SPECTRAL ANALYSIS OF FLOWS AROUND COMPACT OBJECTS

Presented by

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Working with

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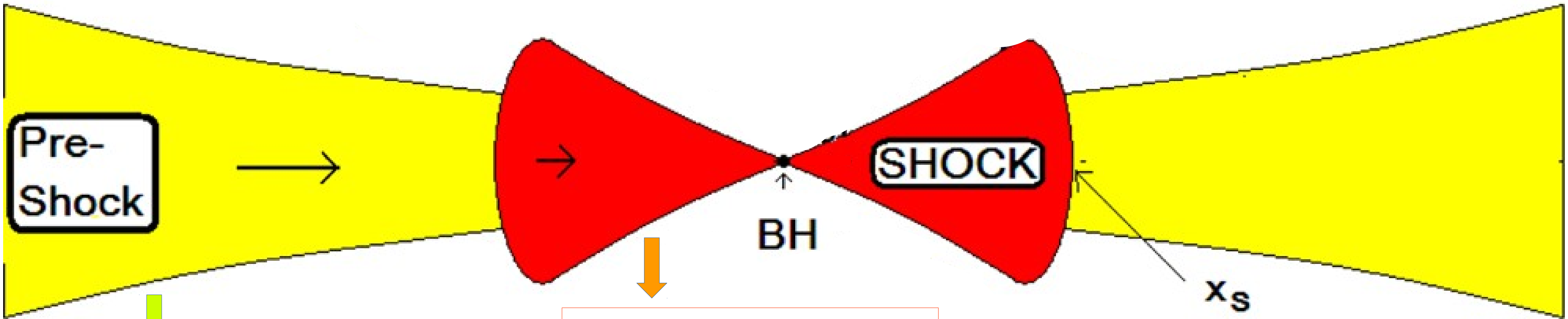
OUTLINE OF THE TALK :

- Introduction to the model used
- Assumptions and equations of motions used
  - Relativistic equation of state
  - Relativistic Coulomb coupling term
  - Optical Depth for moving media
- Spectral analysis
- Conclusion

INTRODUCTION :



$$\Gamma_p = 0.0$$
$$\Lambda_p = \Gamma_{ep}$$
$$\Gamma_e = \Gamma_{ep}$$
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Cartoon diagram for a sub-Keplerian flow with a shock, at the center of which is a black hole (BH)

Picture credit : *Kumar et.al 2014*

ASSUMPTIONS AND EQUATIONS OF MOTIONS USED

Relativistic Equation Of State (EoS) :

- Most commonly used EoS (fixed Γ) : $e = \rho c^2 + \frac{p}{(\Gamma - 1)}$

- Approximate EoS (which reproduces the relativistically correct EoS) :

$$e = \rho c^2 + p \left(\frac{9p + 3\rho c^2}{3p + 2\rho c^2} \right)$$

Ryu et.al. 2006

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Chattopadhyay & Ryu, 2009

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$$\Rightarrow\quad e=nm_e c^2 f_p+\frac{1}{\eta}nm_e c^2 f_e$$

$$\text{where},\ \eta=\frac{m_e}{m_p},\ f_i=1+\Theta_i\left(\frac{9\Theta_i+3}{3\Theta_i+2}\right),\ \Theta_i=\left(\frac{kT_i}{m_i c^2}\right)$$

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$$\Rightarrow e = n m_e c^2 f_p + \frac{1}{\eta} n m_e c^2 f_e$$

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- <u>Two-temperature equations :</u>

The proton and electron temperature profiles are described by two energy balance equations :

– Non-Relativistic :

$$\frac{1}{(\gamma_p - 1)} \frac{dT_p}{dx} - \frac{T_p}{n} \frac{dn}{dx} + \frac{\Delta Q_p}{u n c^2} = 0 \qquad \frac{1}{(\gamma_e - 1)} \frac{dT_e}{dx} - \frac{T_e}{n} \frac{dn}{dx} + \frac{\Delta Q_e}{u n c^2} = 0$$

– Relativistic :

$$\frac{df_p}{d\Theta_p} \frac{d\Theta_p}{dr} - \frac{\Theta_p}{n} \frac{dn}{dr} + \frac{\Delta Q_p \eta}{u \rho_e c^2} = 0 \qquad \frac{df_e}{d\Theta_e} \frac{d\Theta_e}{dr} - \frac{\Theta_e}{n} \frac{dn}{dr} + \frac{\Delta Q_e}{u \rho_e c^2} = 0$$

Relativistic Coulomb coupling term :

- Ions being at higher temperature than the electrons will transfer its energy to electrons through Coulomb collisions. The transfer rate is given by :
  - Non-Relativistic : $\Gamma_{ep} = \frac{3}{2} nk \frac{(T_p - T_e)}{t_{ep}}$ $\quad where, t_{ep} = \frac{3k^{3/2}}{4(2\pi)^{1/2} e^4 2n \ \ln\Lambda_0} m_e m_p \left(\frac{T_e}{m_e} + \frac{T_p}{m_p} \right)^{3/2}$
  - Relativistic : *Used from Stepney & Guilbert, 1983*

$$\Lambda_p = \frac{3}{2} \frac{m_e}{m_p} n^2 \sigma_T c \frac{(kT_p - kT_e)}{K_2\left(\frac{1}{\Theta_e}\right) K_2\left(\frac{1}{\Theta_p}\right)} \ln\Lambda \left[\frac{2(\Theta_e + \Theta_p)^2 + 1}{(\Theta_e + \Theta_p)} K_1\left(\frac{\Theta_e + \Theta_p}{\Theta_e \Theta_p}\right) + 2K_0\left(\frac{\Theta_e + \Theta_p}{\Theta_e \Theta_p}\right) \right]$$

$where, K's = modified\ Bessel\ functions, \ln\Lambda = Coulomb\ logarithm = 20$

Optical depth for moving media :

- Optical depth change with the relative direction of the photon with respect to the direction of the flow.

