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INTERPRETING THE SPLIT PATTERNS IN THE TERNARY LUMINOSITY DIAGRAM

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(work in progress by the Bari group)

Plan of the talk

Introduction and notation

The Ternary Luminosity diagram

Single split case

Double splits

Summary

Introduction

We assume a two neutrino scenario with inverted hierarchy

Approximately three regimes can be identified:

- Synchronized oscillations
- Bipolar oscillations
- “Split” regime - where the split fully develops until the end of collective effects

The pendulum analogy allows us to understand many features of the collective oscillations

Still lacking a full understanding of multiple split cases

Notation

Bloch vectors

$$\mathbf{P} = \mathbf{P}(E, r)$$

$$\omega = \Delta m^2 / 2E$$

$$\dot{\mathbf{P}} = (+\omega \mathbf{B} + \lambda \mathbf{z} + \mu \mathbf{D}) \times \mathbf{P}$$

$$\dot{\overline{\mathbf{P}}} = (-\omega \mathbf{B} + \lambda \mathbf{z} + \mu \mathbf{D}) \times \overline{\mathbf{P}}$$

Global vectors

$$\mathbf{J} = \sum \mathbf{P} \qquad \overline{\mathbf{J}} = \sum \overline{\mathbf{P}} \qquad \mathbf{D} = \mathbf{J} - \overline{\mathbf{J}}$$

$$\mathbf{W} = \sum \omega \mathbf{P} \qquad \overline{\mathbf{W}} = \sum \omega \overline{\mathbf{P}}$$

Potential energy of the system $U \sim W_z + \overline{W}_z$

In this analysis we use an artificially "more adiabatic" scenario

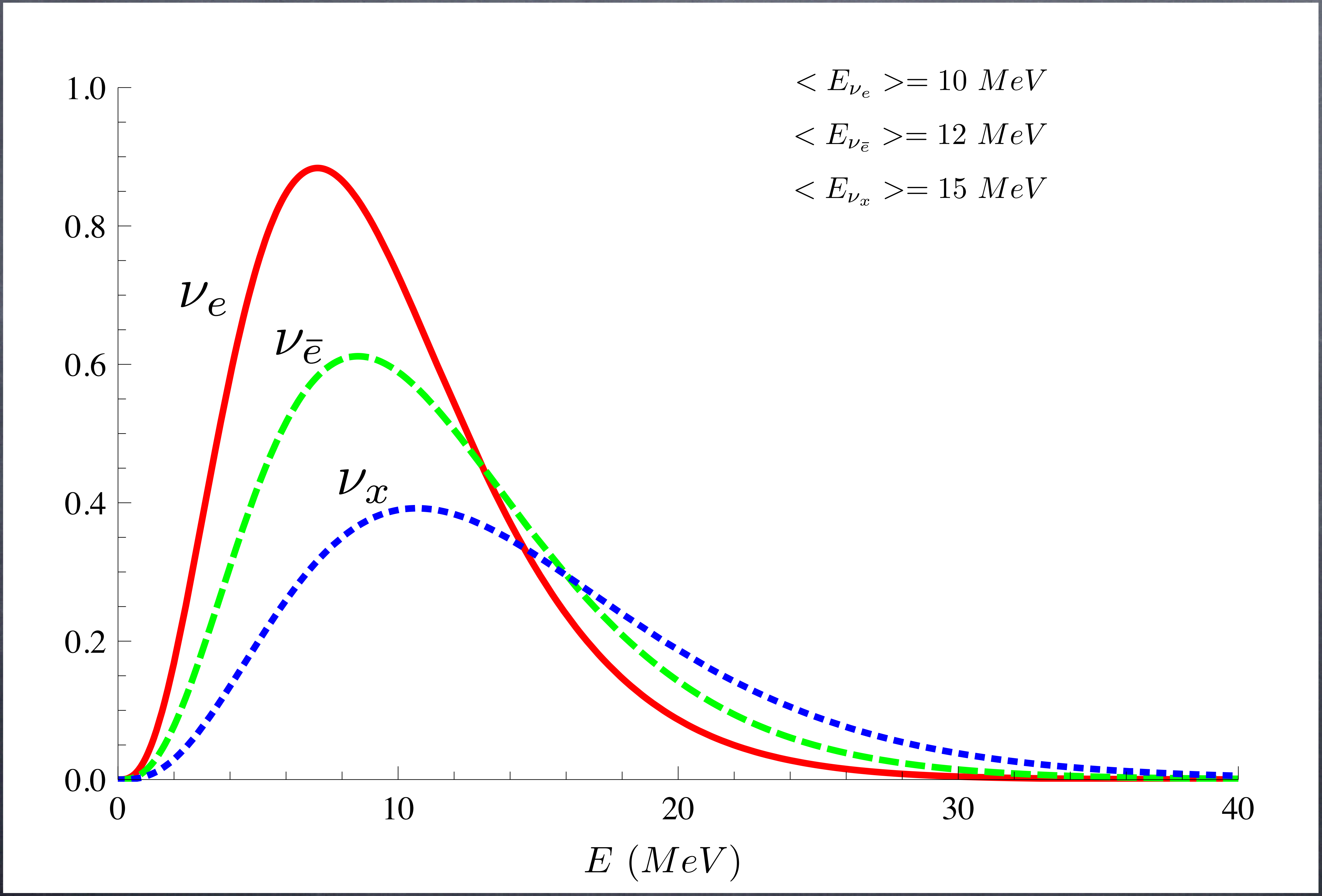
$$\mu(r) \to \mu(r) \times \frac{r}{R_n}$$

R_n Radius of the neutrinosphere

Vacuum + neutrino self interactions

$$\lambda = 0$$

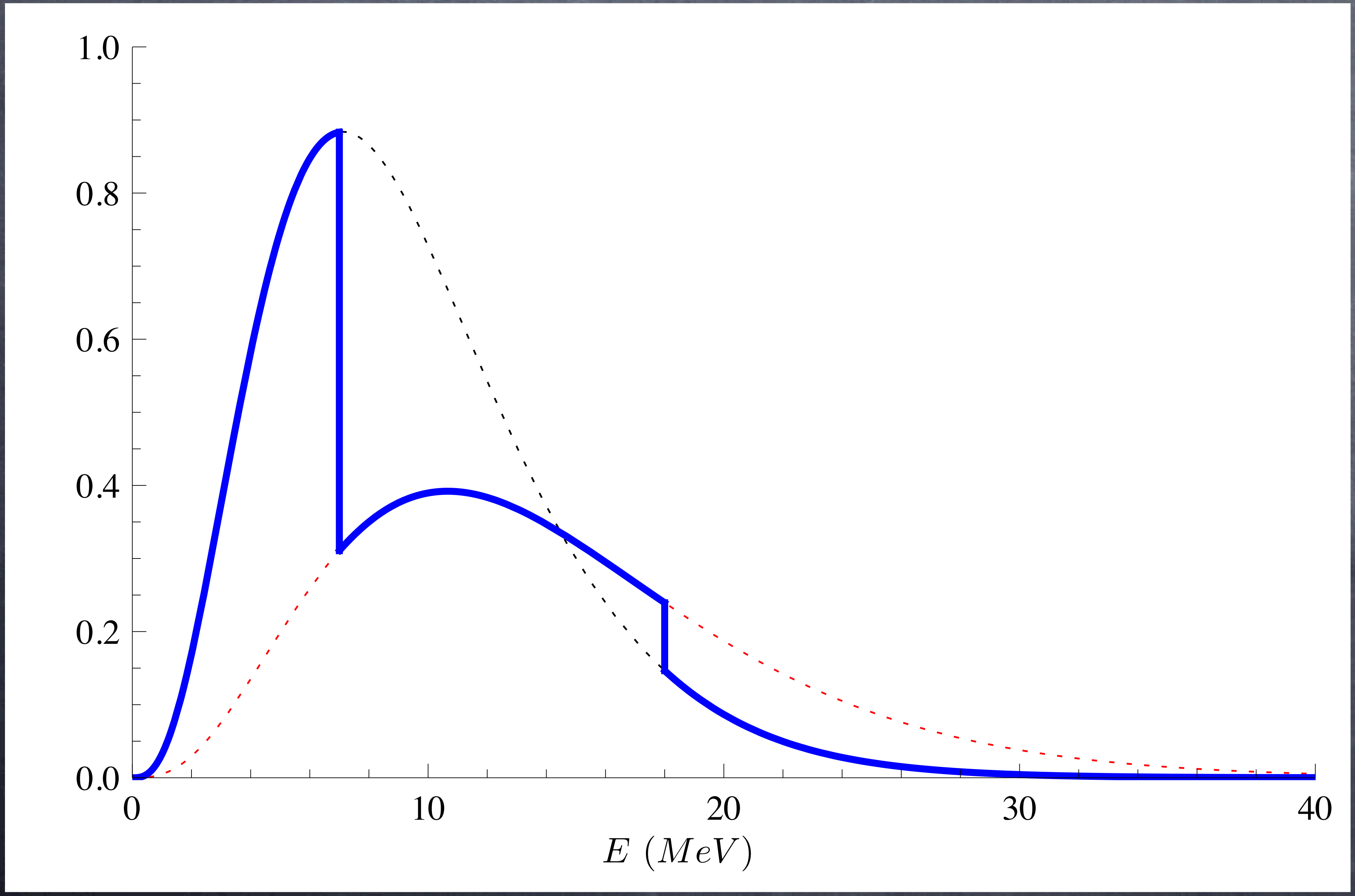
Initial Spectra for this analysis



The parametrization

$$n_\alpha(E) = a_\alpha E^3 e^{-b_\alpha E} \times \ell_\alpha$$

Allows analytical integration of $\int n_\alpha(E)dE$ and $\int n_\alpha(E)\frac{dE}{E}$



