Jigsaw 10
TIFR, Mumbai - 24 February 2010

Loop-level differences in the Standard Model ν_μ - ν_τ refraction & possible effects on the collective ν oscillation phenomenology

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Basics of Neutrino Oscillations

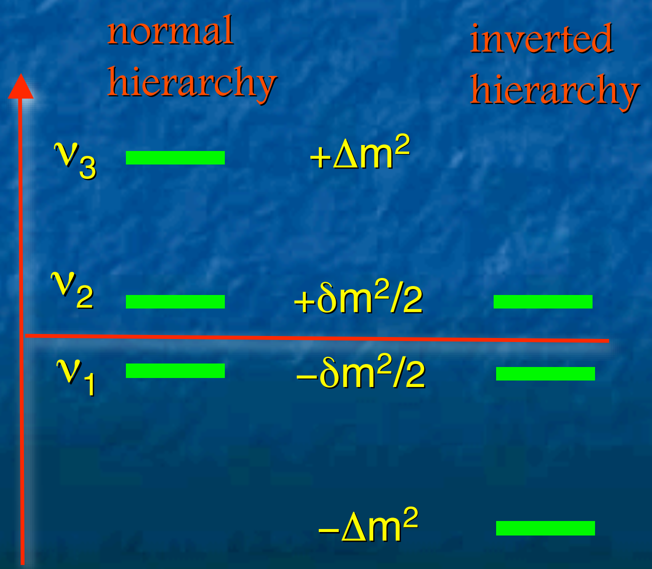
We have now compelling evidence that the Hamiltonian of ν evolution is non-diagonal in flavour space. (Almost?) all data are consistent with a 3ν oscillation framework

$$H = \frac{U\,M^2 U^\dagger}{2\,p} + \mathrm{diag}(V, 0, 0)$$

Vacuum mixing term

Mixing parameters: $U(\theta_{12}, \theta_{23}, \theta_{13}, \delta)$

(as for CKM matrix)



MSW term (matter potential)

Energy shift due to different interactions of different flavours

$$V = \sqrt{2} G_F \left(n_{e^-} - n_{e^+}\right)$$

arises at tree level due to the "extra" charged current interaction for ν_e in medium (– for anti)

[Wolfenstein, PRD 17, 2369 (1978), Mikheev & Smirnov, Sov. J. Nucl. Phys. 42, 913 (1985)]

Beyond the MSW term

The usual MSW potential provides a sufficient description of the matter effect in many practical cases (most notably solar ν's) but care must be taken in some circumstances where either

→ the leading term vanishes ($n_e \simeq 0$, as in the Early Universe)

→ subtle three-flavour effects come into play (possibly in SNae)

In particular, the refraction index difference in the ν_μ–ν_τ sector, which vanishes at leading order in G_F in an ordinary medium not containing μ and/or τ, gets a contribution at higher order.

- Where exactly this result comes from?
- Is it relevant at all?

The BLM result

At 1 loop-level, the ν_μ - ν_τ degeneracy is broken even in an ordinary medium due to charged lepton mass circulating in the loop

Botella, Lim, Marciano PRD 35 (1987) 896

In the low-energy limit, the correction has the form of a four-fermion operator, hence effectively one finds a "MSW-like" term with an effective τ density:

$$\Delta E_{\nu_\tau \nu_\mu} = \sqrt{2}\, G_F\, n_\tau^{\rm eff} \simeq 2.6 \times 10^{-5}\, n_B,$$

$$n_\tau^{\rm eff} = \frac{3}{2} \frac{G_F m_\tau^2}{\sqrt{2}\, \pi^2} \left[\log\left(\frac{m_W^2}{m_\tau^2} \right) - 1 + \frac{Y_n}{3} \right] n_B\,,$$

It is instructive to sketch the steps leading to this result

The 'assumptions' of the calculation

- Treat flavour states (as opposed as mass states) as asymptotic ones, later accounting for mixing. Just "a trick", no real need anywhere for initial or final flavour states.