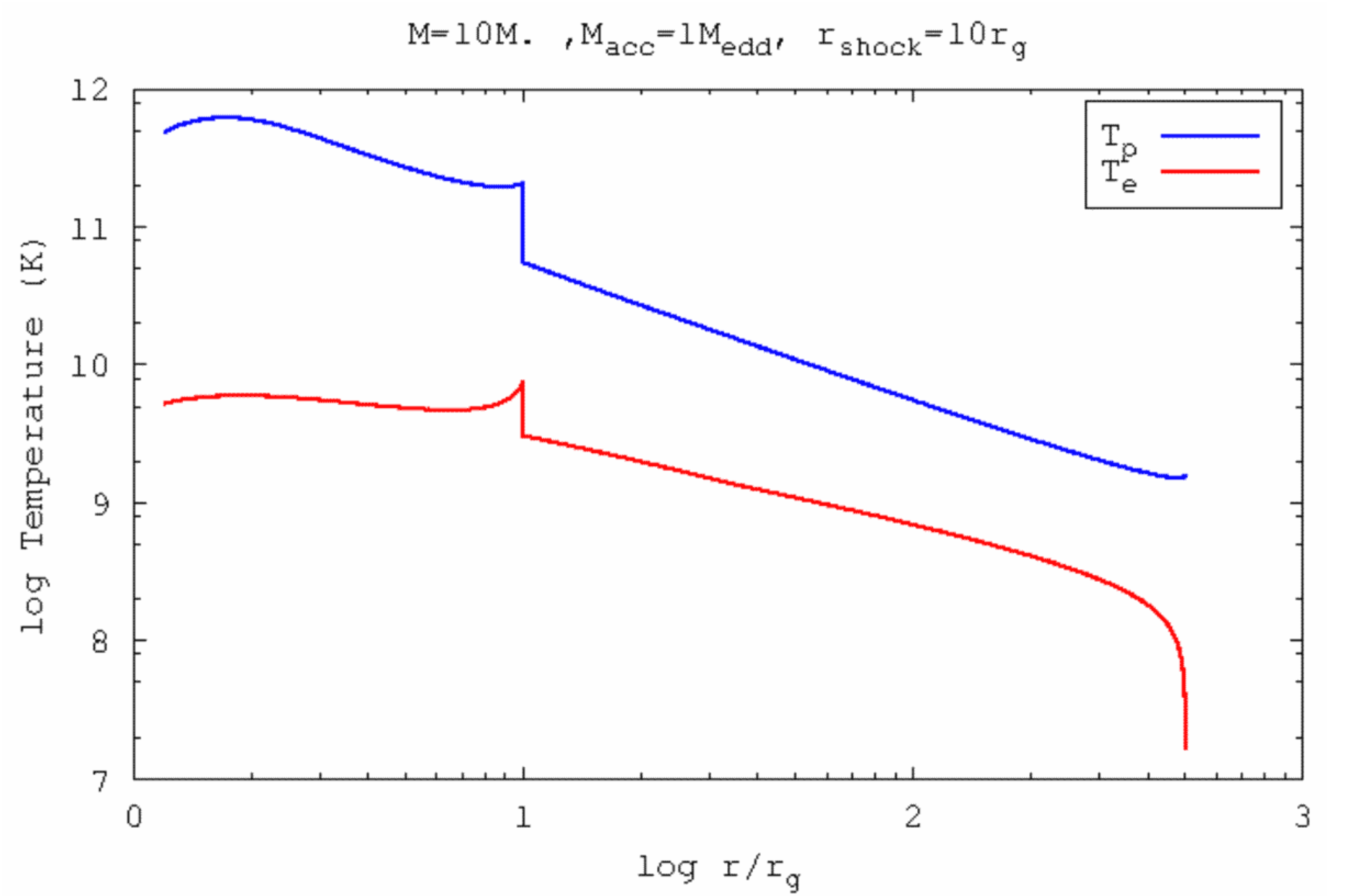
  - Stationary case : $\tau = \int n \sigma_T dx$

  - Moving media : $\tau = \int n \left[\sigma c \left(1 - \frac{v}{c} \cos \varphi \right) \right] dx$

