Synchronization and partial decoherence at intermediate neutrino densities

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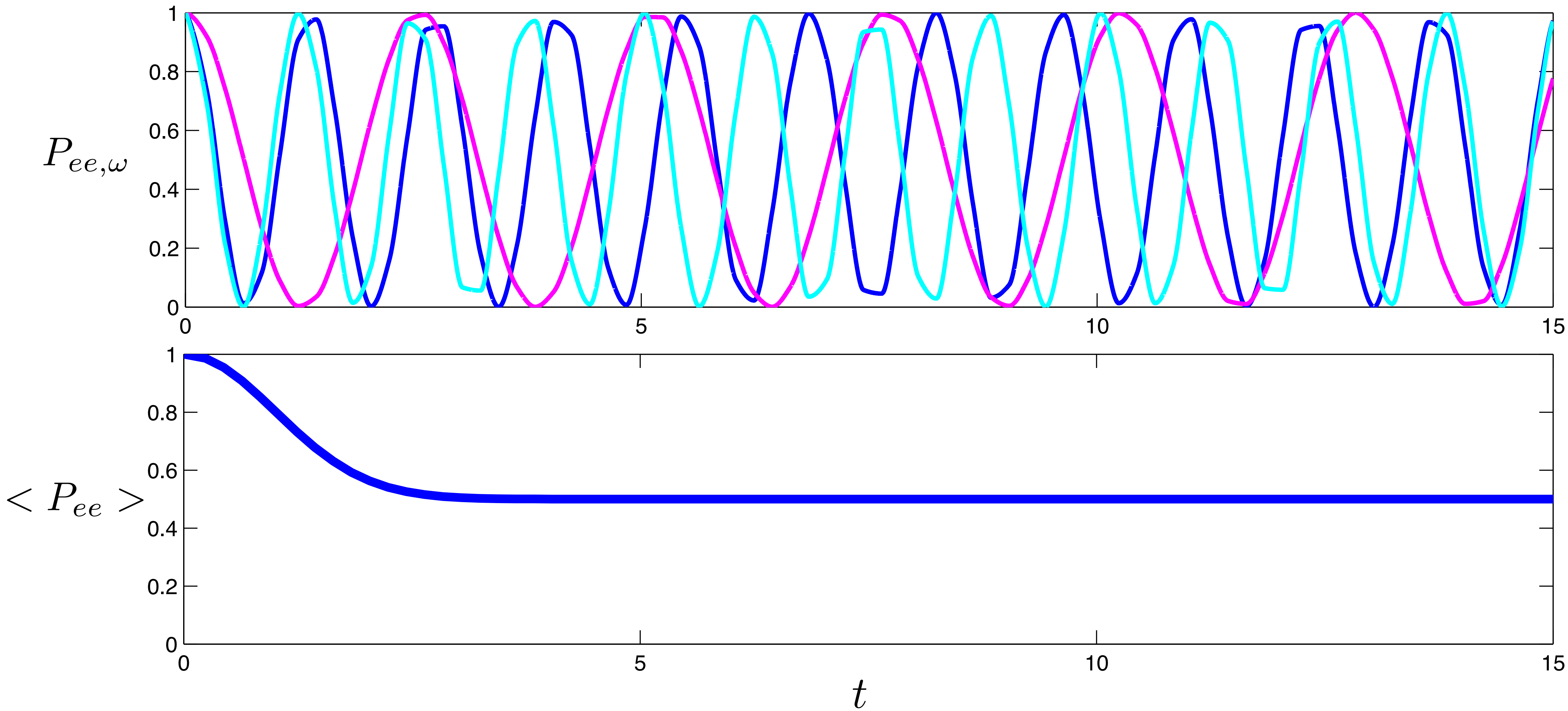
Outline

- Introduction
- Formalism for collective effects
- Weakly interacting case for a Gaussian neutrino spectrum and for two frequency modes
- Weakly interacting case: general scenario
- Conclusions

This talk is based on work in collaboration with G.G. Raffelt (pre-print to appear).

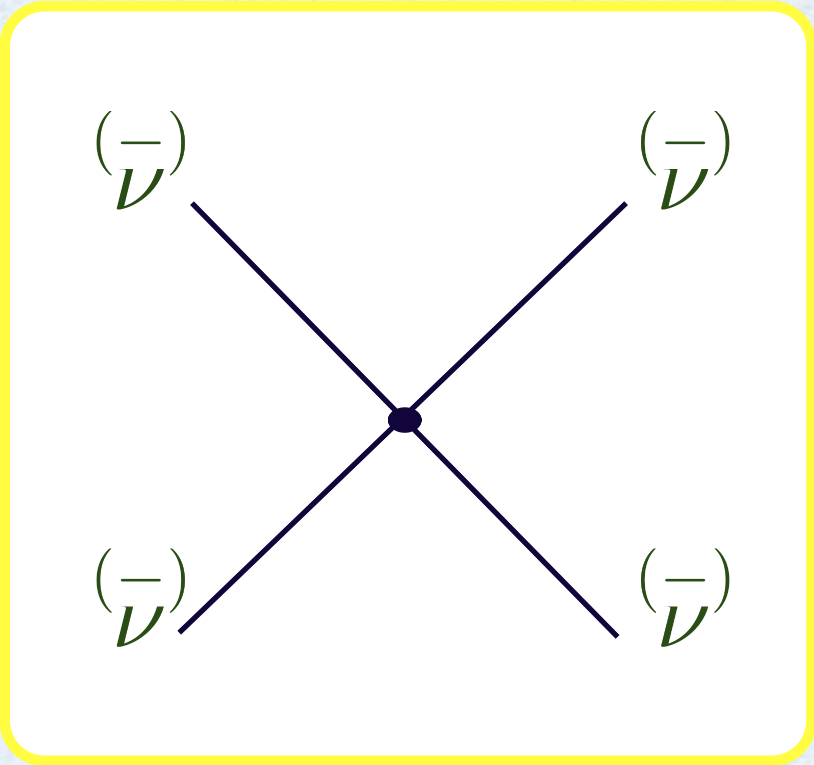
Introduction: Neutrinos in vacuum



Let's consider a neutrino ensemble with many frequency modes $\omega = \Delta m^2/2E$. These modes can develop growing relative phases so that **the mean survival probability for the ensemble quickly becomes equal to 1/2, and the overall flavor content "decoheres"**.

Introduction: $\nu - \nu$ interactions

When the neutrino density is high neutrino-neutrino interactions can not be neglected.



$\nu - \nu$ neutral current interactions are described by means of a potential:

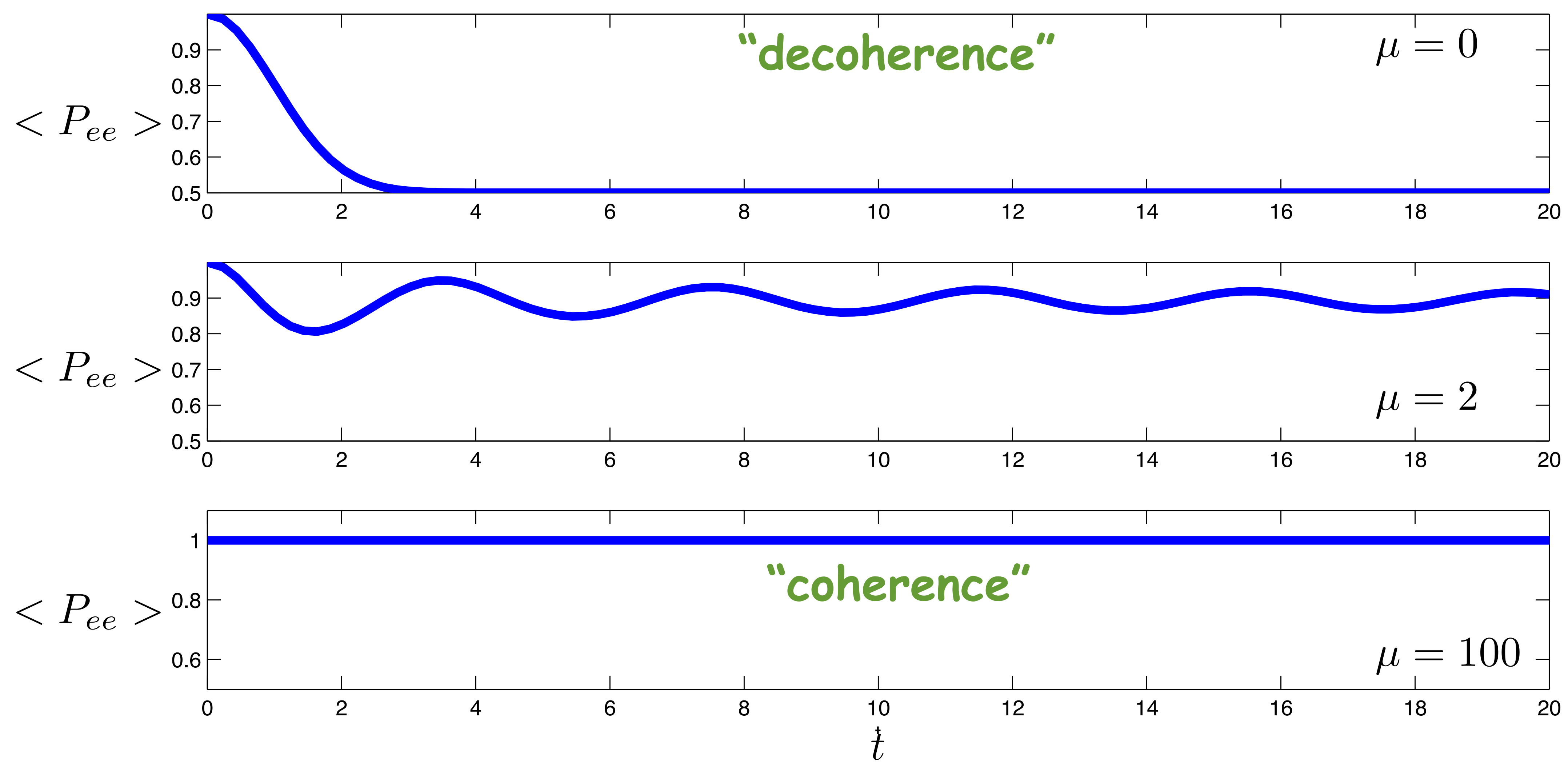
$$\mu = \sqrt{2} G_F n_\nu$$

with n_ν the total effective neutrino number density.

In what follows, **we always assume μ constant**. Where not specified, the self-interaction potential and the frequencies are measured in km^{-1}.

Introduction: Effect of $\nu - \nu$ interactions

The vacuum behavior changes dramatically when we switch-on the potential μ due to the presence of the other neutrinos.



