Infrared Structure, QCD, LHC and All That

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Plan

- Form Factors in Gauge Theories
- Infrared Structure
  - Soft
  - Collinear
- Multi-leg, Multi-loop amplitudes
  - K+G equation
  - Catani's proposal
- Factorisation and Resummation
- Casimir Duality
- UV from IR

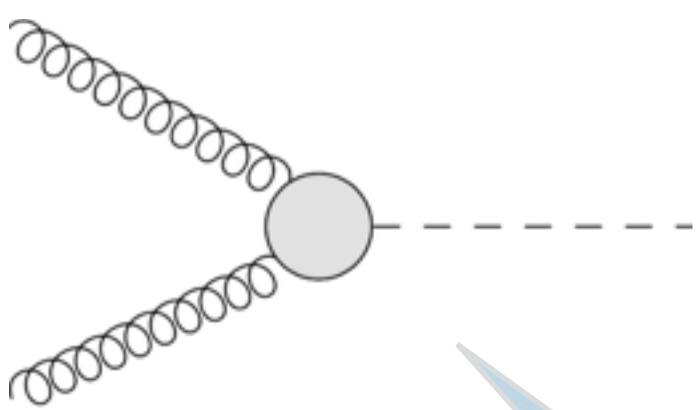
Form Factor

Form Factor : On-shell matrix elements of composite operators

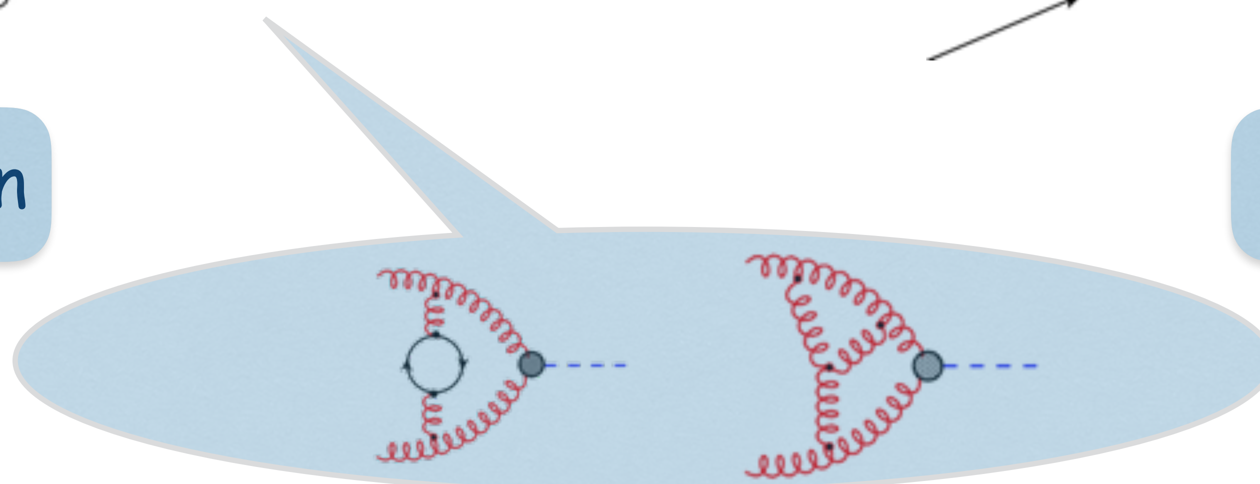
$$< p'|\mathcal{O}|p >$$

Gauge boson form factor

$$< g(p')|G^a_{\mu\nu}G^{\mu\nu a}|g(p) >$$

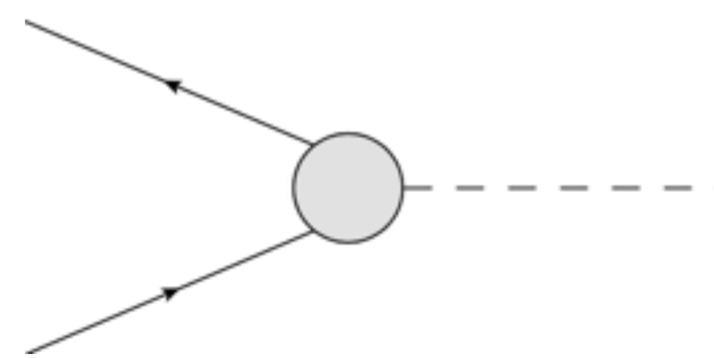


Higgs production



Fermion form factor

$$< e(p')|\overline{\psi}\gamma_\mu\psi|e(p) >$$

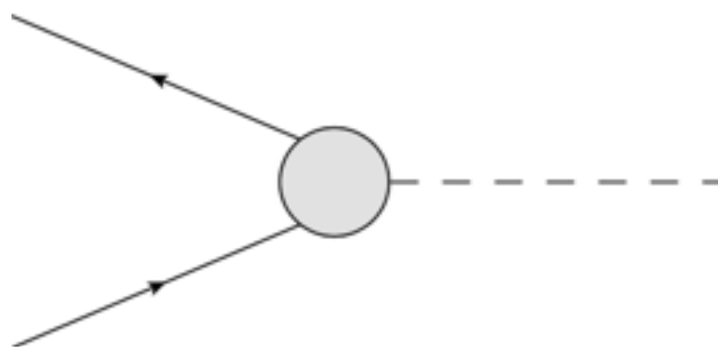


Drell-Yan

Sudakov Form Factor

[Sudakov, Sen, Sterman, Collins, Magnea]

Large $q^2 = (p+p')^2$ behaviour of form factors



Sudakov: $< e(p')|\overline{\psi}\gamma_\mu\psi|e(p) >$

$$\exp\left(-\frac{g^2}{8\pi^2}\ln^2(\frac{q^2}{m^2})\right)$$

Sen: QCD Leading and subleading logs exponentiate:

$$\left(\frac{g_s^2}{8\pi^2}\right)^n \ln^\nu\left(\frac{q^2}{m^2}\right), \qquad 2\nu \leq n$$

$< q(p')|\overline{\psi}\gamma_\mu\psi|q(p) >$

Infrared divergences

[Bloch, Nordsieck, Kinoshita,: Lee, Nauenberg]

Quantum Field Theories with massless particles encounter two kinds of divergences:

Soft :

On-shell amplitudes in gauge theories contain Soft divergences due to massless gauge bosons.

Collinear :

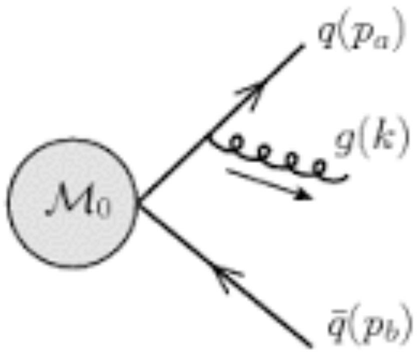
If the matter fields in the theory are light (mass of the particles are negligible compared to hard scale of the process), there will be mass singularities, called Collinear divergences

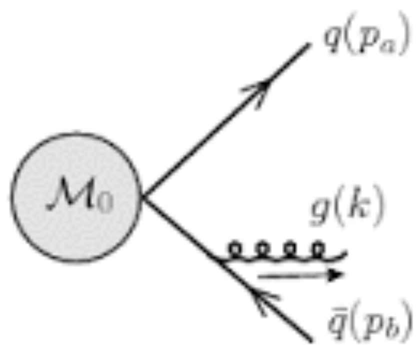
Infrared divergences

[Bloch, Nordsieck, Kinoshita,: Lee, Nauenberg]

In the Limit $k \to p \quad (p_a \text{ or } \mathrm{p_b})$ $\qquad m_a, m_b << Q$

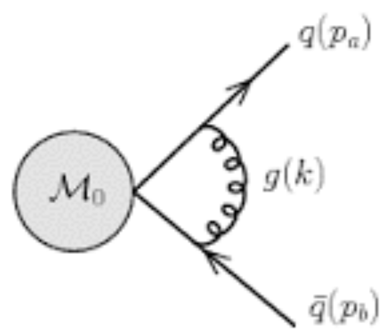
Real emission





$$\frac{1}{(p+k)^2} = \frac{1}{2p^0 k^0 (1 - \cos\theta)}$$

Virtual



$k^0 \to 0$ Soft divergence

$\cos\theta \to 0$ Collinear divergence

Infrared divergences

[S Weinberg]

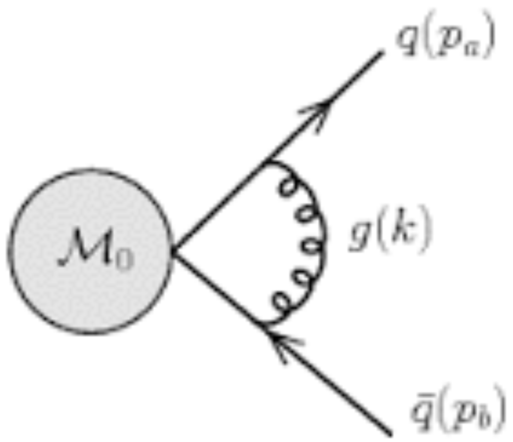
“In [Yang-Mills theory] a soft photon (**gluon**) *emitted from an external line can itself emit a pair of soft charged massless particles, which themselves emit soft photons* (**gluons**), *and so on, building up a cascade of soft massless particles each of which contributes an infra-red divergence. The elimination of such complicated interlocking infra-red divergences would certainly be a Herculean task, and might not even be possible. ”*

S. Weinberg, Phys. Rev. 140B (1965)

Sudakov Form Factor

One loop on-shell form factor

$$(p-k)^2 = 0,$$
$$p_a^2 = p_b^2 = m^2 << q^2$$



Soft

$k \to 0$

Collinear

$p_a || k \ or \ p_b || k$

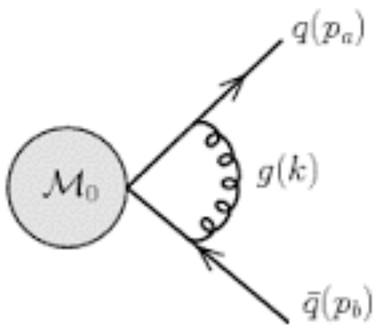
$$\int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2((p_a+k)^2 - m^2)((p_b-k)^2 - m^2)} \to \infty$$

Ill defined

Virtual effect

One loop on-shell form factor

$$\int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2((p_a+k)^2-m^2)((p_b-k)^2-m^2)} \to \infty$$



$$p_a^2 = p_b^2 = m^2 << q^2$$

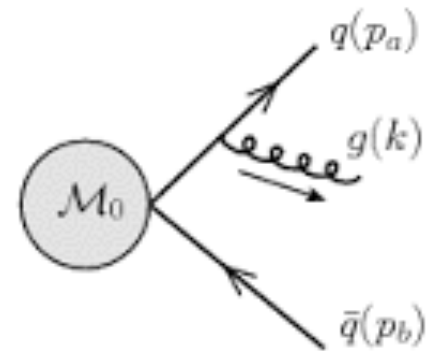
Summing to all orders in g^2

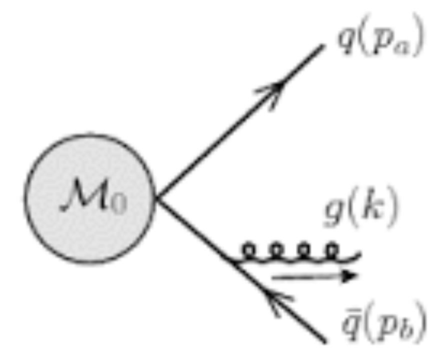
$$1 - g^2\left(\infty\right) + \frac{1}{2!}g^4\left(\infty\right) - \frac{1}{3!}g^6\left(\infty\right) + \cdots = exp(-g^2\infty)$$

Probability to happen this is ZERO

Real emission

Real photon emission:





$$\int \frac{d^4k}{(2\pi)^4} \frac{\delta^+(k^2)}{((p_a+k)^2-m^2)((p_b-k)^2-m^2)} \to \infty$$

Summing multiple emissions

$$p_a^2 = p_b^2 = m^2 << q^2$$

$$1 + g^2\infty + g^4\infty + \cdots$$

Probability grows uncontrollably

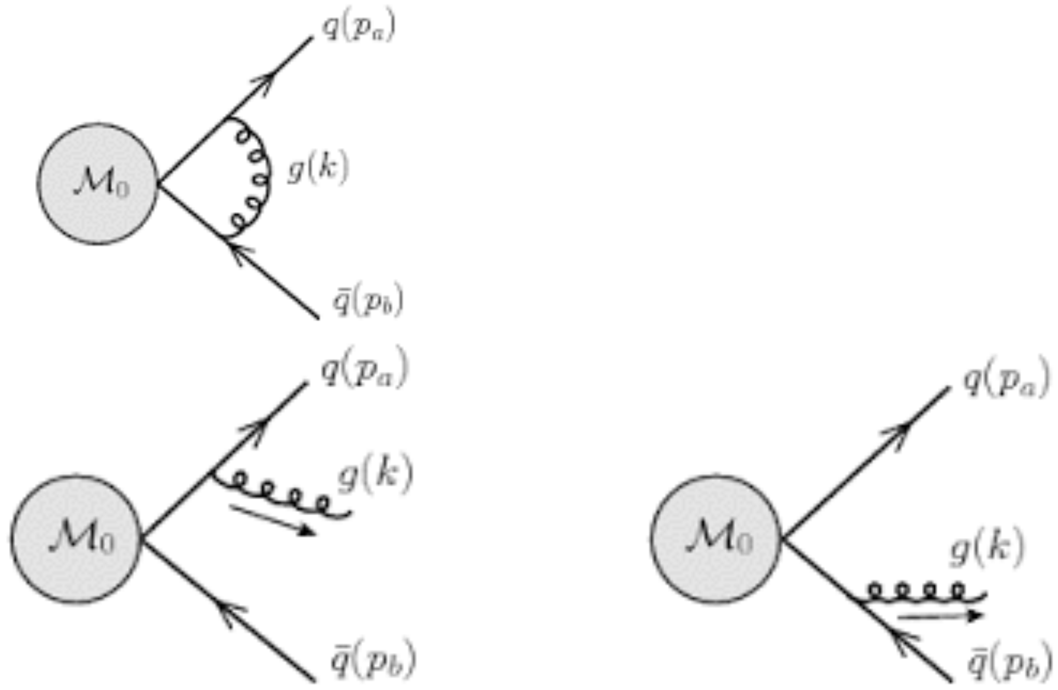
"Weinberg Fear"