Niedźwiecki et.al 2006

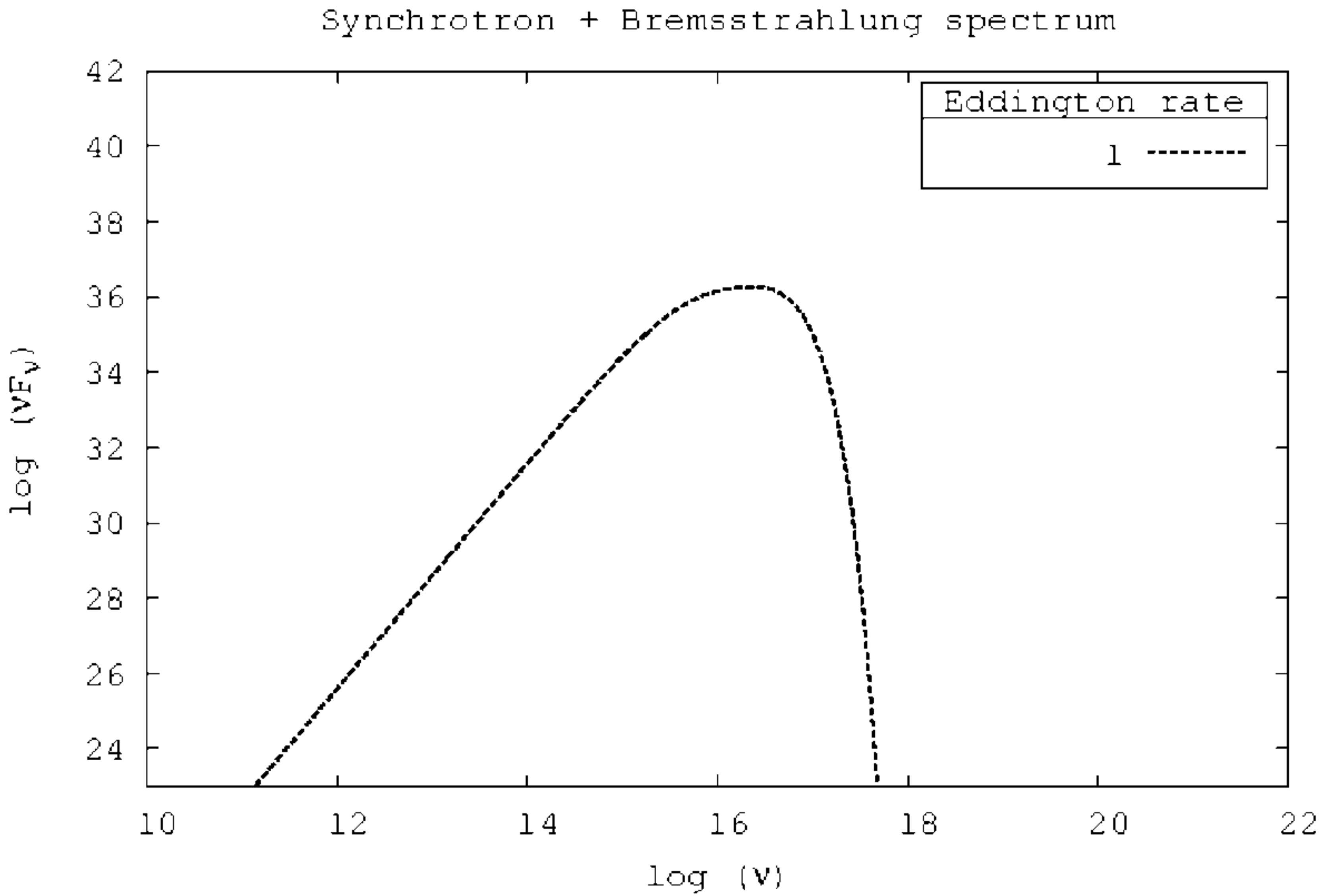
Spectral Analysis

Two-temperature solution :



$M=10M_\odot$, $\dot{M}=1\dot{M}_{edd}$, $Tp_\infty=1.6x10^9K$, $Te_\infty=1.6x10^7K$, $r_{out}=500\ r_g$, $r_{shock}=10\ r_g$

Variation of spectral shape with accretion rate :



