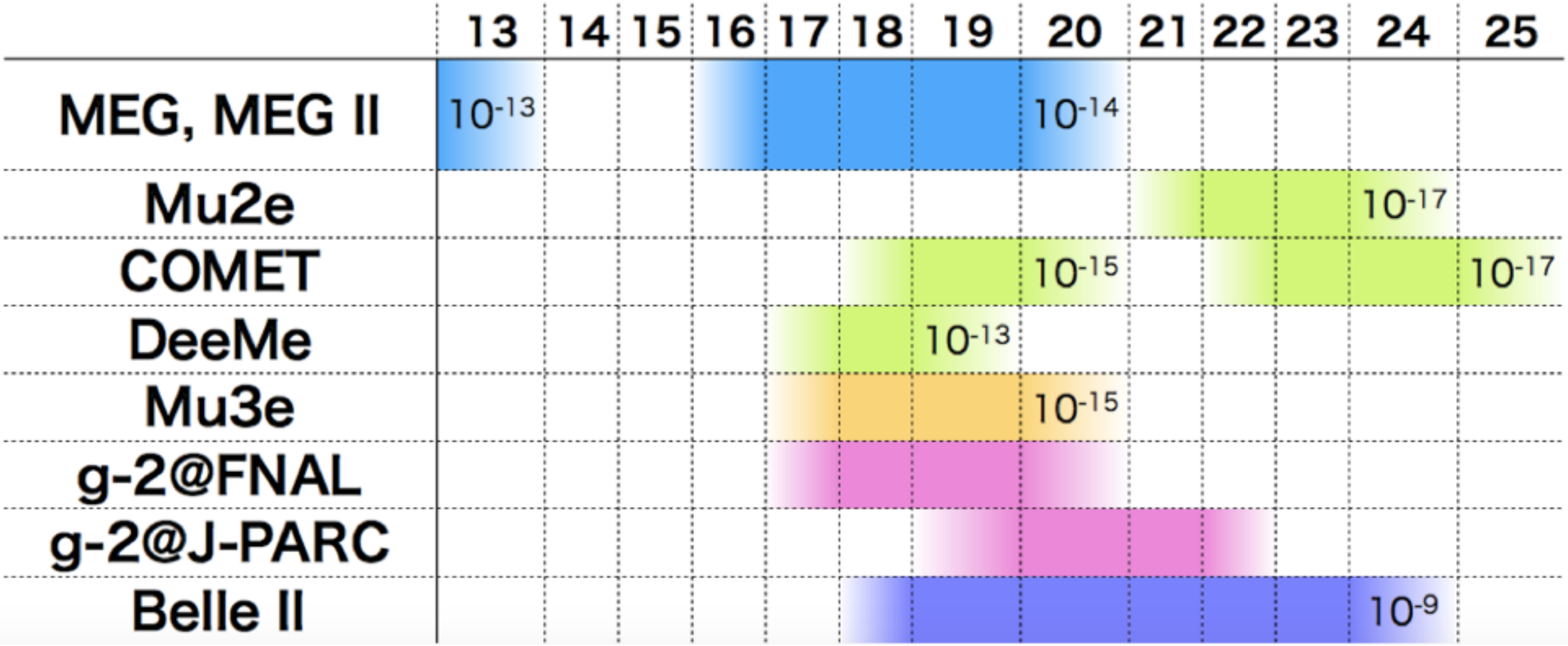
- maybe a global 3 sigma effect

[Altmannshofer & Straub (15); Descotes-Genon et al. (15)]

KAON2016

Aoki

Timelines



Work in collaboration with A. Crivellin, L.Tunstull and M. Hoferichter

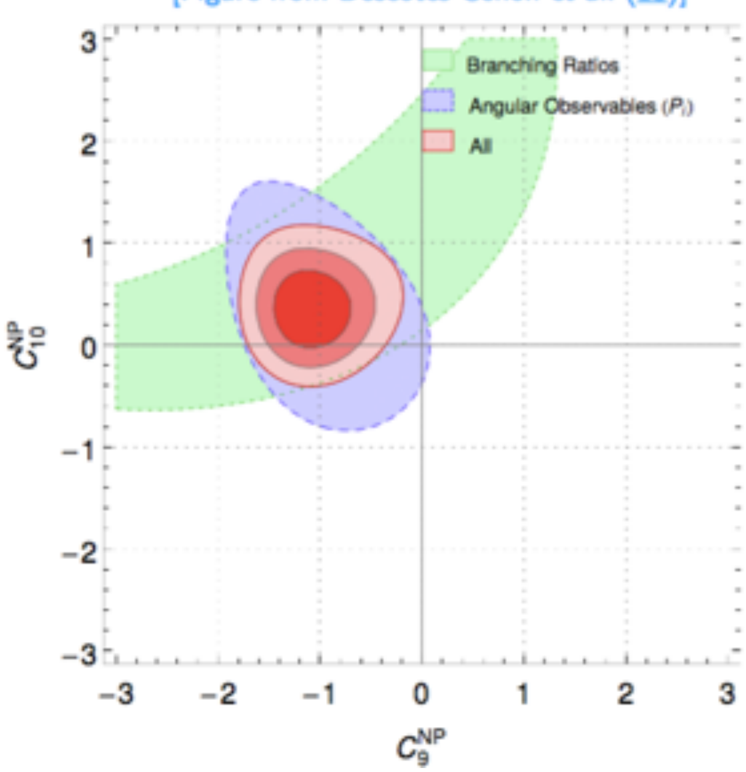
Addressing LFUV in kaon decays

$$\mathcal{H}_{\rm eff}^{|\Delta B|=1} = -\frac{4G_F}{\sqrt{2}} V_{tb}V_{ts}^* \sum_i C_i^B(\mu) Q_i^B(\mu)$$

$$Q_9^B = \frac{e^2}{32\pi^2}\left[\bar{s}\gamma^\mu(1-\gamma_5)b\right] \sum_{\ell=e,\mu}\left[\bar{\ell}\gamma_\mu\ell\right],$$

$$Q_{10}^B = \frac{e^2}{32\pi^2}\left[\bar{s}\gamma^\mu(1-\gamma_5)b\right] \sum_{\ell=e,\mu}\left[\bar{\ell}\gamma_\mu\gamma_5\ell\right].$$

[Figure from Descotes-Genon et al. (15)]



$C_{9,10}^{NP}$ ~O(1)

Assuming MFV: what we expect in kaon decays ?

$$\mathcal{L}_{\rm eff}^{|\Delta S|=1} = -\frac{G_F}{\sqrt{2}} V_{ud}V_{us}^* \sum_{i=1}^{13} C_i(\mu) Q_i(\mu)$$

$$Q_{11} \equiv Q_{7V} = \left[\bar{s}\gamma^\mu(1-\gamma_5)d\right] \sum_{\ell=e,\mu}\left[\bar{\ell}\gamma_\mu\ell\right],$$

$$Q_{12} \equiv Q_{7A} = \left[\bar{s}\gamma^\mu(1-\gamma_5)d\right] \sum_{\ell=e,\mu}\left[\bar{\ell}\gamma_\mu\gamma_5\ell\right].$$

LFUV: Kaons

Channel	a_+	b_+	Reference
ee	-0.587 ± 0.010	-0.655 ± 0.044	E865
ee	-0.578 ± 0.016	-0.779 ± 0.066	NA48/2
$\mu\mu$	-0.575 ± 0.039	-0.813 ± 0.145	NA48/2

$$a_+^{\rm NP} = \frac{2\pi\sqrt{2}}{\alpha} V_{ud} V_{us}^* * C_{7V}^{\rm NP}$$

$$C_{7V}^{\mu\mu} - C_{7V}^{ee} = \alpha \frac{a_+^{\mu\mu} - a_+^{ee}}{2\pi\sqrt{2} V_{ud} V_{us}^*} \quad \overset{MFV}{\Longrightarrow} \quad C_{9V}^{B,\mu\mu} - C_{9V}^{B,ee} = \alpha \frac{a_+^{\mu\mu} - a_+^{ee}}{2\pi\sqrt{2} V_{td} V_{ts}^*} = -19 \pm 79$$

NA62 PLEASE!!