Indistinguishable states

[Bloch, Nordsieck, Kinoshita,: Lee, Nauenberg]

If the detector is not sensitive to photons below certain energy Es (soft ones)

Below this energy the Detector can not distinguish these two processes when the gluons are soft/collinear



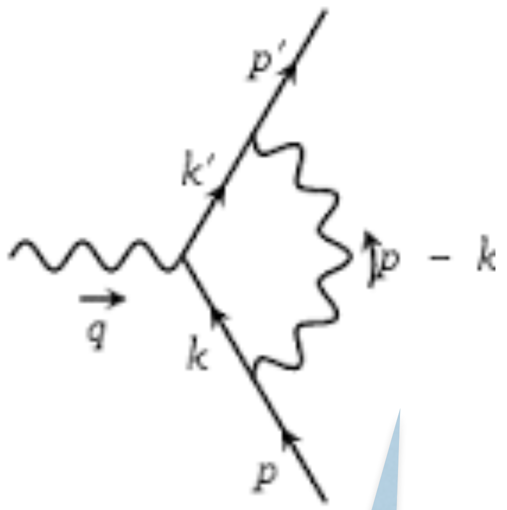
Indistinguishable when soft or collinear

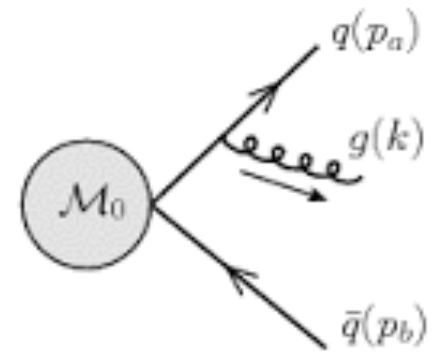
Sum their contributions and it is finite but dependent on Es !

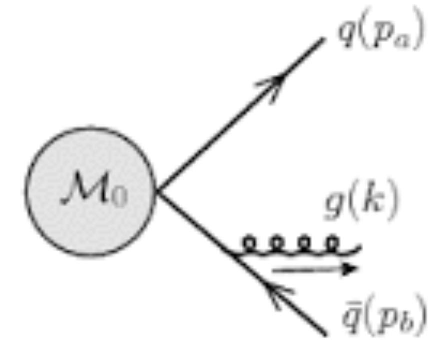
IR contribution

[Bloch, Nordsieck,Yannie, Suura]

If the detector is not sensitive to photons below certain energy Es (soft ones)







$$|\mathcal{B}|^2 \exp\left\{-\frac{\alpha}{\pi}\mathcal{K}\ln\left(\frac{E}{\lambda}\right)\right\}$$

$$\exp\left\{+\frac{\alpha}{\pi}\mathcal{K}\ln\left(\frac{\Delta E}{\lambda}\right)\right\}$$

$$\exp\left\{-\frac{\alpha}{\pi}\mathcal{K}\ln\left(\frac{E}{\lambda}\right)\right\}\exp\left\{+\frac{\alpha}{\pi}\mathcal{K}\ln\left(\frac{\Delta E}{\lambda}\right)\right\} \longrightarrow \left(\frac{\Delta E}{E}\right)^{\alpha\mathcal{K}/\pi}$$

Probability with no energy loss is Zero

Infrared Safety

[Bloch, Nordsieck, Kinoshita,: Lee, Nauenberg]

Physical processes that happen at Long distances are responsible for these divergences.

Measurable quantities are not sensitive to soft and Collinear divergences

REASON

Long distance physics is associated to configurations that are experimentally indistinguishable

Infrared Safety

[Bloch, Nordsieck, Kinoshita,: Lee, Nauenberg]

Bloch and Nordsieck Theorem

Soft Singularities cancel between real and virtual processes when one adds up all states which are indistinguishable by virtue of the energy resolution of the apparatus.

$$\exp\left\{-\frac{\alpha}{\pi}\mathcal{K}\ln\left(\frac{E}{\lambda}\right)\right\}\exp\left\{+\frac{\alpha}{\pi}\mathcal{K}\ln\left(\frac{\Delta E}{\lambda}\right)\right\} \longrightarrow \left(\frac{\Delta E}{E}\right)^{\alpha\mathcal{K}/\pi}$$

Kinoshita, Lee and Nauenberg Theorem

Both soft and collinear singularities cancel when the summation is carried out among all the mass degenerate states.

Infrared Safety

[Kulish, Fadeev]

Alternate formalism in QED was proposed by Kulish and Fadeev:

Evolution operator can be factorised into Asymptotic and Regular ones

Fock states are dressed with soft photons giving Coherent states.

S-matrix elements between these Coherent states give IR finite results.

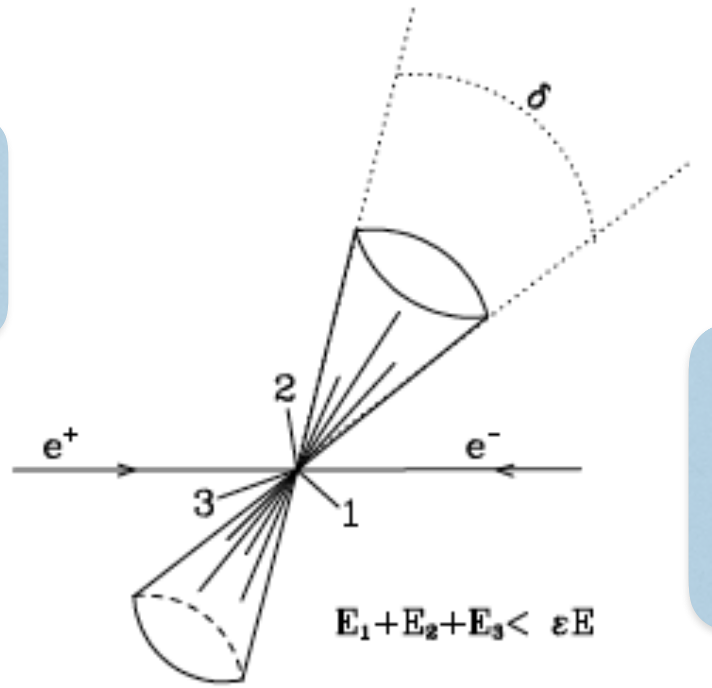
Sterman-Weinberg Jet in QCD

[Sterman, Weinberg]

Any event in electron-positron collision containing

Two cones of opening angle δ that contain all the energy of the event, excluding atmost ϵ fraction of the total.

Infra-red Safe

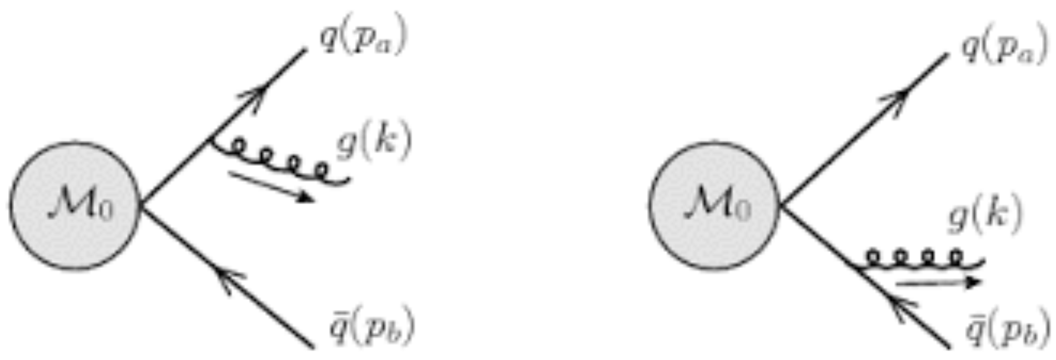


$$= \sigma_0 \left(1 - \frac{4\alpha_s C_F}{2\pi} \log \epsilon \log \delta^2 \right)$$

Infrared divergences

[Yennie, Frautschi, Suura, Weinberg]

Eikonal approximation:



$$g_s T^a \frac{p_\mu}{p \cdot k + i\epsilon} \mathcal{M}_0^{\mu a}$$

Universal current

Born amplitude

Infrared divergences FACTORISE

Infrared divergences

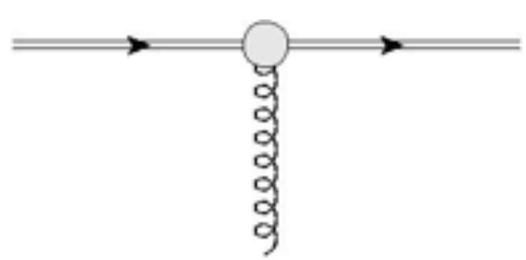
[Yennie, Frautschi, Subram; Weinberg]

Phase space integrals Diverge

$$\int \frac{dk_0}{k_0} |M|^2_{s,c} \to \infty$$

Loop integrals Diverge

$$|M|^2_{s,c} \approx (g_s T^a \frac{p_\mu}{p \cdot k + i\epsilon}) \ \ (g_s T^a \frac{p^\mu}{p \cdot k + i\epsilon})^* |\mathcal{M}_0|^2$$



$$i g_s \, \mathbf{T}^a \beta^\mu \times \frac{i}{\beta \cdot k + i\varepsilon}$$

Wilson Line Captures IR:

$$\mathcal{W}_\beta(\infty, 0) = \mathcal{P} \exp \left[i g_s \int_0^\infty \beta_\mu \, A^\mu(\lambda\beta) \, d\lambda \right] \qquad \beta^\mu = \frac{p^\mu}{\sqrt{p^2}}$$

Sudakov Equation (K+G Eqn.)

$$\mathcal{F}^{\lambda}_{\beta} = <\beta|\mathcal{O}^{\lambda}|\beta>$$

[Sen, Sterman, Moch, Vogt, Vermaseren; Ravindran; Magnea]

$$d = 4 + \varepsilon$$

$$Q^2 \frac{d}{dQ^2} \ln \mathcal{F}^{\lambda}_{\beta}(\hat{a}_s, Q^2, \mu^2, \epsilon) = \frac{1}{2}\left[K^{\lambda}_{\beta}(\hat{a}_s, \frac{\mu_R^2}{\mu^2}, \epsilon) + G^{\lambda}_{\beta}(\hat{a}_s, \frac{Q^2}{\mu_R^2}, \frac{\mu_R^2}{\mu^2}, \epsilon)\right]$$

poles

No poles

RG invariance

$$\mu_R^2 \frac{d}{d\mu_R^2} K^{\lambda}_{\beta}(\hat{a}_s, \frac{\mu_R^2}{\mu^2}, \epsilon) = -\mu_R^2 \frac{d}{d\mu_R^2} G^{\lambda}_{\beta}(\hat{a}_s, \frac{Q^2}{\mu_R^2}, \frac{\mu_R^2}{\mu^2}, \epsilon) = -A^{\lambda}_{\beta}(a_s(\mu_R^2))$$

Cusp (soft) Anomalous dim.

Casimir Duality

$$A_q = \frac{C_F}{C_A} A_g$$

Upto 3 loops

Single pole mystery

[Moch, Vogt, Vermaseren; Ravindran; Magnea]

Solution in $4+\epsilon$ dim:

$$\ln \mathcal{F}^{\lambda}_{\beta}(\hat{a}_s, Q^2, \mu^2, \epsilon) = \sum_{i=1}^{\infty} \hat{a}_s^i \left(\frac{Q^2}{\mu^2}\right)^{i\frac{\epsilon}{2}} S_\epsilon^i \hat{\mathcal{L}}^{\lambda}_{\beta,i}(\epsilon)$$

with

$$\hat{\mathcal{L}}^{\lambda}_{\beta,1}(\epsilon) = \frac{1}{\epsilon^2}\Big\{-2A^{\lambda}_{\beta,1}\Big\} + \frac{1}{\epsilon}\Big\{G^{\lambda}_{\beta,1}(\epsilon)\Big\}$$

$$\hat{\mathcal{L}}^{\lambda}_{\beta,2}(\epsilon) = \frac{1}{\epsilon^3}\Big\{\beta_0 A^{\lambda}_{\beta,1}\Big\} + \frac{1}{\epsilon^2}\Big\{-\frac{1}{2}A^{\lambda}_{\beta,2} - \beta_0 G^{\lambda}_{\beta,1}(\epsilon)\Big\} + \frac{1}{\epsilon}\Big\{\frac{1}{2}G^{\lambda}_{\beta,2}(\epsilon)\Big\}$$

$$\begin{aligned}\hat{\mathcal{L}}^{\lambda}_{\beta,3}(\epsilon) =& \frac{1}{\epsilon^4}\Big\{-\frac{8}{9}\beta_0^2 A^{\lambda}_{\beta,1}\Big\} + \frac{1}{\epsilon^3}\Big\{\frac{2}{9}\beta_1 A^{\lambda}_{\beta,1} + \frac{8}{9}\beta_0 A^{\lambda}_{\beta,2} + \frac{4}{3}\beta_0^2 G^{\lambda}_{\beta,1}(\epsilon)\Big\} \\ &+ \frac{1}{\epsilon^2}\Big\{-\frac{2}{9}A^{\lambda}_{\beta,3} - \frac{1}{3}\beta_1 G^{\lambda}_{\beta,1}(\epsilon) - \frac{4}{3}\beta_0 G^{\lambda}_{\beta,2}(\epsilon)\Big\} + \frac{1}{\epsilon}\Big\{\frac{1}{3}G^{\lambda}_{\beta,3}(\epsilon)\Big\}\end{aligned}$$

Cusp Anomalous dim.

