

On the resonant and non-resonant effects in the $B \rightarrow K^* \nu \bar{\nu}$

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Work in progress

- $B \rightarrow K^*(\rightarrow K\pi)\nu\bar{\nu}$ is governed by $b \rightarrow s\bar{\nu}\nu$ FCNC transition and therefore sensitive to new physics
- The form factors are the only source of hadronic uncertainty in $B \rightarrow K^*\nu\bar{\nu}$
- The standard model short distance physics of $b \rightarrow s\bar{\nu}\nu$ transition is very well known
- Experimentally challenging, expected to be seen in the near future (Belle II)
- For new physics searches, resonant $B \rightarrow (K_0^*, \kappa)(\rightarrow K\pi)\nu\bar{\nu}$ and non-resonant $B \rightarrow K\pi\nu\bar{\nu}$ backgrounds become important

The $b \rightarrow s\bar{\nu}\nu$ effective Hamiltonian is

$$\mathcal{H}_{\text{eff}} = -\frac{4G_F}{\sqrt{2}}\lambda_t\frac{\alpha}{8\pi}\left[(C_L + C_R)(\bar{s}\gamma_\mu b) + (C_R - C_L)(\bar{s}\gamma_\mu\gamma_5 b)\right]\sum_i\bar{\nu}_i\gamma^\mu(1 - \gamma_5)\nu_i.$$

[Altmannshofer et al. JHEP 0904 \(2009\) 022](#), [Buras et al. JHEP 1502, 184 \(2015\)](#)

$C_L = -X(m_t^2/m_W^2)/\sin^2\theta$, including NLO QCD corrections and two-loop electroweak corrections

[Buchalla et al. Nucl. Phys. B 400 \(1993\) 225/Nucl. Phys. B 548 \(1999\) 309](#), [Misiak Phys. Lett. B 451 \(1999\) 161](#), [Brod et al. Phys. Rev. D 83, 034030 \(2011\)](#)

$X(t) = 1.469 \pm 0.017$ [Girrbach Noe arXiv:1410.3367](#)

$C_R = 0$ in the Standard Model

The hadronic matrix elements of the vector and axial vector currents between the B and the final K^* are

$$\langle K^*(k, n) | \bar{s} \gamma_\mu b | B(p_B) \rangle = \epsilon_{\mu\nu\alpha\beta} \epsilon_n^{*\nu} p_B^\alpha k^\beta \frac{2iV(q^2)}{m_B + m_{K^*}},$$

$$\begin{aligned} \langle K^*(k, n) | \bar{s} \gamma_\mu \gamma_5 b | B(p_B) \rangle = & -\epsilon_n^* (m_B + m_{K^*}) A_1(q^2) + (p_{B\mu} + k_\mu) \frac{\epsilon_n^* \cdot q}{m_B + m_{K^*}} A_2(q^2) + \\ & + q_\mu (\epsilon_n^* \cdot q) \frac{2m_{K^*}}{q^2} (A_3(q^2) - A_0(q^2)), \end{aligned}$$

we use the form factors from combined fit to results in Lattice QCD and light-cone sum rules [Bharucha et al. JHEP 08 \(2016\) 098](#), [Horgan et al. Phys. Rev. D 89, 094501 \(2014\)](#), [Ball et al. Phys. Rev. D 71, 014029 \(2005\)](#)

the B to K_0^* hadronic matrix element is

$$\langle K_0^*(k) | \bar{s} \gamma_\mu \gamma_5 b | B(p_B) \rangle = (p_B + k)_\mu f_+(q^2) + q_\mu f_-(q^2)$$

f_+ is known from calculations in the QCD sum rules [Aliiev et al. Phys. Rev. D 76, 074017 \(2007\)](#)

$$f_+(\hat{q}^2) = \frac{f_+(0)}{1 - a_+ \hat{q}^2 + b_+ \hat{q}^4}, \quad \hat{q}^2 = q^2 / m_B^2$$

the parameters are $f_+(0) = 0.31 \pm 0.08$, $a_+ = 0.81$, $b_+ = -0.21$

we limit the region of validity to 14 GeV^2

The $B \rightarrow K^* \nu \bar{\nu}$ transversity amplitudes

$$H_{\perp}(q^2) = \frac{\sqrt{2}(C_L + C_R)\lambda^{1/2}(m_B^2, q^2, m_{K^*}^2)}{m_B + m_{K^*}} V(q^2),$$