Useful for hypothetically sharp splits

TERNARY LUMINOSITY DIAGRAM

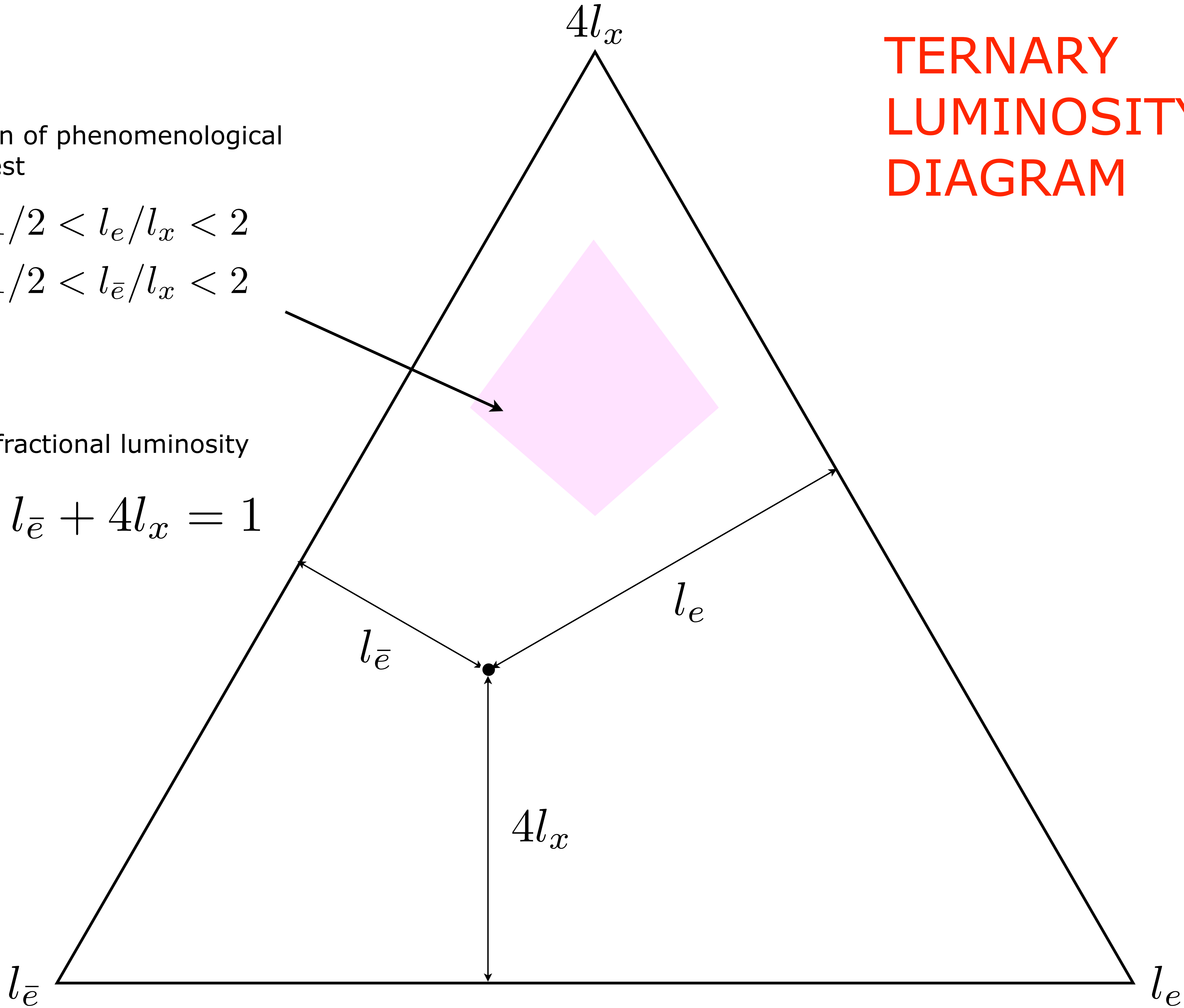
Region of phenomenological interest

$$1/2 < l_e/l_x < 2$$

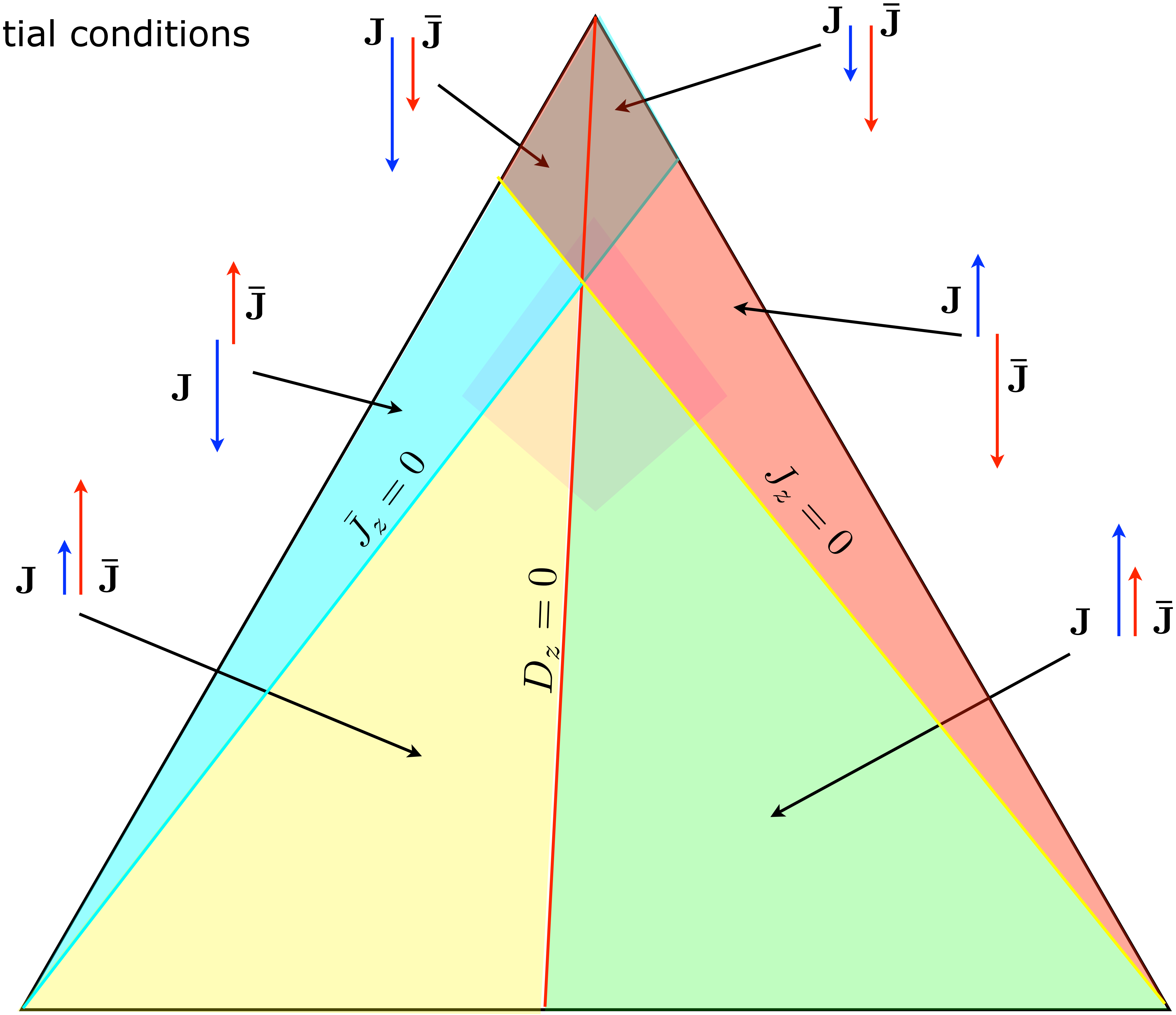
$$1/2 < l_{\bar{e}}/l_x < 2$$

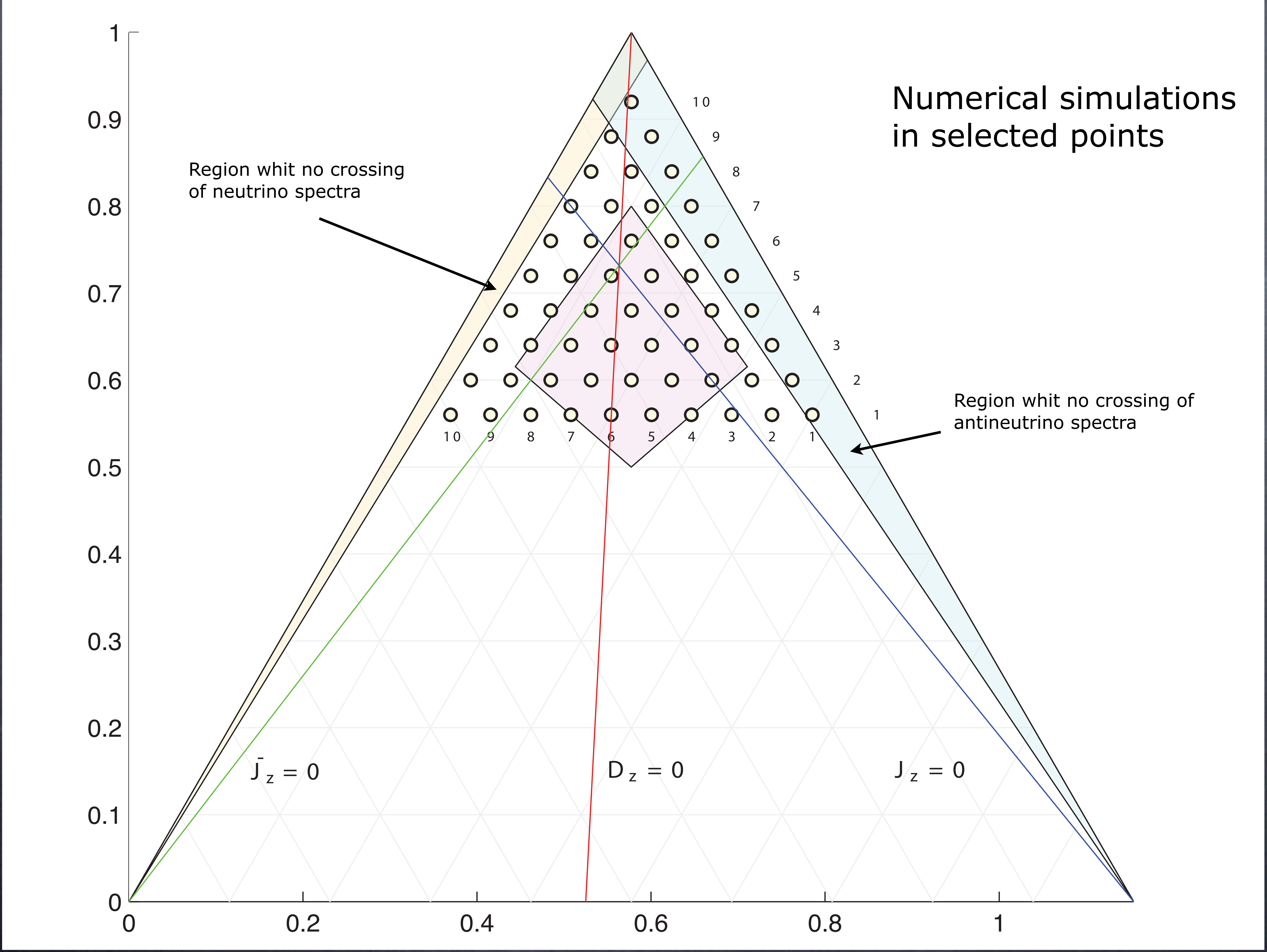
l_α fractional luminosity

$$l_e + l_{\bar{e}} + 4l_x = 1$$



Initial conditions

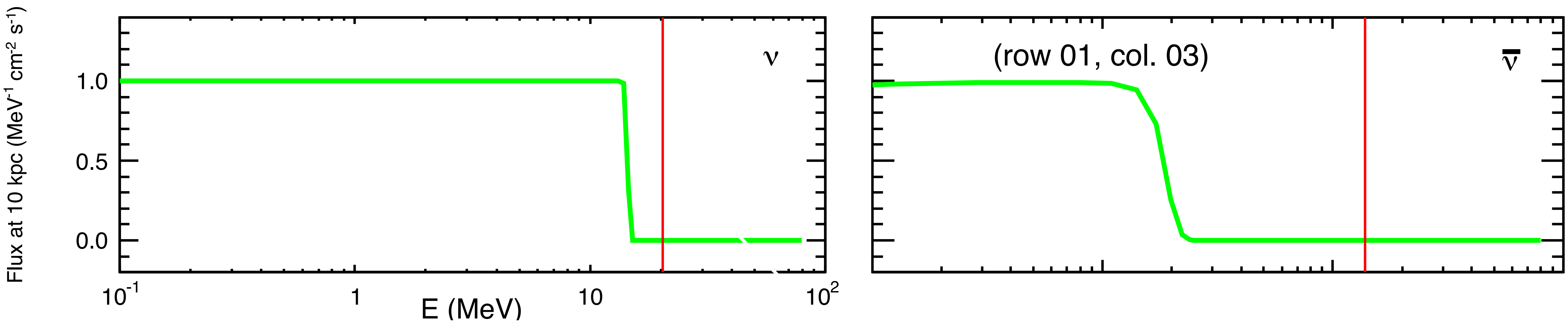




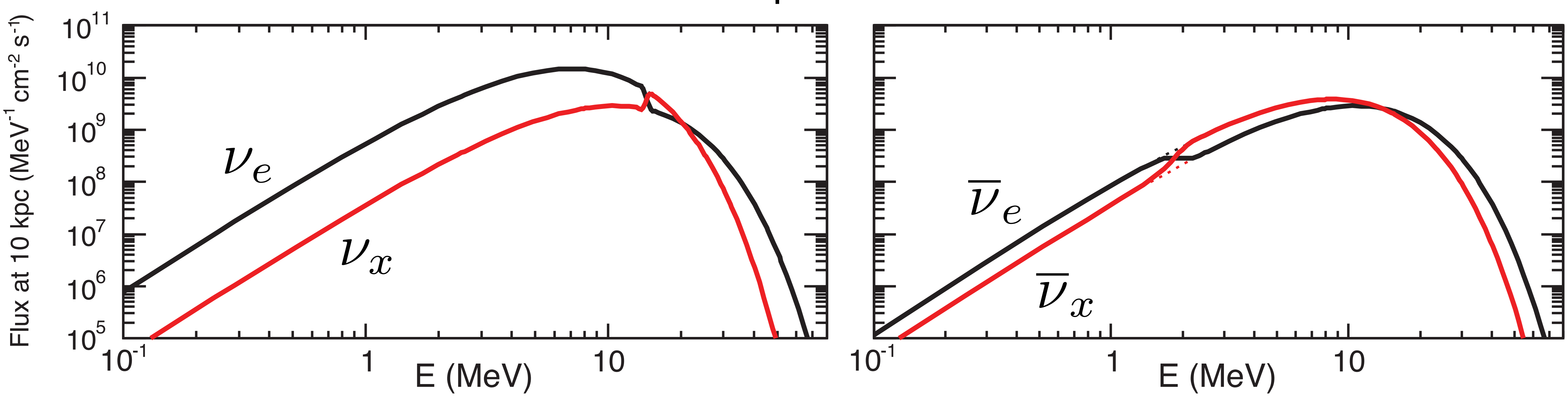
Numerical simulations in selected points
Region whit no crossing of neutrino spectra
Region whit no crossing of antineutrino spectra
$\bar{J}_z = 0$
$D_z = 0$
$J_z = 0$
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
0.2
0.4
0.6
0.8
1
10 9 8 7 6 5 4 3 2 1
10 9 8 7 6 5 4 3 2 1