Single Pole Mystery is solved

$$G^{\lambda}_{\beta,i}(\epsilon) = 2\left(B^{\lambda}_{\beta,i} - \gamma^{\lambda}_{\beta,i}\right) + f^{\lambda}_{\beta,i} + C^{\lambda}_{\beta,i} + \sum_{k=1}^{\infty} \epsilon^k g^{\lambda,k}_{\beta,i}$$

Single Pole mystery

[Ravindran, Smith, van Neerven; Moch et. al.]

UV Anomalous dim.

$$C^{\lambda}_{\beta,i} = \sum_j s_j C^{\lambda}_{\beta,j}, j < i$$

$$G^{\lambda}_{\beta,i}(\epsilon) = 2\left(B^{\lambda}_{\beta,i} - \gamma^{\lambda}_{\beta,i}\right) + f^{\lambda}_{\beta,i} + C^{\lambda}_{\beta,i} + \sum_{k=1}^{\infty} \epsilon^k g^{\lambda,k}_{\beta,i}$$

Collinear Anomalous dim.

Soft Anomalous dim.

Casimir Duality

$$f_q = \frac{C_F}{C_A} f_g$$

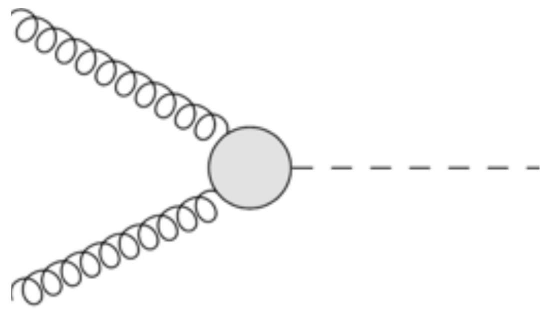
Upto 3 loops

Form Factor

[Moch, Vogt, Vermaseren,VR, Smith, v Neerven]

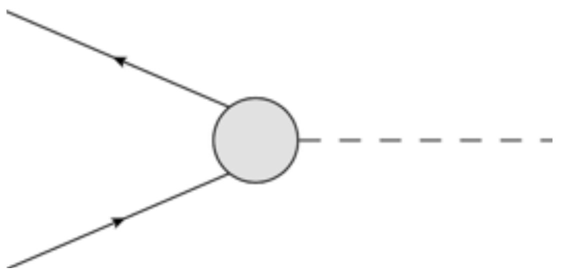
Gluon form factor

$$< g(p')|G^a_{\mu\nu}G^{\mu\nu a}|g(p) >$$



Quark form factor

$$< q(p')|\overline{\psi}\gamma_\mu\psi|q(p) >$$



Anomalous dimension

$$\gamma_q, \qquad \gamma_g$$

$$A_q = \frac{C_F}{C_A} A_g$$

$$f_q = \frac{C_F}{C_A} f_g$$

$$B_q, \qquad B_g$$

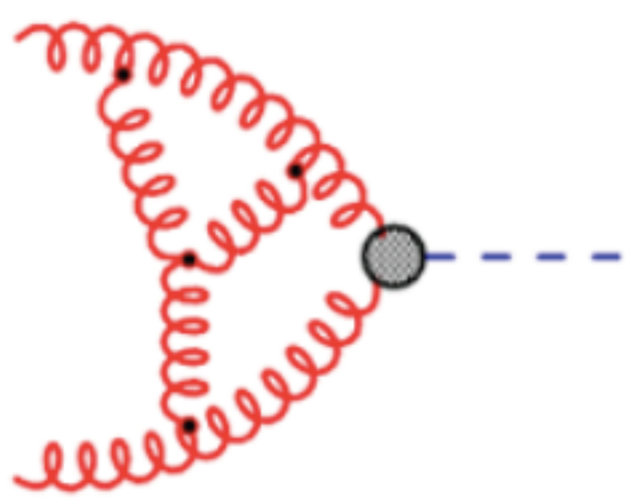
UV

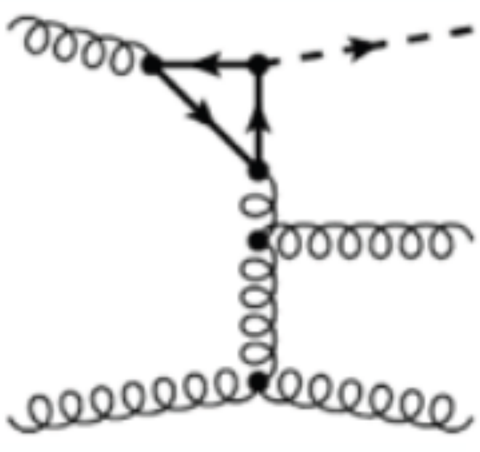
Cusp

Soft

Collinear

Multi-loops and Multi-legs



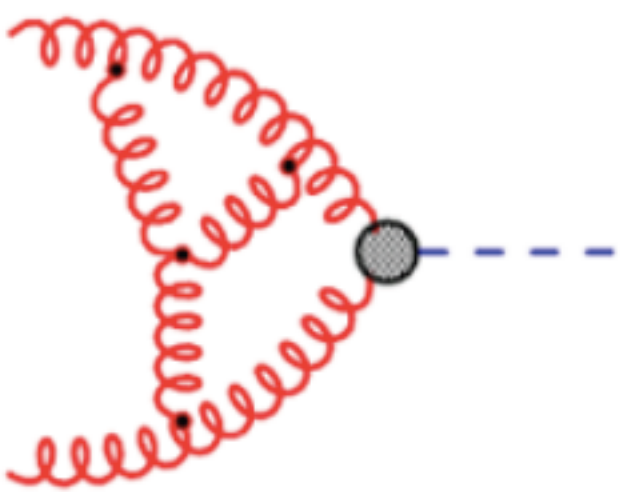


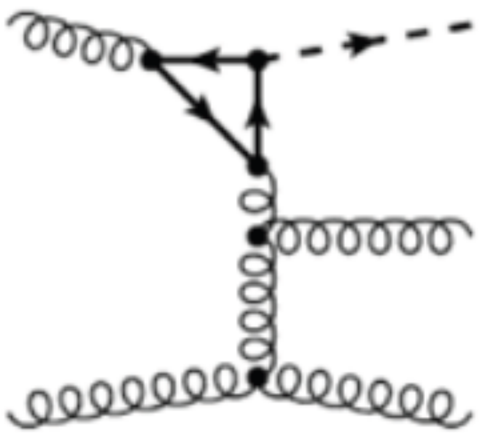
Catani's proposal

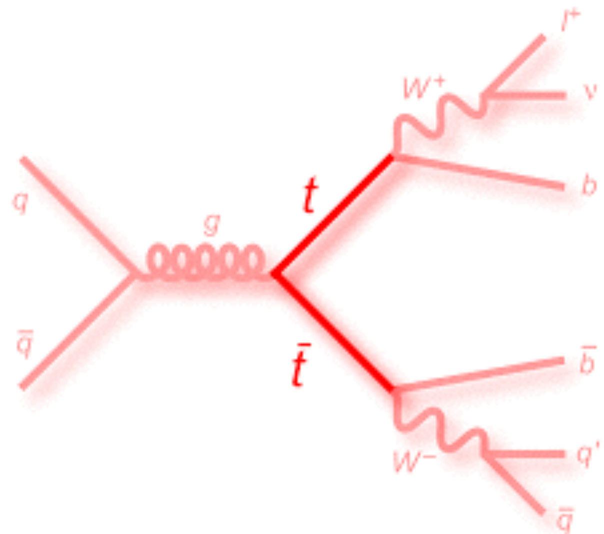
[Yennie, Frautschi, Subram; Weinberg]

UV Renormalised on-shell QCD amplitudes

$$|\mathcal{M}_n(\epsilon, \{p\})\rangle$$







Universal Infrared Structure

Catani's proposal

[S. Catani]

Universal IR Subtraction Operator

Up to Two loop !

$$\left[1 - \frac{\alpha_s}{2\pi} \boldsymbol{I}^{(1)}(\epsilon) - \left(\frac{\alpha_s}{2\pi}\right)^2 \boldsymbol{I}^{(2)}(\epsilon)\right] |\mathcal{M}_n(\epsilon, \{p\})\rangle$$

→ IR Finite

$$\boldsymbol{I}^{(1)}(\epsilon) = \frac{e^{\epsilon\gamma_E}}{\Gamma(1-\epsilon)} \sum_i \left(\frac{1}{\epsilon^2} + \frac{g_i}{\boldsymbol{T}_i^2}\frac{1}{\epsilon}\right) \sum_{j\neq i} \frac{\boldsymbol{T}_i \cdot \boldsymbol{T}_j}{2} \left(\frac{\mu^2}{-s_{ij}}\right)^{\epsilon}$$

$$\boldsymbol{I}^{(2)}(\epsilon) = \frac{e^{-\epsilon\gamma_E}\,\Gamma(1-2\epsilon)}{\Gamma(1-\epsilon)} \left(K + \frac{\beta_0}{2\epsilon}\right) \boldsymbol{I}^{(1)}(2\epsilon) - \frac{1}{2}\boldsymbol{I}^{(1)}(\epsilon)\left(\boldsymbol{I}^{(1)}(\epsilon) + \frac{\beta_0}{\epsilon}\right) + \boldsymbol{H}^{(2)}_{\rm R.S.}(\epsilon)$$

Colour matrices satisfy $\sum_i \boldsymbol{T}_i\, |\mathcal{M}_n(\epsilon, \{p\})\rangle = 0.$

Catani's proposal

[Catani]

Upto Two loop !

$$\left[1 - \frac{\alpha_s}{2\pi} \boldsymbol{I}^{(1)}(\epsilon) - \left(\frac{\alpha_s}{2\pi}\right)^2 \boldsymbol{I}^{(2)}(\epsilon)\right] |\mathcal{M}_n(\epsilon, \{p\})\rangle$$

Universal IR Subtraction Operators
depend only on
Process independent

Soft and Collinear

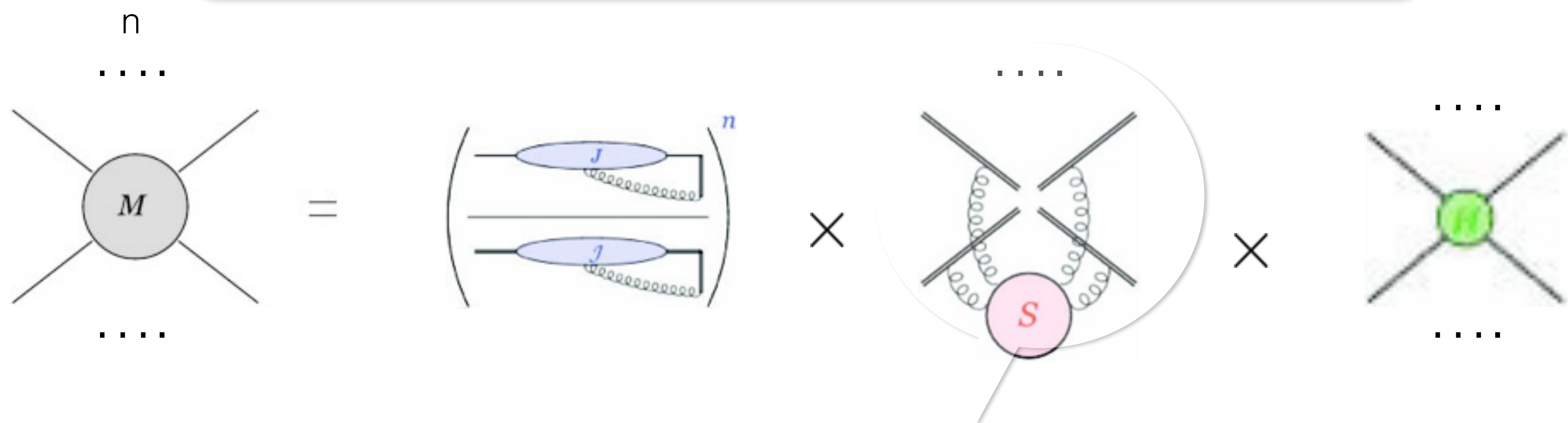
Anomalous Dimensions

Sterman's proof using factorisation

On-shell QCD amplitude in color basis:

[G. Sterman, M Tejeda-Yeomans]

$$\mathcal{M}^{[\mathrm{f}]}_{\{r_i\}}\left(\beta_j, \frac{Q^2}{\mu^2}, \alpha_s(\mu^2), \epsilon\right) = \sum_{L=1}^{N^{[\mathrm{f}]}} \mathcal{M}^{[\mathrm{f}]}_{L}\left(\beta_j, \frac{Q^2}{\mu^2}, \alpha_s(\mu^2), \epsilon\right) (c_L)_{\{r_i\}}$$



$$|\mathcal{M}_n(\epsilon, \{p\})\rangle = \prod_{i=1}^{n+2} J^{[i]}\left(\frac{Q'^2}{\mu^2}, \alpha_s(\mu^2), \epsilon\right) S^{[\mathrm{f}]}_{LI}\left(\beta_j, \frac{Q'^2}{\mu^2}, \frac{Q'^2}{Q^2}, \alpha_s(\mu^2), \epsilon\right) H^{[\mathrm{f}]}_{I}\left(\beta_j, \frac{Q^2}{\mu^2}, \frac{Q'^2}{Q^2}, \alpha_s(\mu^2)\right)$$

Collinear Soft Hard

Renormalisation Group for IR

[Sterman]

Factorisation of IR singularities:

IR singular

$$\mathcal{M}\left(\frac{p_i \cdot p_j}{\mu^2}, \alpha_s(\mu^2), \epsilon\right) = \mathcal{Z}\left(\frac{p_i \cdot p_j}{\mu_f^2}, \alpha_s(\mu_f^2), \epsilon\right) \times \mathcal{H}\left(\frac{p_i \cdot p_j}{\mu^2}, \frac{\mu^2}{\mu_f^2}, \alpha_s(\mu^2)\right)$$

Introduces Arbitrary Factorisation Scale μ_F

Amplitudes are independent of this scale

Renormalisation Group Invariance (RGE)

$$\mu_f \frac{d}{d\mu_f} \mathcal{Z}\left(\frac{p_i . p_j}{\mu_f^2}, \alpha_s(\mu_f^2), \epsilon\right) = -\mathcal{Z}\left(\frac{p_i . p_j}{\mu_f^2}, \alpha_s(\mu_f^2), \epsilon\right) \Gamma\left(\frac{p_i . p_j}{\mu_f^2}, \alpha_s(\mu_f^2)\right)$$

Three loop conjecture in QCD

[Becher, Neubert, Gardi, Magnea]

Matrix valued solution

$$\mathcal{Z}\left(\frac{p_i\cdot p_j}{\mu_f^2}, \alpha_s(\mu_f^2), \epsilon\right) = \mathcal{P}exp\left[-\int_0^{\mu_f^2}\frac{d\lambda}{\lambda}\Gamma\left(\frac{p_i\cdot p_j}{\lambda}, \alpha_s(\lambda)\right)\right]$$

Conjecture for IR anomalous dimension in QCD

$$\boldsymbol{\Gamma} = \sum_{(i,j)}\frac{\boldsymbol{T}_i\cdot\boldsymbol{T}_j}{2}\,\gamma_{\text{cusp}}(\alpha_s)\,\ln\frac{\mu^2}{-s_{ij}} + \sum_i \gamma^i(\alpha_s)$$

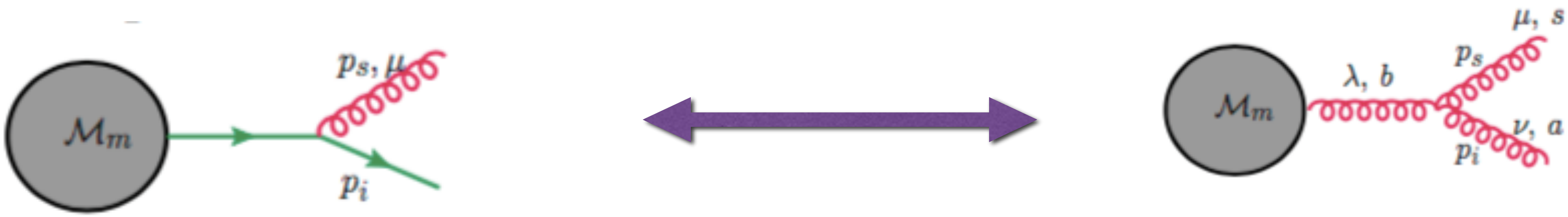
Di-pole

Soft

Soft +Collinear

Only Di-pole part Depends on Kinematics

Casimir Duality



Cusp Anomalous Dimension

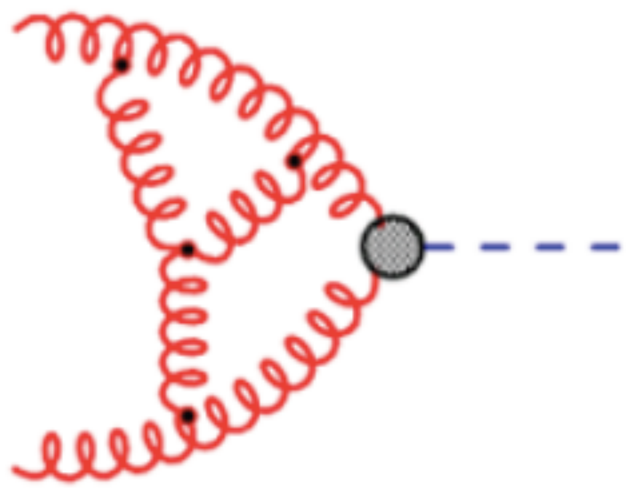
$$A_q = \frac{C_F}{C_A} A_g$$

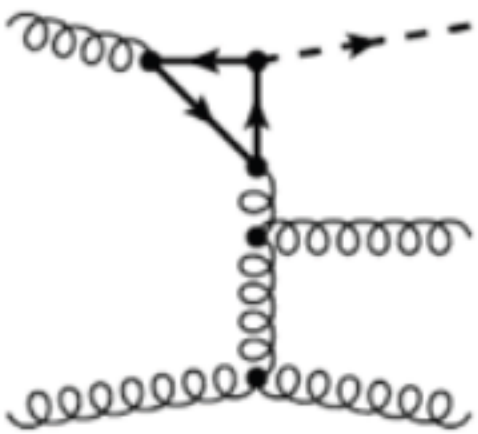
Soft Anomalous Matrix

$$\Gamma_q = \frac{C_F}{C_A} \Gamma_g$$

Upto 3- loops in QCD!

Multi-parton amplitude





Anomalous dimension

$\gamma_q, \quad \gamma_g$

$$A_q = \frac{C_F}{C_A} A_g$$

$$\Gamma_q = \frac{C_F}{C_A} \Gamma_g$$

$B_q, \quad B_g$

UV

Cusp

Soft Matrix

Collinear

Three loop conjecture

All order Conjecture of QCD:

$\gamma^i(\alpha_s)$ are independent of s_{ij}

$$\mathcal{Z}\left(\frac{p_i.p_j}{\mu_f^2},\alpha_s(\mu_f^2),\epsilon\right)=\mathcal{P}exp\left[-\int_0^{\mu_f^2}\frac{d\lambda}{\lambda}\Gamma\left(\frac{p_i.p_j}{\lambda},\alpha_s(\lambda)\right)\right]$$

known

$$\frac{1}{4}\sum_{L=1}^{\infty}\alpha^L\left[\frac{\gamma_c^{(L)}}{L^2\epsilon^2}\mathbf{D_0}-\frac{\gamma_c^{(L)}}{L\epsilon}\mathbf{D}+\frac{4}{L\epsilon}\gamma_J^{(L)}\mathbb{I}+\frac{1}{L\epsilon}\mathbf{\Delta}^{(L)}\right]$$

Three loop non-planar in $\mathcal{N}=4$ SYM

$$\mathbf{\Delta}^{(1)}=\mathbf{\Delta}^{(2)}=\mathbf{0}$$

$$\mathbf{\Delta}_4^{(3)}=\frac{1}{4}f_{abe}f_{cde}\Big[\mathbf{T}_1^a\mathbf{T}_2^b\mathbf{T}_3^c\mathbf{T}_4^d\,\mathcal{S}(x)+\mathbf{T}_4^a\mathbf{T}_1^b\mathbf{T}_2^c\mathbf{T}_3^d\,\mathcal{S}(1/x)\Big],$$

$$\mathbf{\Delta}_3^{(3)}=-C\,f_{abe}f_{cde}\sum_{\substack{i=1...4\\1\le j<k\le 4\\ j,k\neq i}}\{\mathbf{T}_i^a,\mathbf{T}_i^d\}\,\mathbf{T}_j^b\mathbf{T}_k^c\,.$$

$\mathcal{S}(x)$ are dependent on s_{ij} at three loop level

Breakdown of Conjucture

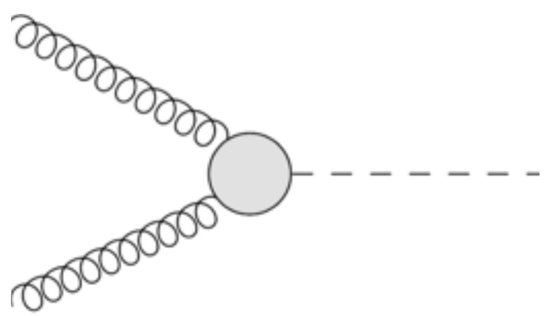
Universality of IR structure

Form Factor

[Moch, Vogt, Vermaseren,VR, Smith, v Neerven]

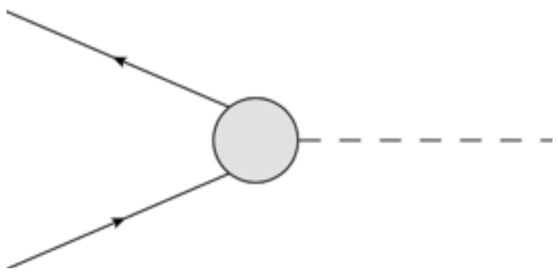
Gluon form factor

$$< g(p')|G^a_{\mu\nu}G^{\mu\nu a}|g(p) >$$



Quark form factor

$$< q(p')|\overline{\psi}\gamma_\mu\psi|q(p) >$$



$A_q, \quad A_g$

$f_q, \quad f_g$

$B_q, \quad B_g$

Anomalous dimension

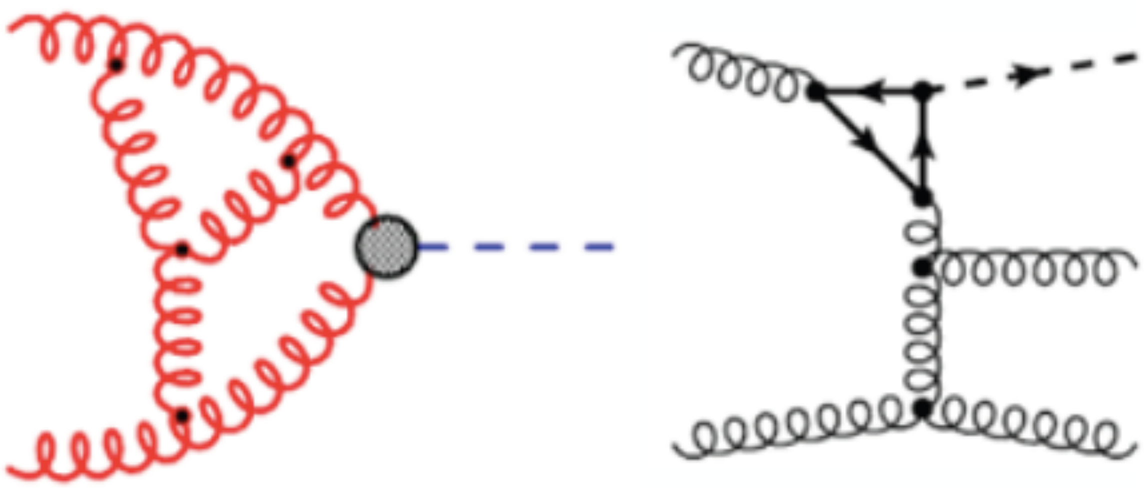
Cusp

Soft

Collinear

Universal independent of Operators, depends only external legs

Multi-parton amplitude



Anomalous dimension

$A_q, \quad A_g$

$\Gamma_q, \quad \Gamma_g$

$B_q, \quad B_g$

Cusp

Soft Matrix

Collinear

Infrared to Ultraviolet

UV renormalisation of Composite operators

[Taushif,Narayan,VR]

- Even in Renormalised Quantum Field Theories Composite operators are often UV divergent:

$$\mathcal{O}(x) = \overline{\psi}(x)\psi(x),$$

$$\mathcal{O}(x) = G^a_{\mu\nu}(x)G^{a\mu\nu}(x)$$

- Multiple of fields at the same space time point gives additional short distance (UV) singularities

- Overall UV renormalisation Z is required for each composite operator

$$\mathcal{O}^R(x,\mu_R^2) = Z_{\mathcal{O}}(\alpha_s(\mu_R^2),\epsilon)\ \mathcal{O}(x)$$

Conventional Method

[Taushif,Narayan,VR]

- Composite Operator Renormalisation

$$\mathcal{O}^R(x,\mu_R^2) = Z_{\mathcal{O}}(\alpha_s(\mu_R^2),\epsilon)\ \mathcal{O}(x)$$

- Sandwich between off-shell quark or gluon states (to avoid IR divergences)

$$< a(p')|O^R|a(p) >= Z_{\mathcal{O}}(\alpha_s(\mu_R^2),\epsilon) < a(p')|O|a(p) >$$

$$p^2, p'^2 \neq m^2, \quad m_{gluon} \neq 0$$

- $Z_{\mathcal{O}}(\alpha_s(\mu_R^2),\epsilon)$ contains only UV divergences

- Technically Challenging due to off-shell terms in loop integrals at higher loops

UV and IR poles mix

[Taushif,Narayan,VR]

On-shell matrix elements between quark and gluon fields are relatively easy to compute, BUT

$$< a(p')|O|a(p) >, \quad a = q, \overline{q}, g$$

UV and IR poles mix in n-dimensions

Trick!