$$H_{\parallel}(q^2) = \sqrt{2}(C_L - C_R)(m_B + m_{K^*})A_1(q^2),$$

$$H_0(q^2) = -\frac{(C_L - C_R)}{2m_{K^*}\sqrt{q^2}} \left[(m_B + m_{K^*})(m_B^2 - m_{K^*}^2 - q^2)A_1(q^2) - \frac{\lambda(m_B^2, q^2, m_{K^*}^2)}{m_B + m_{K^*}}A_2(q^2) \right].$$

The $B \rightarrow K_0^* \nu \bar{\nu}$ transversity amplitude

$$H_0'(q^2) = (C_R - C_L) \frac{\lambda^{1/2}(m_B^2, q^2, m_{K_0^*}^2)}{\sqrt{q^2}} f_+(q^2).$$

finite width of the K^* and the K_0^* , κ are taken into account by Breit-Wigner functions

$$\widetilde{BW}_{K^*}(p^2) = \frac{1}{p^2 - m_{K^*}^2 + im_{K^*}\Gamma_{K^*}},$$

$$\widetilde{BW}_{\text{scalar}}(p^2) = -\frac{g_{\kappa}}{p^2 - (m_{\kappa} - i\Gamma_{\kappa}/2)^2} + \frac{1}{p^2 - (m_{K_0^*} - i\Gamma_{K_0^*}/2)^2}$$

$0 \lesssim |g_{\kappa}| \lesssim 0.2$ and $\arg \in [\pi, \pi/2]$

Becirevic et al. Nucl. Phys. B **868** (2013) 368

Denote $H_{0,\parallel,\perp} \rightarrow \widetilde{H}_{0,\parallel,\perp}$ and $H_0' \rightarrow \widetilde{H}_0'$

the $B \rightarrow K\pi$ matrix elements for vector and the axial vector currents are [Lee et al. Phys. Rev. D 46 \(1992\) 5040](#)

$$\begin{aligned}\langle K\pi | \bar{s}\gamma_\mu b | B \rangle &= h \epsilon_{\mu\nu\alpha\beta} p_B^\nu (p_K^\alpha + p_\pi^\alpha) (p_K^\beta - p_\pi^\beta) \\ \langle K\pi | \bar{s}\gamma_\mu \gamma_5 b | B \rangle &= -i w_+ (p_{K\mu} + p_{\pi\mu}) - i w_- (p_{K\mu} - p_{\pi\mu}) - i r q_\mu.\end{aligned}$$

at leading order in heavy hadron chiral perturbation theory (HH χ PT) [Lee et al. Phys. Rev. D 46 \(1992\) 5040](#), [Buchalla et al. Nucl. Phys. B 525 \(1998\) 333](#)

$$\begin{aligned}w_\pm(q^2, p^2, \theta_K) &= \pm \frac{g f_{B_d}}{2f^2} \frac{m_B}{v \cdot p_\pi + \Delta}, \quad v = p_B/m_B, \quad \Delta = m_{B^*} - m_B, \quad f^2 = f_\pi f_K, \\ h(q^2, p^2, \theta_K) &= \frac{g^2 f_{B_d}}{2f^2} \frac{1}{(v \cdot p_\pi + \Delta)(v \cdot p_{K\pi} + \Delta + \mu_s)}, \quad \mu_s = m_{B_s} - m_B.\end{aligned}$$

HH χ PT is valid when the 3-momentum of final state pseudoscalars are soft
(we take the region of validity from 14 GeV² to the kinematic endpoint)