- Neglect all terms of $O(m_f/m_W)^2$, f=e, μ, u, d, retain only $(m_\tau/m_W)^2$

- Neglect all the momenta of the scattering particles wrt the relevant mass scales probed in the loop (m_τ , m_W).

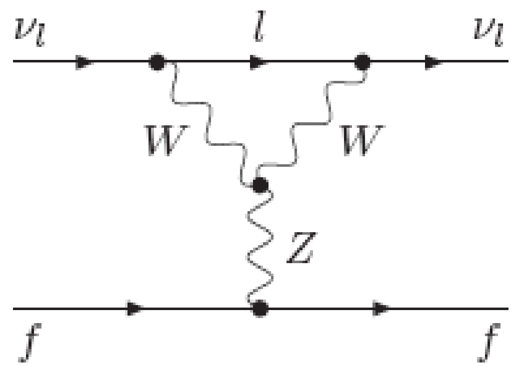
- Assumed neutral & unpolarized background medium of e, n, & p (this also eliminates weak corrections to γff vertex, for example)

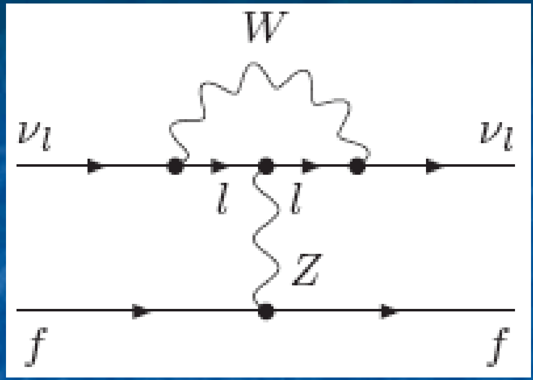
- Useful to classify the diagrams as box vs. non-box

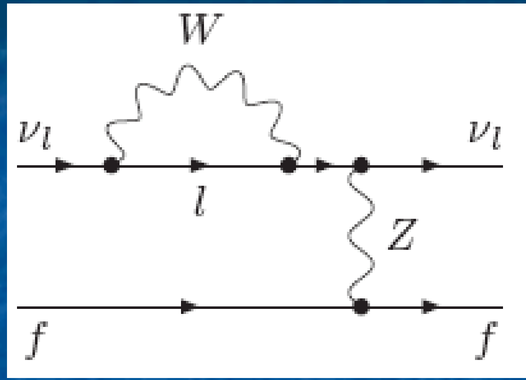
$$\mathcal{H}_{\nu_l f} = \frac{G_{\rm F}}{\sqrt{2}}[\bar{\nu}_l \gamma_\alpha (1-\gamma_5)\nu_l][\bar{f}\gamma^\alpha (c_V^f - c_A^f \gamma_5) f]$$

$$V_{\nu_l f} = \sqrt{2}\, G_{\rm F}\, n_f\, c_V^f \sum_s \int {\rm d}^3\mathbf{k}\, F_f(\mathbf{k}, s)\,(1 - \mathbf{v_p}\cdot\mathbf{v_k}) = \sqrt{2}\, G_{\rm F}\, n_f\, c_V^f$$

Non-box diagrams







ZWW vertex

Zll vertex

leg self-energy

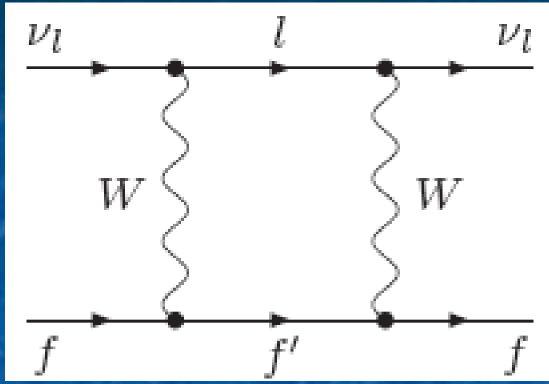
(include would-be-Goldstone bosons ϕ & account for final states, too…)