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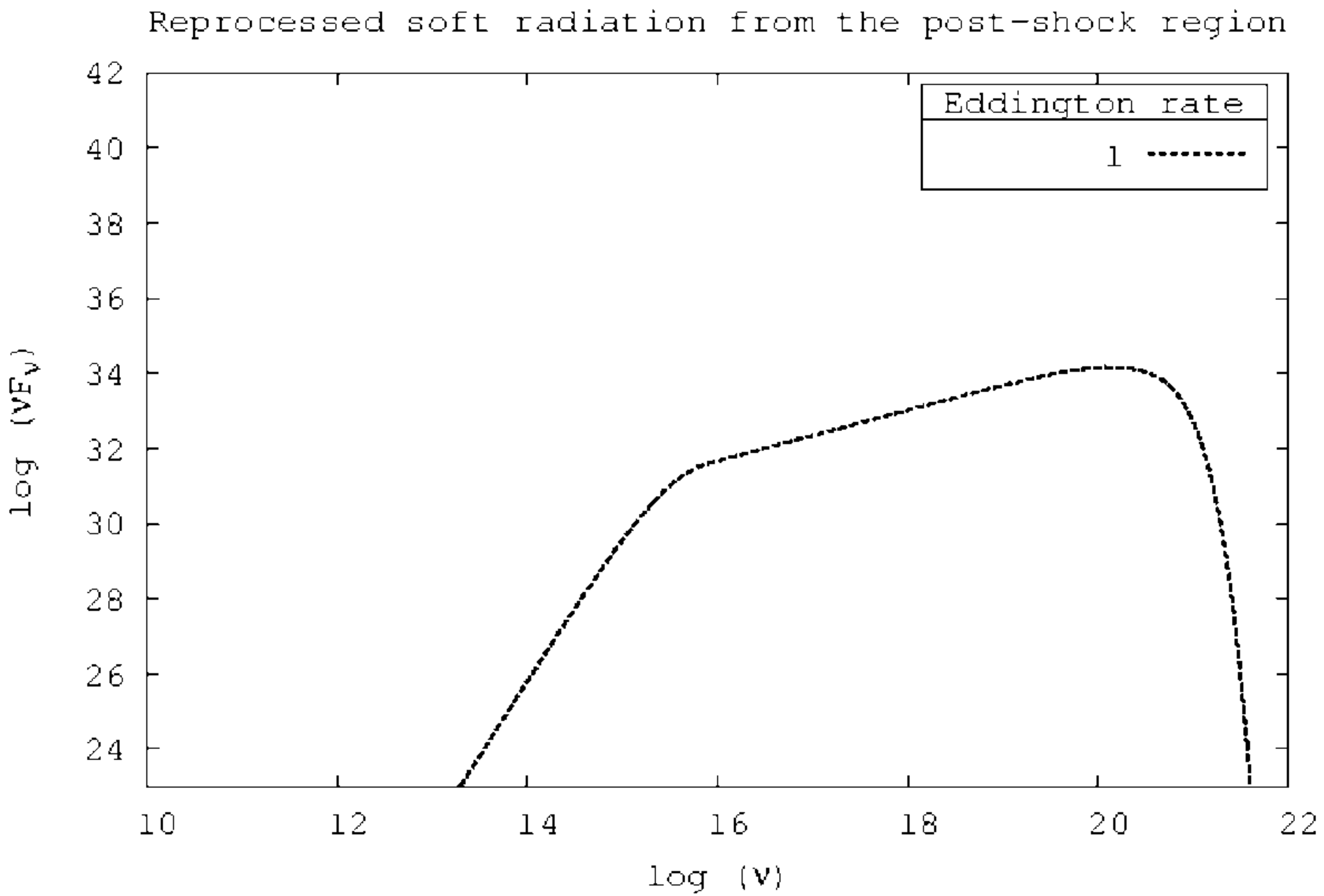

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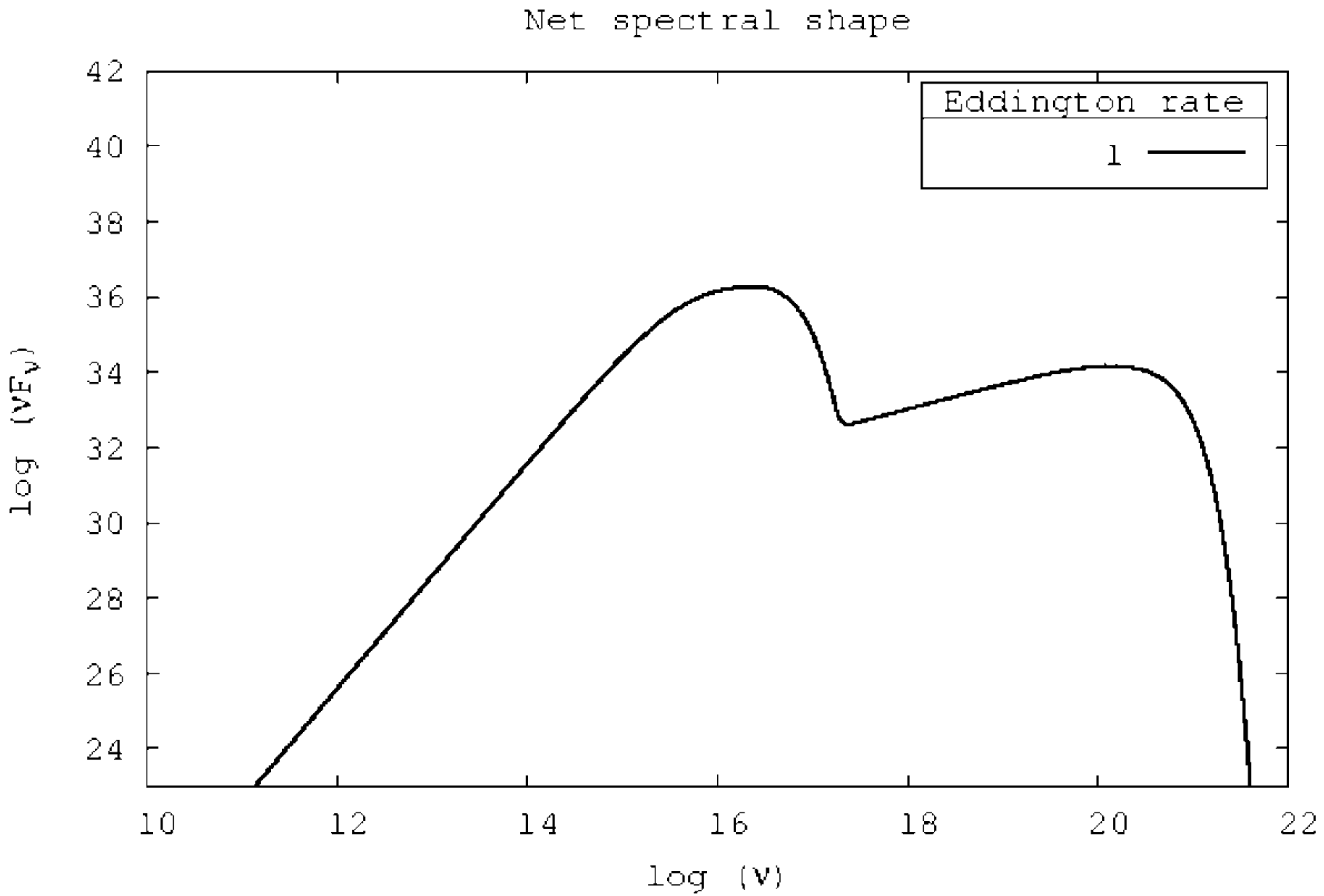