HH χ PT coupling $g = 0.569 \pm 0.076$ [Flynn et al. 2013](#)

we add 20% uncertainty on w_\pm and h

the $B \rightarrow K\pi\nu\bar{\nu}$ transversity amplitudes are

$$H_{0,\parallel}^{\text{nr}} = (C_L - C_R)F_{0,\parallel}^{\text{nr}}, \quad H_{\perp}^{\text{nr}} = (C_L + C_R)F_{\perp}^{\text{nr}}.$$

the $B \rightarrow K\pi\nu\bar{\nu}$ transversity form factors read

$$F_{\perp}^{\text{nr}} = \frac{\lambda^{1/2}(m_{K\pi}^2, m_K^2, m_{\pi}^2)\lambda^{1/2}(m_B^2, p^2, q^2)}{2\sqrt{p^2}} h \sin \theta_K,$$

$$F_{\parallel}^{\text{nr}} = -\sin \theta_K \frac{\lambda^{1/2}(p^2, m_K^2, m_{\pi}^2)}{\sqrt{p^2}} w_-,$$

$$F_0^{\text{nr}} = \frac{i}{2\sqrt{q^2}} \left[w_+ \lambda^{1/2}(m_B^2, q^2, p^2) + w_- \frac{1}{p^2} \left((m_K^2 - m_{\pi}^2) \lambda^{1/2}(m_B^2, q^2, p^2) \right. \right. \\ \left. \left. - (m_B^2 - p^2 - q^2) \lambda^{1/2}(m_K^2, m_{\pi}^2, p^2) \cos \theta_K \right) \right].$$

the transversity amplitudes can be expanded in terms of associated Legendre polynomials P_{ℓ}^m

$$F_0^{\text{nr}} = \sum_{\ell=0} a_0^{\ell}(q^2, p^2) P_{\ell}^{m=0}(\cos \theta_K),$$

$$F_{\parallel}^{\text{nr}} = \sum_{\ell=1} a_{\parallel}^{\ell}(q^2, p^2) \frac{P_{\ell}^{m=1}(\cos \theta_K)}{\sin \theta_K},$$

$$F_{\perp}^{\text{nr}} = \sum_{\ell=1} a_{\perp}^{\ell}(q^2, p^2) \frac{P_{\ell}^{m=1}(\cos \theta_K)}{\sin \theta_K}.$$

the three fold differential distribution is

$$\frac{d^3\Gamma}{dq^2 dp^2 d \cos \theta_K} = 3\mathcal{N}(q^2) \left[|\widetilde{H}_\perp + e^{i\delta_{K^*}} H_\perp^{\text{nr}}|^2 + |\widetilde{H}_\parallel + e^{i\delta_{K^*}} H_\parallel^{\text{nr}}|^2 + |\widetilde{H}_0 + e^{i\delta_{K^*}} H_0^{\text{nr}} + \widetilde{H}_0'|^2 \right].$$

δ_{K^*} is the relative strong phase between the resonant and the non-resonant modes

the two-fold differential distribution in the presence of K^* and scalars (κ, K_0^*)

$$\frac{d^2\Gamma}{dq^2 d \cos \theta_K} = a(q^2) + b(q^2) \cos \theta_K + c(q^2) \cos^2 \theta_K.$$

for a pure K^* contribution only $a(q^2), c(q^2)$ are nonzero and the longitudinal polarization fraction is

$$F_L = \frac{d\Gamma_L/dq^2}{d\Gamma/dq^2}, \quad \frac{d\Gamma_L}{dq^2} = \frac{2}{3}(a(q^2) + c(q^2)), \quad \langle F_L \rangle = \frac{\int_{q_{\min}^2}^{q_{\max}^2} d\Gamma_L/dq^2}{\int_{q_{\min}^2}^{q_{\max}^2} d\Gamma/dq^2}$$

in presence of non-resonant $B \rightarrow K\pi\nu\bar{\nu}$ the two-fold distribution is more complicated

$$\tilde{F}_L = \frac{d\tilde{\Gamma}_L/dq^2}{d\tilde{\Gamma}/dq^2}, \quad \frac{d\tilde{\Gamma}_L}{dq^2} = \int_{-1}^1 \frac{d^2\Gamma}{dq^2 d \cos \theta_K} \left(\frac{1}{3} P_0^0 + \frac{2 \cdot 2 + 1}{3} P_2^0 \right) d \cos \theta_K.$$

– our analysis is done in the following two p^2 windows

P-window or signal window: $[(m_{K^*} - 0.1\text{GeV})^2, (m_{K^*} + 0.1\text{GeV})^2]$ [Aaij et al. JHEP 1308 \(2013\) 131](#)