SINGLE SPLIT - an example

Survival probability

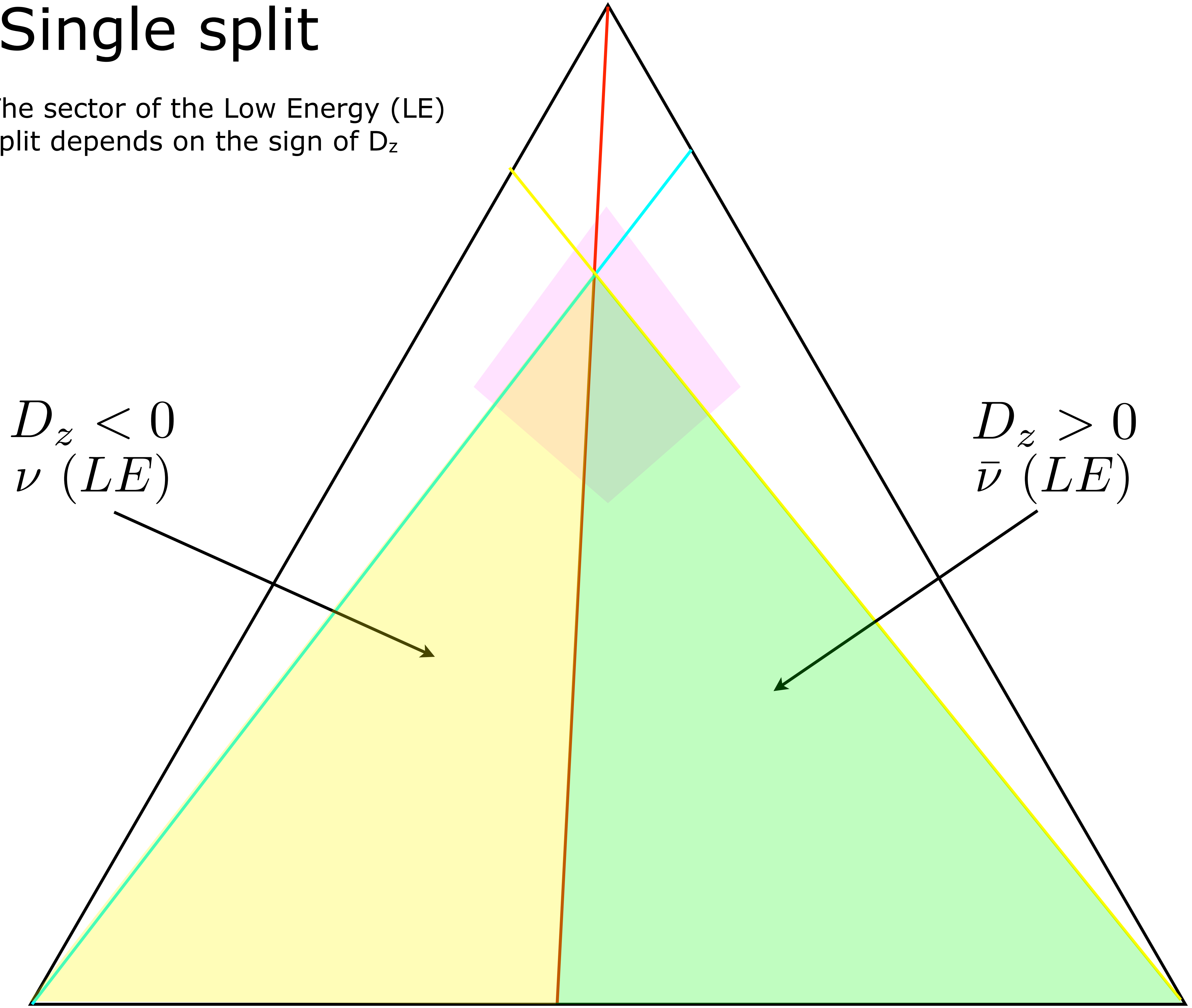


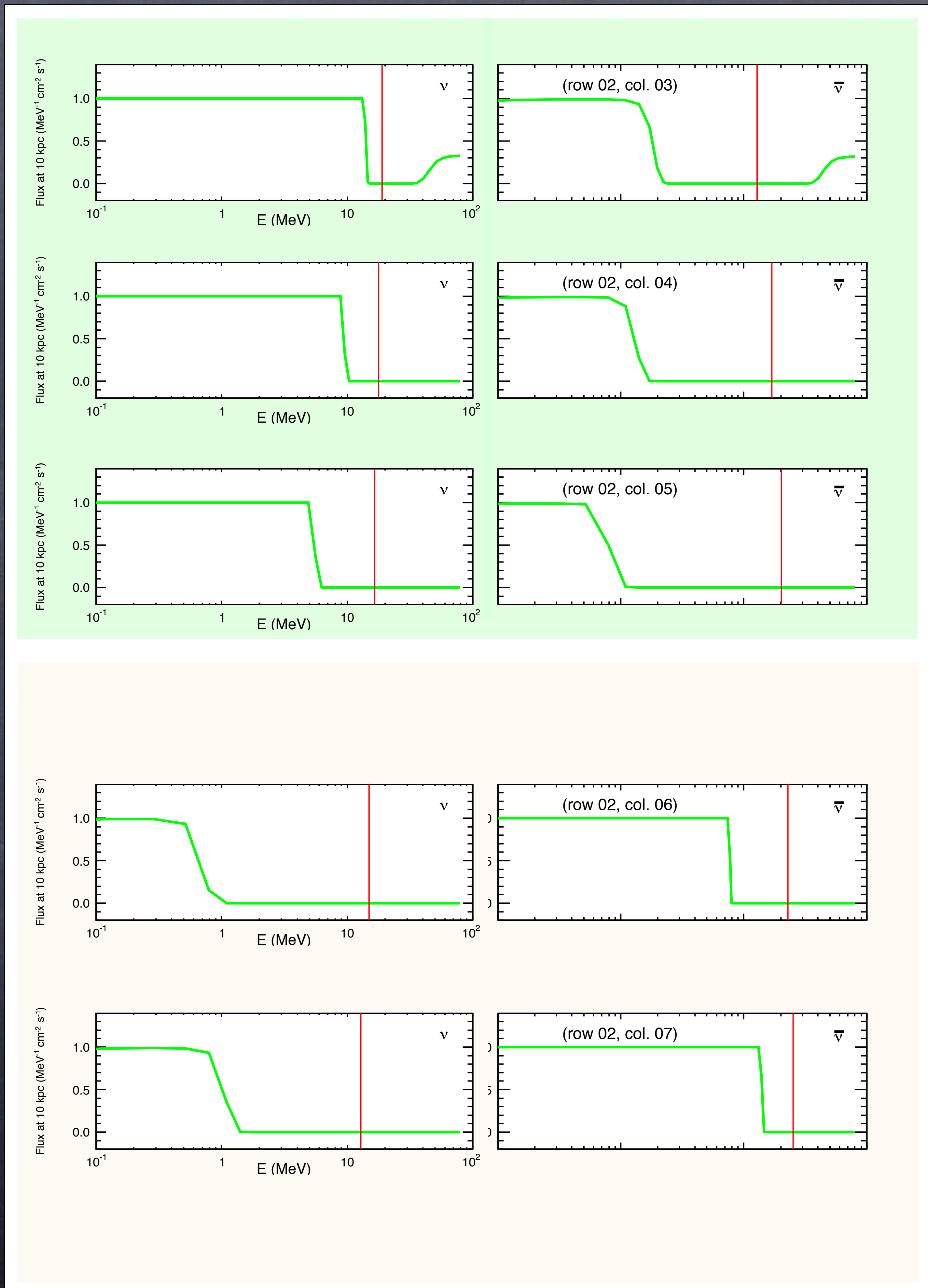
Spectra



Single split

The sector of the Low Energy (LE) split depends on the sign of D_z

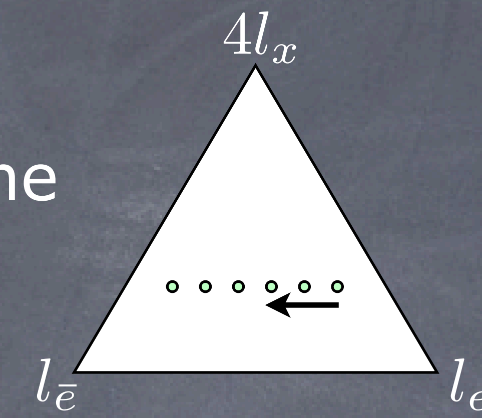




SINGLE SPLIT

Moving across the line

$l_x = const$



Low energy split for antineutrinos

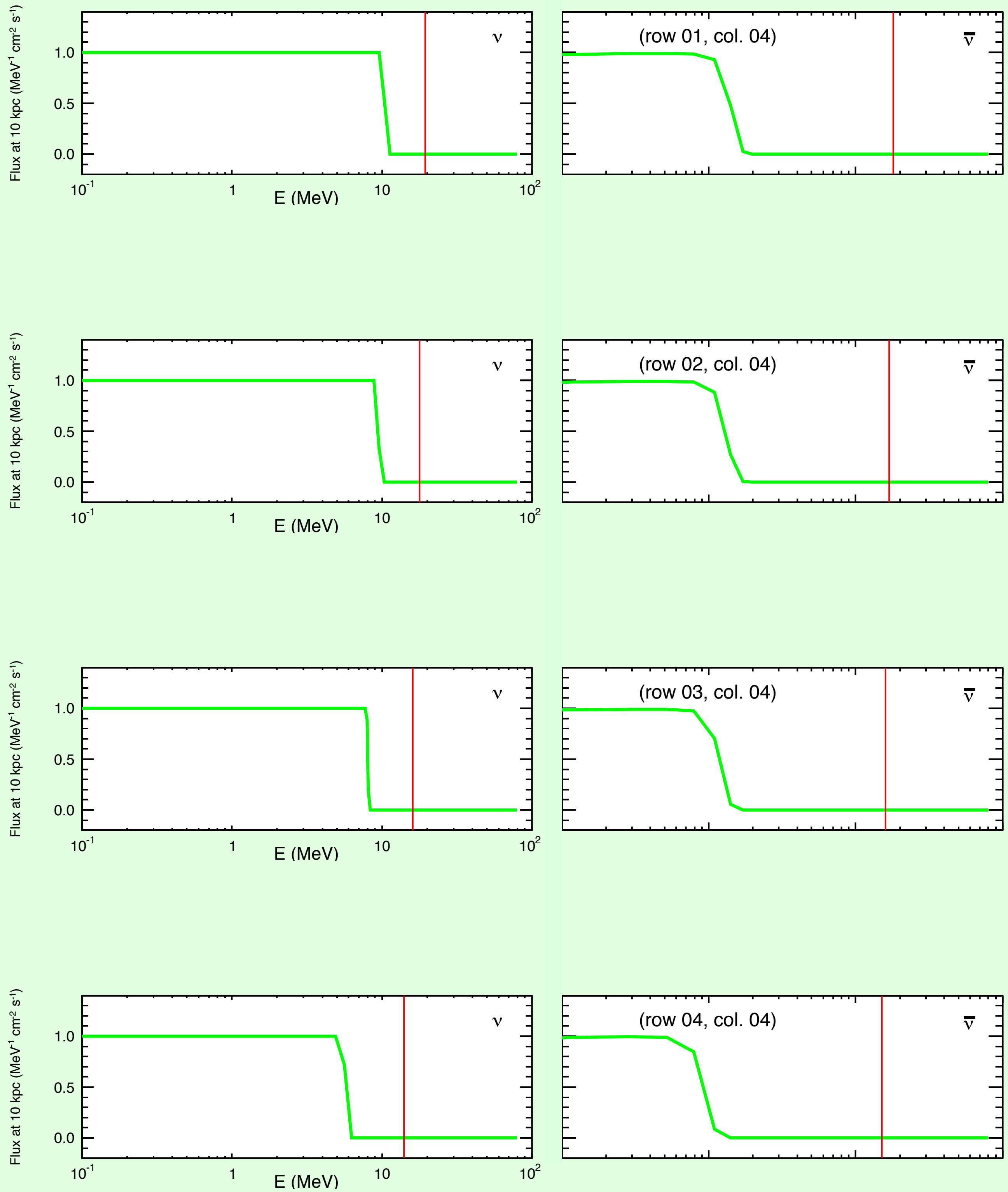
Low energy split broader than the high energy one

LE split and HE split move to lower energy when

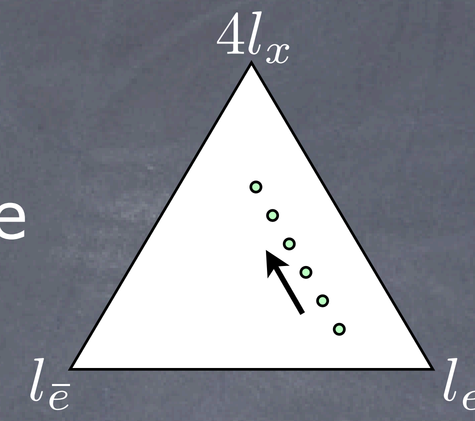
$$D_z \to 0$$

When D_z changes sign

$$\nu \leftrightarrow \bar{\nu}$$



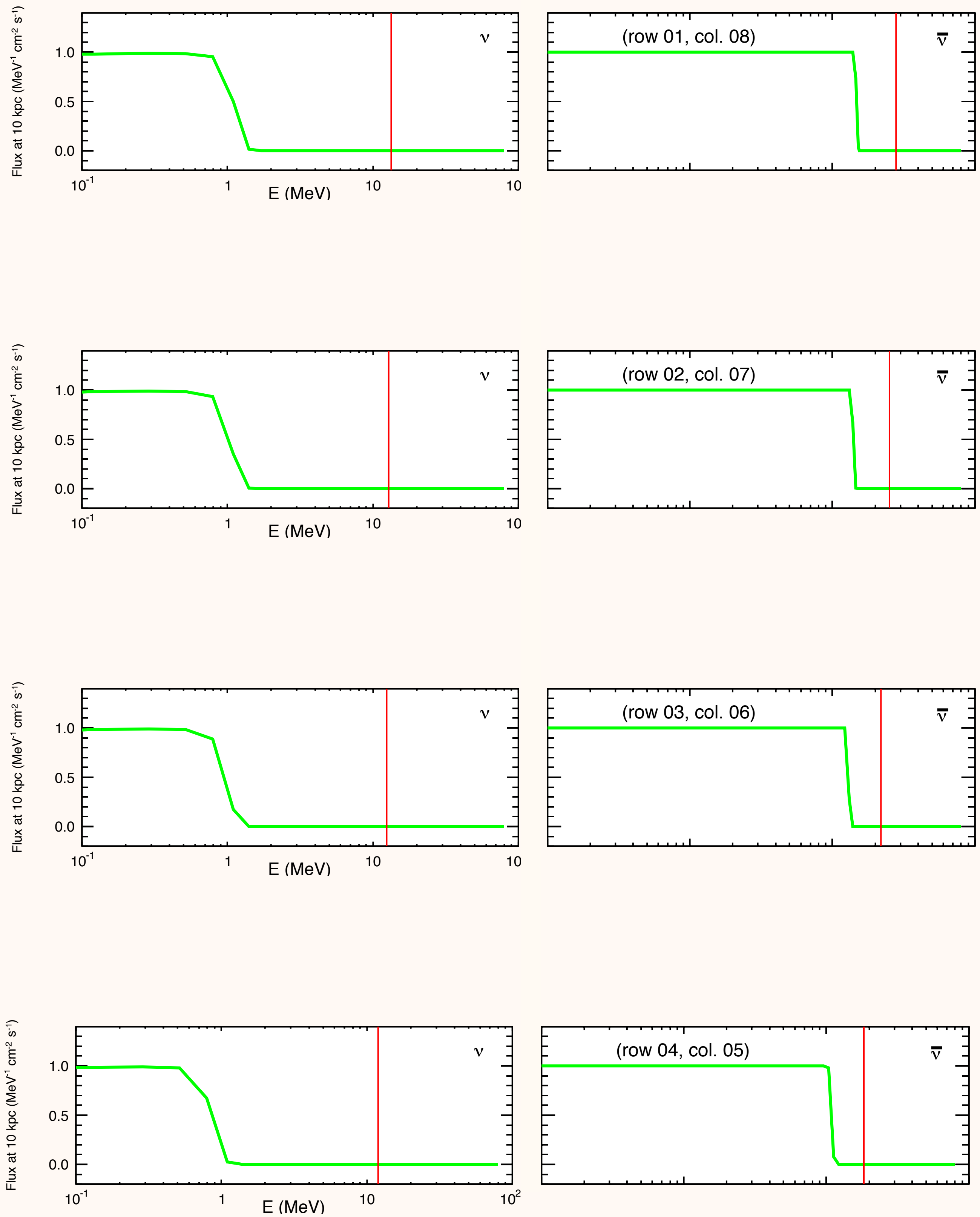
SINGLE SPLIT



Moving across the line
$l_{\bar{e}} = const$

Low energy split
for antineutrinos

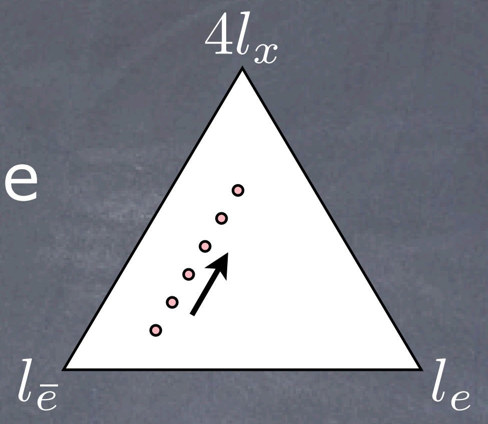
Same behavior as before
since the point is moving