Amplitudes independent of the nature of the background fermions f
(the tree-level ffZ vertex factorizes)

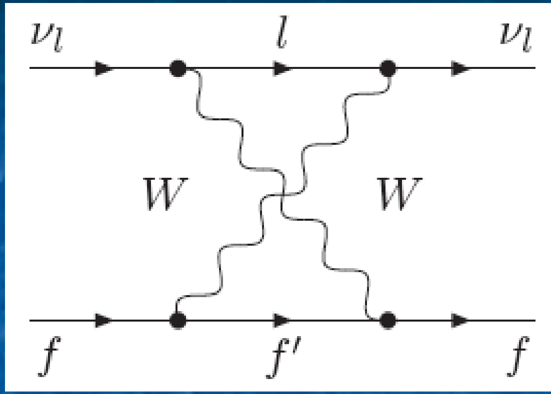
This means that by writing the correction as $c^f_{V/A} \to c^f_{V/A} + \Delta c^{\nu_l f}_{V/A}$

the relative correction $\Delta c^f / c^f$ is universal (f-independent)

Box diagrams



Ladder box
(for f=e,d)



Crossed box
(for f=u)

These diagrams do depend on the nature of the background fermions.
At loop level, it's like having a new kind of "effective neutral current"!

$$\Delta c_{V/A}^{\nu_l d} = -\frac{g^2}{(4\pi)^2}\frac{m_l^2}{M_W^2}\left[\left(3\,c_{V/A}^d+2\right)+\left(c_{V/A}^d+2\right)\log\frac{m_l^2}{M_W^2}\right],$$

$$\Delta c_{V/A}^{\nu_l u} = -\frac{g^2}{(4\pi)^2}\frac{m_l^2}{M_W^2}\left[\left(3c_{V/A}^u-\frac{1}{2}\right)+\left(c_{V/A}^u-\frac{1}{2}\right)\log\frac{m_l^2}{M_W^2}\right]$$

Self-induced loop corrections

Besides "ordinary matter", a SN also contains large densities of ν's.
At loop-level the ν background itself induces a different shift for ν_μ and ν_τ !

$$\mathcal{H}_{\nu\nu} = \sum_{l,l'} \mathcal{H}_{\nu_l \nu_{l'}} = \frac{G_F}{\sqrt{2}} \sum_{l,l'} (1 + \kappa^{\nu_l \nu_{l'}}) \left[\bar{\nu}_l \gamma_\alpha \omega_L \nu_l\right] \left[\bar{\nu}_{l'} \gamma^\alpha \omega_L \nu_{l'}\right]$$

Within same approximations, only coefficients $\kappa_{\tau\beta}$ are non-vanishing.