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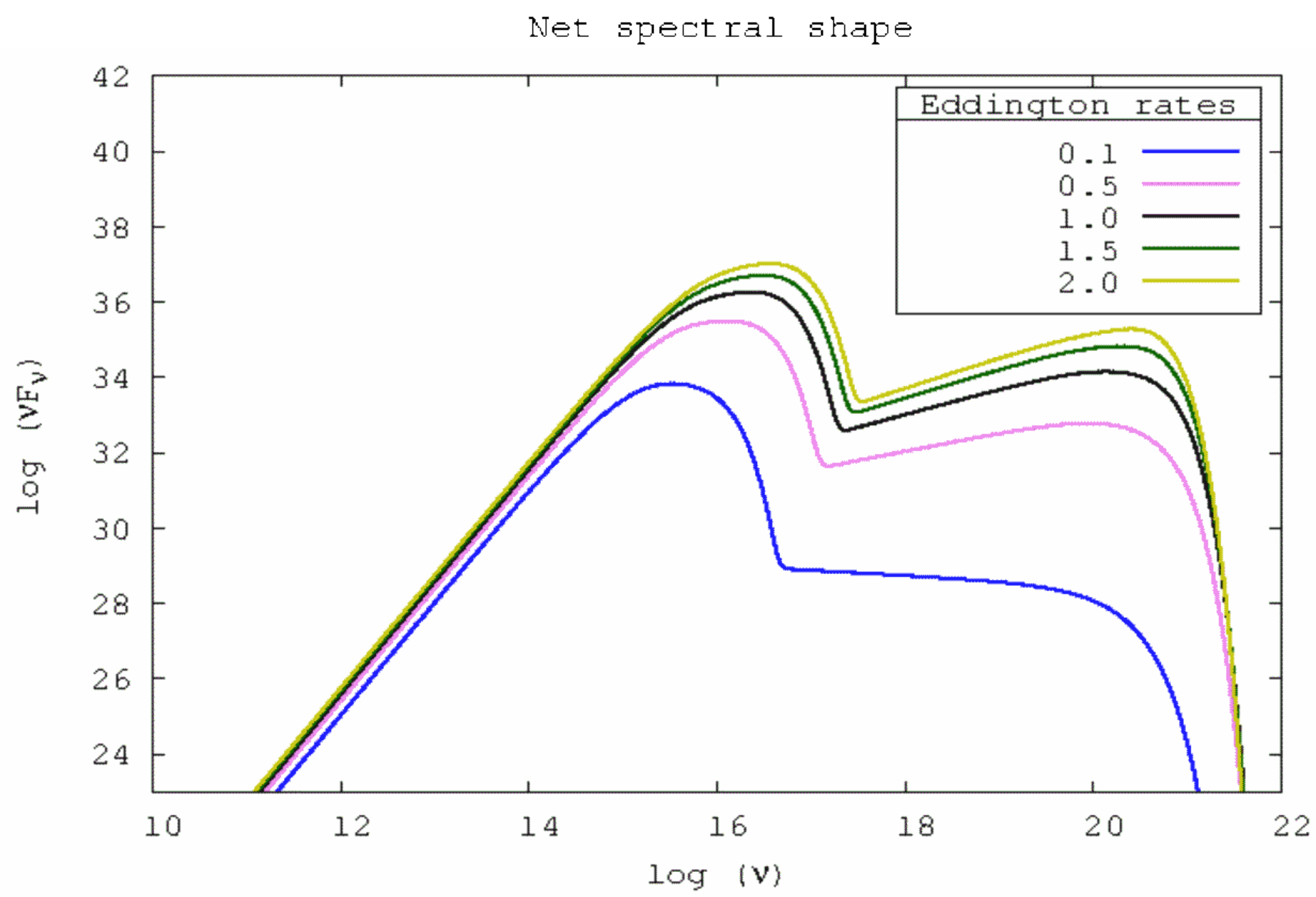