S+P-window: $[(m_K + m_\pi)^2, 1.44 \text{ GeV}^2]$

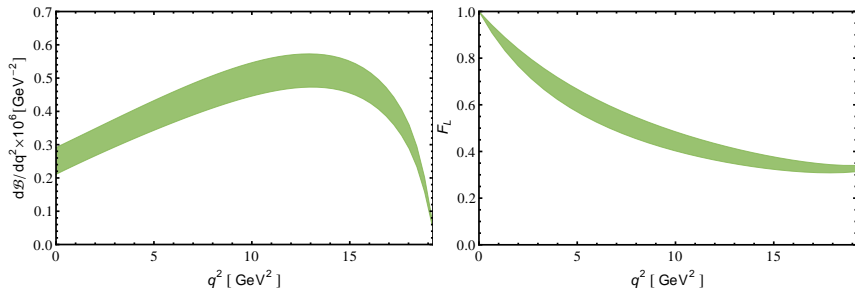


Figure: Differential branching ratio in q^2 (left) and longitudinal polarization fraction F_L (right) for a pure $B \rightarrow K^* (\rightarrow K\pi)\nu\bar{\nu}$ in the P-wave signal window with the K^* taken at finite width.

Linshapes

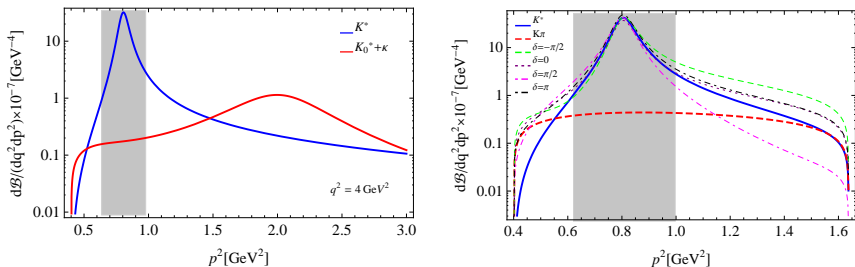


Figure: Left: Comparison of the p^2 -lineshapes of the resonant K^* -contribution (solid blue line) and scalar meson contributions (K_0^* , κ) (red line) at $q^2 = 4 \text{ GeV}^2$. The input parameters are set to the central values, and $g_\kappa = 0.2$, $\arg(g_\kappa) = \pi/2$. Right plot: Comparison of the p^2 -lineshapes of the resonant K^* -contribution (solid blue), purely non-resonant contribution (dashed red) and the resulting lineshapes that also include the interferences, for several chosen values of the strong phase δ , at $q^2 = 16 \text{ GeV}^2$. The vertical shaded region corresponds to the P -window region in p^2 .

Impact of scalar background on branching ratio

– ratio of the integrated branching fractions as a function of the bin size in q^2 :

$$R_{Br} = \frac{\int_{q_{\min}^2}^{q_{\max}^2} \int_{p_{\text{cut}}^2} d^2 \mathcal{B}(B \rightarrow K\pi\nu\bar{\nu}) / (dq^2 dp^2)}{\int_{q_{\min}^2}^{q_{\max}^2} \int_{p_{\text{cut}}^2} d^2 \mathcal{B}(B \rightarrow (K^* \rightarrow K\pi)\nu\bar{\nu}) / (dq^2 dp^2)}$$

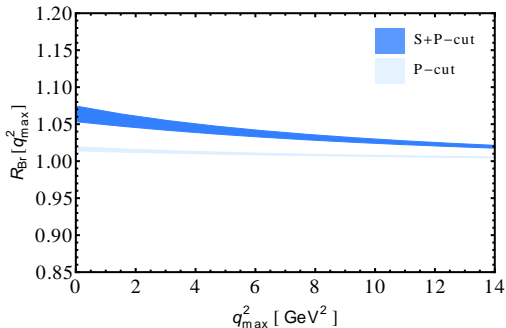


Figure: The $R_{Br}(q_{\max}^2)$ is shown in the P- and S+P-wave signal window at large recoil. The bands correspond to the uncertainties due to form factors and input parameters.