Questions:

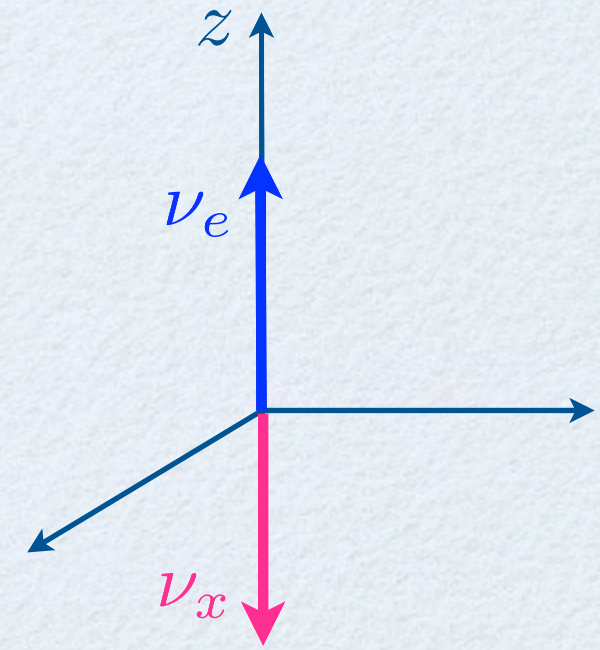
★ What happens at intermediate densities?

★ How does the transition (decoherece-coherence) take place?

★ Does the system decohere completely on larger and larger time scale, or does it partially decohere reaching a stationary state?

Collective effects: Formalism

For each neutrino frequency mode ω, decompose the 2 x 2 neutrino density matrix over Pauli matrices to get the **Bloch 3-vector**. In the flavor basis, the Bloch vectors associated to ν_e and ν_x are respectively aligned and anti-aligned with the z axis.



Considering the Bloch vector's evolution equation for each frequency mode, one has to solve a **large system of non-linear differential equations**:

$$\dot{\mathbf{P}}_\omega = (\omega \mathbf{B} + \mu \mathbf{P}) \times \mathbf{P}_\omega$$

vacuum term (with $\mathbf{B}$ function of the mixing angle θ_{13})

self-interaction term with $\mathbf{P} = \int d\omega \ \mathbf{P}_\omega$

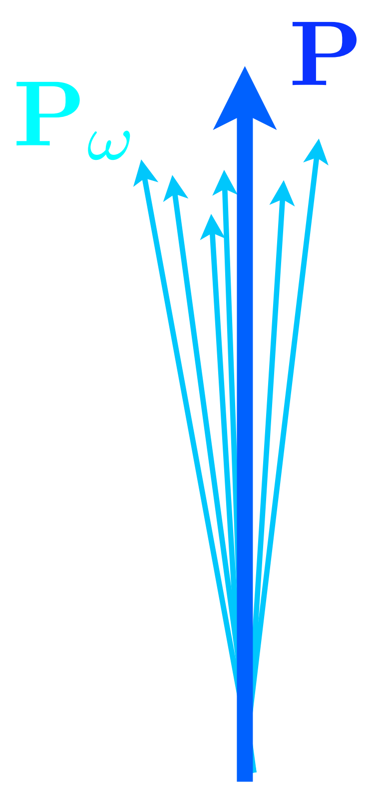
The third component of the Bloch vector is related to the survival probability:

$$P(\nu_e \rightarrow \nu_e) = \frac{1}{2}\left(1 + \frac{P_z^f}{P_z^i}\right)$$

with P_z^i and P_z^f the initial and the final value of the third component of $\mathbf{P}$.

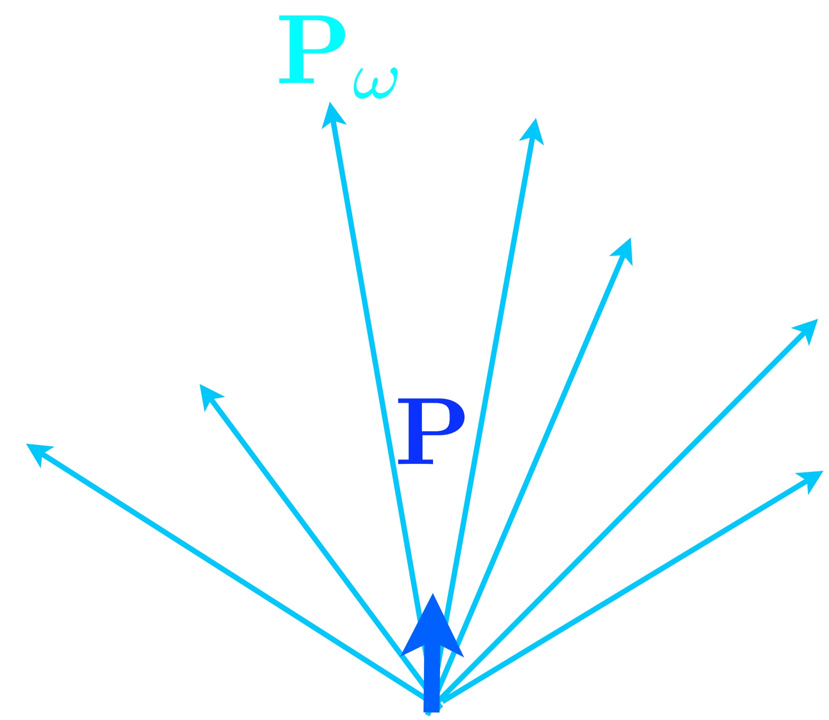
Collective effects: Formalism

High neutrino density:
synchronized oscillations*



All the $\mathbf{P}_\omega$ are sticked together.
The length of $\mathbf{P}$ is conserved.

Negligible neutrino density:
"decoherence"



Each $\mathbf{P}_\omega$ looses the memory of its initial state, and carries no correlation with the phases of the others.
The length of $\mathbf{P}$ quickly tends to zero.

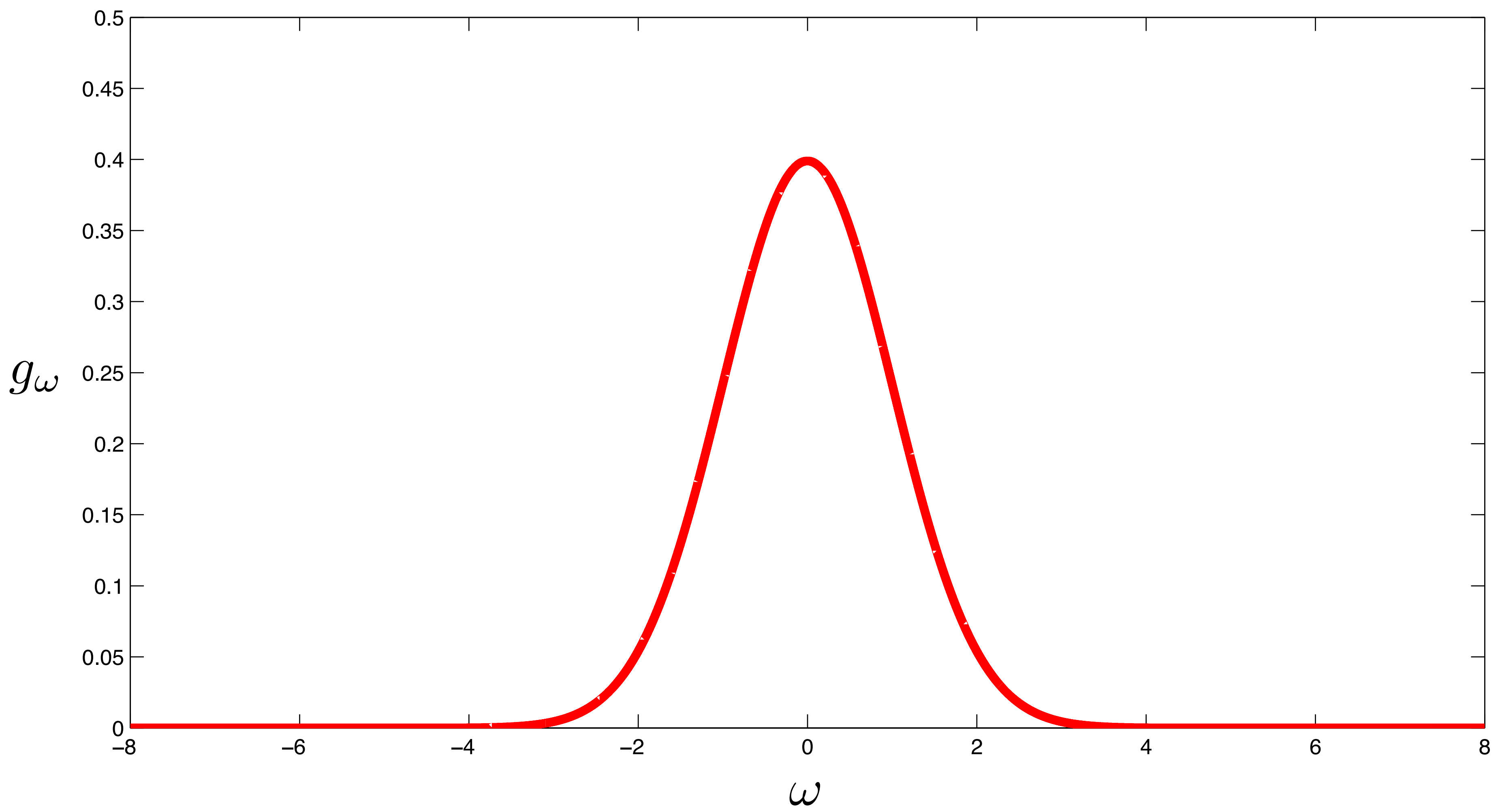
The variation of the module of the global vector is an index of the degree of the kinematical coherence in the neutrino system.

*S. Pastor, G.G. Raffelt and D.V. Semikoz, arXiv: hep-ph/0109035.