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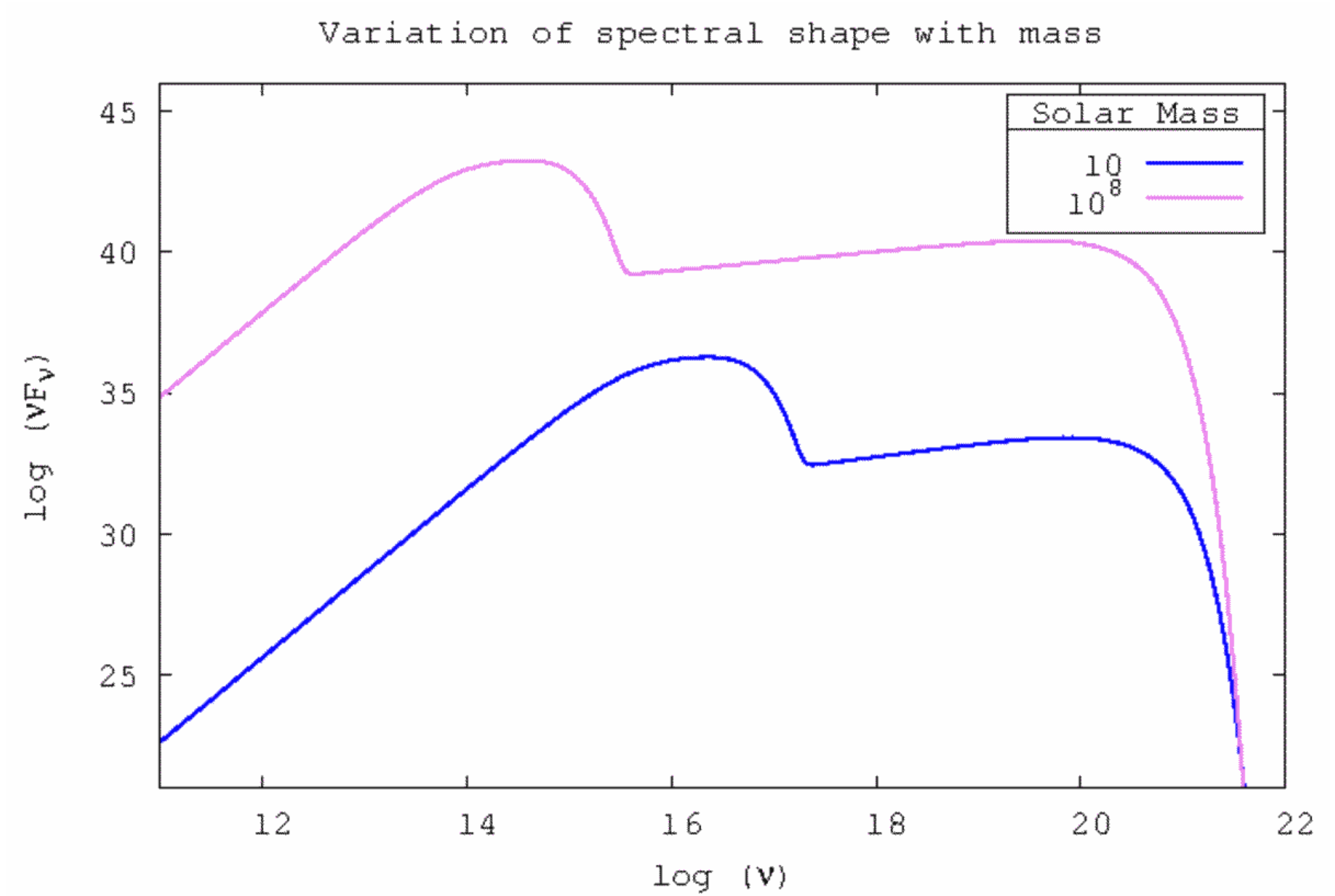
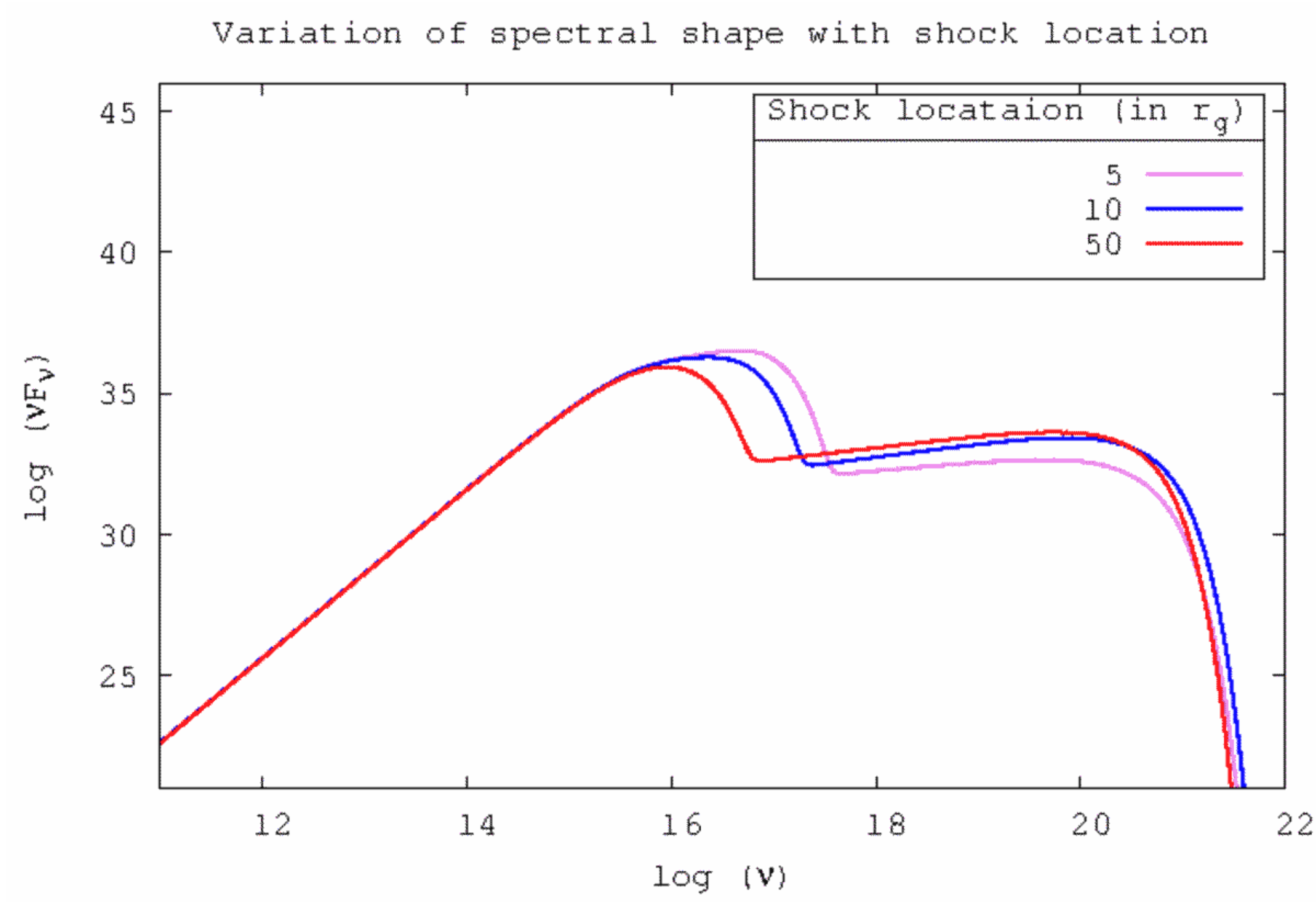
Variation of spectral shape with mass of the object :