Exploit Univerality of IR poles → UV poles

IR to UV

[Taushif,Narayan,VR]

From K+G equation of Form factors

Collinear Anomalous dim.

Soft Anomalous dim.

$$C^\lambda_{\beta,i} = \sum_j s_j C^\lambda_{\beta,j}, j < i$$

$$G^\lambda_{\beta,i}(\epsilon) = 2\left(B^\lambda_{\beta,i} - \gamma^\lambda_{\beta,i}\right) + f^\lambda_{\beta,i} + C^\lambda_{\beta,i} + \sum_{k=1}^{\infty} \epsilon^k g^{\lambda,k}_{\beta,i}$$

UV Anomalous dim.

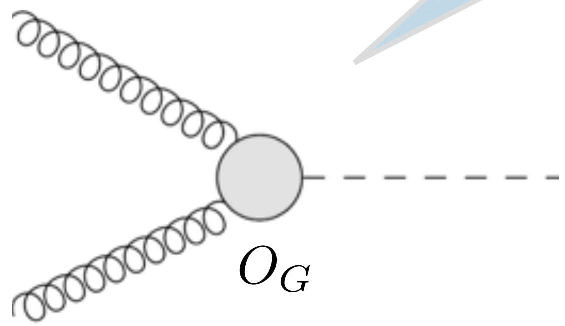
Subtract
of
Universal parts

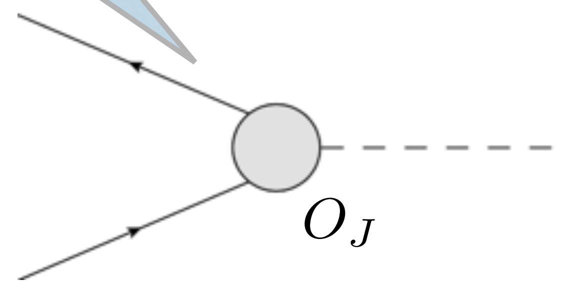
$\gamma^\lambda_{\beta,i}$ → $Z^\lambda_{\beta,i}$

$$\mu_R^2 \frac{d}{d\mu_R^2} \ln Z^\lambda(a_s, \mu_R^2, \epsilon) = \sum_{i=1}^{\infty} a_s^i \gamma_i^\lambda$$

Effective Couplings

$$\mathcal{L}_{\rm eff}^{A} = \Phi^{A}\left[-\frac{1}{8}C_G O_G - \frac{1}{2}C_J O_J\right]$$

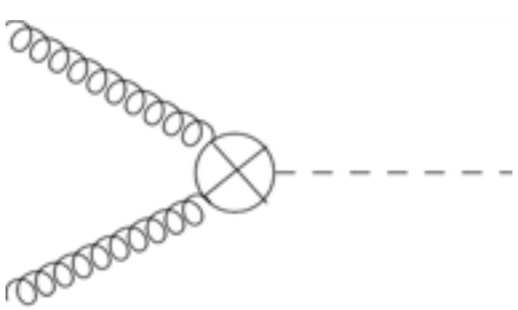


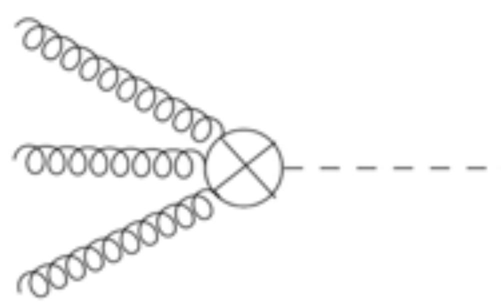


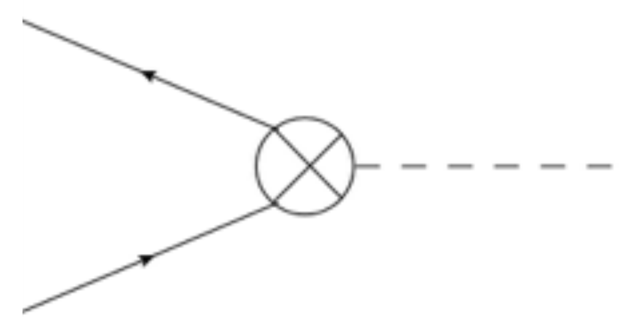
$$O_G(x) = G_a^{\mu\nu}\tilde{G}_{a,\mu\nu} \equiv \epsilon_{\mu\nu\rho\sigma}G_a^{\mu\nu}G_a^{\rho\sigma}$$

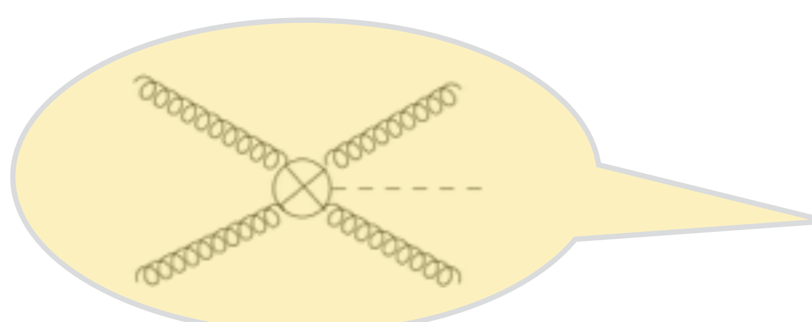
$$O_J(x) = \partial_\mu\left(\bar{\psi}\gamma^\mu\gamma_5\psi\right)$$

Feynman Rules:









Vanishes

UV renormalisation

[Larin]

- O_G mixes under renorm with O_J

Renormalised

Bare

$$[O_G]_R = Z_{GG}\,[O_G]_B + Z_{GJ}\,[O_J]_B$$

O_J needs finite renormalisation Z_5^s

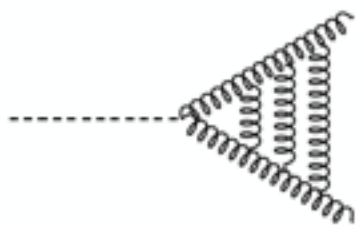
$$[O_J]_R = Z_5^s Z_{\overline{MS}}^s\,[O_J]_B$$

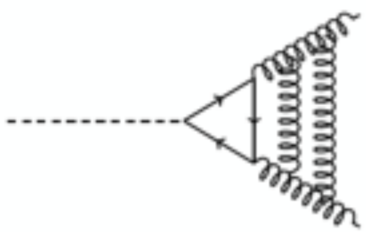
needs finite renormalisation

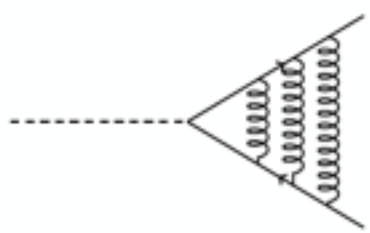
Qgraf and FORM …

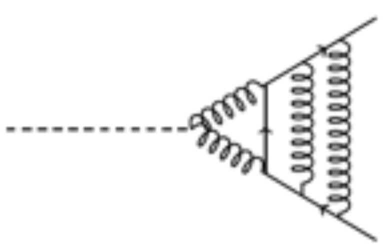
[Nogueira,Vermaseren]

Form Factors at Three loops in QCD

1. 

2. 

3. 

4. 

# of diagrams
1586
447
244
400

Qgraf

Diagram generation

FORM

Feynman rules

SU(N) color algebra

Lorentz contraction

Mathematica

IBP reductions

γ_5 & $\epsilon_{\mu\nu\lambda\sigma}$ in n-dimensions

Defining γ_5 & $\epsilon_{\mu\nu\lambda\sigma}$ in $n \neq 4$ dimension ?

Many Prescriptions exist $\{\gamma_5, \gamma^\mu\} \neq 0$ $n \neq 4$

['t Hooft and Veltman]

We follow:

$$\gamma_5 = \frac{i}{4!}\epsilon_{\mu_1\mu_2\mu_3\mu_4}\gamma^{\mu_1}\gamma^{\mu_2}\gamma^{\mu_3}\gamma^{\mu_4}$$

n-dim

$$\epsilon_{\mu_1\mu_2\mu_3\mu_4}\epsilon^{\nu_1\nu_2\nu_3\nu_4} = 4!\,\delta^{\nu_1\cdots\nu_4}_{[\mu_1\cdots\mu_4]}$$

[Larin]

Breaks Chiral Ward identity !

Remedy: Finite renormalisation

Method

Unphysical degrees of Freedom of gluons:

1. Feynman Gauge for internal gluons
2. Physical Polarisation for external gluons

Large number of 3- loop Integrals:

[Chetyrkin, Tkachov; Gehrmann, Remiddi]

1. IBP reduction
2. Lorentz Invariant (LI) Identities

*Reduze2 and **LiteRed*** → *22 Master Integrals*

Integration By Parts (IBP)

[Tkachov, Chetyrkin]

- Generalization of **Gauss's theorem** in d dimension.
- Within dimensional regularization, all integrals in d dimension are well-defined and convergent.

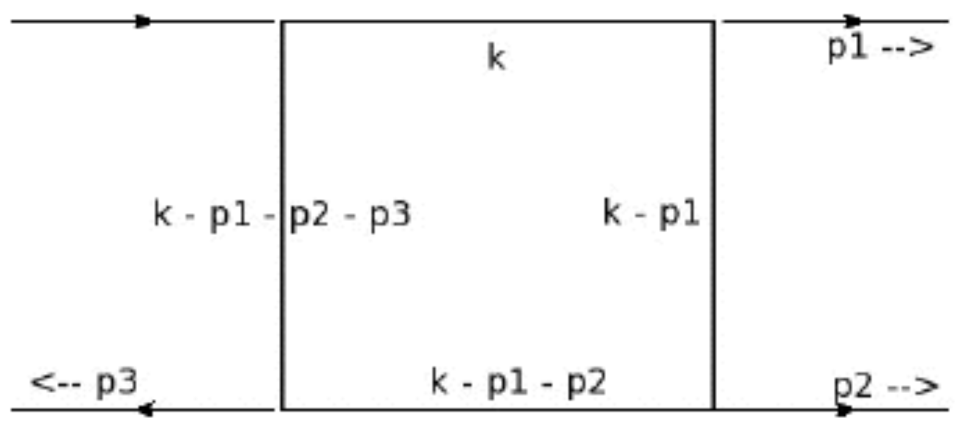
$$\Downarrow$$

the integrand must be zero at boundary
(*necessary condition for convergence*)

- to make it free from Lorentz index

$$\int \prod_{i=1}^{l} \mathcal{D}^d k_l \frac{\partial}{\partial k_j^\mu} \left(\frac{v^\mu}{D_1^{n_1} \ldots D_m^{n_m}} \right) = 0 \qquad \Big|_{v \equiv k_i, p_i}$$

IBP at one-loop



Consider

$$\mathcal{I}(n) = \int \mathcal{D}^d k \frac{1}{\mathcal{D}_0 \; \mathcal{D}_1 \; \mathcal{D}_{12}^n} \qquad \{n = 1, 2, \ldots\}$$

$$\mathcal{D}_0 = k^2, \mathcal{D}_1 = (k - p_1)^2, \mathcal{D}_{12} = (k - p_1 - p_2)^2, \\ \mathcal{D}_{123} = (k - p_1 - p_2 - p_3)^2 \; .$$

IBP

$$0 = \int \mathcal{D}^d k \frac{\partial}{\partial k^\mu} \left(\frac{k^\mu}{\mathcal{D}_0 \; \mathcal{D}_1 \; \mathcal{D}_{12}^n} \right) \\ = \Big(d - (n+3) \Big) \; \mathcal{I}(n) + n \; s \; \mathcal{I}(n+1)$$

Master Integral

$$\mathcal{I}(n+1) = -\frac{d - (n+3)}{n \; s} \; \mathcal{I}(n) \\ = (-1)^n \; \frac{(d - (n+3)) \ldots (d-5) \; (d-4)}{n! \; s^n} \; \mathcal{I}(1)$$

Lorentz Invariance

[Gehrmann, Remiddi]

- Under Lorentz transformation of external momenta

$$p_i^\mu \to p_i^\mu + \delta p_i^\mu = p_i^\mu + \omega^\mu_\nu p_i^\nu \quad \text{with } \omega^\mu_\nu = -\omega^\nu_\mu$$

the integrals are invariant i.e.

$$\mathcal{I}(p_i) = \mathcal{I}(p_i + \delta p_i) = \mathcal{I}(p_i) + \omega^\nu_\mu \sum_j p_j^\mu \frac{\partial}{\partial p_j^\nu} \mathcal{I}(p_i)$$

- from which the identity can be derived

$$\sum_j \left(p_{j,\mu} \frac{\partial}{\partial p_j^\nu} - p_{j,\nu} \frac{\partial}{\partial p_j^\mu} \right) \mathcal{I}(p_i) = 0$$