toward the line $D_z = 0$



SINGLE SPLIT

Moving across the line

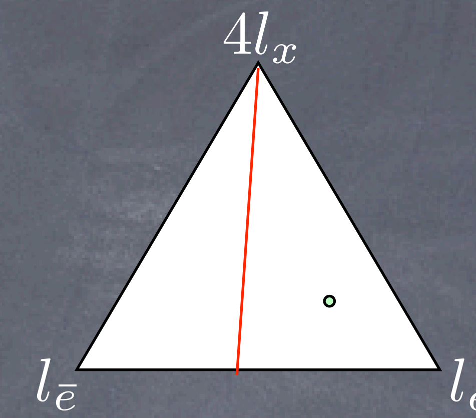
$l_e = const$



Low energy split for neutrinos

Same as before with $\nu \leftrightarrow \bar{\nu}$

Consider a typical case with $D_z > 0$



The High Energy split for neutrinos fixed by lepton number conservation and minimization of the potential energy

Final vectors

Initial vectors

Assuming complete antineutrino spectral swap, one gets a good approximation for the high energy neutrino split by solving the integral equation

$$\int_{E_c}^{\infty} (n_e^i(E) - n_x^i(E))dE = \int_0^{\infty} (n_{\bar{e}}^f(E) - n_x^f(E))dE$$

Low Energy split

$$\dot{\mathbf{P}} = (+\omega\mathbf{B} + \mu\mathbf{D}) \times \mathbf{P}$$
$$\dot{\overline{\mathbf{P}}} = (-\omega\mathbf{B} + \mu\mathbf{D}) \times \overline{\mathbf{P}}$$
$$\uparrow \quad \uparrow$$