$\dot{M}=1\dot{M}_{edd}$, $Tp_\infty=1.6x10^9K$, $Te_\infty=1.6x10^7K$, $r_{out}=500\ r_g$, $r_{shock}=10\ r_g$

Variation of spectral shape with shock location :



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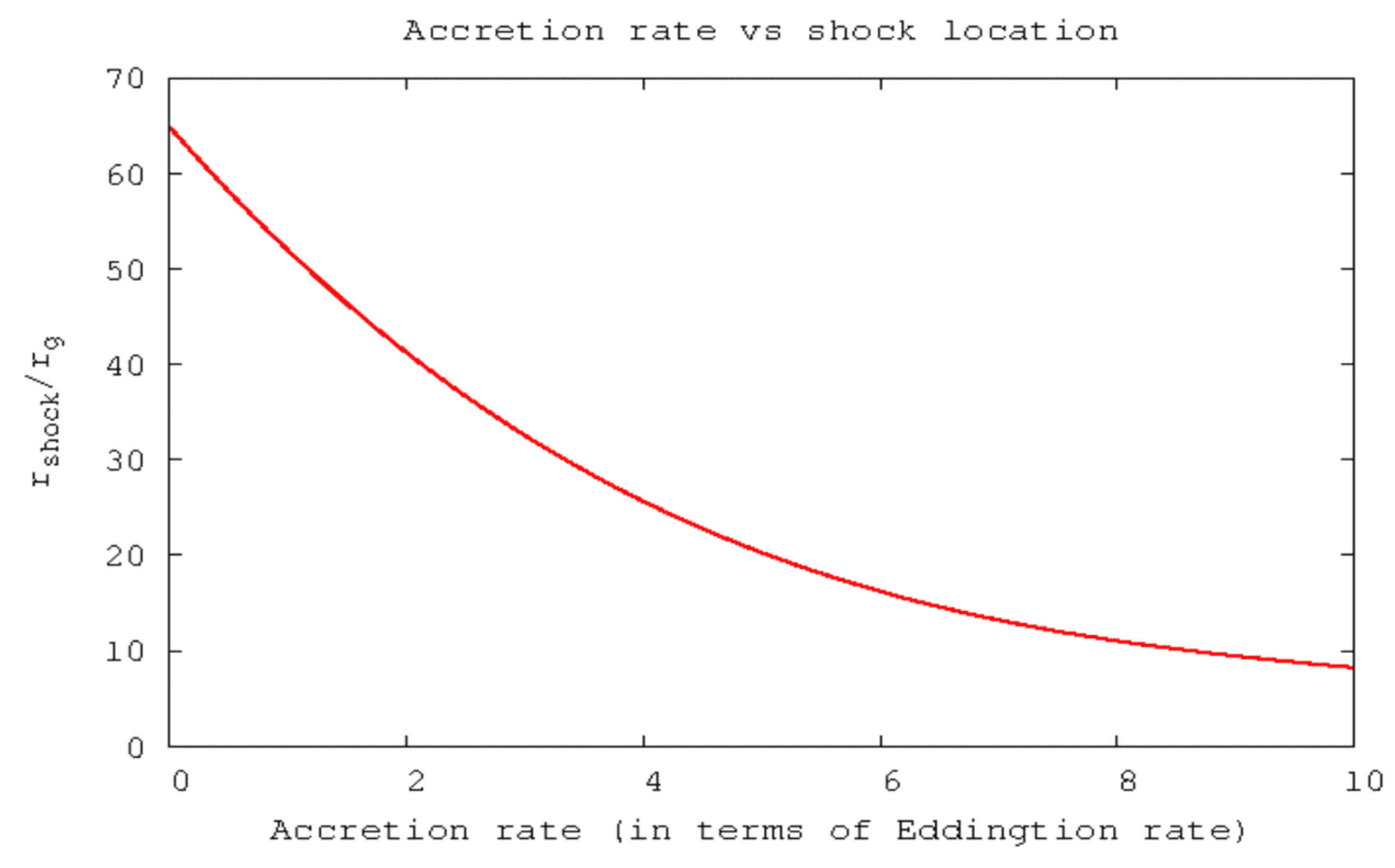
Accretion rate as a function of shock location :