Impact of non-resonant background on branching ratio

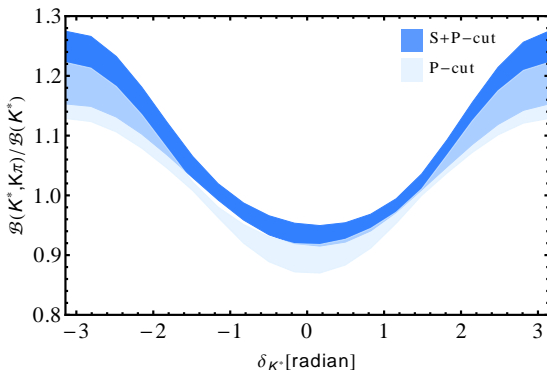


Figure: The ratio of total integrated branching ratio $\mathcal{B}(B \rightarrow (K^*, K\pi)(\rightarrow K\pi)\nu\bar{\nu})$ to $\mathcal{B}(B \rightarrow (K^*)(\rightarrow K\pi)\nu\bar{\nu})$ with respect to the relative strong phase δ_{K^*} . The bands indicate uncertainties coming from form factors and input parameters. The light and dark bands correspond to P- and S+P-window cuts.

\Rightarrow the strong can be fixed from $B \rightarrow K^* \ell\ell$ and $B \rightarrow K\pi\ell\ell$ interference effects [Das et al. 1506.06699](#)

Total integrated branching ratio

	$\mathcal{B} \times 10^{-6}$ in [0-14] GeV ²	$\mathcal{B} \times 10^{-6}$ in [14-19] GeV ²
$\mathcal{B}(B \rightarrow K^*(\rightarrow K\pi)\nu\bar{\nu}) _{\text{narrow}}$	6.97 ± 0.77	2.48 ± 0.26
$\mathcal{B}(B \rightarrow K^*(\rightarrow K\pi)\nu\bar{\nu}) _{\text{P-cut}}$	5.91 ± 0.66	2.08 ± 0.21
$\mathcal{B}(B \rightarrow K^*(\rightarrow K\pi)\nu\bar{\nu}) _{\text{P+S-cut}}$	6.51 ± 0.72	2.26 ± 0.23
$\mathcal{B}(B \rightarrow (K^*, K_0^*)(\rightarrow K\pi)\nu\bar{\nu}) _{\text{P-cut}}$	5.95 ± 0.66	—
$\mathcal{B}(B \rightarrow (K^*, K_0^*)(\rightarrow K\pi)\nu\bar{\nu}) _{\text{S+P-cut}}$	6.64 ± 0.72	—
$\mathcal{B}(B \rightarrow (K^*, \text{nonres})(\rightarrow K\pi)\nu\bar{\nu}) _{\text{P-cut}}$	—	$2.12 \pm 0.22^{+0.44}_{-0.33}$
$\mathcal{B}(B \rightarrow (K^*, \text{nonres})(\rightarrow K\pi)\nu\bar{\nu}) _{\text{S+P-cut}}$	—	$2.39 \pm 0.24^{+0.59}_{-0.27}$

⇒ less sensitive to resonant scalars

Impact of non-resonant background on longitudinal polarization fraction

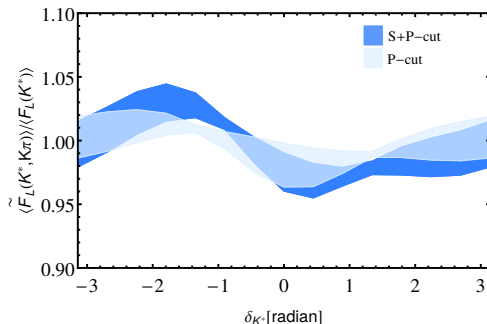


Figure: The total integrated longitudinal polarization fraction $\langle \tilde{F}_L(K^*, K\pi) \rangle$ to $\langle F_L(K^*) \rangle$ with respect to the relative strong phase δ_{K^*} . The bands indicate uncertainties coming from form factors and input parameters. The light and dark bands correspond to P- and S+P-window cuts.

Summary

we have studied the impacts of resonant and non-resonant backgrounds on $B \rightarrow K^* \nu \bar{\nu}$

depending on the relative strong phase there could be up to 30% effect on \mathcal{B}

F_L is less sensitive to the backgrounds, can be used to test form factors

Thank You