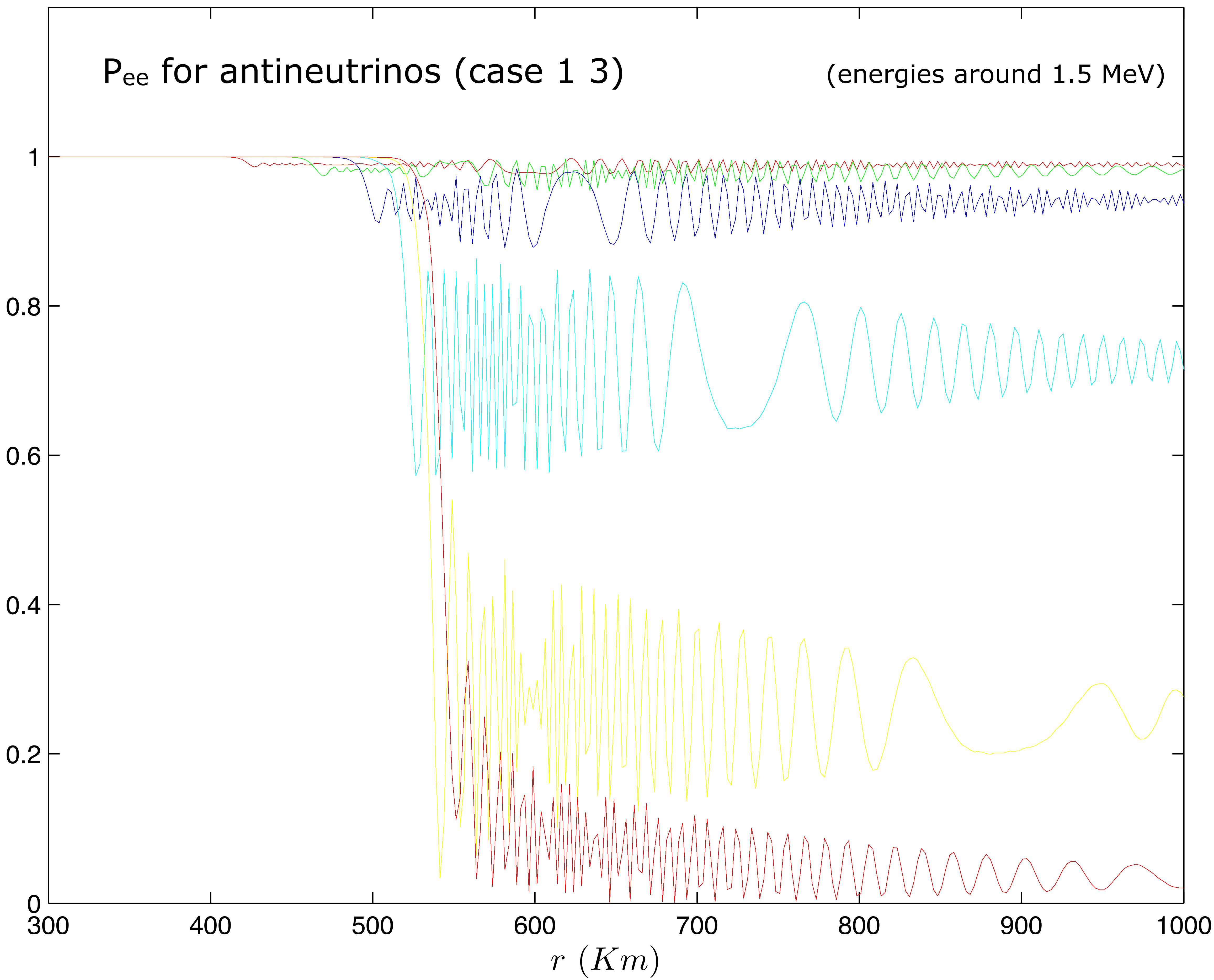
Depending on the sign of D_z there can be a cancelation for neutrinos or antineutrinos due to the different sign of omega in the two equations

When $D_z > 0$ antineutrinos can experience a MSW-like resonance on the self-interaction potential. The resonance can happen for neutrinos when $D_z < 0$

If the crossing probability P_c at the resonance

$$P_c = e^{-2\pi\omega \sin^2\theta|\mu/\dot{\mu}|}$$

is close to one, the the survival probability for neutrinos or antineutrinos is close to one



Pee for antineutrinos (case 1 3)
(energies around 1.5 MeV)
1
0.8
0.6
0.4
0.2
0
300
400
500
600
700
800
900
1000
r (Km)

The lower split energy can be estimated solving the resonance condition

$$\omega = |\mu D_z|$$

and imposing

$$P_c = P^*$$

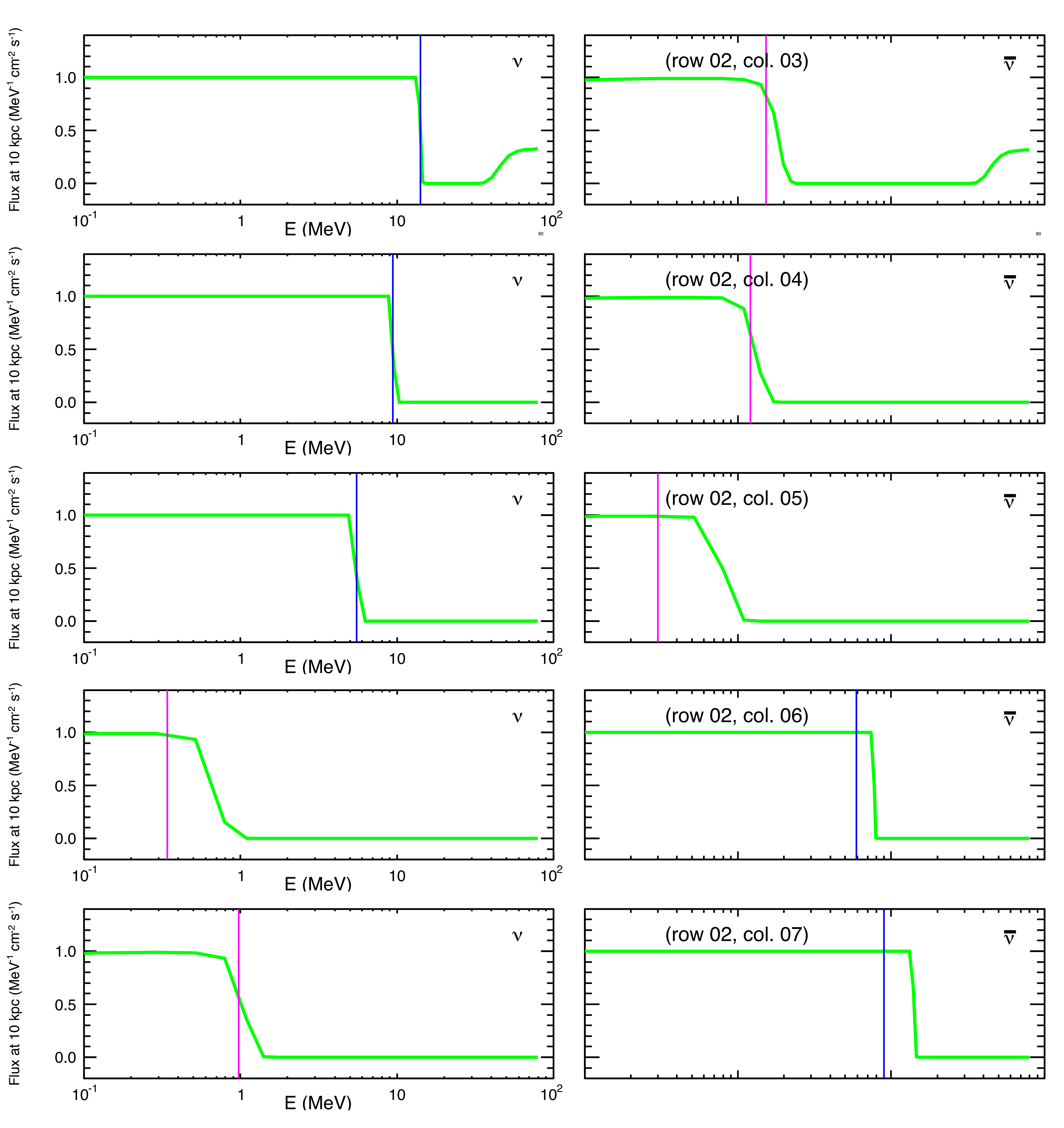
where P^* is a fixed number close to one.

We find a reasonable agreement with the simulations if we use

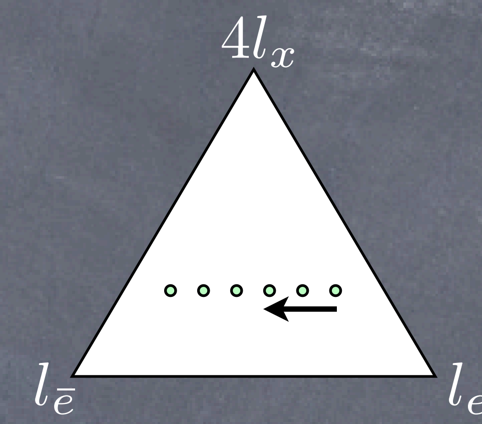
$$P^* = 0.97$$

Caveat

Since the resonance happens at different radii for different modes, and since the P_c changes for different modes, strictly speaking, both the split energy and the resonance radius are not very well defined