High statistics: nominal # of decays 50 times greater than NA48/2

KLOE-2!!

Also analyzed LFV Kaon decays and $K_L \to \mu\mu \quad (C_{10}^{NP})$

$$K(p_K) \to \pi(p_1)\pi(p_2)\gamma(q)$$

- Lorentz + gauge invariance $\Rightarrow$ Electric (E) and Magnetic(M) amplitude

$$A(K \to \pi\pi\gamma) = F^{\mu\nu}\left[E\partial_\mu K\partial_\nu\pi + M\varepsilon_{\mu\nu\rho\sigma}\partial^\rho K\partial^\sigma\pi\right]$$

- Unpolarizated photons

$$\frac{d^2\Gamma}{dz_1 dz_2} \sim |E|^2 + |M|^2$$

$$|E^2| = |E_{IB}|^2 + 2Re(E^*_{IB}E_D) + |E_D|^2$$

$$\downarrow$$

Low Theorem $\Rightarrow E_{IB} \sim \frac{1}{E^*_\gamma} +$ c $\qquad E_D$, M chiral tests

We need FIGHT DE/IB$\sim 10^{-3}$

	IB	DE_{exp}	
$K_S \to \pi^+\pi^-\gamma$	10^{-3}	$< 9 \cdot 10^{-5}$	$E1$
$K^+ \to \pi^+\pi^0\gamma$	10^{-4} $(\Delta I = \frac{3}{2})$	$(0.599 \pm 0.037)10^{-5}$ NA48/2	$M1$,$E1$
$K_L \to \pi^+\pi^-\gamma$	10^{-5} (CPV)	$(2.92 \pm 0.07)10^{-5}$ KTeVnew	$M1$, VMD

CPV is only from IB K_L (also measured in $K_L \to \pi^+\pi^-e^+e^-$)

BUT IB suppressed in K^+ and K_L.

$$K^+ \to \pi^+\pi^0\gamma$$

$$A(K \to \pi\pi\gamma) = F^{\mu\nu}\left[E\partial_\mu K\partial_\nu\pi + M\varepsilon_{\mu\nu\rho\sigma}\partial^\rho K\partial^\sigma\pi\right]$$

$E1$ and $M1$ are measured with Dalitz plot

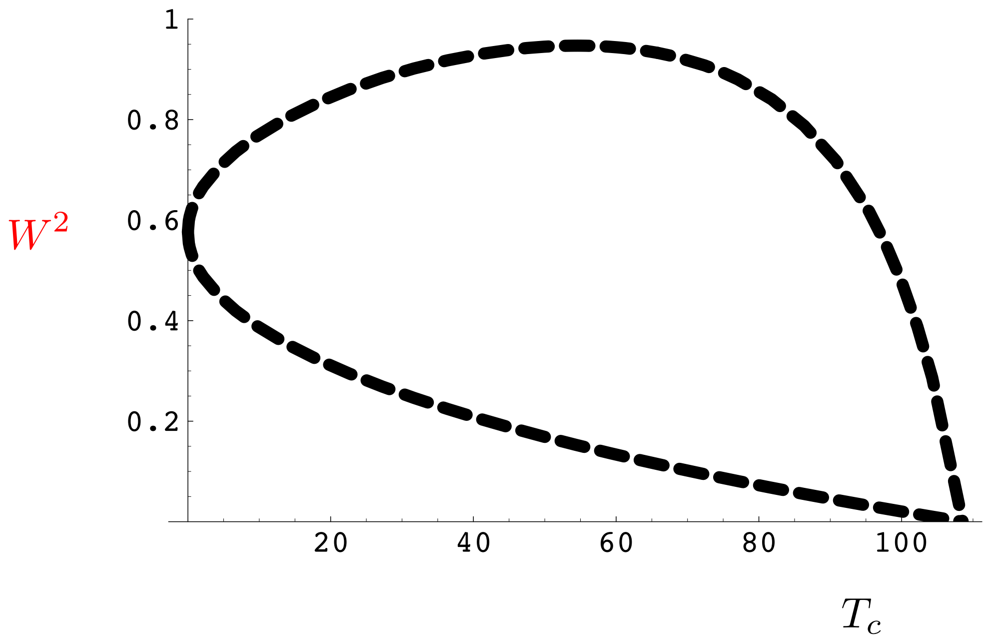
$$\frac{\partial^2\Gamma}{\partial T_c^*\partial W^2} = \frac{\partial^2\Gamma_{IB}}{\partial T_c^*\partial W^2}\left[1 + \frac{m_{\pi+}^2}{m_K}2Re\left(\frac{E1}{eA}\right)W^2 + \frac{m_{\pi+}^4}{m_K^2}\left(\left|\frac{E1}{eA}\right|^2 + \left|\frac{M1}{eA}\right|^2\right)W^4\right]$$

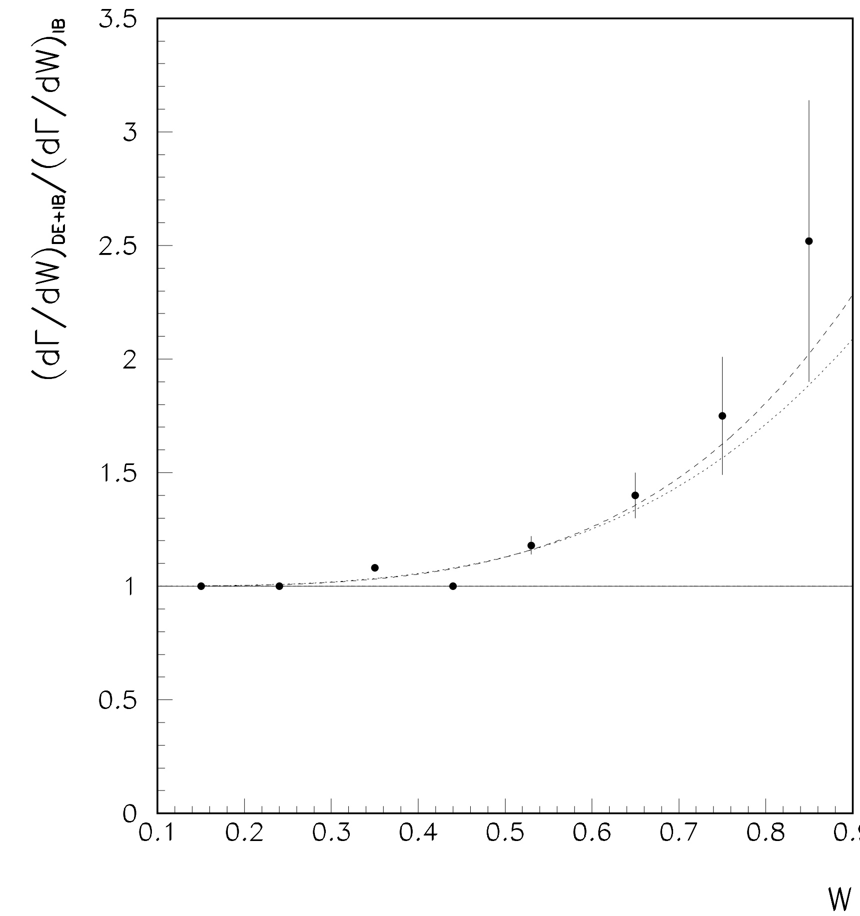
$$W^2 = (q \cdot p_K)(q \cdot p_+)/(m_\pi^2 m_K^2)$$