Weakly-interacting case: Setup of the problem for a Gaussian spectrum

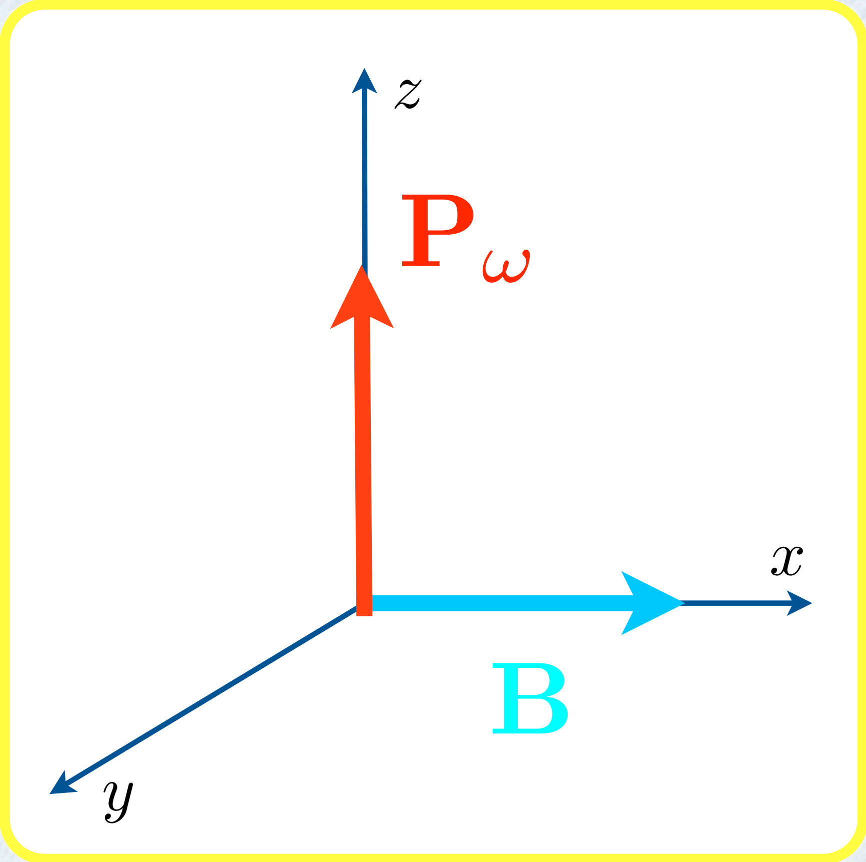
We choose a Gaussian spectrum. For sake of simplicity, we shift the spectrum in ω centering it on $\omega = 0$, ...

$$g_\omega = \sqrt{\frac{1}{2\pi}} \exp\left(-\frac{\omega^2}{2}\right)$$



Weakly-interacting case: Setup of the problem

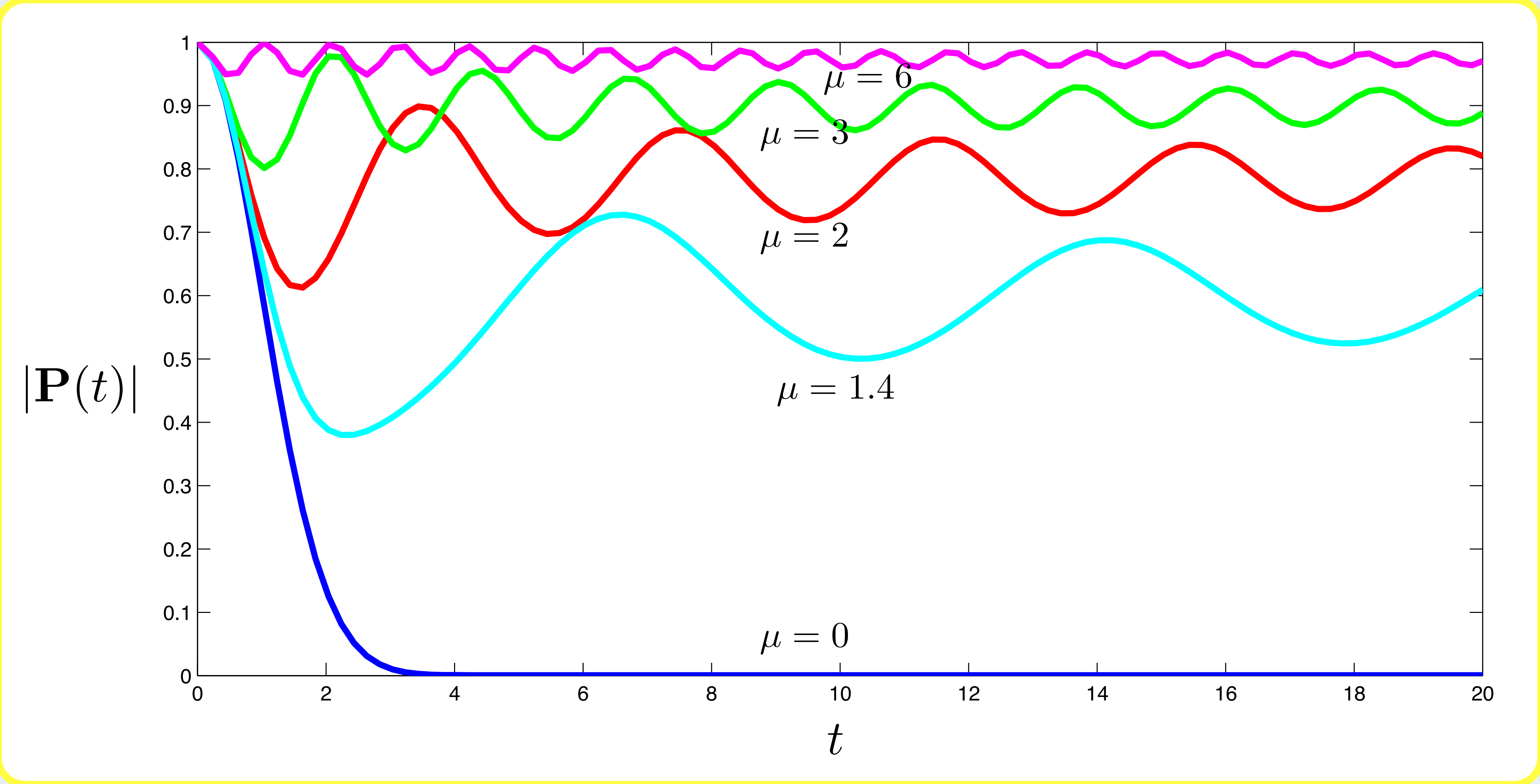
... and choose a symmetric initial configuration ($\theta_{13} = \pi/4$):



Since our spectrum is centered on $\omega = 0$, $\mathbf{P}$ remains parallel to the z-axis ($< P_x > = < P_y > = 0$), except for changing its partial length by partial or complete decoherence.

Weakly-interacting case: Numerical results

For intermediate densities of neutrinos, and Gaussian spectra we found **partial decoherence** (the final length of $\mathbf{P}$ is non null, but finite), and asimptotically **a "stationary" state is reached on finite time scales**.

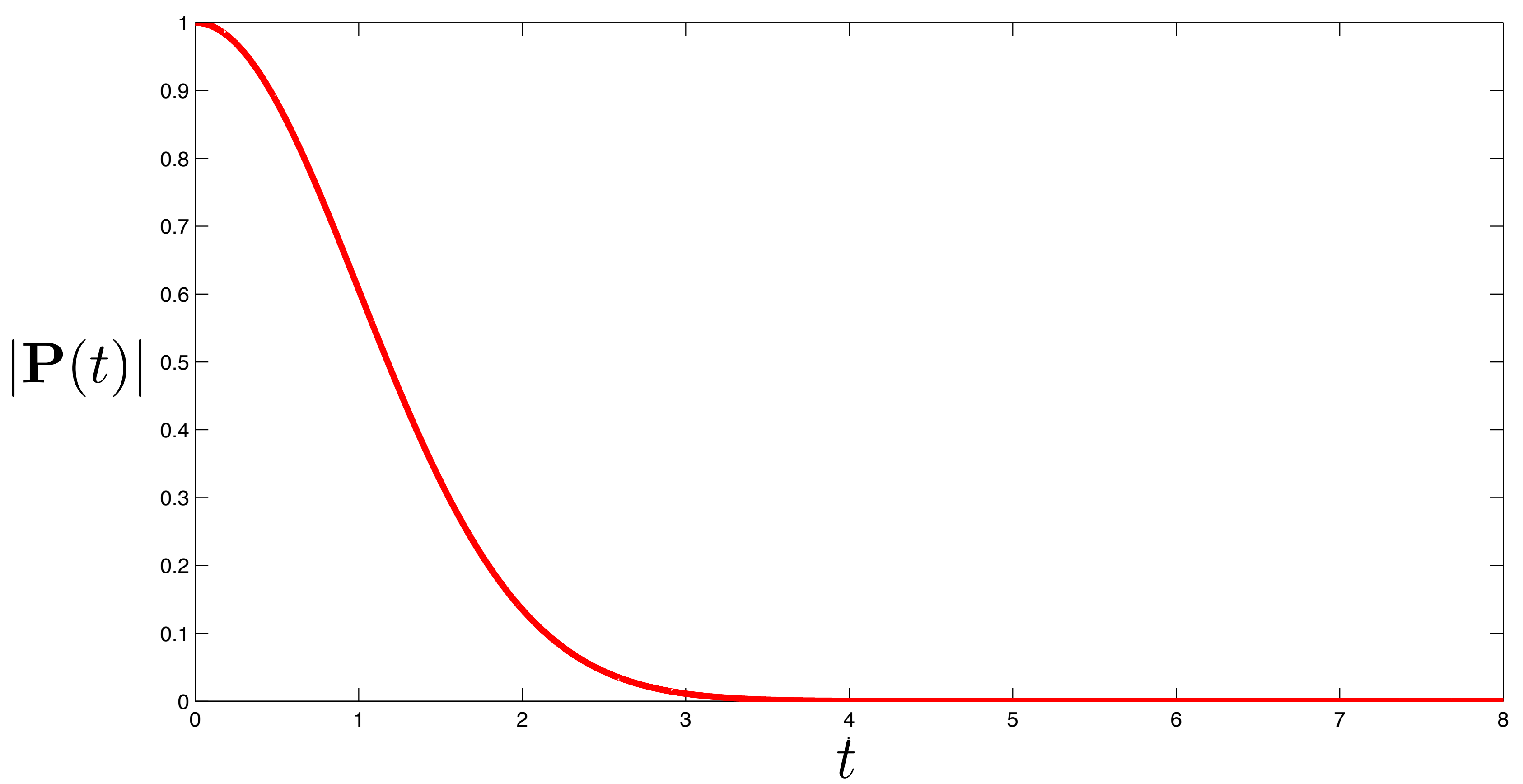


How to explain these results?