Differential eqns. for Integrals

Simplest Integral :

$$\mathcal{I} = \int \mathcal{D}^d k \frac{1}{k^2\,(k-p_1-p_2)^2}$$

Satisfies differential equation $\Downarrow$

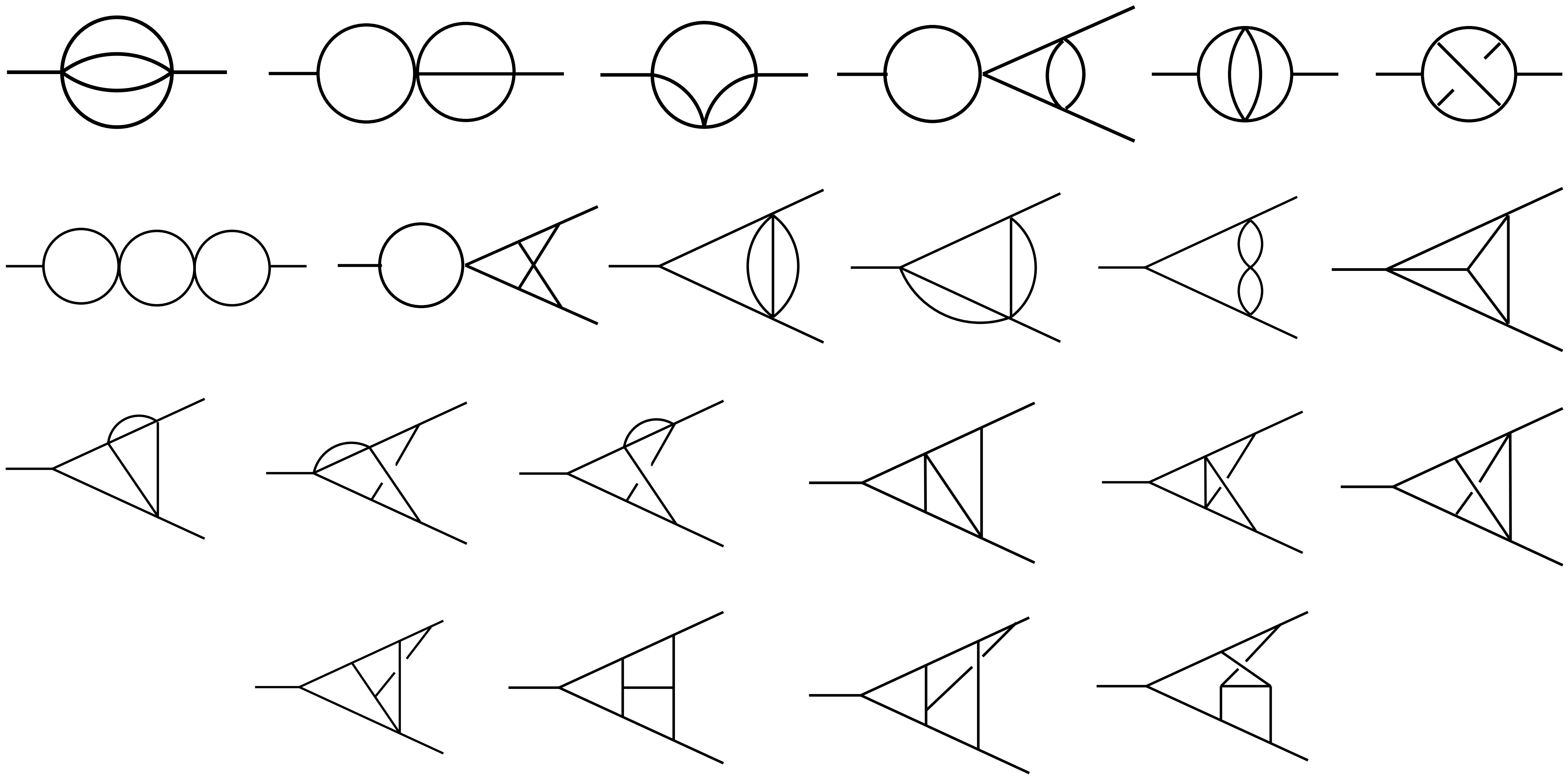
$$\frac{d}{ds_{12}}\mathcal{I} = \frac{(d-4)}{2s_{12}}\mathcal{I}$$

A linear homogeneous differential equation to solve, so

$$\mathcal{I} = \mathbf{C}\,(s_{12})^{\frac{(d-4)}{2}}$$

Master Integrals

[T.Gehrmann,T.Huber,D.Maitre,G.Heinrich,C.Studerus,D.A.Kosower,V.A.Smirnov,A.V.Smirnov, R.N.Lee]



UV Renormalisation Constant

$$\mu_R^2 \frac{d}{d\mu_R^2} \ln Z^\lambda(a_s, \mu_R^2, \epsilon) = \sum_{i=1}^{\infty} a_s^i \gamma_i^\lambda$$

Solution to third order

$$Z^\lambda = 1 + a_s\left[\frac{1}{\epsilon}2\gamma_1^\lambda\right] + a_s^2\left[\frac{1}{\epsilon^2}\left\{2\beta_0\gamma_1^\lambda + 2(\gamma_1^\lambda)^2\right\} + \frac{1}{\epsilon}\gamma_2^\lambda\right] + a_s^3\left[\frac{1}{\epsilon^3}\left\{8\beta_0^2\gamma_1^\lambda + 4\beta_0(\gamma_1^\lambda)^2 + \frac{4(\gamma_1^\lambda)^3}{3}\right\} + \frac{1}{\epsilon^2}\left\{\frac{4\beta_1\gamma_1^\lambda}{3} + \frac{4\beta_0\gamma_2^\lambda}{3} + 2\gamma_1^\lambda\gamma_2^\lambda\right\} + \frac{1}{\epsilon}\left\{\frac{2\gamma_3^\lambda}{3}\right\}\right].$$

$$[O_J]_R = Z_5^s Z_{\overline{MS}}^s [O_J]_B$$

[Larin,Zoller]

$$Z_5^s = 1 + a_s\{-4C_F\} + a_s^2\left\{22C_F^2 - \frac{107}{9}C_A C_F + \frac{31}{18}C_F n_f\right\}$$

All UV Z_{IK} agree with those in the literature

Adler-Bell-Jackie Anomaly

CP odd operators

$O_J(x) = \partial_\mu \left(\bar{\psi}\gamma^\mu \gamma_5 \psi\right)$ and $O_G(x) = G_a^{\mu\nu} \tilde{G}_{a,\mu\nu} \equiv \epsilon_{\mu\nu\rho\sigma} G_a^{\mu\nu} G_a^{\rho\sigma}$

related by

ABJ Anomaly

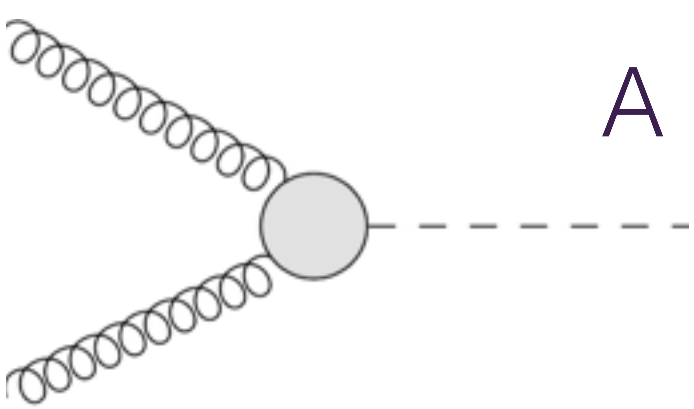
$$[O_J]_R = a_s \frac{n_f}{2} [O_G]_R$$

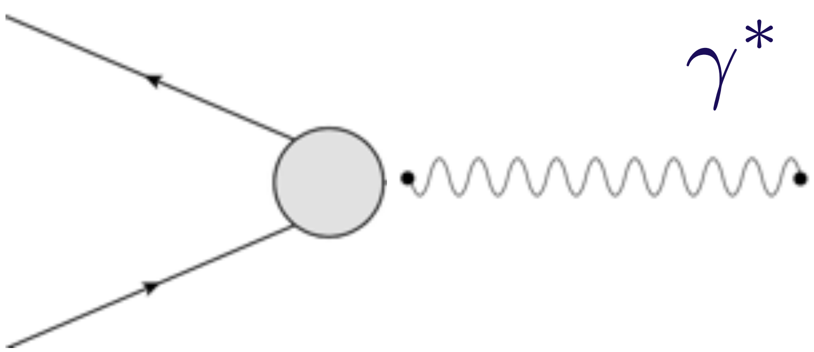
Renormalisation Group Invariance

$$\gamma_{JJ} = \frac{\beta}{a_s} + \gamma_{GG} + a_s \frac{n_f}{2} \gamma_{GJ}$$

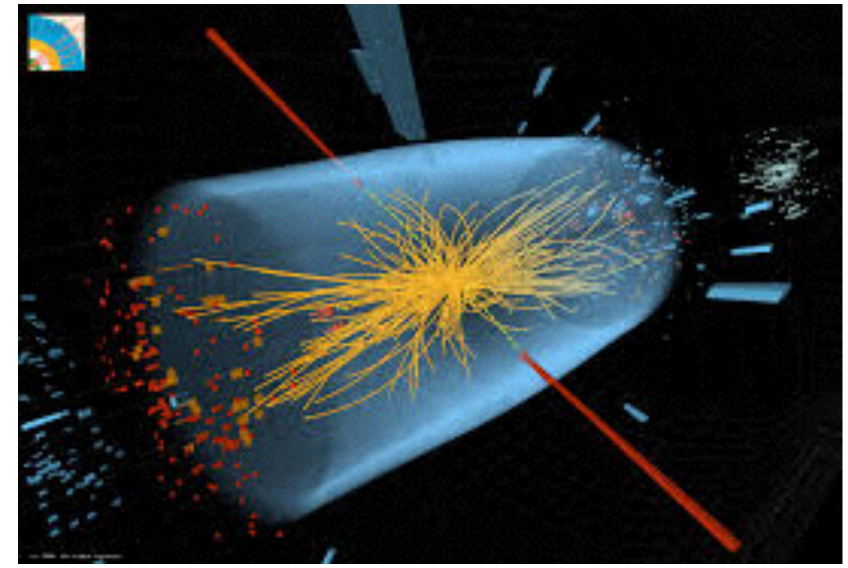
Check !

Duality between Higgs and DY



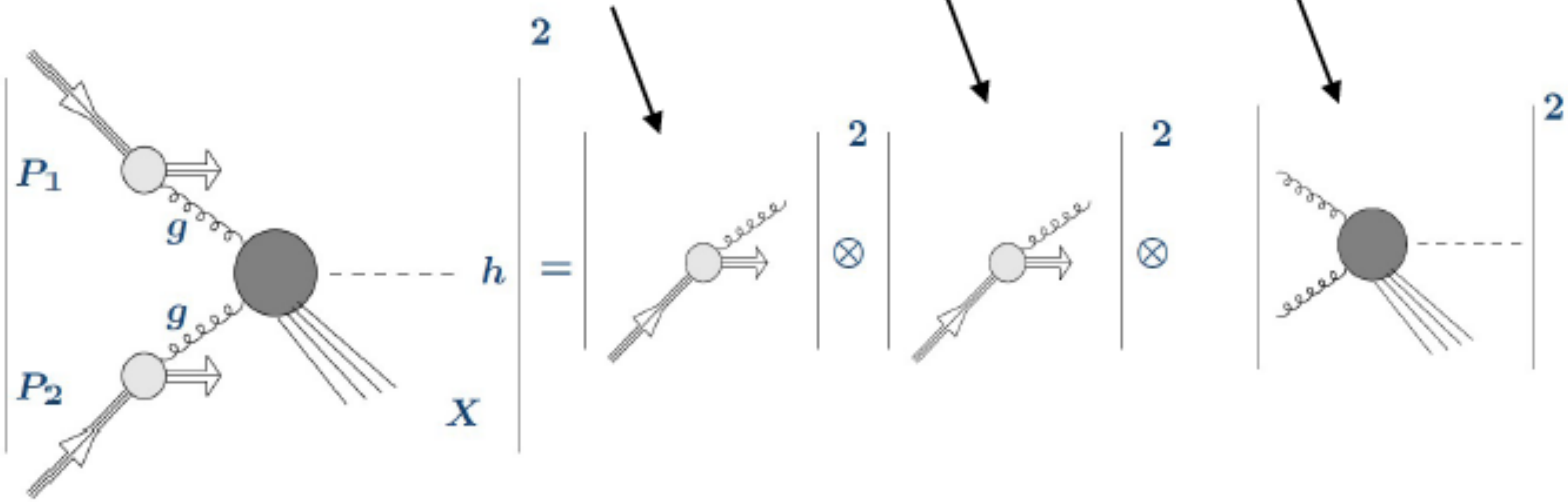


Physics at the LHC



$$P_1 + P_2 \rightarrow higgs + X$$

$$d\sigma^{P_1 P_2} = \sum_{i} \int dx_1 \int dx_2 f_{\frac{a}{P_1}} \left(x_1, \mu_F^2\right) f_{\frac{b}{P_2}} \left(x_2, \mu_F^2\right) d\hat{\sigma}^{ab} \left(x_1, x_2, \{p_i\}, \mu_F^2\right),$$



Parton Model in QCD

Inclusive cross section:

$$\sigma^A(\tau, m_A^2) = \sigma^{A,(0)}(\mu_R^2) \sum_{a,b=q,\bar{q},g} \int_\tau^1 dy\, \Phi_{ab}(y, \mu_F^2) \Delta_{ab}^A\left(\frac{\tau}{y}, m_A^2, \mu_R^2, \mu_F^2\right)$$

Partonic Flux:

$$\Phi_{ab}(y, \mu_F^2) = \int_y^1 \frac{dx}{x} f_a(x, \mu_F^2) f_b\left(\frac{y}{x}, \mu_F^2\right) ,$$

Partonic cross section:

$$\Delta_{ab}^A(z, q^2, \mu_R^2, \mu_F^2) = \Delta_{ab}^{A,\mathrm{SV}}(z, q^2, \mu_R^2, \mu_F^2) + \Delta_{ab}^{A,\mathrm{hard}}(z, q^2, \mu_R^2, \mu_F^2)$$