$$A = A(K^+ \to \pi^+\pi^0)$$

Departure from IB

$W^2 = (q \cdot p_K)(q \cdot p_+)/(m_\pi^2 m_K^2)$

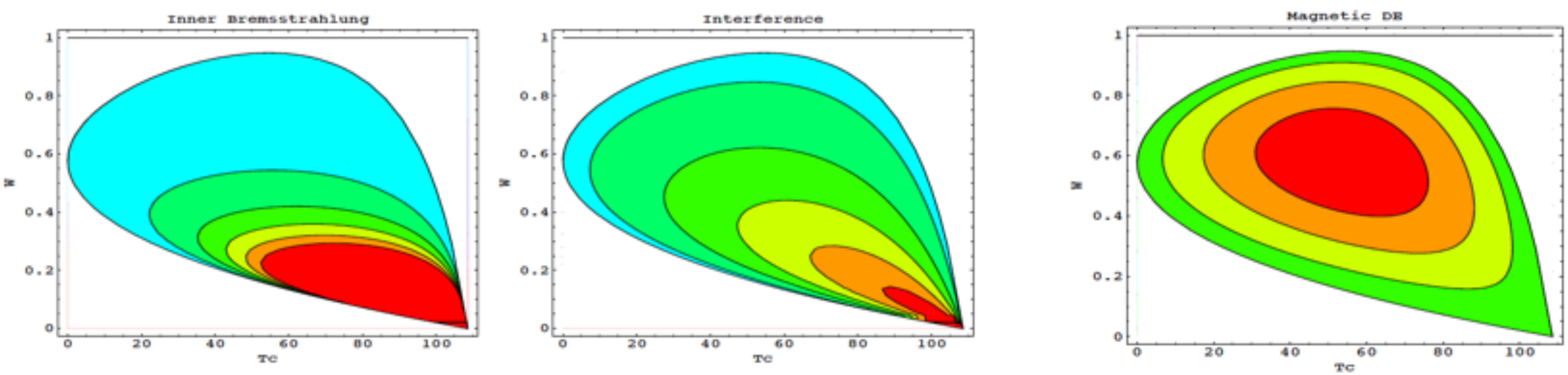




$$\frac{\partial^2\Gamma}{\partial T_c^* \partial W^2} = \frac{\partial^2\Gamma_{IB}}{\partial T_c^* \partial W^2}\left[1 + \frac{m_{\pi^+}^2}{m_K} 2Re(\frac{E1}{eA}) W^2 + \frac{m_{\pi^+}^4}{m_K^2}\left(|\frac{E1}{eA}|^2 + |\frac{M1}{eA}|^2\right) W^4\right]$$

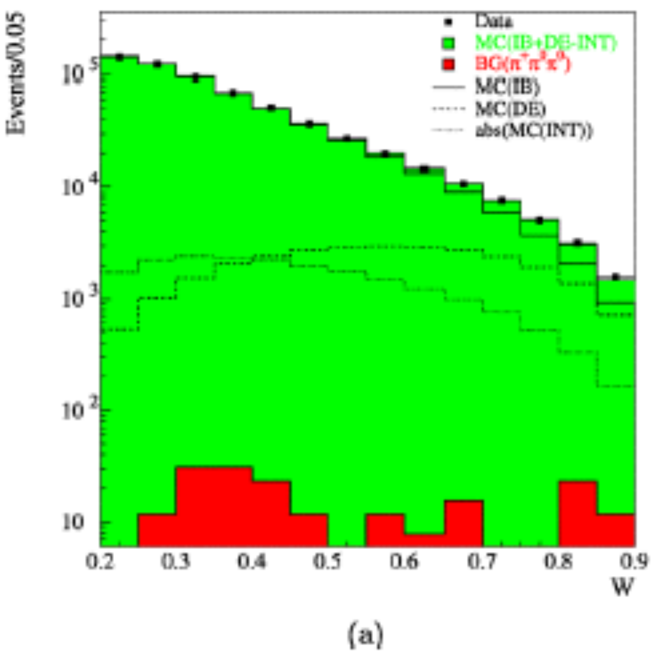
IB from Low theorem

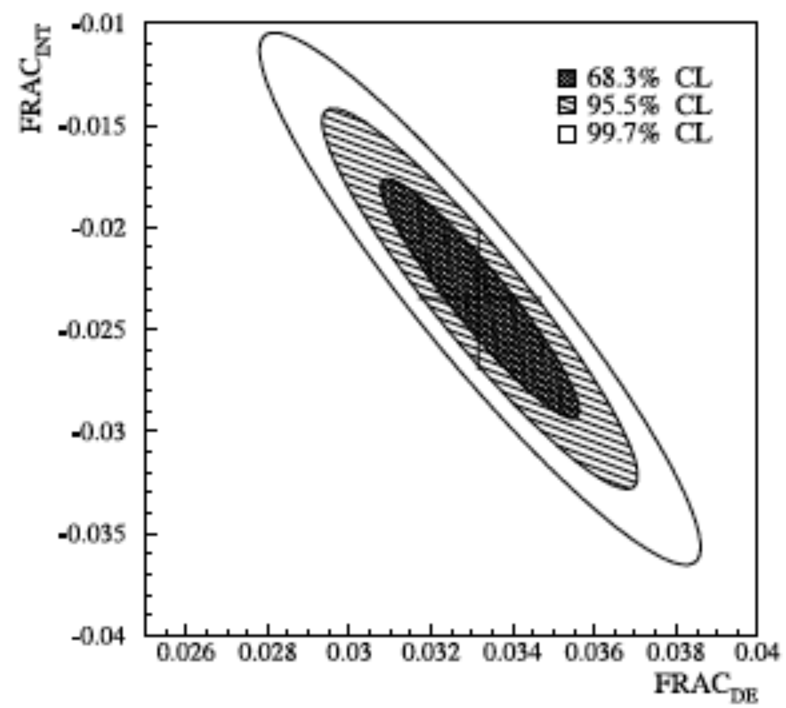
Dalitz plot

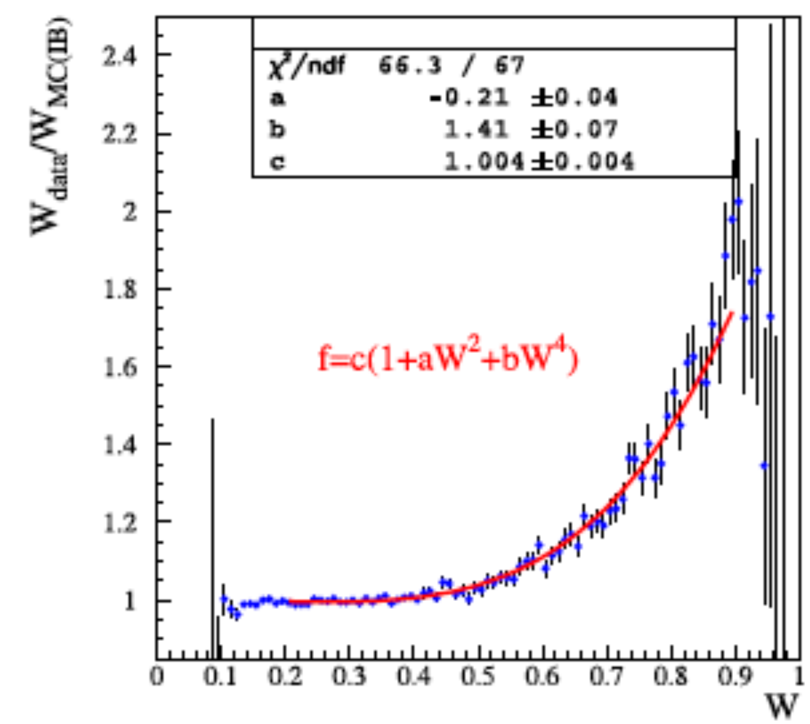


$$\frac{\partial^2 \Gamma}{\partial T_c^* \partial W^2} = \frac{\partial^2 \Gamma_{IB}}{\partial T_c^* \partial W^2} \left[1 + \frac{m_{\pi^+}^2}{m_K} 2Re\left(\frac{E1}{eA}\right) W^2 + \frac{m_{\pi^+}^4}{m_K^2} \left(\left|\frac{E1}{eA}\right|^2 + \left|\frac{M1}{eA}\right|^2 \right) W^4 \right]$$