The simplest scenario: Non-interacting case

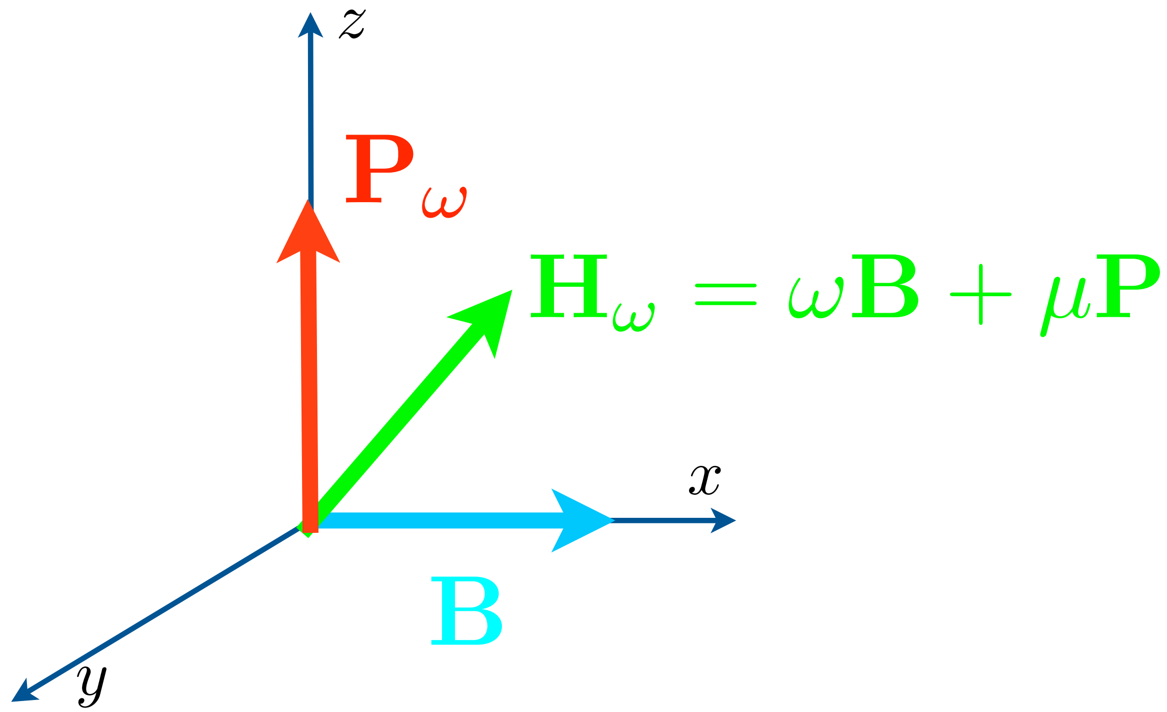
Let's analyze first of all the most simple case: the vacuum case.
In the non-interacting case, **the global length of the polarization vector is the cosine transform of the initial spectrum**, and can be analytically computed

$$|\mathbf{P}(t)| = \exp\left(-\frac{t^2}{2}\right)$$



The length of $\mathbf{P}$ exponentially tends to zero: Complete decoherence.

Weakly-interacting case: Analytical explanation for a Gaussian spectrum



Each $\mathbf{P}_\omega$ has a precession around $\mathbf{H}_\omega$.
We can assume that the mean of the transverse components for each $\mathbf{P}_\omega$ is equal to zero.
This means that we can approximate the global vector

$$\mathbf{P} = \int d\omega\ \mathbf{P}_\omega \longrightarrow \mathbf{A} = \int d\omega\ \langle \mathbf{P}_\omega \rangle$$

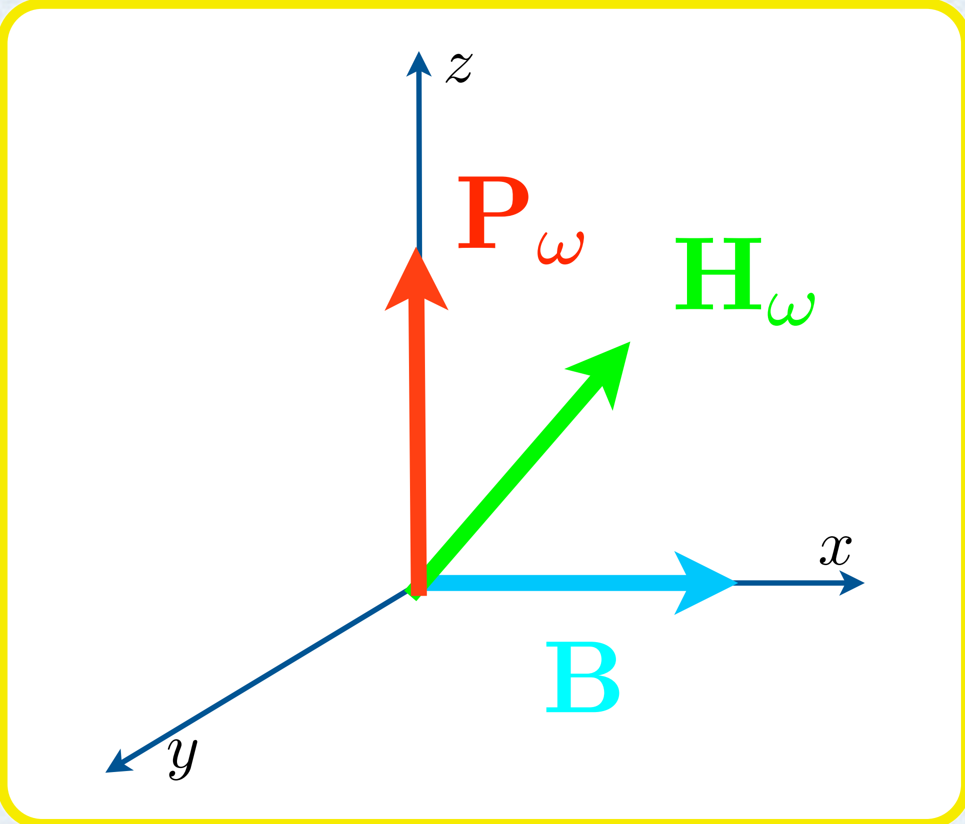
this approximation is equivalent to compute the projection of each $\mathbf{P}_\omega$ on $\mathbf{H}_\omega$.
We get an implicit equation

$$\mathbf{A} = \int d\omega \frac{\mathbf{P}_\omega \cdot \mathbf{H}_\omega}{\mathbf{H}_\omega^2} \mathbf{H}_\omega = \int d\omega \frac{\mathbf{P}_\omega \cdot (\omega\mathbf{B} + \mu\mathbf{A})}{(\omega\mathbf{B} + \mu\mathbf{A})^2} (\omega\mathbf{B} + \mu\mathbf{A})$$

To solve this equation, we need some further approximation. Let's check the sudden approximation.

Weakly-interacting case: Sudden approximation

In the sudden approximation, we assume that the $\mathbf{P} \to \mathbf{A}$ transition is not adiabatic.



each $\mathbf{P}_\omega$ is always parallel to the z-azis. The initial $\mathbf{P}$ shrinks instantaneously to the final $\mathbf{A}$.

From the previous implicit equation follows:

$$\frac{1}{\mu} = \int d\omega \; g_\omega \frac{\mu A}{\omega^2 + \mu^2 A^2}$$

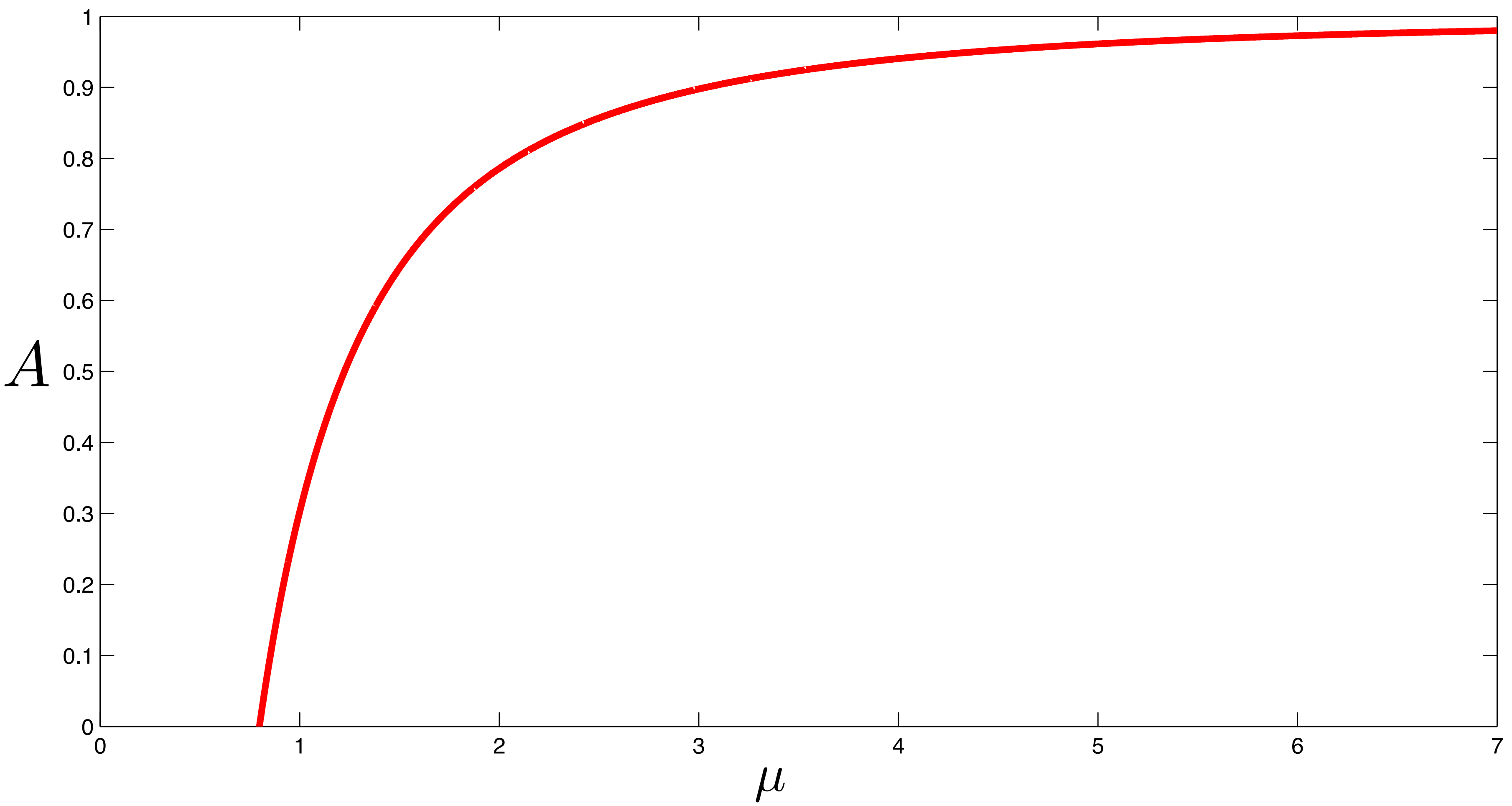
This equation can be solved to find $A(\mu)$.