For $\beta \neq \tau$ the result follows from previous calculations

$$\kappa^{\tau\beta} = -\epsilon \equiv \frac{G_F\, m_\tau^2}{\sqrt{2}\pi^2}$$

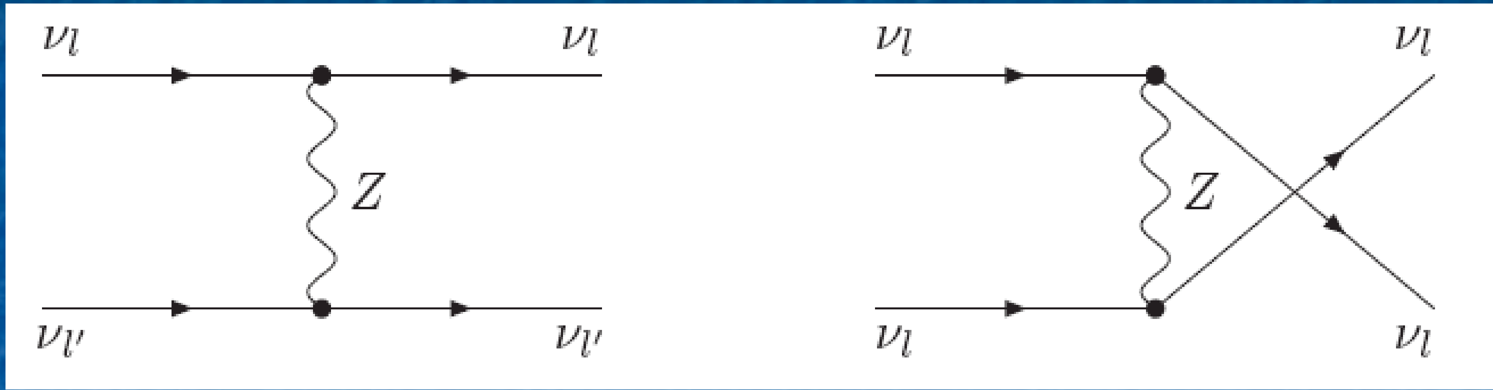
For $\beta = \tau$ several differences arise…

Note: Original U(3) symmetry of the tree-level Hamiltonian is broken!

Mirizzi, Pozzorini, Raffelt, PS JHEP 10 (2009) 020 [arxiv: 0907.3674]

Correction to ν_τ–ν_τ refraction index

➢ Must include also u-channel exchanges (already at tree level) *"trivial"*



➢ One must also correct the "lower" vertex and legs *"trivial"*

➢ The box cannot be deduced by the previous computations, but leads to a different results due to the identity of fermions.

As a result, one gets: $\kappa_{\tau\tau}=-3/2\ \varepsilon$. That, at $O(\varepsilon)$, implies the effective H:

$$\mathcal{H}_{\nu\nu} = \frac{G_F}{\sqrt{2}} \left\{ [\bar{\nu}(1-\epsilon \mathsf{T})\gamma^\lambda \omega_L \nu][\bar{\nu}(1-\epsilon \mathsf{T})\gamma_\lambda \omega_L \nu] + \frac{\epsilon}{2}[\bar{\nu}\mathsf{T}\gamma^\lambda \omega_L \nu][\bar{\nu}\mathsf{T}\gamma_\lambda \omega_L \nu] \right\}$$

Mirizzi, Pozzorini, Raffelt, PS JHEP 10 (2009) 020 [arxiv: 0907.3674]

Self-induced ν_μ–ν_τ 'potential'

Naively, the tree-level potential is modified as follows:

$$V_{\nu_\tau \nu_\beta} = \sqrt{2} G_F \left(1 - \frac{G_F m_\tau^2}{\sqrt{2}\,\pi^2}\right) n_{\nu_\beta} \quad (\beta \neq \tau)$$

$$V_{\nu_\tau \nu_\tau} = \sqrt{2} G_F \left(2 - 3\frac{G_F m_\tau^2}{\sqrt{2}\,\pi^2}\right) n_{\nu_\tau}\,.$$

In reality, entanglement implies that the are "off-diagonal" terms…

$$\mathcal{H}_\mathsf{G} = \frac{G_F}{\sqrt{2}} (\bar{\nu} \mathsf{G} \gamma^\lambda \omega_L \nu)(\bar{\nu} \mathsf{G} \gamma_\lambda \omega_L \nu)$$

G. Sigl, G. Raffelt,

Nucl. Phys. B 406 (1993) 423

$$\mathsf{N}_\nu \to \mathsf{N}_\nu^{\mathrm{eff}}(\mathsf{G}) = \int \frac{d^3\mathbf{q}}{(2\pi)^3} \left\{ \mathsf{G}\,(\varrho_\mathbf{q} - \bar{\varrho}_\mathbf{q})\,\mathsf{G} + \mathsf{G}\,\mathrm{Tr}[(\varrho_\mathbf{q} - \bar{\varrho}_\mathbf{q})\mathsf{G}] \right\}$$

Self-induced ν_μ–ν_τ 'potential' (II)