NA48/2 , 600 K candidates







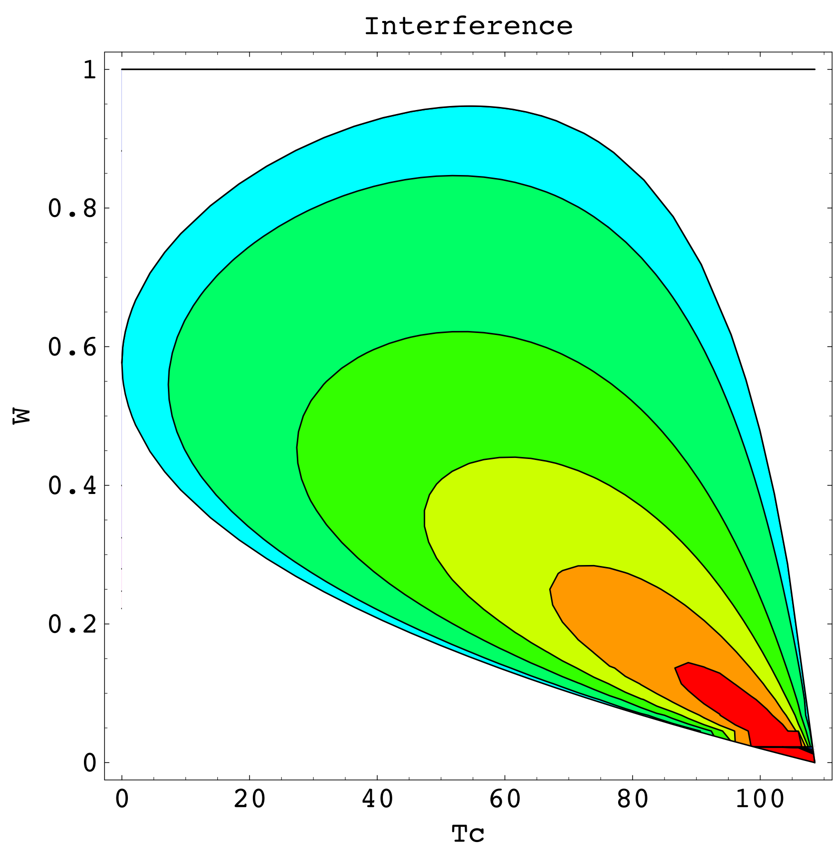
$NA48$	$T_c^* \in [0,80]\, MeV$
$Frac(DE) =$	$(3.32{\pm}0.15 \pm 0.14){\times}10^{-2}$
$Frac(INT) =$	$(-2.35{\pm}0.35 \pm 0.39){\times}10^{-2}$

Frac(DE) ratio to IB

Frac(INT) ratio to IB

first experiment IB from theory

NA48/2 CP violation



Dalitz plot analysis crucial

SM $\leq \mathcal{O}(10^{-5})$ Paver et al.

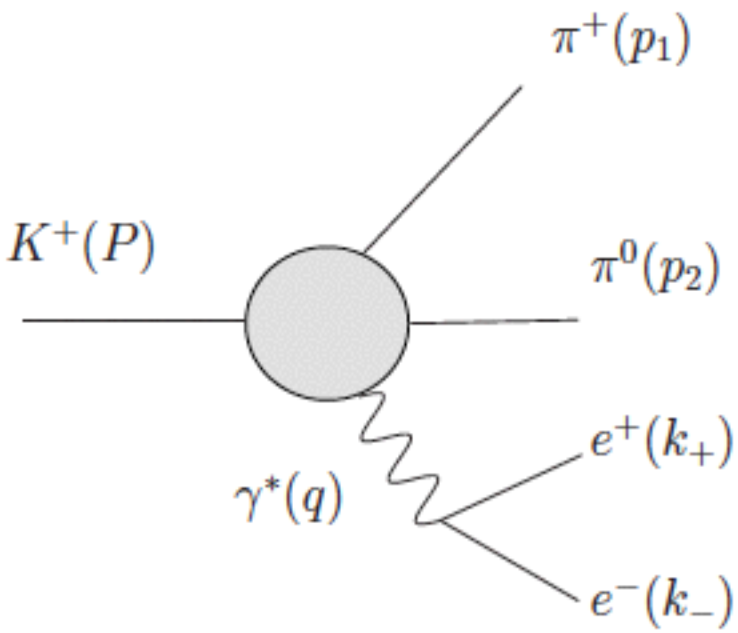
NP $\leq \mathcal{O}(10^{-4})$ Colangelo et al.

NA48/2 $< 1.5 \cdot 10^{-3}$ at 90% CL

BUT NOT in the interesting interf. kin. region (statistics)

$K_L \to \pi^+\pi^-\gamma^* \to \pi^+\pi^-e^+e^-$

Sehgal et al; Savage,Wise et al



- $\mathcal{M}_{LD} = \frac{e}{q^2}\, \bar{e}\gamma^\mu(1-\gamma^5)\, e\, H_\mu$
- $H^\mu = F_1 p_1^\mu + F_2 p_2^\mu + F_3\, \varepsilon^{\mu\nu\alpha\beta} p_{1\nu} p_{2\alpha} q_\beta$
- $F_{1,2} \sim E \quad F_3 \sim M$

- Interference $E \quad M$ novel compared to $K_L \to \pi^+\pi^-\gamma$
- $E \quad M$ known from $K_L \to \pi^+\pi^-\gamma$ (IB and DE)

$K^+ \to \pi^+\pi^0\gamma^* \to \pi^+\pi^0 e^+e^-$ Cappiello, Cata,G.D. and Gao,

- the asymm. , $\frac{\Re(E_B\ M^*)}{|E_B|^2 + |M|^2}$, not as lucky $E_B >> M$:

- $B(K^+)_{IB} \sim 3.3 \times 10^{-6}\ \sim\ 50\ B(K^+)_M$

- Short distance info without having simultaneously K^+ and K^-, asymm. in phase space, (P-violation) interesting! No ϵ-contamination