Weakly-interacting case: Sudden approximation

The previous equation has solution:

$$\frac{1}{\mu} = \sqrt{\frac{\pi}{2}} \exp\left(\frac{y^2}{2}\right) \operatorname{erfc}\left(\sqrt{\frac{1}{2}}\, y\right)$$

with $y = \mu A$



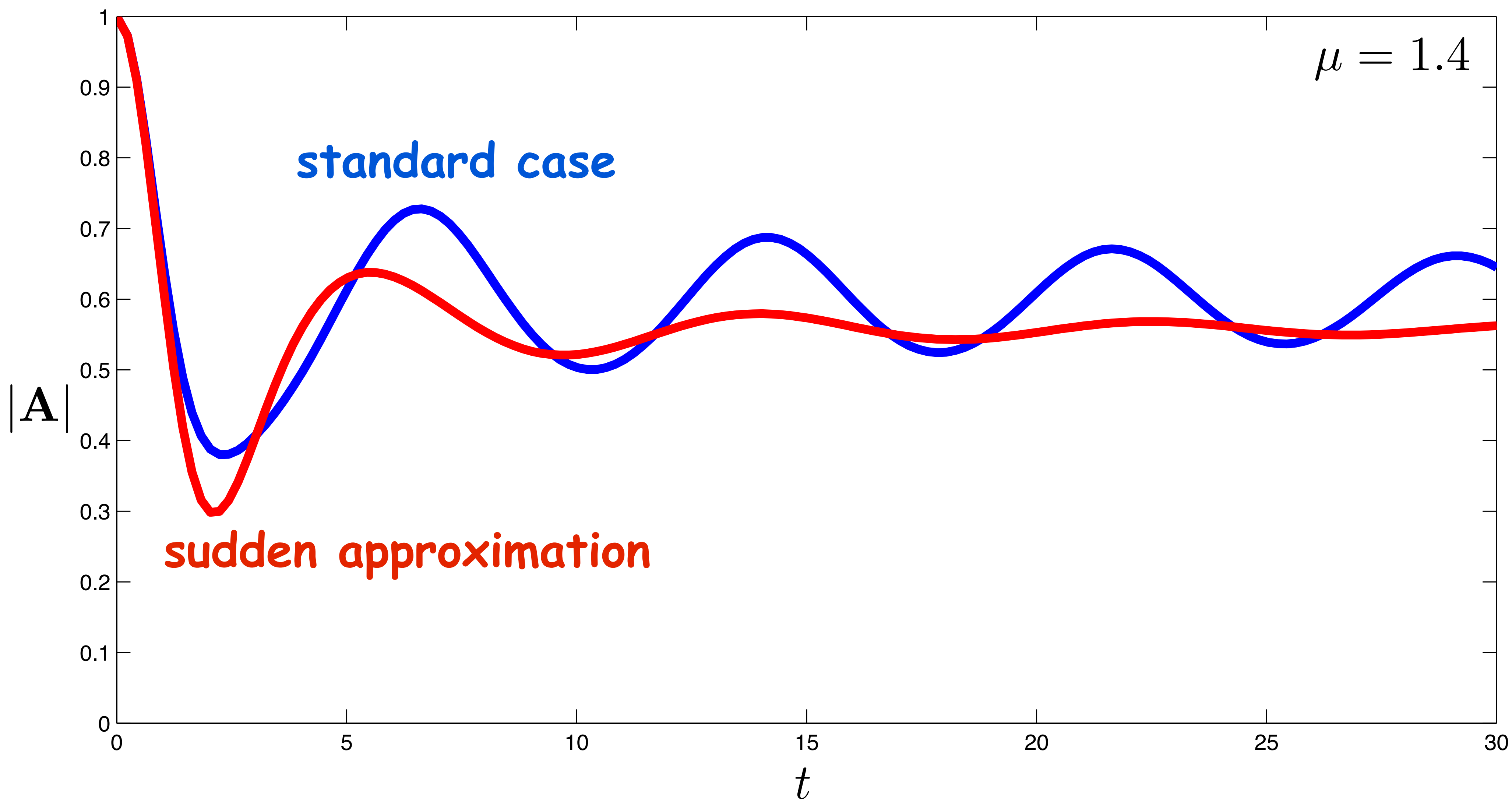
The transition to the complete decoherent regime ($A \to 0$) is sharp and it is expected for

$$\mu \to \sqrt{2/\pi} \simeq 0.8$$

this behavior is reproduced by numerical simulations.

Weakly-interacting case: Sudden approximation

The sudden approximation reproduces quite well the final stationary value of $|\mathbf{A}|$. But not completely the dynamics. In fact, if in the equations of motion we substitute the final stationary value of $\mathbf{A}$ the evolution rate is different. Using the sudden approximation the "stationary" final value is reached faster.



Weakly-interacting case: Summary for a Gaussian spectrum

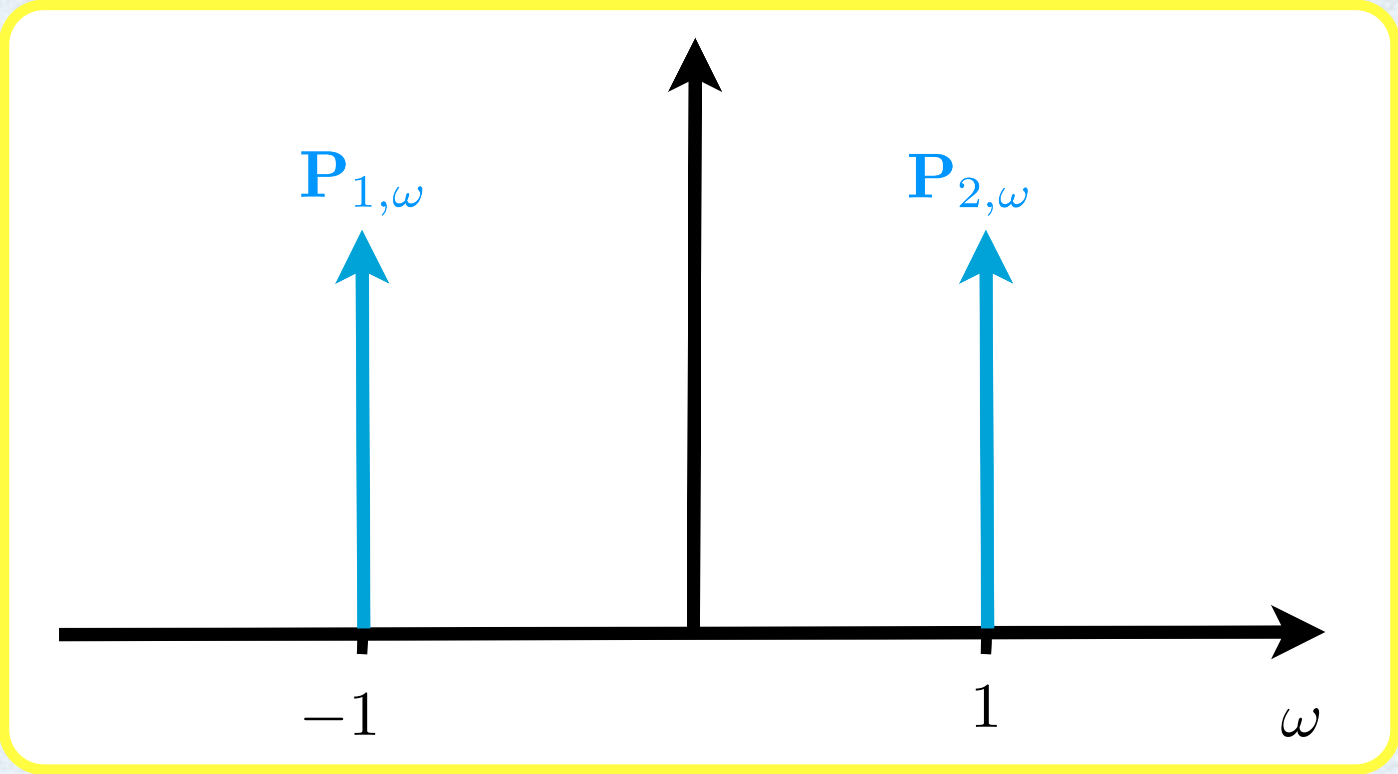
Until now we have seen that with a Gaussian spectrum

★ in the vacuum case and for very small values of μ, the system completely decoheres ($\mathbf{A} = 0$).

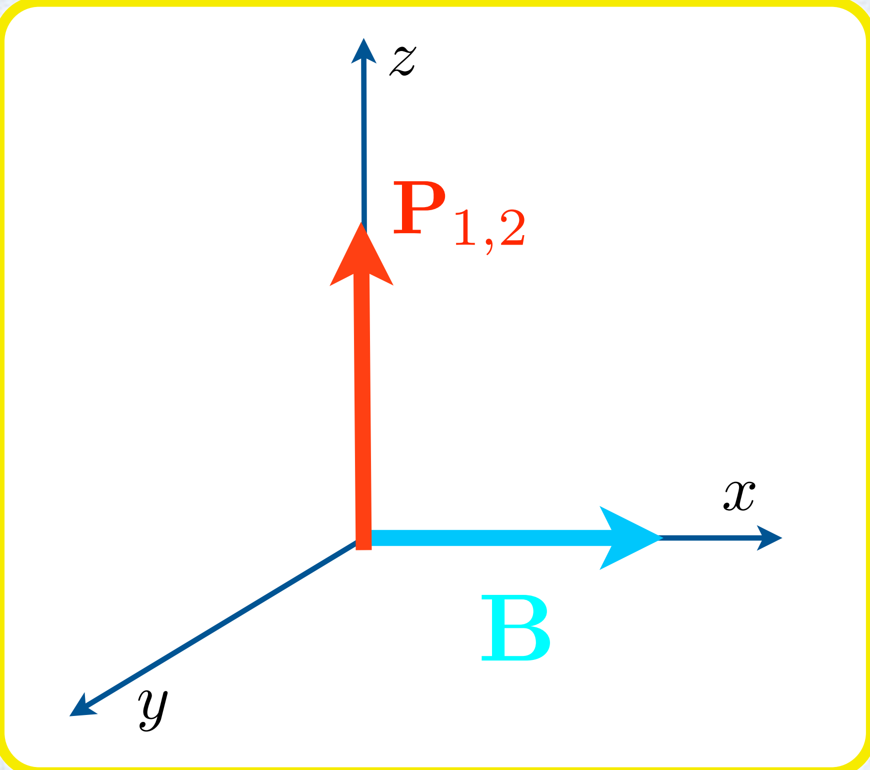
★ for intermediate neutrino densities, the system partially decoheres ($\mathbf{A} \neq 0$).

★ in both the cases, after a finite time-scale, $\mathbf{A}$ reaches a stationary value.