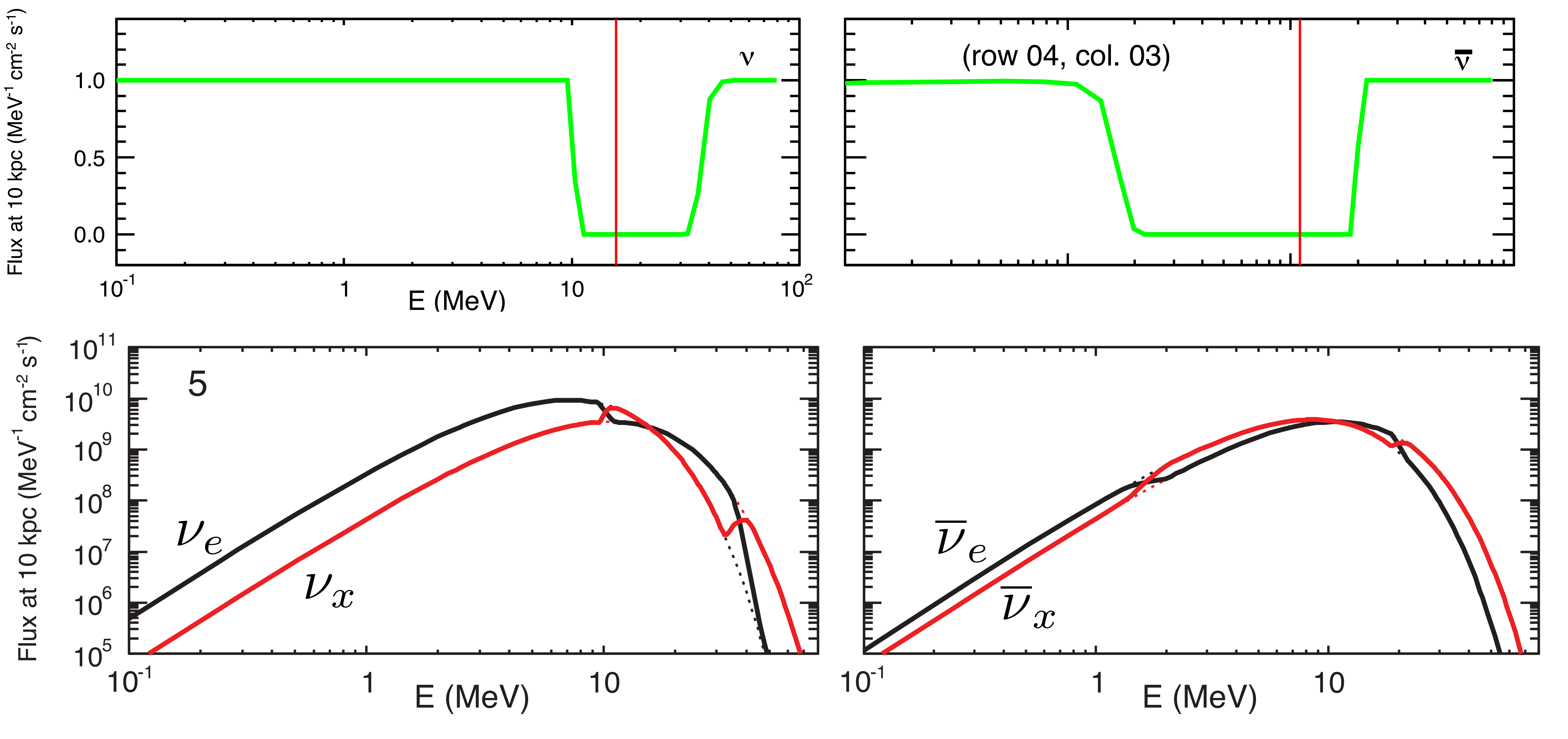


SINGLE SPLIT

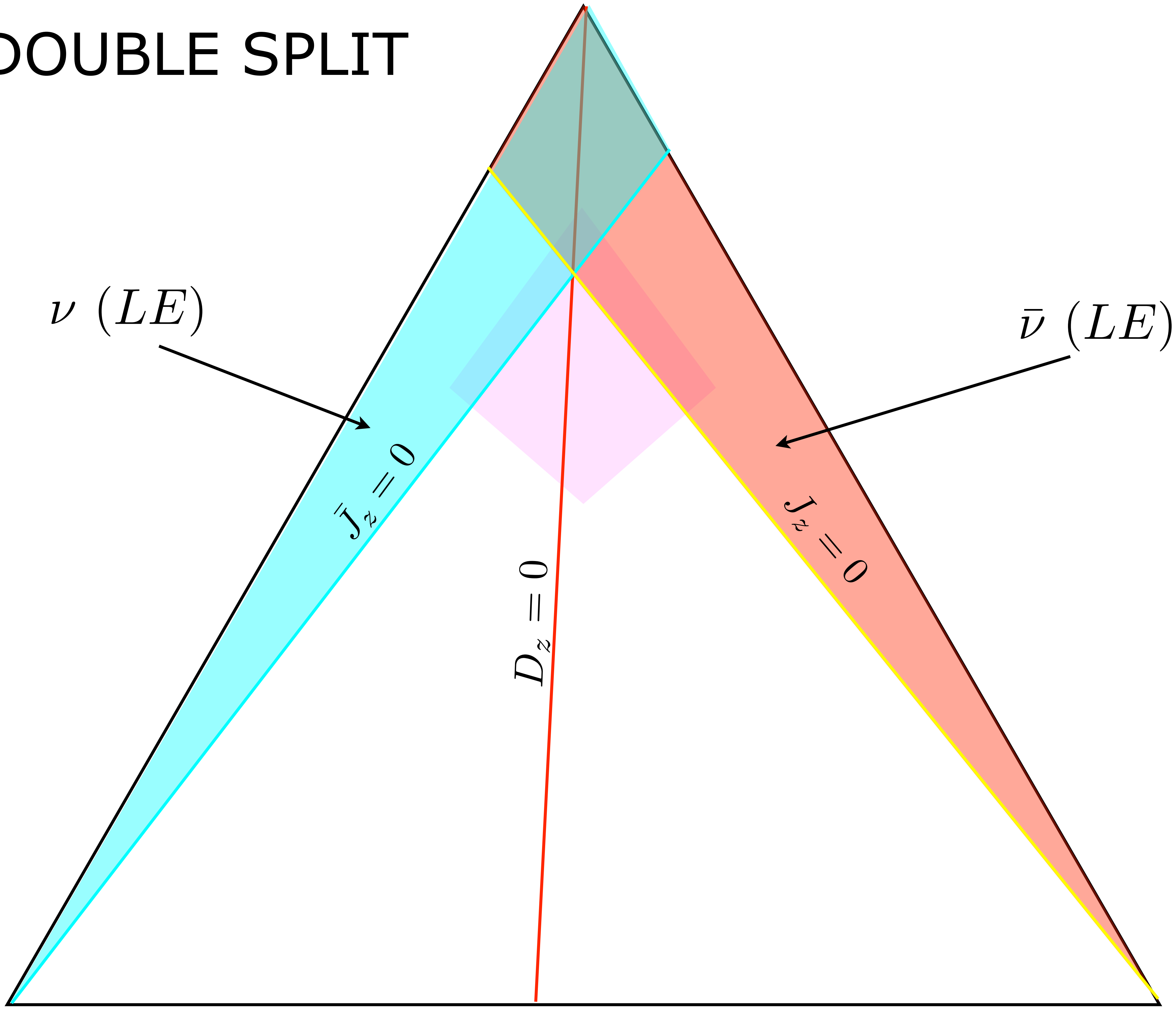


Agreement between estimated energies and simulation

DOUBLE SPLIT - an example



DOUBLE SPLIT

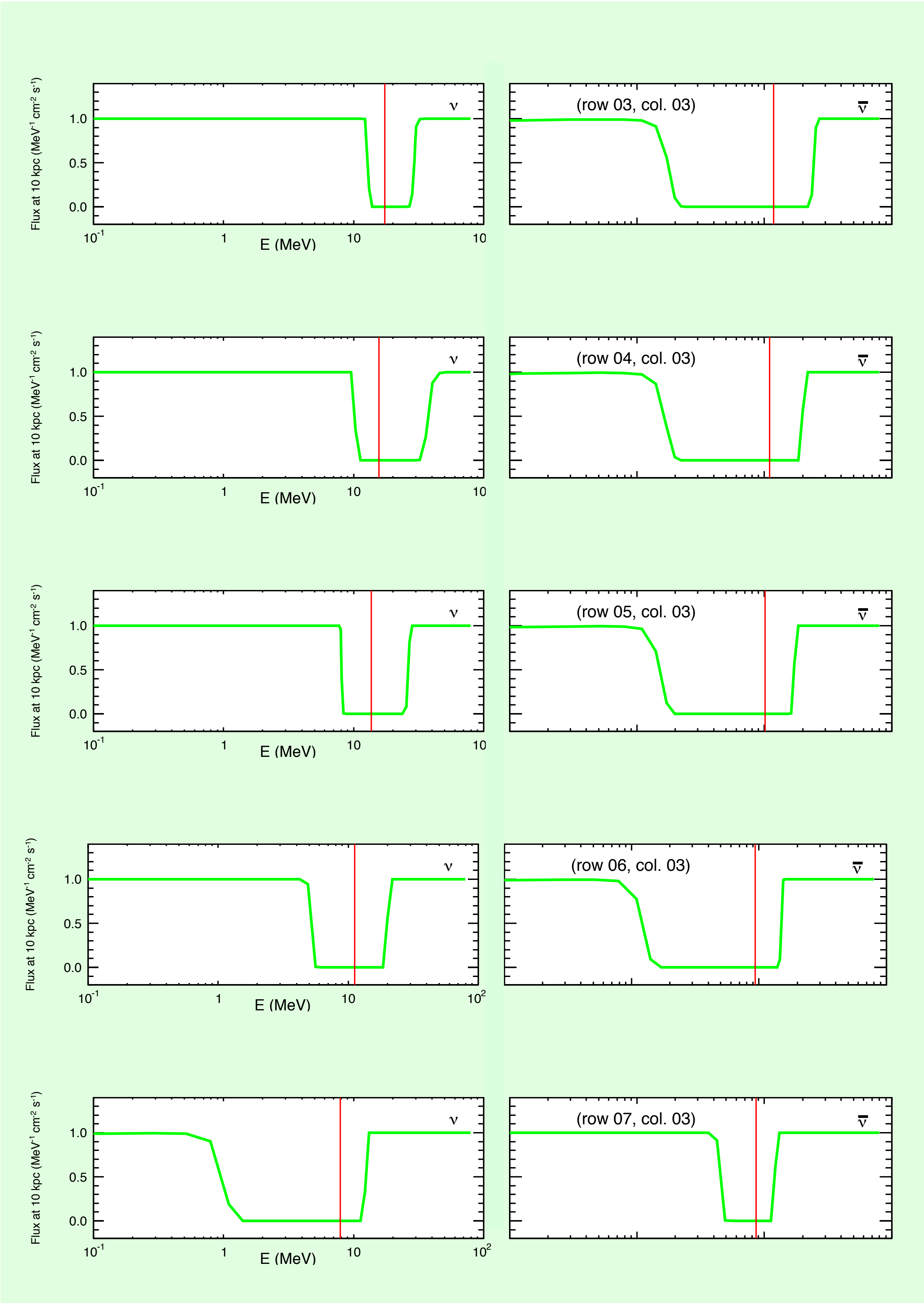


The features of the double split can be interpreted by means of the following arguments

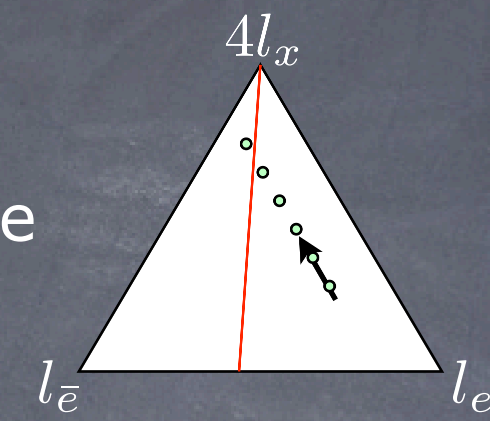
Conservation laws

Resonance on the self-interaction potential

Minimization of the energy



DOUBLE SPLIT



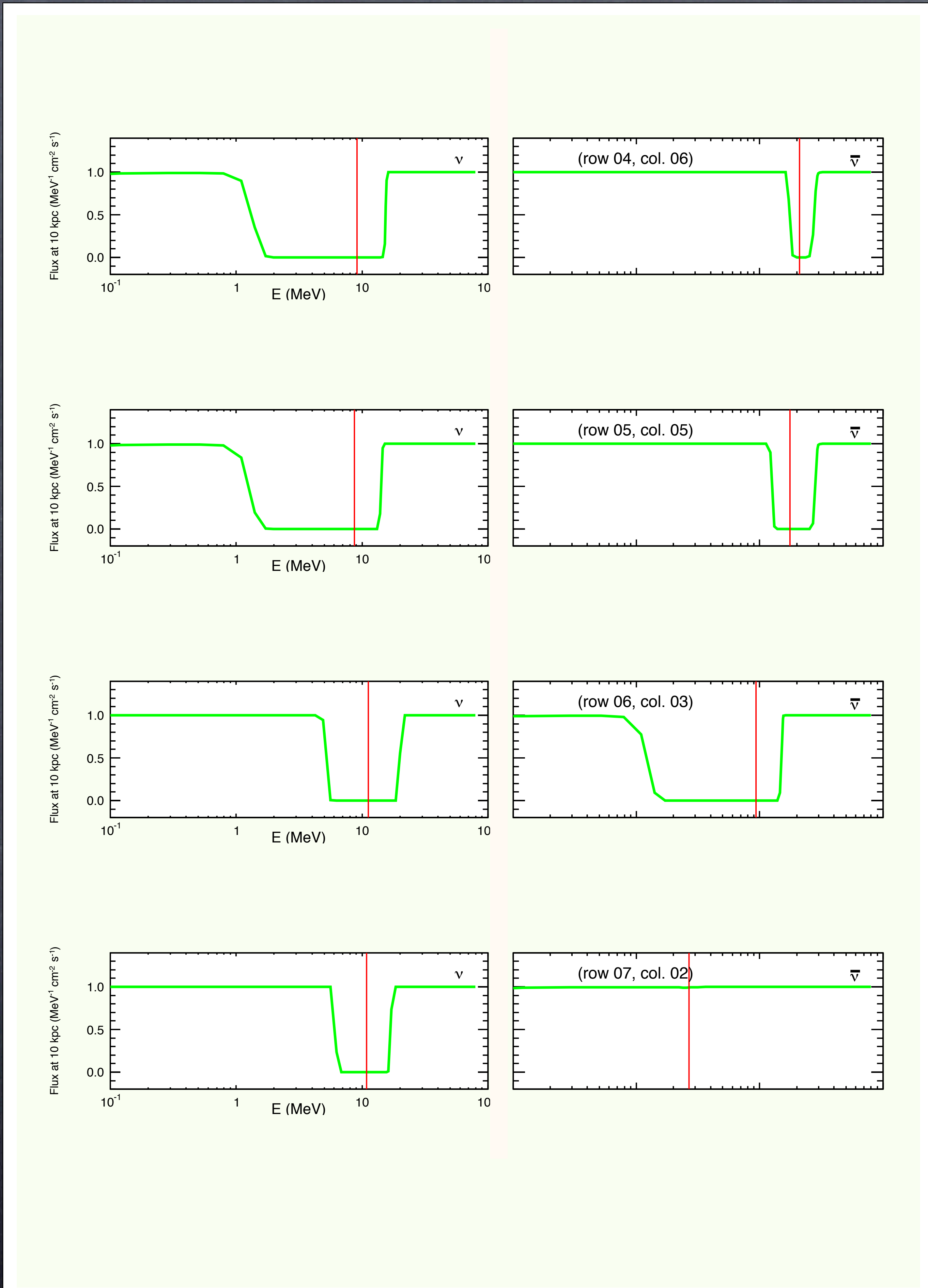
Moving across the line

$l_{\bar{e}} = const$

$(J_z > 0, \bar{J}_z < 0)$

For both neutrinos and antineutrinos split energies are placed on opposite sides with respect to the crossing energy