Soft + Virtual

Hard

Mass Factorisation

Bloch and Nordsiek:

Final state Soft and Collinear singularities cancel

KLN theorem:

Initial state collinear singularities are removed by

MASS FACTORISATION

Factorise the collinear part of Partonic cross section:

$$\sigma^B(z) = \Delta(z, \mu_F) \otimes \Gamma^2\left(z, \mu_F, \frac{1}{\epsilon}\right)$$

Renormalise the Bare parton distribution functions

$$f_a(z, \mu_F) = \Gamma\left(z, \mu_F, \frac{1}{\epsilon}\right) \otimes f_a^B(z)$$

Soft+Virtual

[VR]

Soft+Virtual: $$\Delta_{g,i}^{A,\mathrm{SV}} = \Delta_{g,i}^{A,\mathrm{SV}}|_{\delta}\delta(1-z) + \sum_{j=0}^{2i-1} \Delta_{g,i}^{A,\mathrm{SV}}|_{\mathcal{D}_j}\mathcal{D}_j\,.$$

$$\mathcal{D}_i \equiv \left[\frac{\ln^i(1-z)}{1-z}\right]_+$$

Factorisation of universal IR configuration leads to

Exponentiation

$$\Delta_g^{A,\mathrm{SV}}(z,q^2,\mu_R^2,\mu_F^2) = \mathcal{C}\exp\left(\Psi_g^A\left(z,q^2,\mu_R^2,\mu_F^2,\epsilon\right)\right)\Big|_{\epsilon=0}$$

Mellin Convolution in z-space:

$$\mathcal{C}e^{f(z)} = \delta(1-z) + \frac{1}{1!}f(z) + \frac{1}{2!}f(z)\otimes f(z) + \cdots\,.$$

Exponentiation

[VR]

UV finite cross section $Z^2(\mu_R^2,\epsilon)\ \sigma^B(z,Q^2)$

Factor out Virtual

$$|\hat{F}(Q^2,\epsilon)|^2\ Z^2(\mu_R^2,\epsilon)\sigma^{S+H}(z,Q^2)$$

Mass Factorisation

$$\Gamma^T\left(z,\mu_F,\frac{1}{\epsilon}\right)\otimes\left(|\hat{F}(Q^2,\epsilon)|^2\ Z^2(\mu_R^2,\epsilon)\sigma^{S+H}(z,Q^2)\right)\otimes\Gamma\left(z,\mu_F,\frac{1}{\epsilon}\right)$$

Keeping only $\delta(1-z),\ \mathcal{D}_i(z)$

$$\Delta_I(z,q^2,\mu_R^2,\mu_F^2)=\mathcal{C}exp\left(\Psi_I(z,q^2,\mu_R^2,\mu_F^2,\epsilon)\right)|_{\epsilon=0}$$

$$I=q,g$$

Exponentiation

[VR]

RG invariance, K+G equation, Mass factorisation:

$$\Delta_g^{A,\mathrm{SV}}(z,q^2,\mu_R^2,\mu_F^2) = \mathcal{C}\exp\left(\Psi_g^A\left(z,q^2,\mu_R^2,\mu_F^2,\epsilon\right)\right)\Big|_{\epsilon=0}$$

α_s^3

$$\Psi_g^A\left(z,q^2,\mu_R^2,\mu_F^2,\epsilon\right) = \left(\ln\left[Z_g^A(\hat{a}_s,\mu_R^2,\mu^2,\epsilon)\right]^2 + \ln\left|\mathcal{F}_g^A(\hat{a}_s,Q^2,\mu^2,\epsilon)\right|^2\right)\delta(1-z)$$
$$+ 2\Phi_g^A(\hat{a}_s,q^2,\mu^2,z,\epsilon) - 2\mathcal{C}\ln\Gamma_{gg}(\hat{a}_s,\mu_F^2,\mu^2,z,\epsilon)\,.$$

	Divergences
• Z_g^A is operator renormalisation	UV
• $\mathcal{F}_g^A$ is the Form Factor	UV + Soft + Collinear
• Φ_g^A is the Soft distribution function	Soft
• Γ_{gg} is the Altarelli Parisi kernel	Initial state collinear

Sum Total = Finite

Exponentiation

[VR]

$$\Psi_g^A\left(z,q^2,\mu_R^2,\mu_F^2,\epsilon\right) = \left(\ln\left[Z_g^A(\hat{a}_s,\mu_R^2,\mu^2,\epsilon)\right]^2 + \ln\left|\mathcal{F}_g^A(\hat{a}_s,Q^2,\mu^2,\epsilon)\right|^2\right)\delta(1-z)$$
$$+ 2\Phi_g^A(\hat{a}_s,q^2,\mu^2,z,\epsilon) - 2\mathcal{C}\ln\Gamma_{gg}(\hat{a}_s,\mu_F^2,\mu^2,z,\epsilon)\,.$$

α_s^3

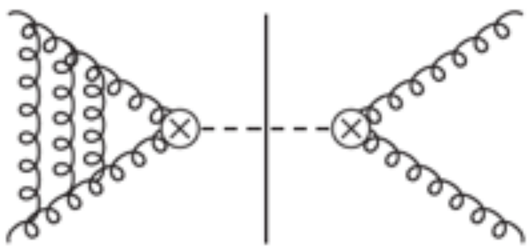
$$q^2\frac{d}{dq^2}\Phi_g^A(\hat{a}_s,\mu^2,z,q^2,\epsilon) = \frac{1}{2}\left[\overline{K}_g(\hat{a}_s,\mu^2,z,\mu_R^2,\epsilon) + \overline{G}_g^A(\hat{a}_s,\mu^2,z,q^2,\mu_R^2,\epsilon)\right]$$

$$\Phi_g^A(\hat{a}_s,\mu^2,z,q^2,\epsilon) = \sum_{i=1}^{\infty}\hat{a}_s^i\left(\frac{q^2(1-z)^2}{\mu^2}\right)^{i\epsilon/2} S_\epsilon^i\left(\frac{i\epsilon}{(1-z)}\right)\hat{\phi}_g^{(i)}(\epsilon)$$

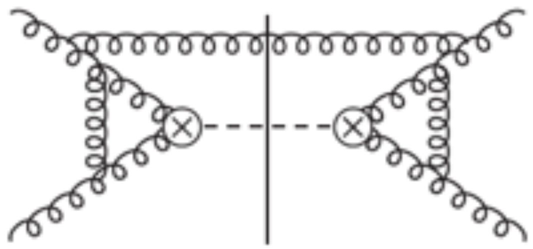
$$\mathcal{D}_i \equiv \left[\frac{\ln^i(1-z)}{1-z}\right]_+$$

$$\frac{1}{1-z}[(1-z)^2]^{i\frac{\varepsilon}{2}} = \frac{1}{i\varepsilon}\delta(1-z) + \sum_{j=0}^{\infty}\frac{(i\varepsilon)^j}{j!}\mathcal{D}_j$$

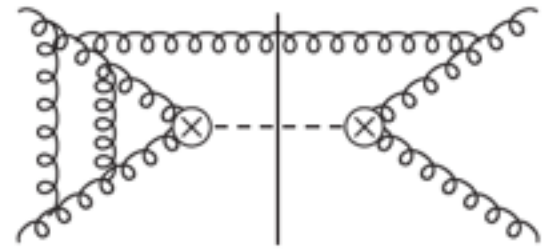
Higgs Studies by theorists



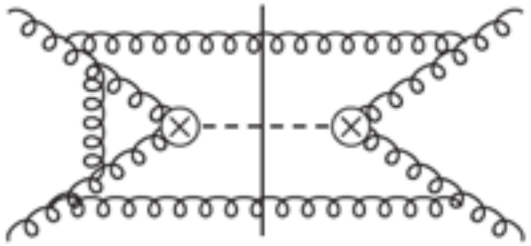
Triple virtual



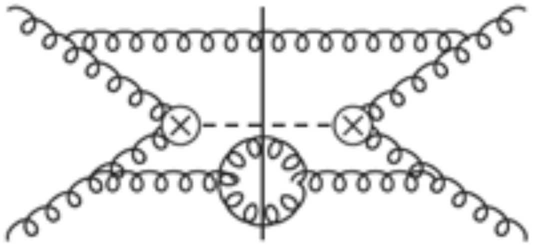
Real-virtual squared



Double virtual real



Double real virtual



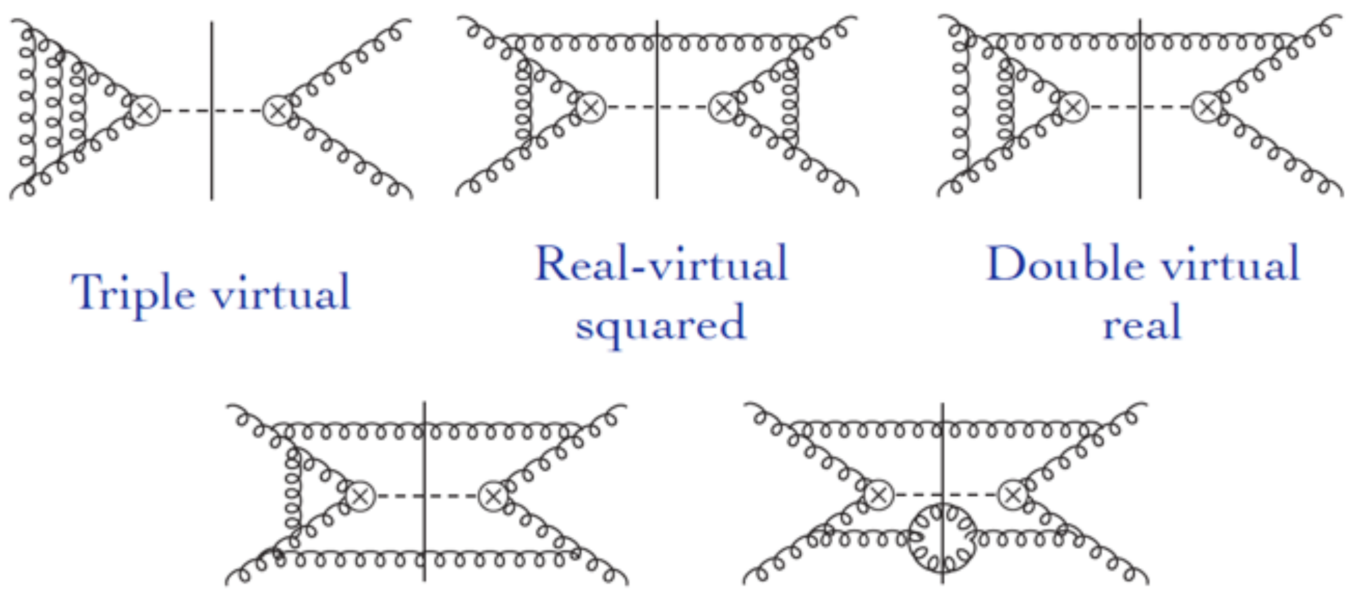
Triple real

100 000 diagrams

	Integrals
NNLO	50 000
N3LO	517 531 178

Higgs production to N^3LO in QCD at the LHC

$Anastasiou, Duhr, Dulat, Furlan, Gehrmann, Herzog, Mistlberger$



100 000 diagrams	Integrals	Master Integrals
NNLO	50 000	27
N3LO	517 531 178	1028

Integration By Parts

$$\int \frac{d^d k_1}{(2\pi)^d} \cdots \int \frac{d^d k_3}{(2\pi)^d} \frac{\partial}{\partial k_i} \cdot \left(v_j \frac{1}{\prod_l D_l^{n_l}} \right) = 0$$

Lorentz Invariance

$$p_i^\mu p_j^\nu \left(\sum_k p_{k[\nu} \frac{\partial}{\partial p_k^{\mu]}} \right) J(\vec{n}) = 0.$$

↓

Master Integrals

Exponentiation

[VR]

RG invariance, K+G equation, Mass factorisation:

$$\Delta_q^{DY,SV}(z, q^2, \mu_R^2, \mu_F^2) = \mathcal{C}exp\left(\Psi_q^{DY}(z, q^2, \mu_R^2, \mu_F^2, \epsilon)\right)\Big|_{\epsilon=0}$$

α_s^3

$$\Psi_q^{DY}(z, q^2, \mu_R^2, \mu_F^2, \epsilon) = \left(\ln\left[Z_q^{DY}(\hat{a}_s, \mu_R^2, \mu^2, \epsilon)\right]^2 + \ln\left|\mathcal{F}_q^{DY}(\hat{a}_s, Q^2, \mu^2, \epsilon)\right|^2\right)\delta(1-z)$$
$$+2\Phi_q^{DY}(\hat{a}_s, q^2, \mu^2, z, \epsilon) - 2\mathcal{C}\ln\Gamma_{qq}(\hat{a}_s, \mu_f^2, \mu^2, z, \epsilon).$$

Upto NNLO (two loop)

$$\Phi_q^{DY^{(i)}} = \frac{C_F}{C_A}\Phi_g^{A(i)} \qquad i = 1, 2$$

Conjecture for N3LO

For Drell-Yan (DY)

[VR]