But there are cases in which the global vector continues to oscillate on infinite time scales, and can reach a final "mean" value finite or null according to the neutrino potential μ. This is the case of two Bloch vectors:



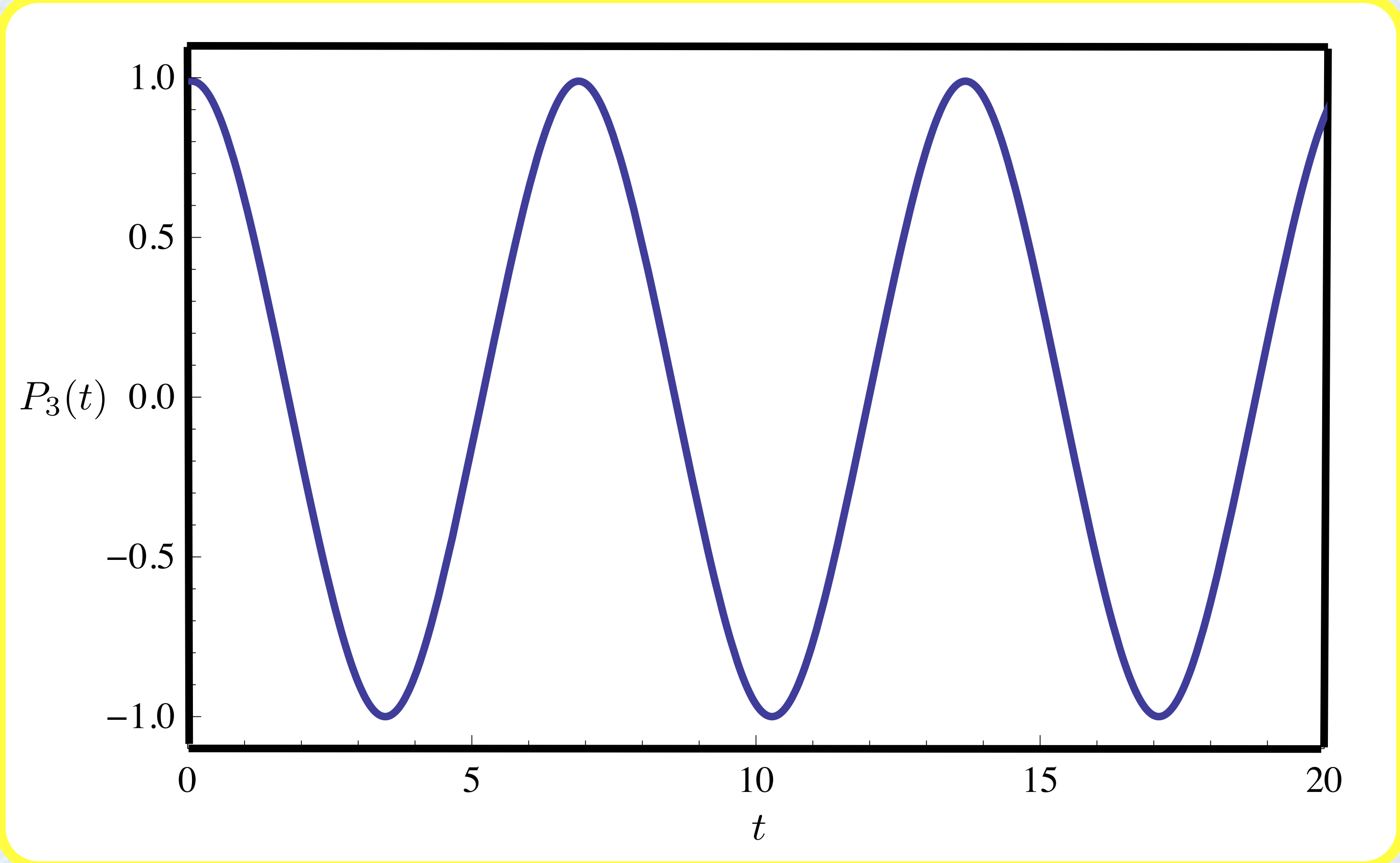
Weakly-interacting case: 2 Bloch vectors scenario



Let's consider the case of two Bloch vectors that interact with a neutrino-neutrino potential

$$\mu = 1$$

In this case the global vector continue to oscillate on infinite time scale and only a "mean" global length can be defined.



Weakly-interacting case: 2 Bloch vectors scenario

In the most general case, we can assume for each time t

$$\mathbf{P}_1 = \frac{1}{2}\begin{pmatrix} x \\ y \\ z \end{pmatrix} \qquad \mathbf{P}_2 = \frac{1}{2}\begin{pmatrix} -x \\ -y \\ z \end{pmatrix}$$

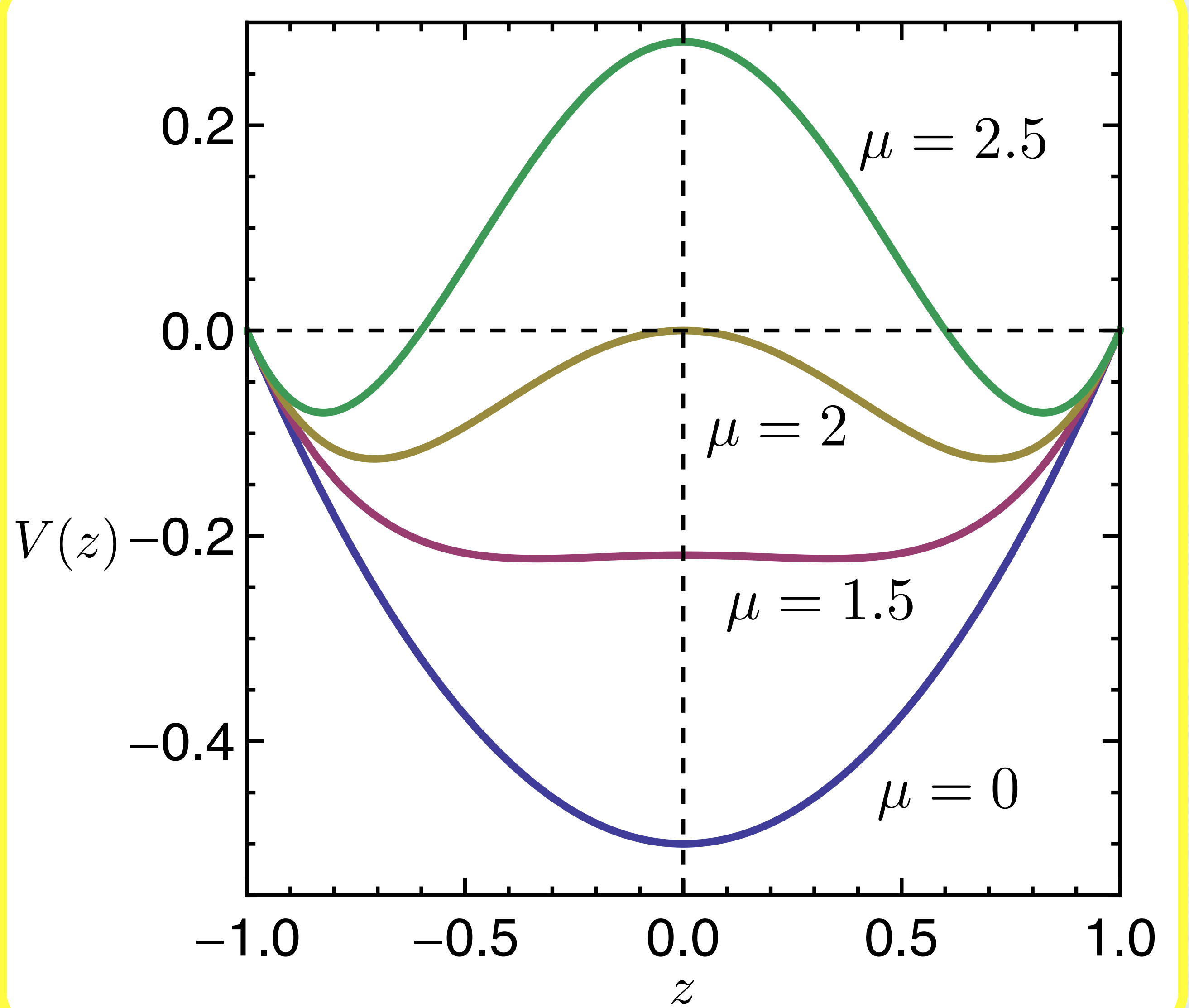
our system has three degrees of freedom. The evolution equations and the conservation laws impose two constraints that leave one free degree of freedom.

Our system is reduced to one equation in the variable z, formally similar to the energy equation for a particle:

$$\frac{1}{2}\dot{z}^2 + V(z) = 0$$

with

$$V(z) = -\frac{1}{2}\left[(1-z^2) - \frac{\mu^2}{4}(1-z^2)^2\right]$$



For $0 \leqslant \mu < 2$ the motion oscillates between $-1 \leqslant z \leqslant 1$ and $< z >= 0$. For $\mu > 2$, the system is trapped in the region of positive and $< z > \rightarrow 1$.

Weakly-interacting case: 2 Bloch vectors scenario

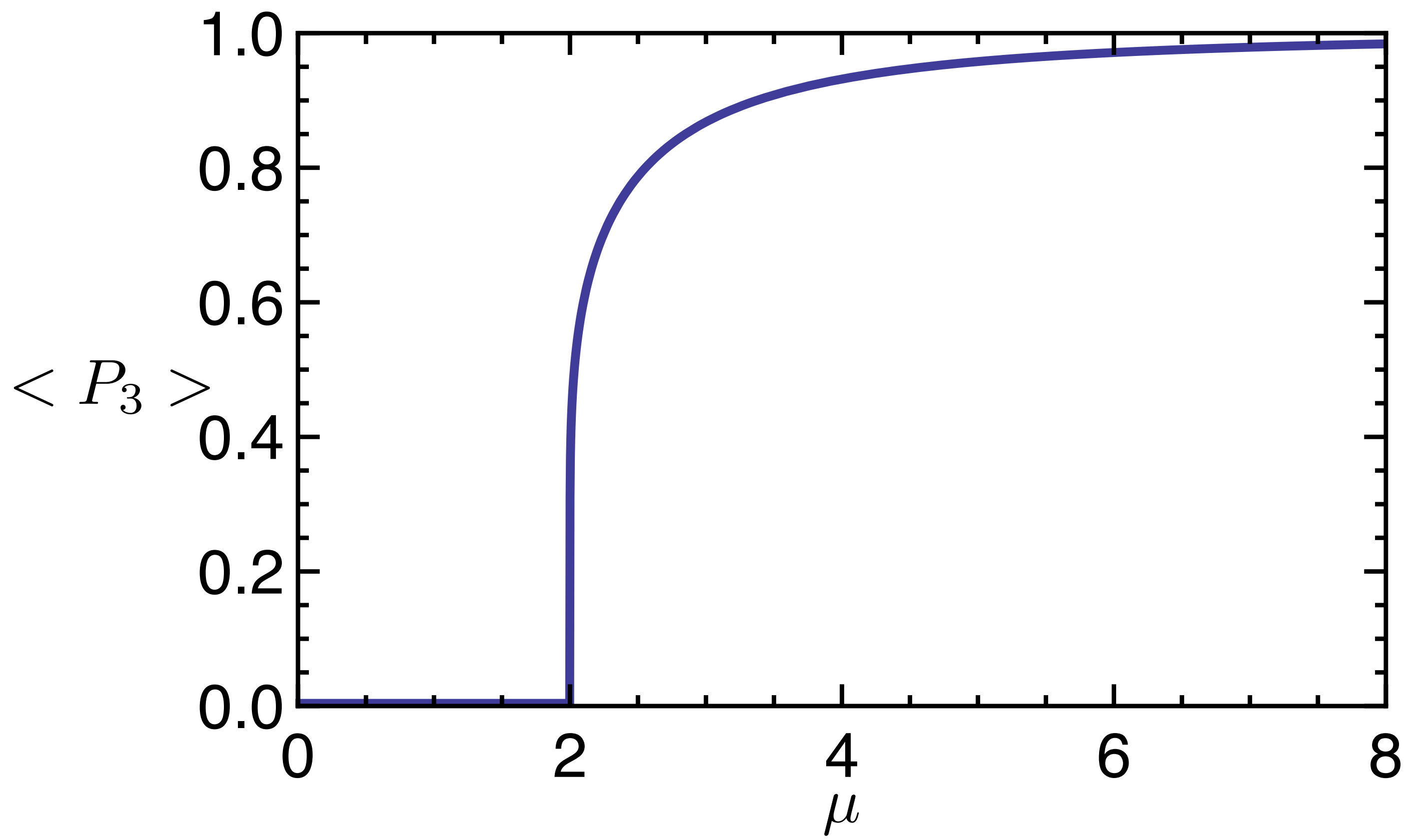
From the "potential behavior" we expect a fairly abrupt transition for

$$\mu \simeq 2$$

We can compute the average final value of the global vector $< P_3 >=< z >$ for arbitrary μ

$$< z >= \frac{1}{T}\int_0^T dt \frac{z(t)}{\dot{z}(t)} = \frac{\pi}{2\mathrm{EllipticK}(4/\mu^2)}$$

where T is half an oscillation period.