- *Kumar & Chattopadhyay (2014)* computed r_{shock} for given values of λ , $\dot{M}_{sk}$, α, r_{out}. *Vyas et. al. (2015)* expressed the r_{shock} as a function of $\dot{M}_{sk}$ using a fitting function for the data generated above. The form of the equation is : $r_{shock} = 64.8735 - 14.1476\,\dot{M}_{sk} + 1.24286\,\dot{M}_{sk}^2 - 0.039467\,\dot{M}_{sk}^3$



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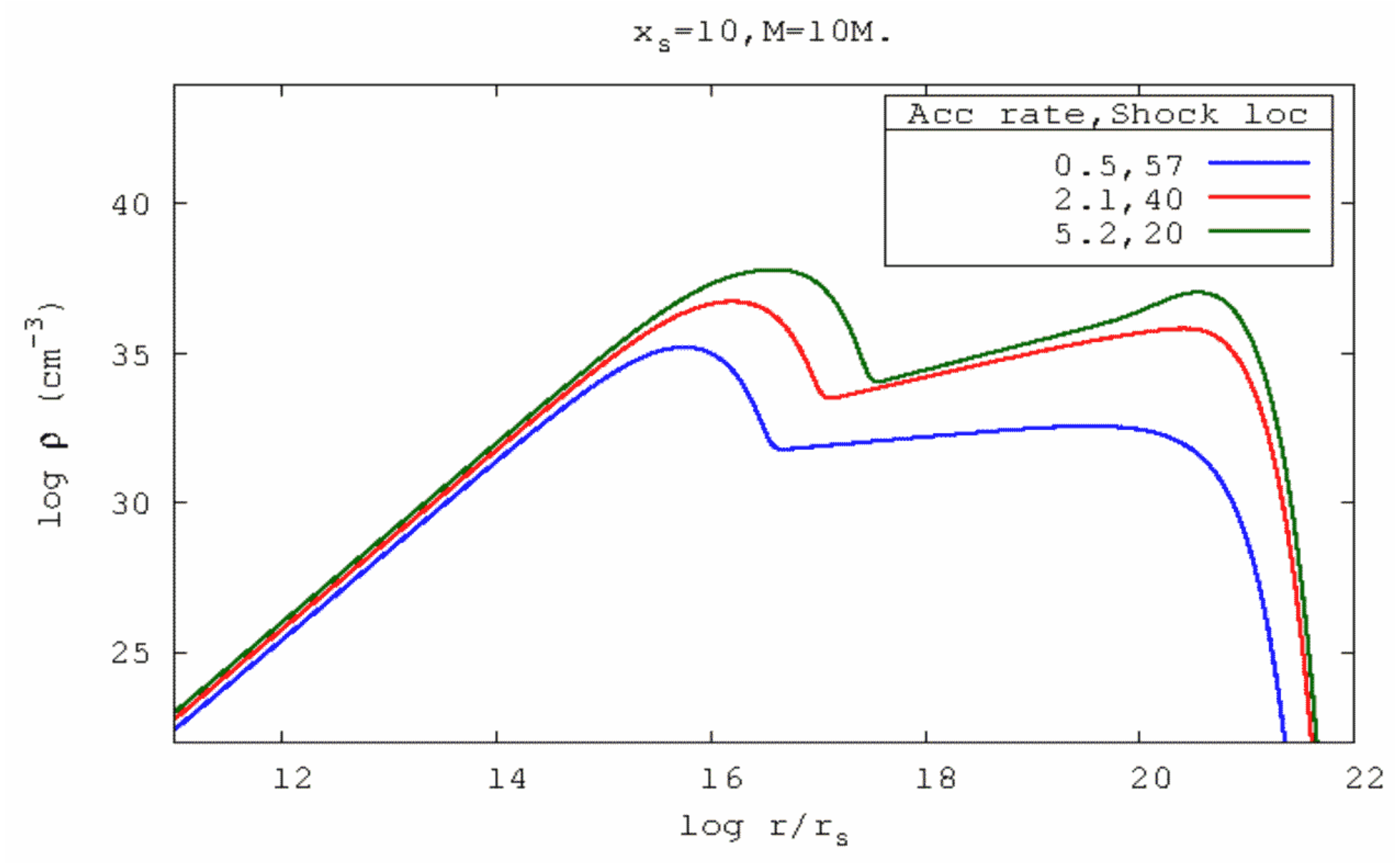

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SUMMARY :

- Improvements include : Inclusion of the relativistic equation of state while calculating the temperature profiles for the proton and the electron, the relativistic Coulomb coupling term, optical depth for moving media in the model.
- We presented the change in spectral shape as functions of different parameters like mass of the compact object, accretion rate of the flow and shock position.

Thank You!