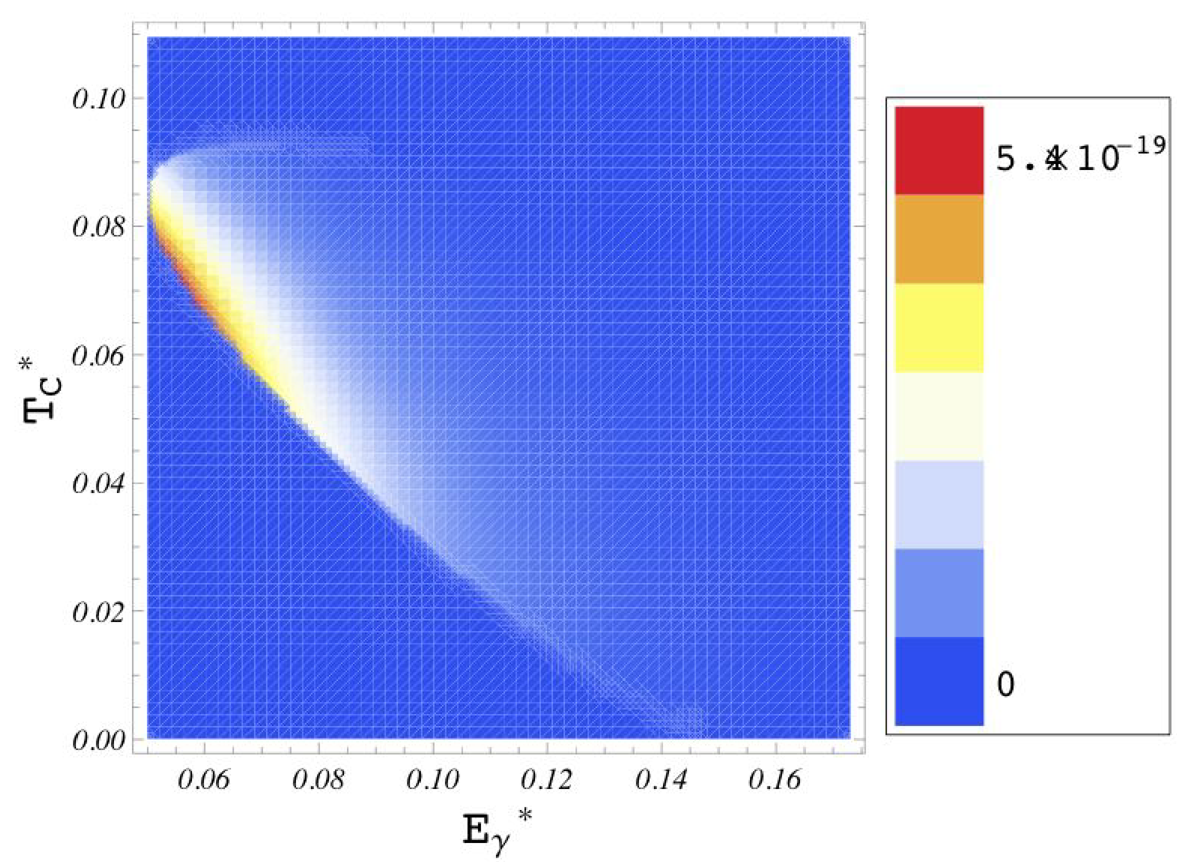
- interesting Dalitz plots (at fixed q^2) to disentangle M from E_B

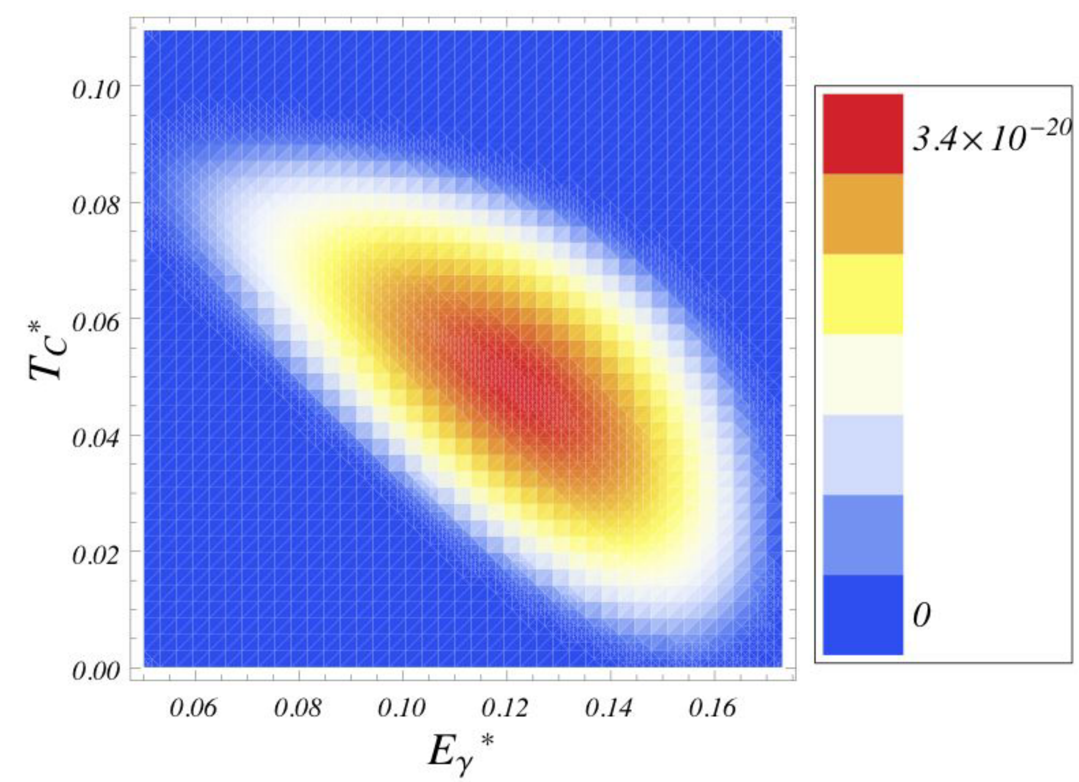
- at $q^2 = 50\mathrm{MeV}$ IB only 10 times larger than DE

$K^+ \to \pi^+ \pi^0 e^+ e^-$

Starting from CP conserving IB, DE

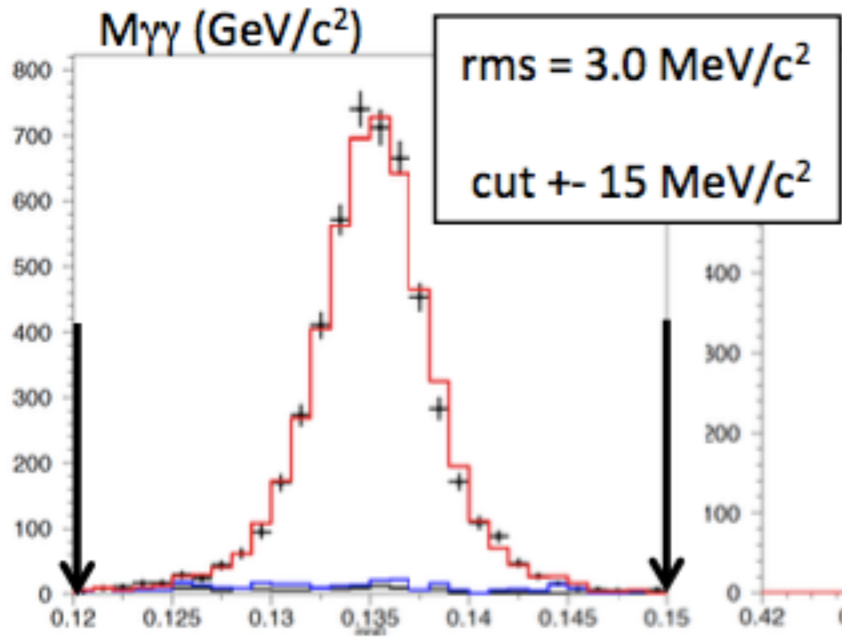
q_c (MeV)	B $[10^{-8}]$	B/M	B/E	B/BE	B/BM
$2m_l$	418.27	71	4405	128	208
55	5.62	12	118	38	44
100	0.67	8	30	71	36
180	0.003	12	5	-19	44