So, the prescription to account for this effect in the EOM for ρ is

$$\mathcal{H}_{\nu\nu} = \frac{G_F}{\sqrt{2}} \left\{ [\bar{\nu}(1-\epsilon\mathsf{T})\gamma^\lambda \omega_L \nu][\bar{\nu}(1-\epsilon\mathsf{T})\gamma_\lambda \omega_L \nu] + \frac{\epsilon}{2}[\bar{\nu}\mathsf{T}\gamma^\lambda \omega_L \nu][\bar{\nu}\mathsf{T}\gamma_\lambda \omega_L \nu] \right\}$$

$$\mathsf{N}_\nu \to \mathsf{N}_\nu - \epsilon \begin{pmatrix} 0 & 0 & \mathsf{N}_\nu^{e\tau} \\ 0 & 0 & \mathsf{N}_\nu^{\mu\tau} \\ \mathsf{N}_\nu^{\tau e} & \mathsf{N}_\nu^{\tau\mu} & \mathsf{N}_\nu^{ee} + \mathsf{N}_\nu^{\mu\mu} + 2\mathsf{N}_\nu^{\tau\tau} \end{pmatrix}$$

Quantitatively, 3 flavour and multi-mode simulations are needed to assess the relevance of this term for realistic situations. Yet…

Application I: Collective effects & θ_{13}

Collective "oscillations" originate from an instability in flavor space of interacting dense ν systems: Energy can be minimized via flavor swaps, provided that the flavour dynamics is non-trivial

While the mixing angle θ_{13} can trigger this instability, by no way this is the only mechanism to produce the phenomenon!

It is important to realize that, no matter how small θ_{13} is:

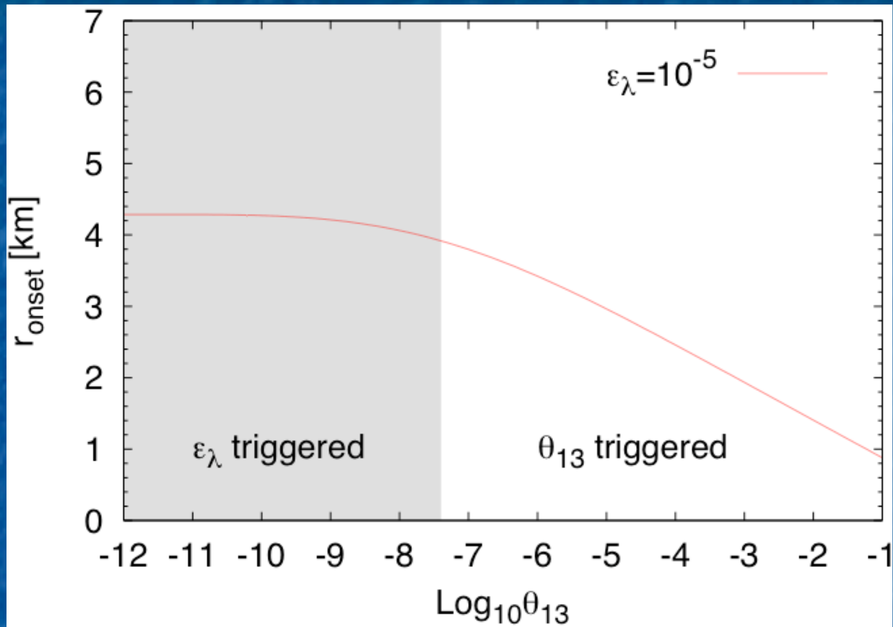
- the instability will be generically triggered.
- Any phenomenological observation linked to collective effects does not tell us anything about the magnitude of θ_{13}

Qualitatively different from "MSW" observables as Earth Matter Effect!

Dasgupta, Raffelt, Tamborra, arxiv: 1001.5396

What triggers the instability?

In the limit of $\Phi_{\nu\mu} = \Phi_{\nu\tau}$ & for $\theta_{13}=0$, loop effects still trigger the instability!