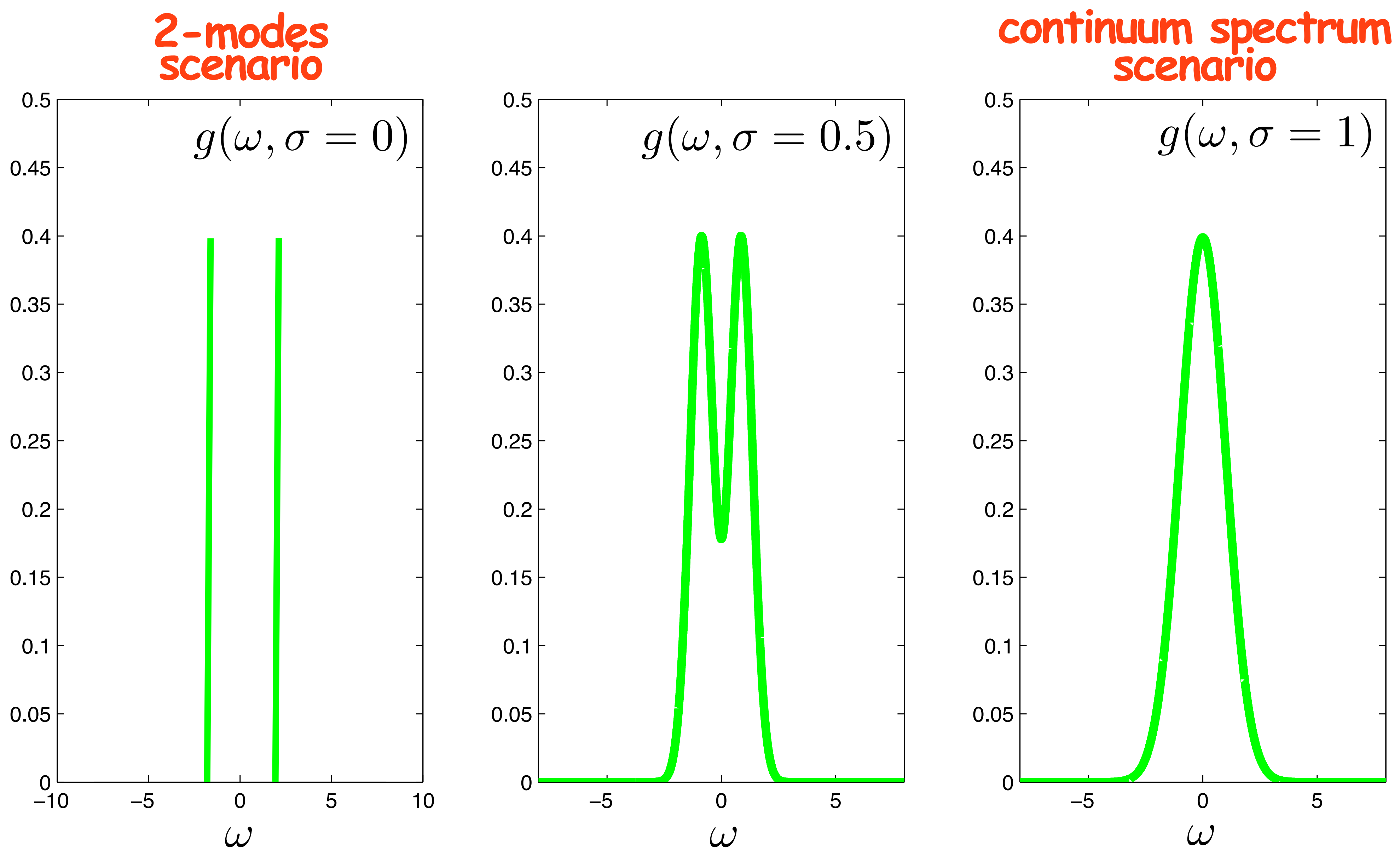


This behavior is numerically well reproduced.

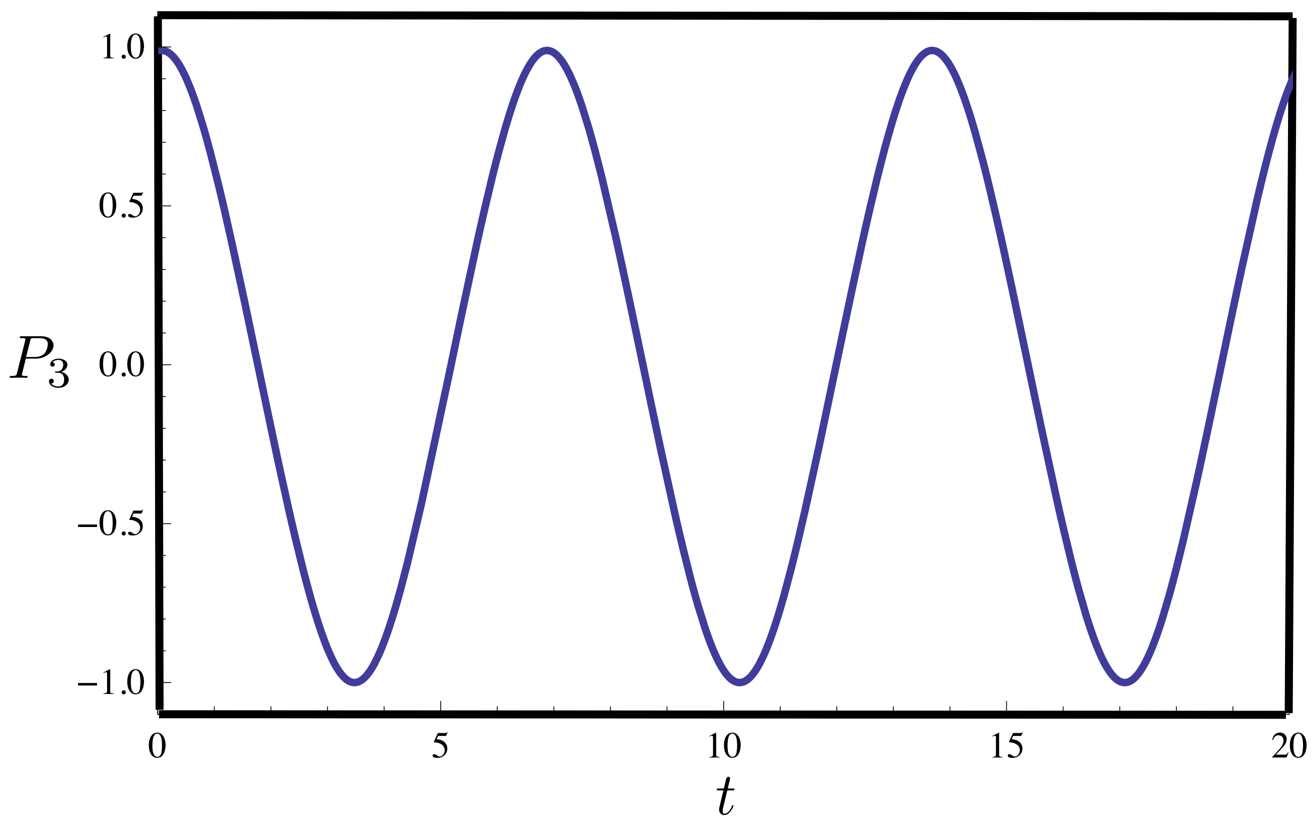
Weakly-interacting case: General case

In the most general case, we can consider a double-gaussian spectrum with $\sigma \in [0,1]$

$$g_\sigma(\omega) = \frac{1}{2\sqrt{2\pi\sigma^2}}\left[\exp\left(-\frac{(\omega-\sqrt{1-\sigma^2})^2}{2\sigma^2}\right)+\exp\left(-\frac{(\omega+\sqrt{1-\sigma^2})^2}{2\sigma^2}\right)\right]$$