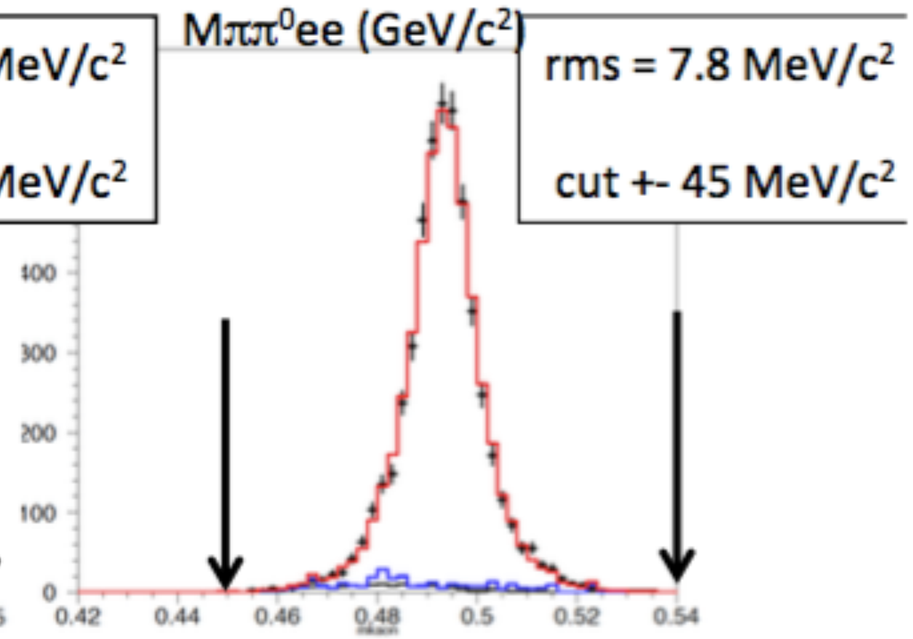


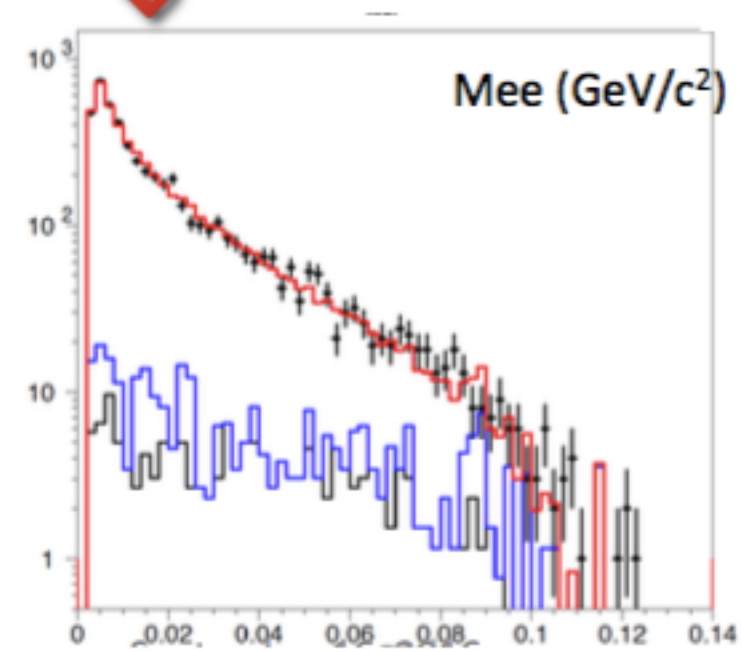


NA48/2: $K^{+/-} \rightarrow \pi^{+/-}\pi^0 e^+ e^-$

Bloch-Devaux







New Result!

$BR = (4.22 \pm 0.06_{stat} \pm 0.04_{syst} \pm 0.13_{ext})\ 10^{-6}$

dominated by external error on BR(π0D)

In perfect agreement with

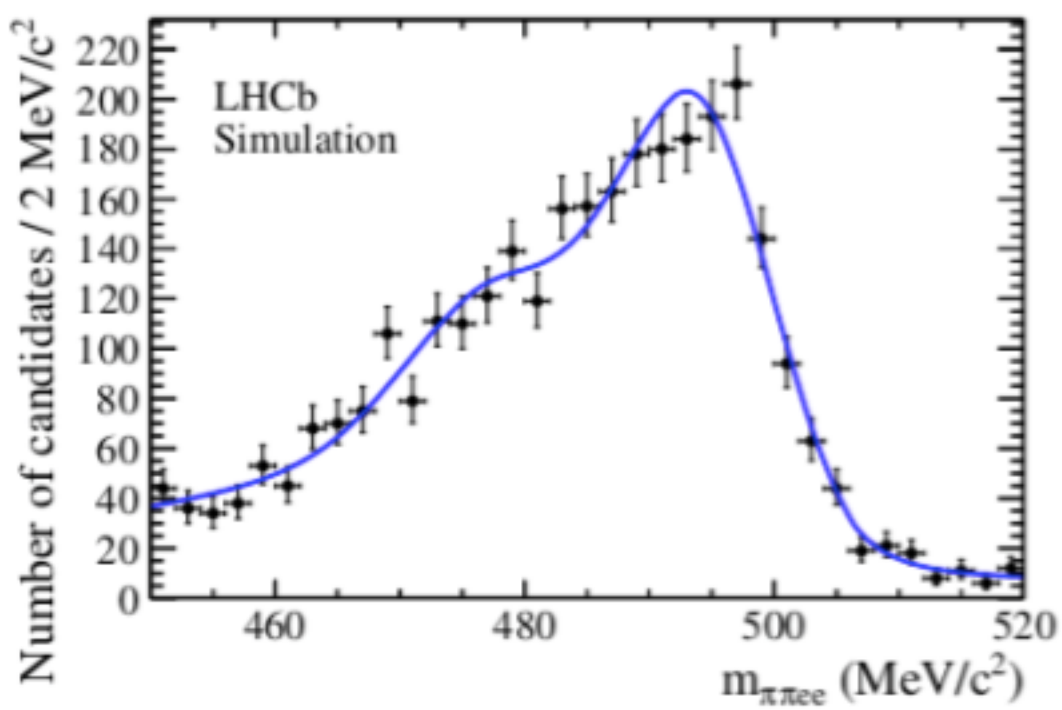
Theory : ChPT calculations EPJ C72 (2012) IB +DE + INT

BR (IB) = 4.19 10^{-6} no Rad Cor, No Isospin breaking Cor Total 4.29 10^{-6}
BR (IB) = 4.10 10^{-6} no Rad Cor, with Isospin breaking Cor** Total 4.19 10^{-6}