Dasgupta, Raffelt, Tamborra, arxiv: 1001.5396

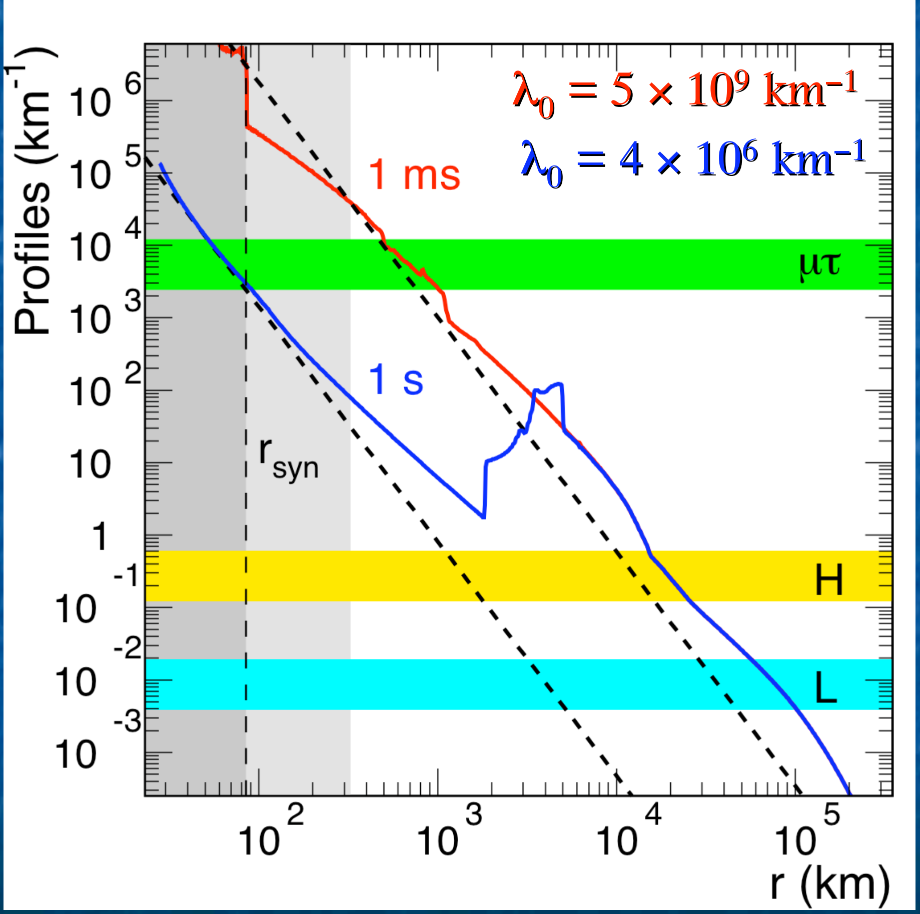
(Alternatively, the effect can be induced by small thermal μ population, slightly different opacities, stochastic fluctuations in the thermal production of ν_μ & ν_τ at the ν–sphere…)