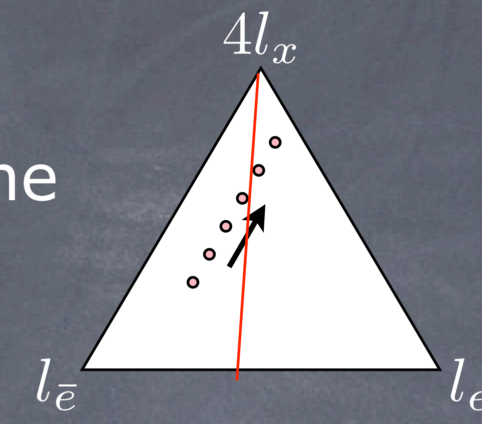
The LE split moves to the left and becomes broader (as the point approaches the line $D_z = 0$)



DOUBLE SPLIT

Moving across the line

$l_e = const$



$$(J_z < 0, \bar{J}_z > 0)$$

Same as before with neutrinos and antineutrinos interchanged

In the last plot the double split is not present (only a very small dip in the probability)

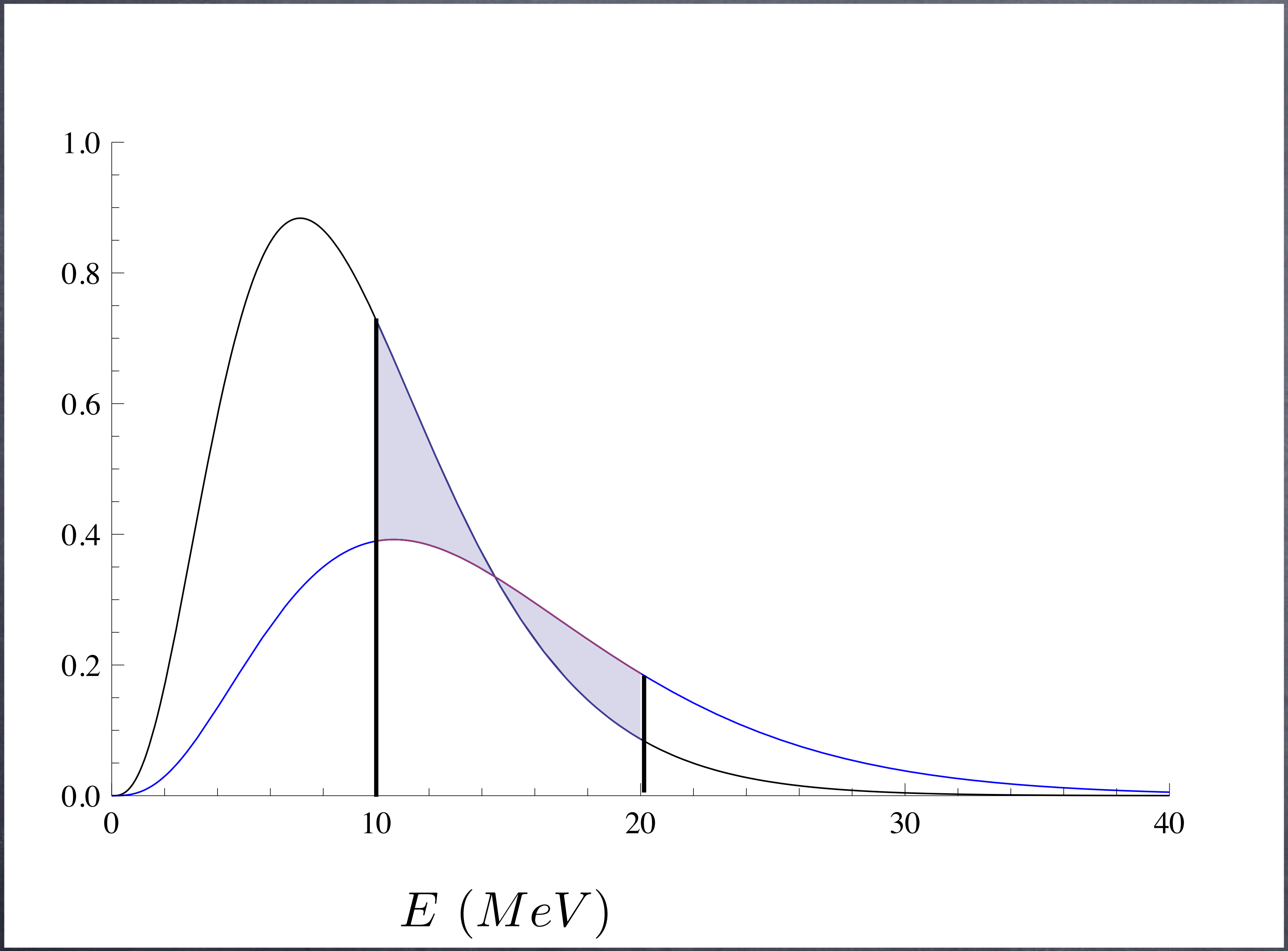
Conservation laws

From the simulations we see that the vectors $\mathbf{J}$ and $\bar{\mathbf{J}}$ are stuck

Therefore D_z, J_z and $\bar{J}_z$ are conserved

What are the implications on the kind of split?

Our choice for initial spectra implies that with only one split J_z cannot be conserved (in the case of neutrinos, analogously for $\bar{J}_z$ and antineutrinos)



The only possibility is a double split, whit two split energies such that the shaded areas are equal (if there is a crossing between the spectra)

Which is the width of the double split?

The width of the split can be understood by using

- Minimization of potential energy
- Resonance on the self-interaction potential
- End of collective effects

Minimization of the potential energy

Consider, for instance, the neutrino case

The two split energies, E_1 and E_2 are linked through the conservation of J_z ⟶ $E_2 = E_2(E_1)$

W_z is an increasing function of E_1

↓

The system prefers the minimum possible value of E_1 and thus the maximum E_2 value