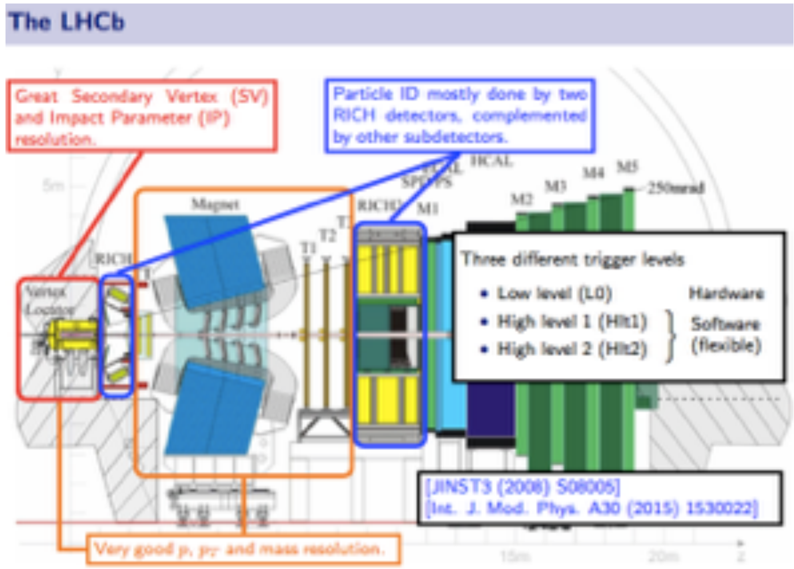
$K^0_S \to \pi^+\pi^- e^+ e^-$

NEW! **LHCb-PUB-2016-016**

- $\mathcal{B} \sim 10^{-5}$, important background for $K^0_S \to 4\ell$ (sensitive to NP)
- $K^0_S \to \pi^+\pi^- e^+ e^-$ interesting for light dark matter searches [PRD 92, 115017 (2015)]
- Expected yield: $N_{sig}/1\,\text{fb}^{-1} \sim 100$ for Run II, up to 5×10^4 for the upgrade
- Evidence/observation feasible with the LHCb Run I dataset



$K_S \to \mu\overline{\mu}$ LHCB

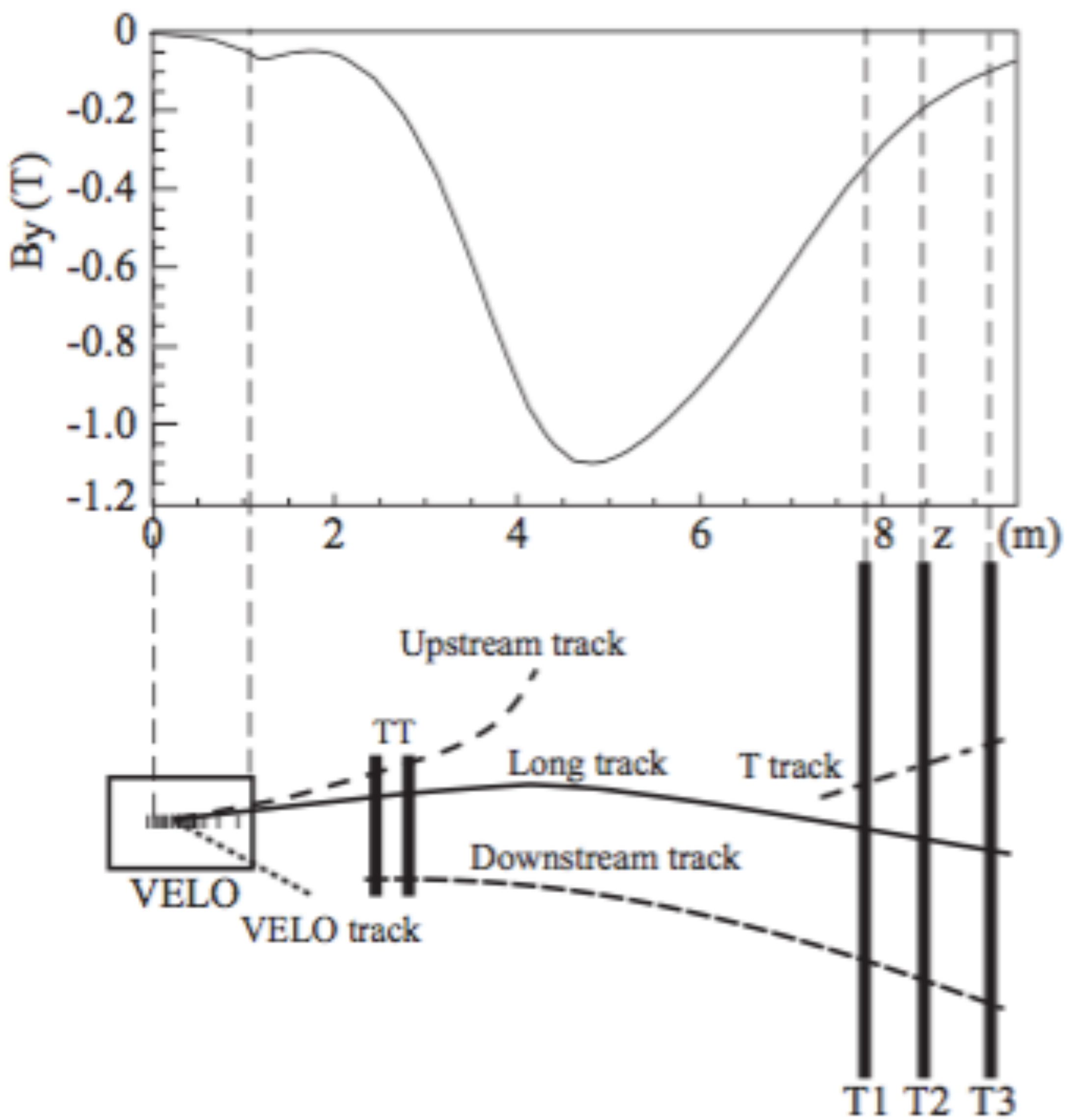


CERN SPS

After 40 years $B(K_S \to \mu^+\mu^-) < 3.1 \times 10^{-7}$at 90%CL

improvement by 3 orders of magnitudes from LHCB

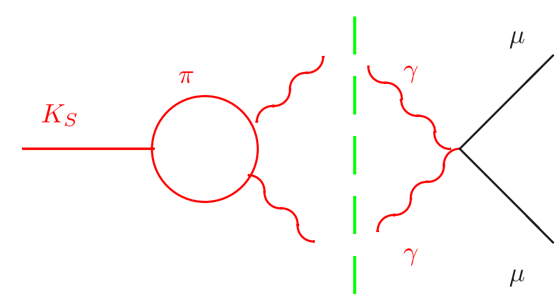
$$B(K_S \to \mu^+\mu^-) < 6.9(5.8) \times 10^{-9}\text{at } 95(90)\%\text{CL}$$



[Int. J. Mod. Phys. A30 (2015) 1530022]