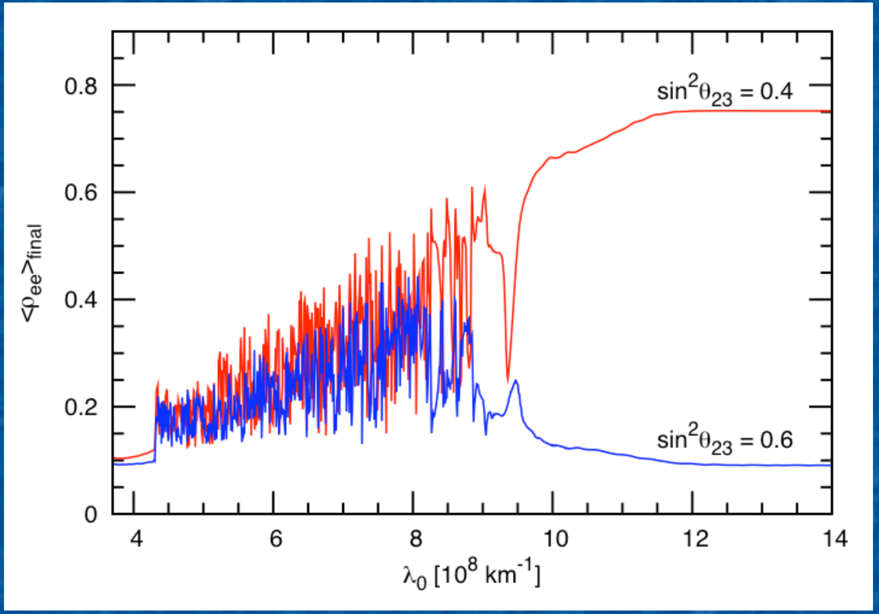
It is also possible that new dynamics as NSI, not necessarily in the ν–charged fermion sector, but even in the ν–ν sector (see *Blennow, Mirizzi, PS PRD 78 (2008) 113 004 [arxiv: 0810.2297]*), has similar effects

For a more general overview of NSI effects, see following talk by R. Tomas

Application II: $\nu_\tau - \nu_\mu$ resonance?

An exploratory investigation suggested that the resonance at density $\rho = 3 \times 10^7$ g cm^{-3} would imply for example sensitivity to the octant θ_{23} : these effects might be relevant in the early phase (accretion) of an iron-core SN.

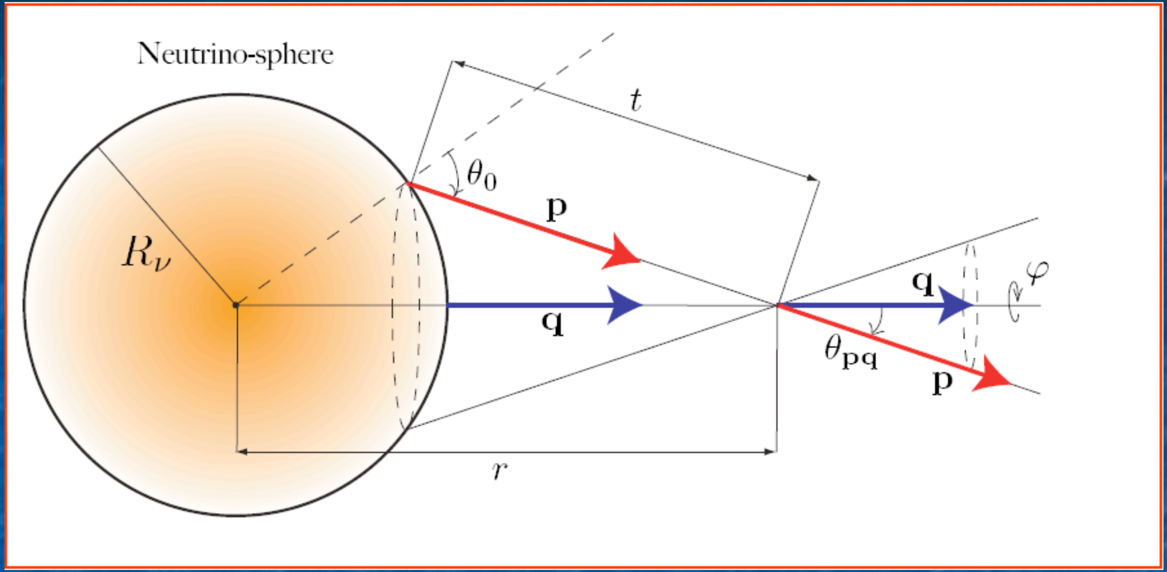




Esteban-Pretel et al, PRD 72 (2008) 065024 [arxiv: 0712.1137]