Higgs Production → Drell-Yan Production

$$\Delta_g^{A,\mathrm{SV}}(z,q^2,\mu_R^2,\mu_F^2) = \mathcal{C}\exp\left(\Psi_g^A\left(z,q^2,\mu_R^2,\mu_F^2,\epsilon\right)\right)\Big|_{\epsilon=0}$$

$$\Psi_g^A \to \Psi_q^{DY}$$

- Z_q^{DY} → Known
- $\mathcal{F}_q^{DY}$ → Known
- Γ_{qq}^{DY} → Known

α_s^3

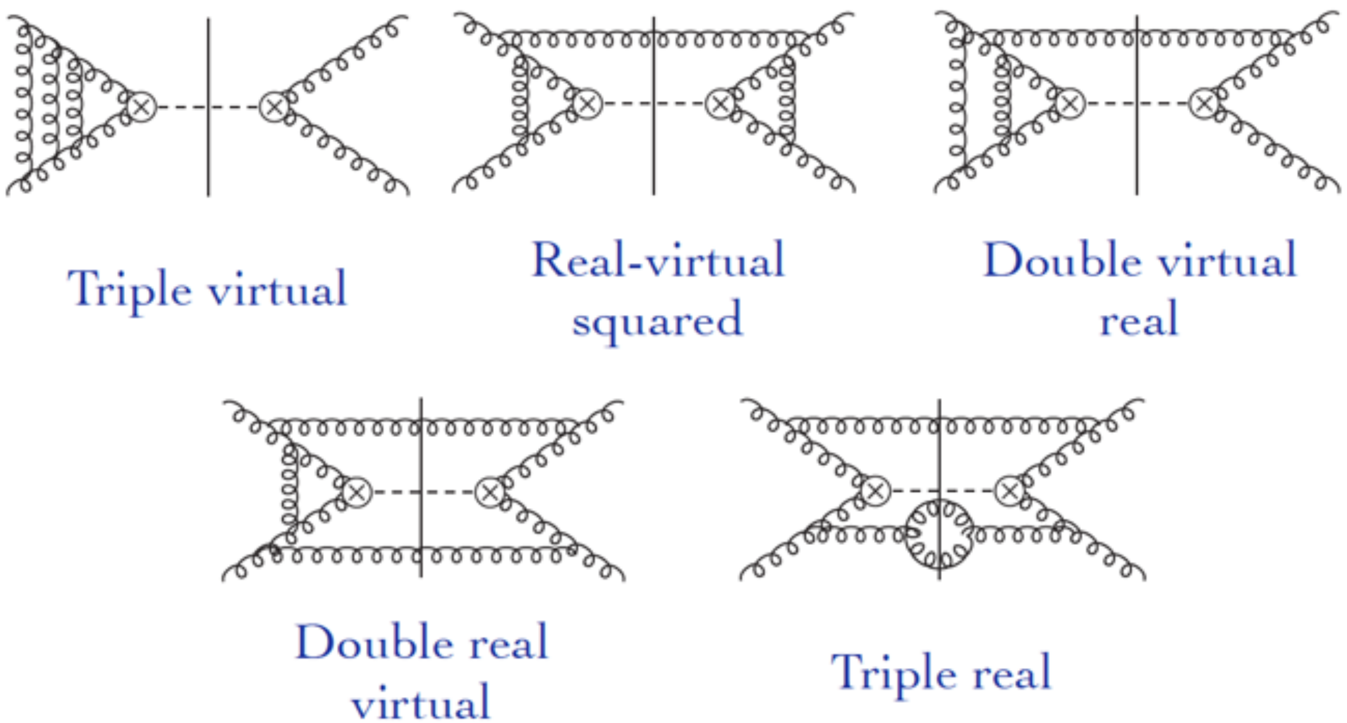
IR Safety

$$\Phi_q^{DY^{(i)}} = \frac{C_F}{C_A}\Phi_g^{A(i)}$$

$i = 3$

Higgs production at 3-loops

[Taushif,Narayan,VR]



$\rightarrow \Phi_g^A$

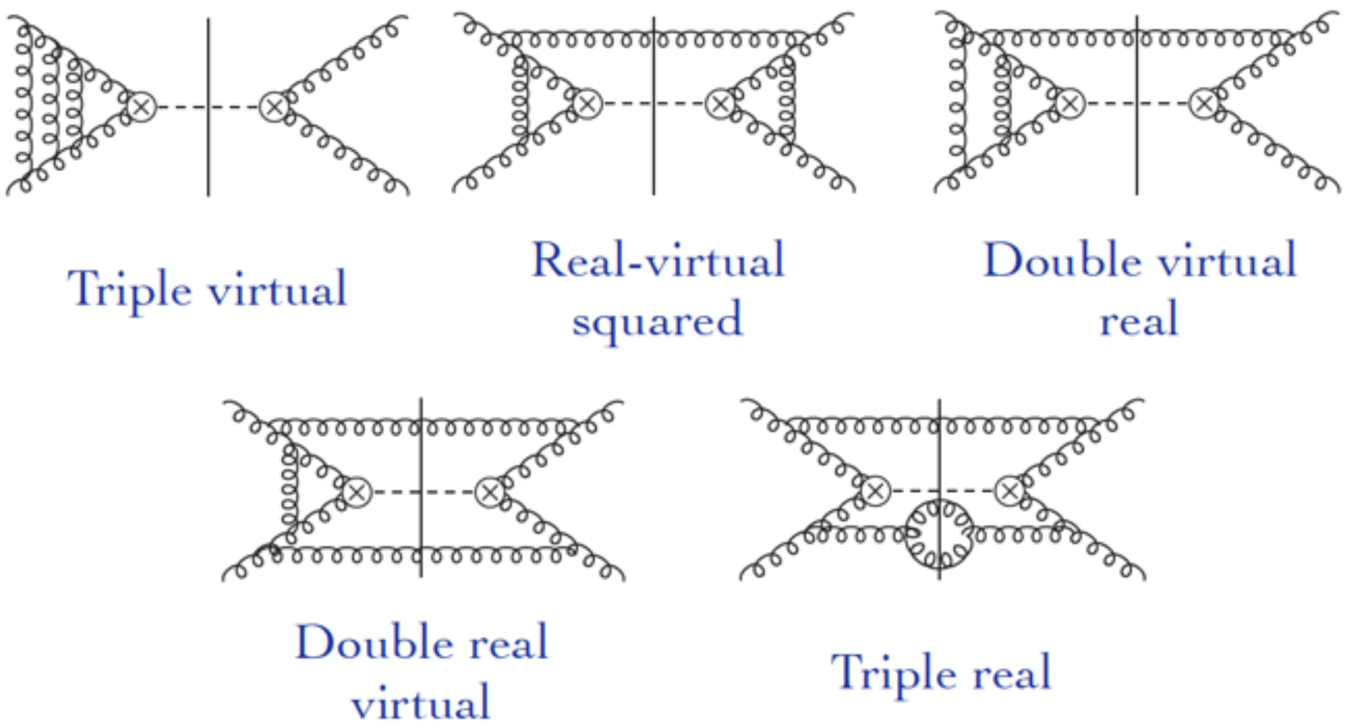
New

α_s^3

$$\overline{\mathcal{G}}_3^{I,1} = C_I\Big\{C_A{}^2\Big(\frac{152}{63}\,\zeta_2{}^3 + \frac{1964}{9}\,\zeta_2{}^2 + \frac{11000}{9}\,\zeta_2\zeta_3 - \frac{765127}{486}\,\zeta_2 + \frac{536}{3}\,\zeta_3{}^2 - \frac{59648}{27}\,\zeta_3 - \frac{1430}{3}\,\zeta_5 + \frac{7135981}{8748}\Big)$$
$$+ C_A n_f\,\Big(-\frac{532}{9}\,\zeta_2{}^2 - \frac{1208}{9}\,\zeta_2\zeta_3 + \frac{105059}{243}\,\zeta_2 + \frac{45956}{81}\,\zeta_3 + \frac{148}{3}\,\zeta_5 - \frac{716509}{4374}\Big) + C_F n_f\,\Big(\frac{152}{15}\,\zeta_2{}^2$$
$$- 88\,\zeta_2\zeta_3 + \frac{605}{6}\,\zeta_2 + \frac{2536}{27}\,\zeta_3 + \frac{112}{3}\,\zeta_5 - \frac{42727}{324}\Big) + n_f{}^2\,\Big(\frac{32}{9}\,\zeta_2{}^2 - \frac{1996}{81}\,\zeta_2 - \frac{2720}{81}\,\zeta_3 + \frac{11584}{2187}\Big)\Big\}\,, \qquad (11)$$

Higgs production at 3-loops

[Taushif,Narayan,VR]



α_s^3 → Φ_g^A

$$\Phi_q^{DY} = \frac{C_F}{C_A}\Phi_g^A$$

New

- $\mathcal{F}_q^{DY}$
- Γ_{qq}^{DY} $\quad + \; \Phi_q^{DY}$ → N^3LO $\quad \Delta^{DY,SV}(z,q^2)$
- Z_q^{DY}

N3LO for DY

[Taushif,Narayan,VR]

Soft+Virtual:

$$\Delta_{g,i}^{A,SV} = \Delta_{g,i}^{A,SV}|_{\delta}\delta(1-z) + \sum_{j=0}^{2i-1} \Delta_{g,i}^{A,SV}|_{\mathcal{D}_j}\mathcal{D}_j \, .$$

New!

known

$$\mathcal{D}_i \equiv \left[\frac{\ln^i(1-z)}{1-z}\right]_+$$

$$\begin{aligned}
\Delta_{q,3}^{SV}|_\delta = {} & C_A^2 C_F\Big(\frac{13264}{315}\,{\zeta_2}^3 + \frac{14611}{135}\,{\zeta_2}^2 - \frac{884}{3}\,\zeta_2\zeta_3 + 843\,\zeta_2 - \frac{400}{3}\,{\zeta_3}^2 + \frac{82385}{81}\,\zeta_3 - 204\,\zeta_5 - \frac{1505881}{972}\Big) \\
& + C_A C_F^2\,\Big(-\frac{20816}{315}\,{\zeta_2}^3 - \frac{1664}{135}\,{\zeta_2}^2 + \frac{28736}{9}\,\zeta_2\zeta_3 - \frac{13186}{27}\,\zeta_2 + \frac{3280}{3}\,{\zeta_3}^2 - \frac{20156}{9}\,\zeta_3 - \frac{39304}{9}\,\zeta_5 + \frac{74321}{36}\Big) \\
& + C_A C_F n_f\,\Big(-\frac{5756}{135}\,{\zeta_2}^2 + \frac{208}{3}\,\zeta_2\zeta_3 - \frac{28132}{81}\,\zeta_2 - \frac{6016}{81}\,\zeta_3 - 8\,\zeta_5 + \frac{110651}{243}\Big) + C_F^3\,\Big(-\frac{184736}{315}\,{\zeta_2}^3 + \frac{412}{5}\,{\zeta_2}^2 \\
& + 80\,\zeta_2\zeta_3 - \frac{130}{3}\,\zeta_2 + \frac{10336}{3}\,{\zeta_3}^2 - 460\,\zeta_3 + 1328\,\zeta_5 - \frac{5599}{6}\Big) + C_F^2 n_f\Big(\frac{272}{135}\,{\zeta_2}^2 - \frac{5504}{9}\,\zeta_2\zeta_3 + \frac{2632}{27}\,\zeta_2 + \frac{3512}{9}\,\zeta_3 \\
& + \frac{5536}{9}\,\zeta_5 - \frac{421}{3}\Big) + C_F n_{f,v}\Big(\frac{N^2-4}{N}\Big)\,\Big(-\frac{4}{5}\,{\zeta_2}^2 + 20\,\zeta_2 + \frac{28}{3}\,\zeta_3 - \frac{160}{3}\,\zeta_5 + 8\Big) + C_F n_f^2\,\Big(\frac{128}{27}\,{\zeta_2}^2 + \frac{2416}{81}\,\zeta_2 \\
& - \frac{1264}{81}\,\zeta_3 - \frac{7081}{243}\Big)\,, \qquad (12)
\end{aligned}$$

Confirmed by Catani et al, Shabinger et al

N3LO for DY

[Taushif,Narayan,VR]

Soft+Virtual:

$$\Delta_{g,i}^{A,\mathrm{SV}} = \Delta_{g,i}^{A,\mathrm{SV}}|_{\delta}\,\delta(1-z) + \sum_{j=0}^{2i-1} \Delta_{g,i}^{A,\mathrm{SV}}|_{\mathcal{D}_j}\,\mathcal{D}_j\,.$$

New!

known $\mathcal{D}_i \equiv \left[\frac{\ln^i(1-z)}{1-z}\right]_+$

Q (GeV)	30	50	70	90	100	200	400
$10^3\delta_{\mathrm{N^3LO}}$ (nb)	11.386	2.561	1.724	140.114	5.410	4.567 10^{-2}	3.153 10^{-3}
$10^3\mathcal{D}_{\mathrm{N^3LO}}$ (nb)	-8.397	-2.053	-1.466	-124.493	-4.865	-4.421 10^{-2}	-3.368 10^{-3}

Large cancellation between them

Relations in $\mathcal{N}=4$ SYM

[A.V.Kotikov,L.N.Lipatov,A.I.Onishchenko,V.N.Velizhanin,T. Gehrmann,J. Henn]

Leading Transcendentality Principle

- Set $C_A = C_F = N, T_f n_f = N/2$ for $SU(N)$
- Leading Transcendental (LT) parts of quark and gluon form factors in QCD are equal upto a factor 2^l
- LT part of quark and gluon form factors are identical to the scalar form factor in $\mathcal{N} = 4$ SYM
- LT part of pseudo scalar form factor is identical to quark and gluon form factors in QCD upto a factor 2^l also to scalar form factor in $\mathcal{N} = 4$ SYM

Conclusions

- Form Factors in Gague Theories
- Infrared Structure
  - Soft
  - Collinear
- Multi-leg, Multi-loop amplitudes
  - K+G equation
  - Catani's proposal
- Factorisation and Resummation
- Casimir Duality
- IR to UV and Drell-Yan