Weakly-interacting case: Numerical results

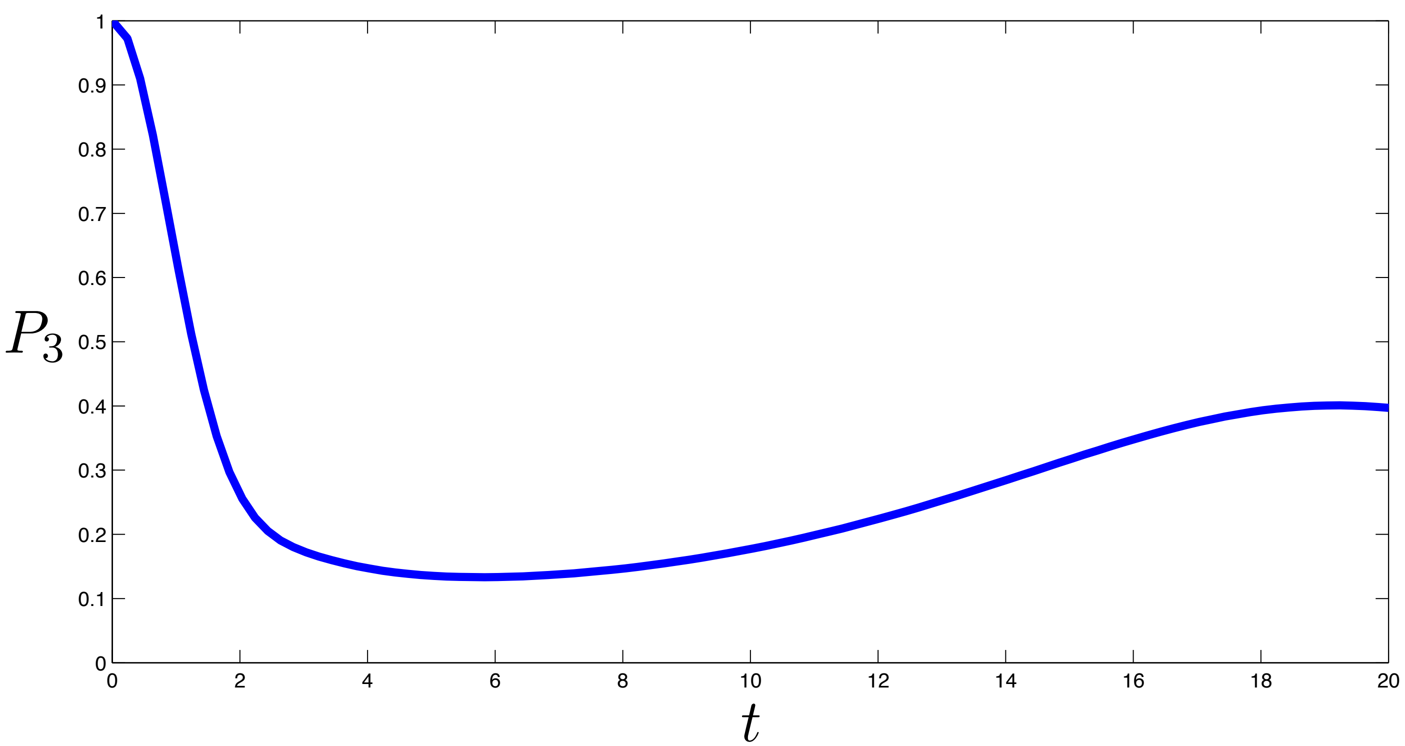


2-modes case ($\sigma = 0$)

P_3 oscillates with constant amplitude, and the **mean length** reaches a finite (partial decoherence) o null (full decoherence) value.

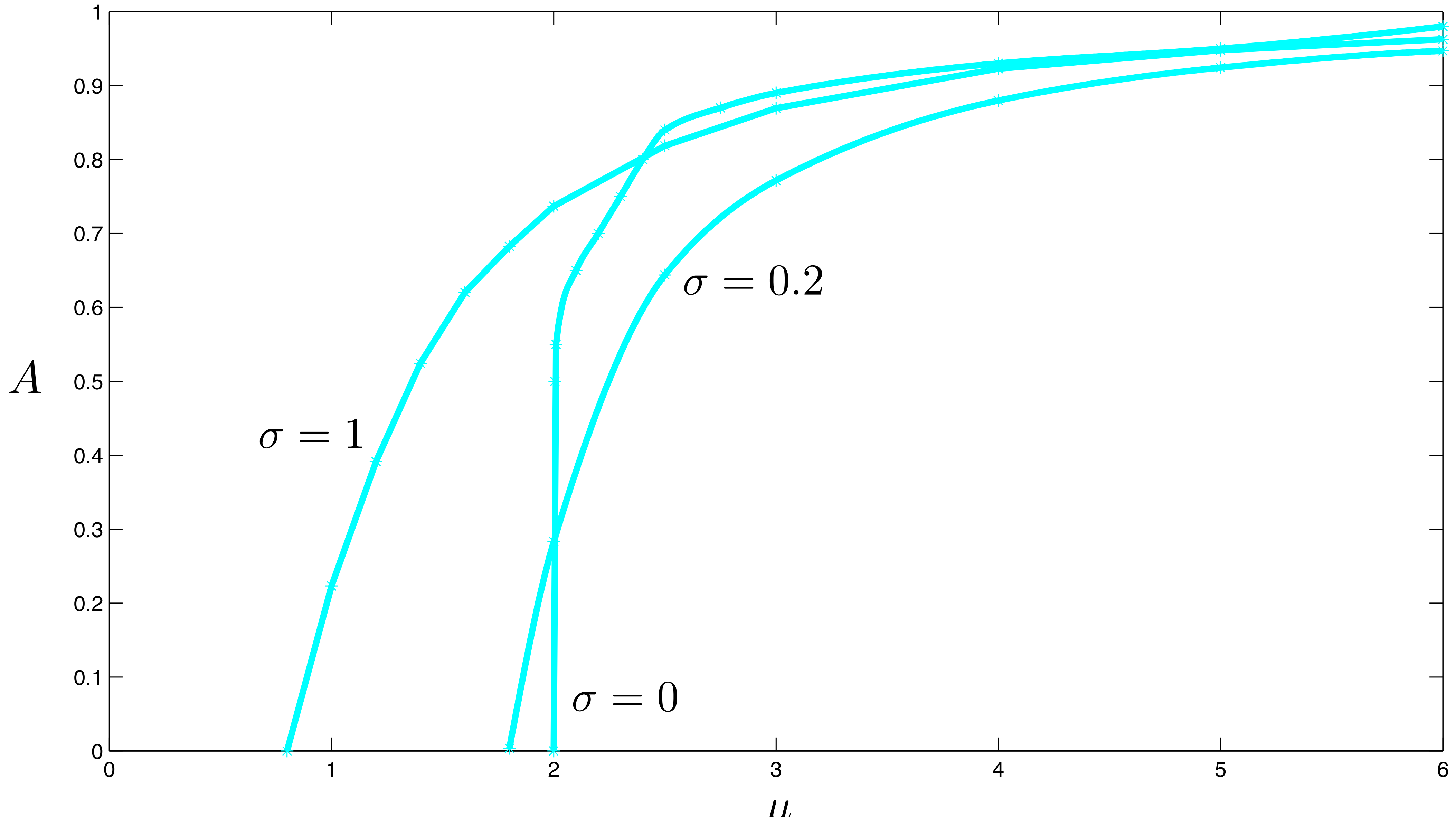
Gaussian case ($\sigma = 1$)

P_3 oscillates with decreasing amplitude and reaches a stationary value.



Weakly-interacting case: Numerical results

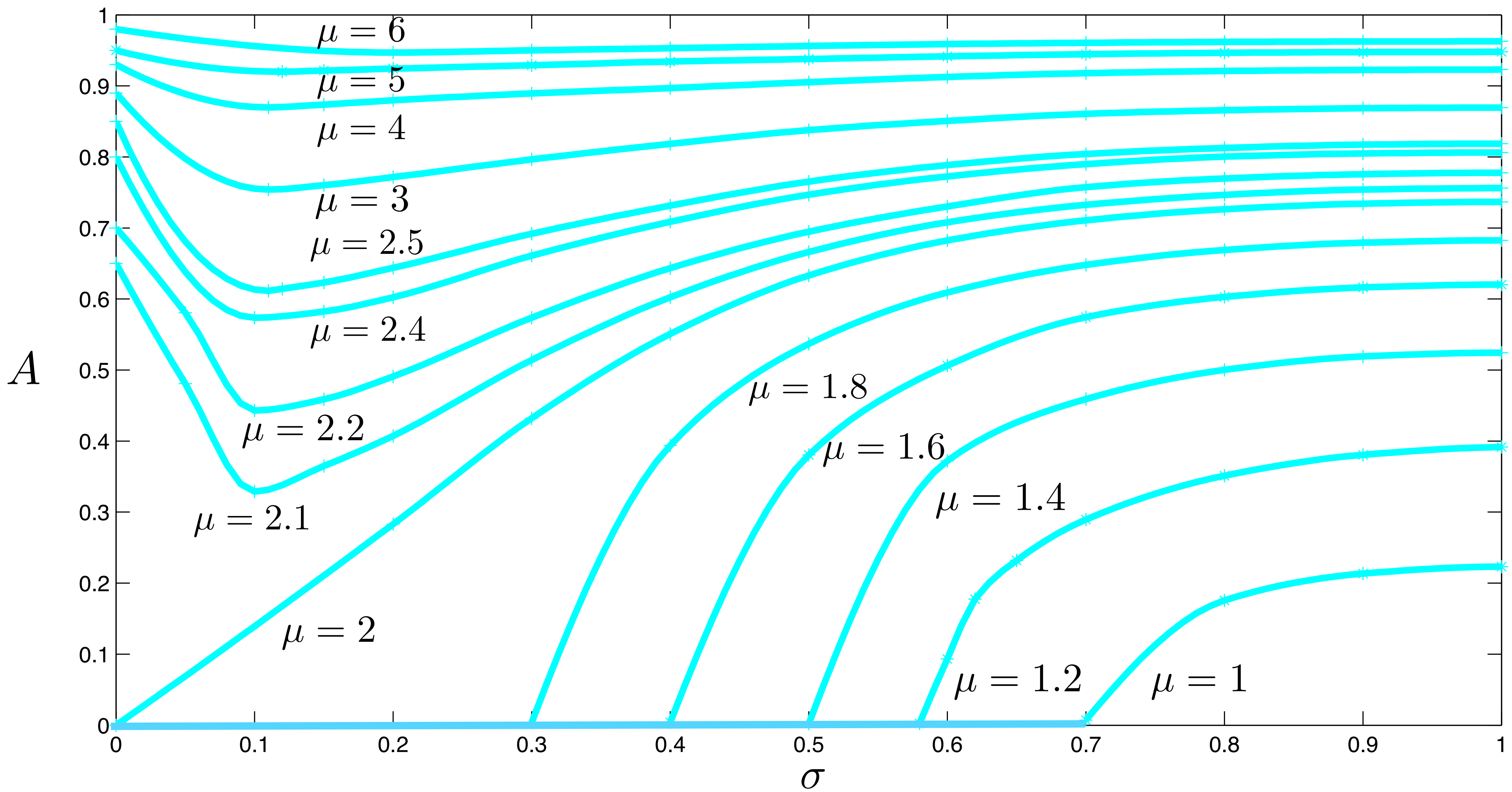
Variation of the final length A as a function of μ for different values of σ



For μ smaller than a critical one, complete decoherence takes place ($A = 0$). The transition is sharp.

Weakly-interacting case: Numerical results

Variation of the final length A as a function of σ for different values of μ



For intermediate values of μ, A reaches a final intermediate length (partial decoherence). The transition is smooth.

Conclusions

★ The strength of the neutrino-neutrino interactions determines the degree of kinematical decoherence.

★ For $\mu = 0$, a complete decoherent regime is reached ($|\mathbf{A}| = 0$). For very high μ, synchronized oscillations take place ($|\mathbf{A}| = 1$). For intermediate μ, a final non-null length is reached ($0 < |\mathbf{A}| < 1$).

★ The final value of the global polarization vector has been analytically understood in the extreme cases: the Gaussian spectrum and the two Bloch vectors scenario.

★ The spectrum shape is responsible of the final behavior. In the 2 Bloch vector limit: stationary state oscillating with constant amplitude is maintained. In the Gaussian limit, a stationary state with decreasing amplitude is reached.