However, sometimes nature likes playing "hide and seek"…

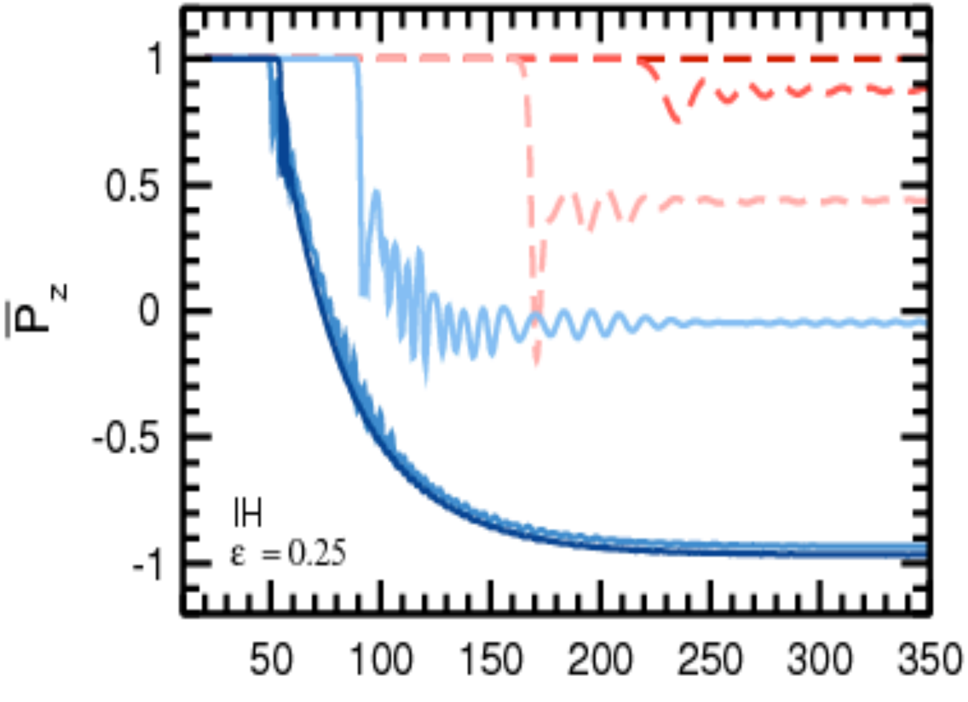
Matter reloaded



- ν's streaming off a spherical source acquire (slightly) different phases at a given radius r, having travelled on different trajectories.
- Matter effects are no longer the same for all modes as in truly isotropic case, & cannot be "rotated away" by a frame transformation in flavour space.

Esteban-Pretel et al, PRD 78 (2008) 085012 [arxiv: 0807.0659]

Consequences



$n_e \gg n_\nu$ No flavour conversion

$n_e \gtrsim n_\nu$ Multiangle decoherence

$n_e \ll n_\nu$ Collective oscillations

- Matter-induced multi-angle decoherence if density is slightly larger than the neutrino one. Likely to happen in early phase (t<300 ms) of CC SN event.
- Inhibits the previously disccused matter-induced 3-flavour effects

Esteban-Pretel et al, PRD 78 (2008) 085012 [arxiv: 0807.0659]

Yet, the jury is still out…

Non-universal loop correction to $\nu_\mu - \nu_\tau$ refraction index are not controlled by the ordinary matter density, so the above mentioned "matter shielding" mechanism is not operational.

It remains to be seen if, in realistic conditions, there is some room for surprises…

Work in Progress…

Summary

- Recap of the steps involved in the computation of flavour non-universal correction to ν refraction indexes in ordinary matter.

- I presented the new results for the similar term in a purely ν background.

- Despite the small magnitude of these corrections, the important role of instabilities in CC SN ν dynamics suggests that it is not necessarily true that they can simply be dismissed

- For example, they could be responsible for triggering the instability (if θ_{13} is very small). Other effects do not appear likely… but might be possible.

- Also, these terms provide the "SM background" to similar effects of larger magnitude that might be induced by new physics in the flavour sector (e.g. Gava & Jean-Louis '09… see also following talk by R. Tomas)

Stay tuned !