SM

$\sim 5 \times 10^{-12}$



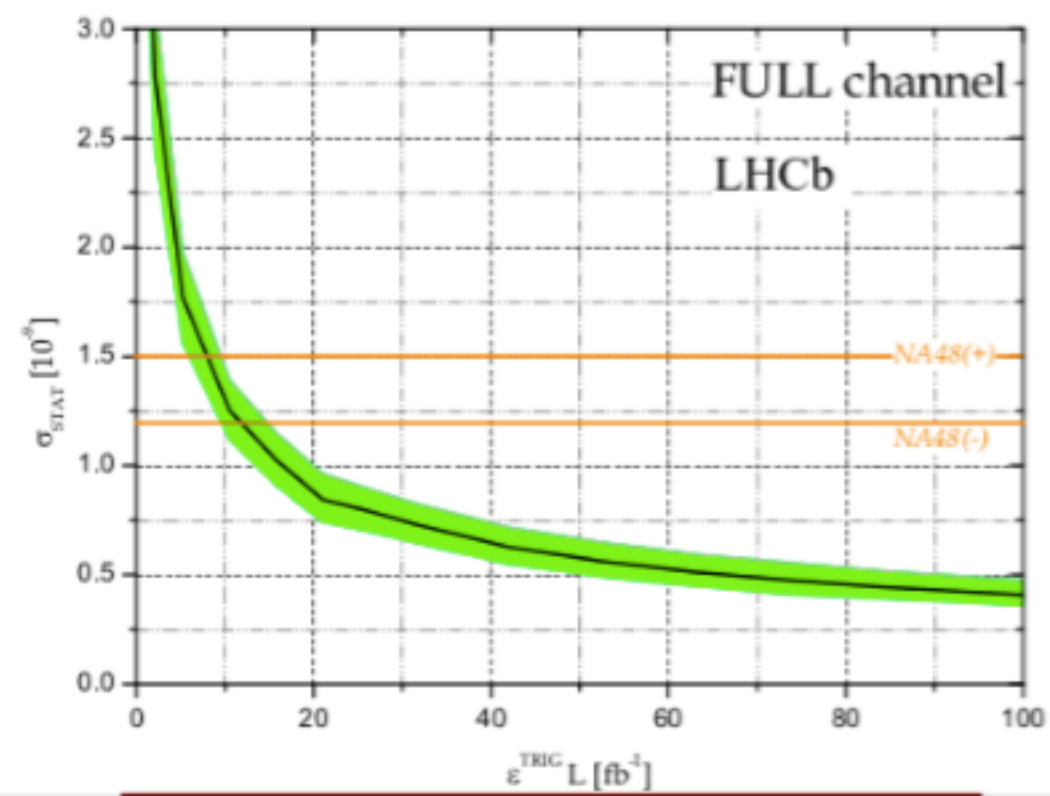
SD 1.5 10^-12

NP 1.5 10^-11
Allowed

NP Limits from CPviol in $K_L \to \mu\mu$

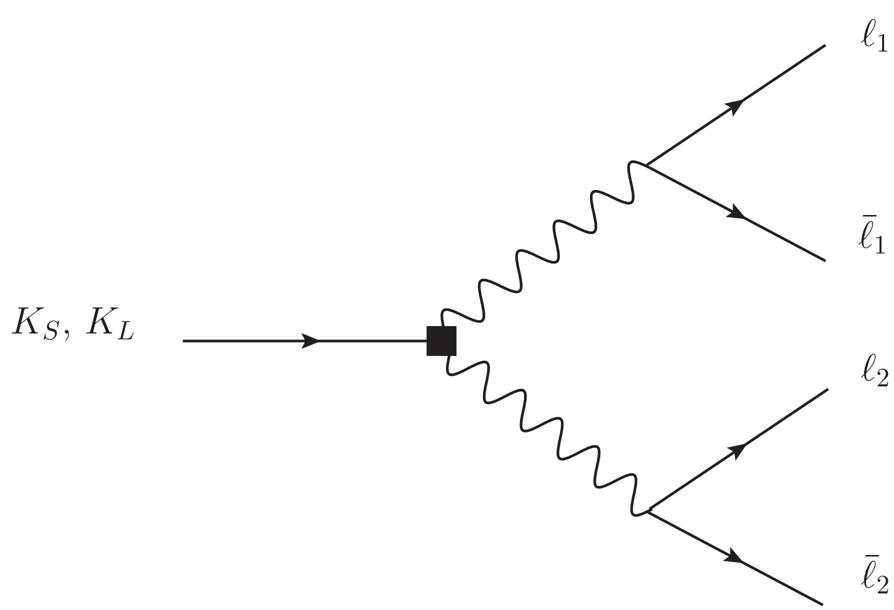
NEW! **LHCb-PUB-2016-017**

- $K_L^0 \to \pi^0 \ell\ell$ sensitive to NP (Extra Dimensions models)
- $\mathcal{B}(K_L^0 \to \pi^0 \mu^+ \mu^-)$ SM prediction unprecise due to a factor originating from $\mathcal{B}(K_S^0 \to \pi^0 \mu^+ \mu^-)$
- NA48 $\mathcal{B}(K_S^0 \to \pi^0 \mu^+ \mu^-) = 2.9^{+1.5}_{-1.2}$ 50% relative error [PLB 599 (2004) 197]
- $K^0 \to \pi^0 \mu^+ \mu^-$ sensitive to $\mathcal{C}_9$ [1601.00970v3]
- Good prospects for the LHCb upgrade



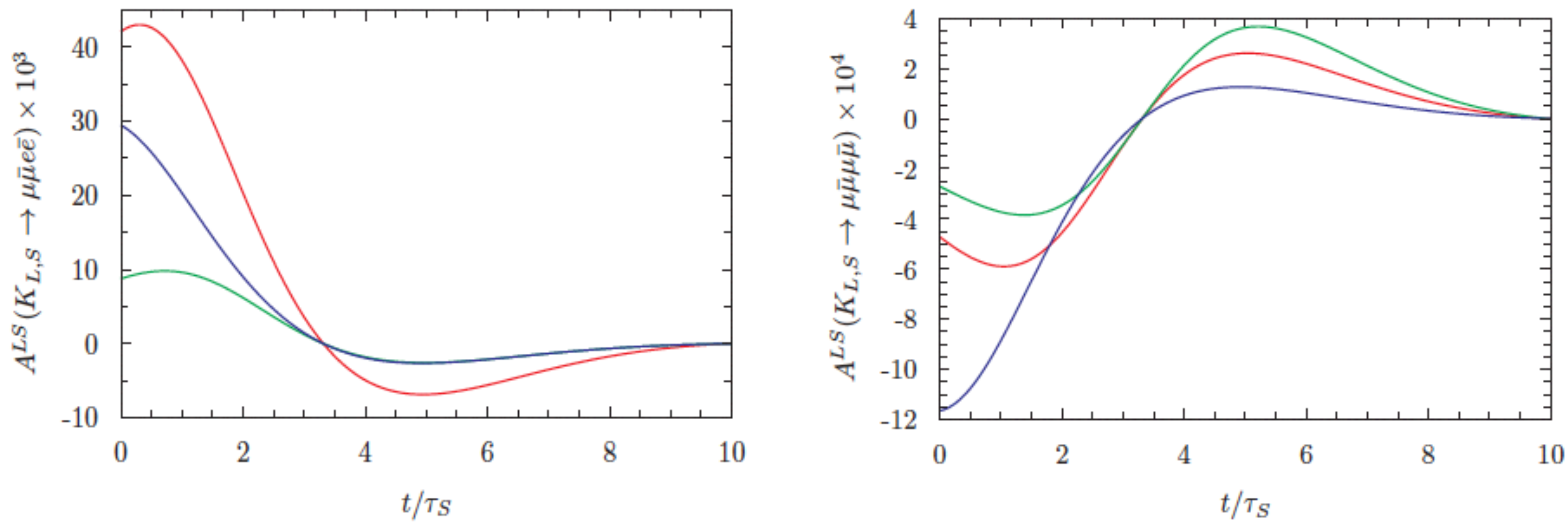
Other interesting channels

$$K_S \to \mu\mu\mu\mu \quad - \quad \text{SM LD} \ \sim 2 \times 10^{-14}$$
$$K_S \to ee\mu\mu \quad - \quad \sim 10^{-11}$$
$$K_S \to eeee \quad - \quad \sim 10^{-10}$$



GD,Greynat, Vulvert

Time interference effects



Interferences between K_L and $K_S \to \ell_1\bar{\ell}_1\ell_2\bar{\ell}_2$. The red line corresponds to the case $\alpha_S = 0$, the green line is $\alpha_S = -3$ while the blue line is $\alpha_S = 3$. As explained in the text we assume the sign $K_L \to \gamma\gamma$. For 4μ's 10^{14} K_S needed , $ee\mu\mu$ 10^{12}

Conclusions

- We have great experiments and strong theoretical motivations
- Short distance physics program and also chiral tests
- NP (LFUV and epsilon') recent motivations