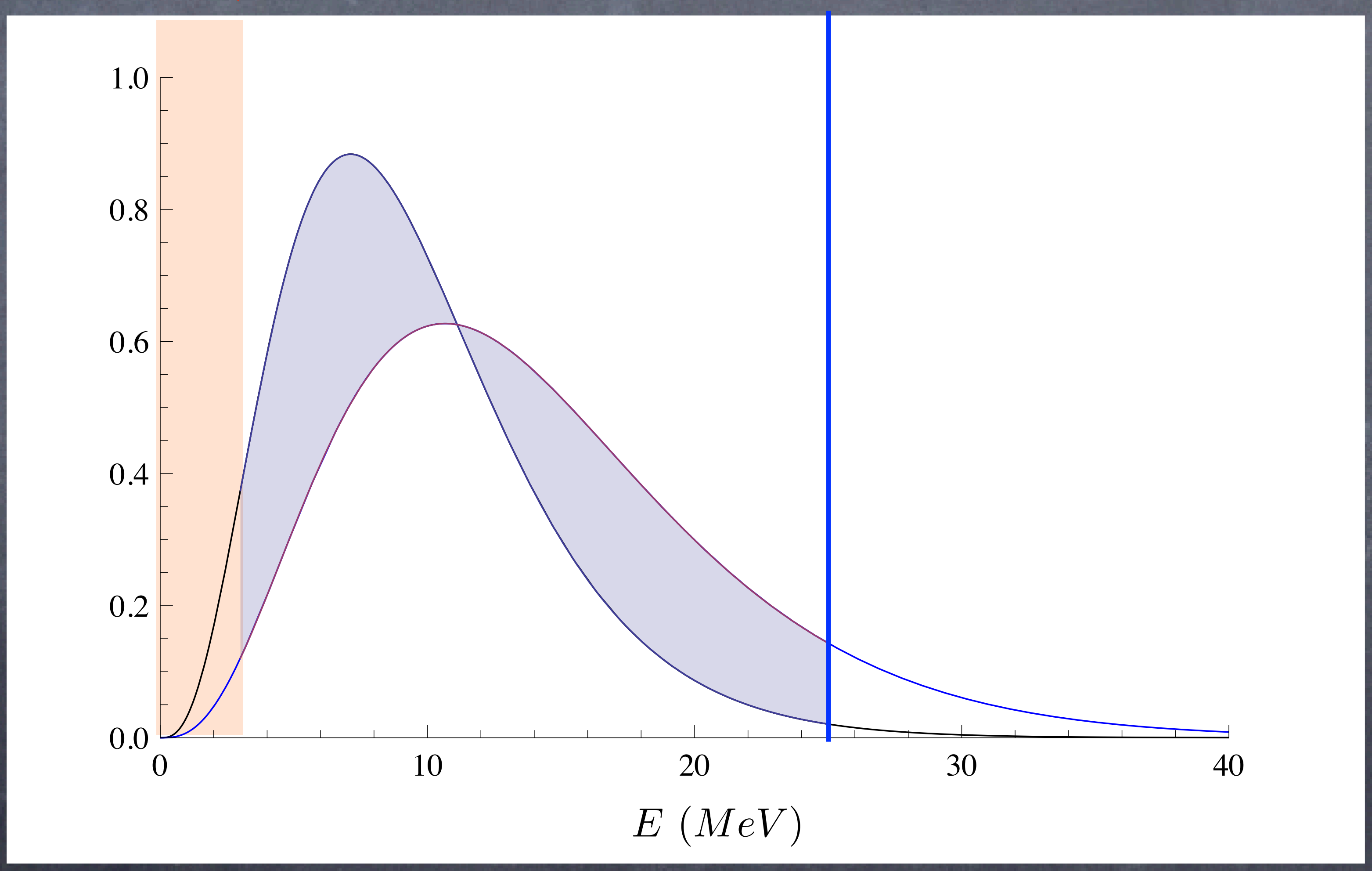
↓

The double split tends to be as large as possible

Consider the neutrino sector and the case $D_z < 0$

There can be no spectral swap below the resonance

Conservation of J_z fixes the second split energy



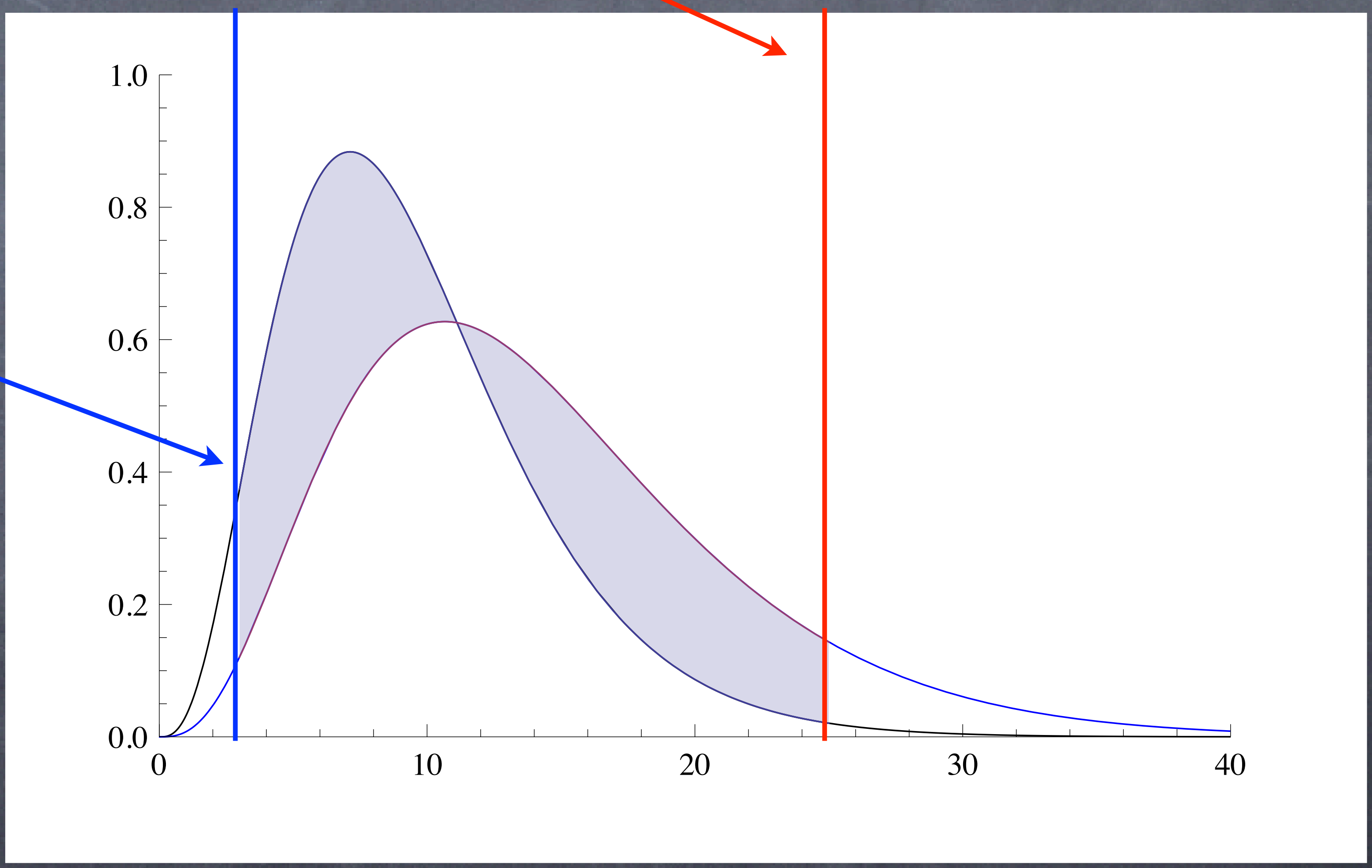
(When $D_z > 0$ the same thing happens for antineutrinos)

Now, consider the neutrino sector but in the case $D_z > 0$

We found an empirical criterium to determine the higher split energy:

We evaluate the frequency $\omega \sim \mu D_z$, at the end of collective effects

Conservation of J_z fixes the lower split energy



When $D_z < 0$ the same thing happens for antineutrinos

DOUBLE SPLIT Summary

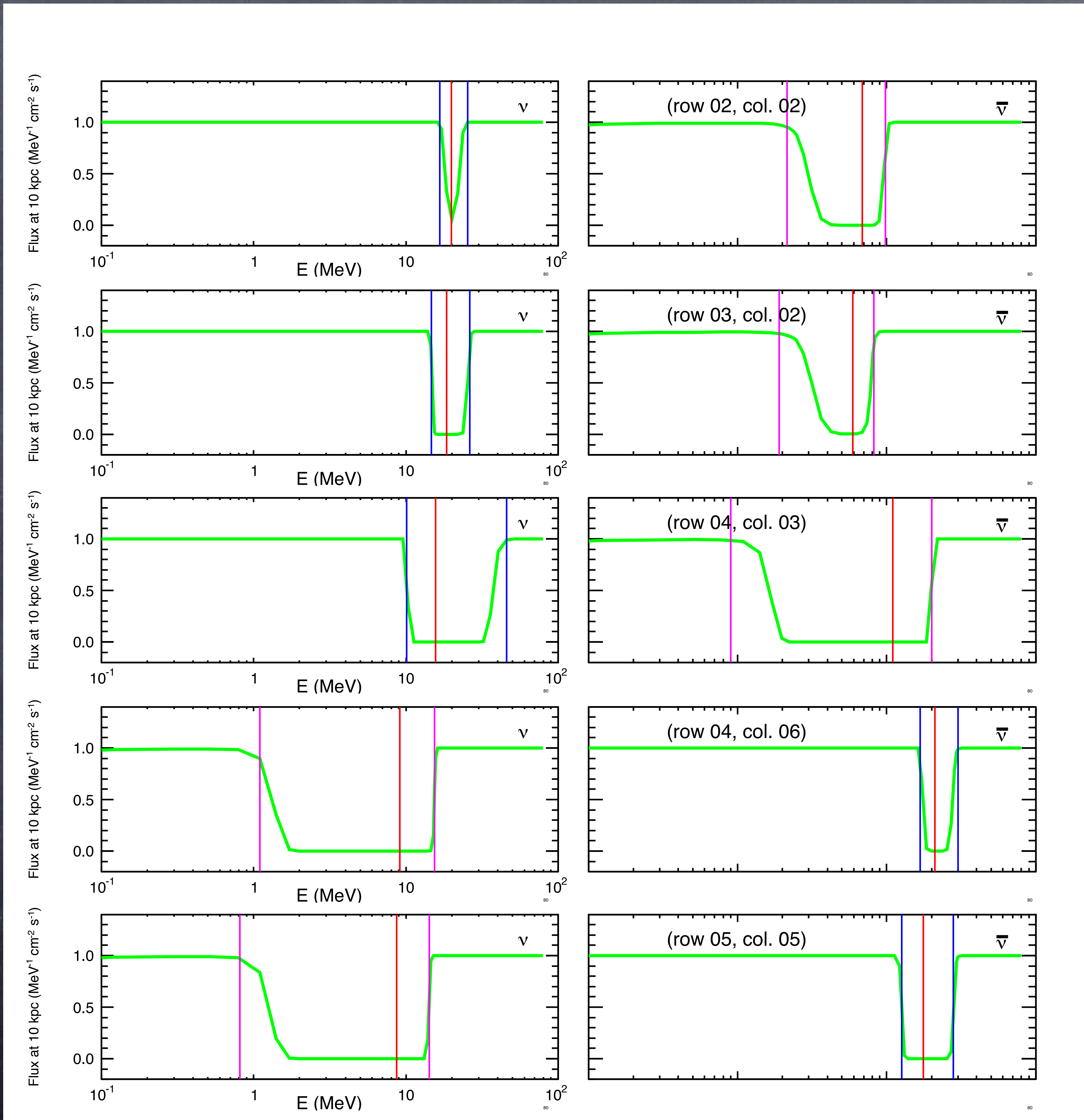
Double split of the largest possible width is favored by the minimization of the energy

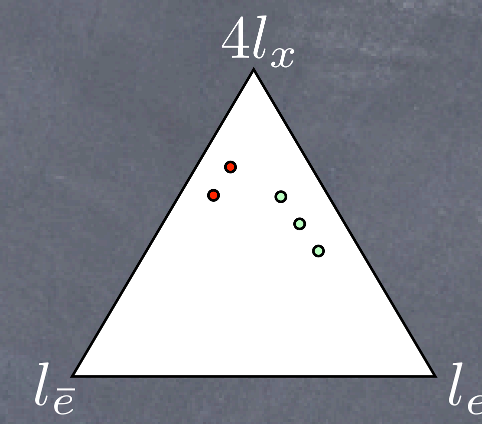
The actual width of the split is determined by

→ the resonance on the neutrino self-interaction potential on one side

→ the end of collective effects on the other side

DOUBLE SPLIT

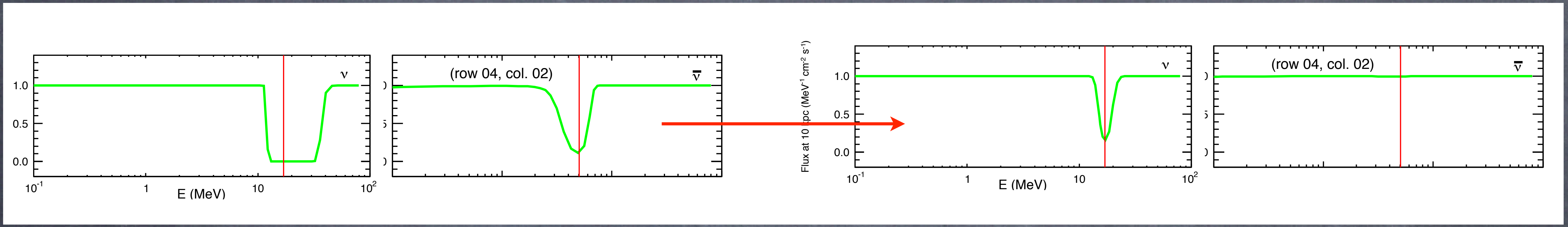




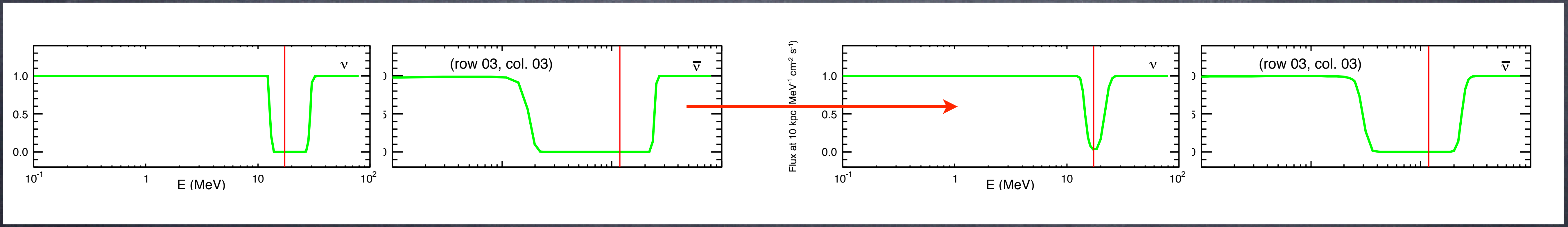
Agreement between estimated energies and simulation

Decreasing the adiabaticity

The number of double splits decreases



The width of the double split decreases



Conclusions

The number of splits depends on the position of the representative point in the ternary luminosity diagram

The system evolves so as to minimize the potential energy and

Single split energies determined by lepton number conservation and resonance on the self-interaction potential

Doble split energies determined by lepton number conservation (+ conservation of J_z), resonance on the self-interaction potential and end of collective effects

Increasing adiabaticity favors double splits and increases their width