
Multiple Splits in Supernova Neutrino Spectra And r-Process Nucleosynthesis

Sovan Chakraborty

Saha Institute of Nuclear Physics, Kolkata

arxiv:0911.1218 [hep-ph]

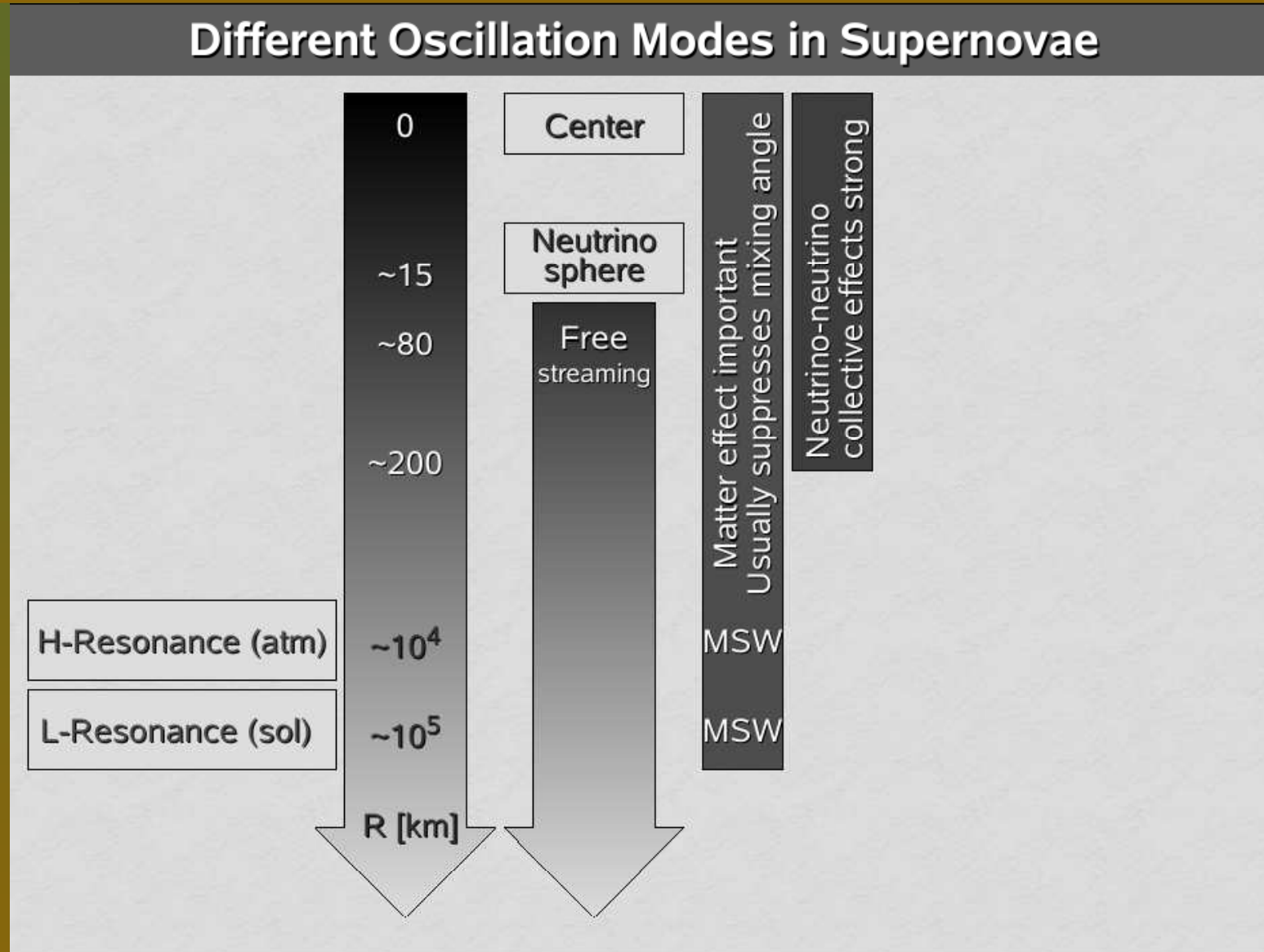
(S.C, S.Choubey, S.Goswami, K.Kar)

Plan of the Talk:

- Collective Effect and Luminosity Variation
- r-Process Nucleosynthesis and Collective Effect
- Remarks .

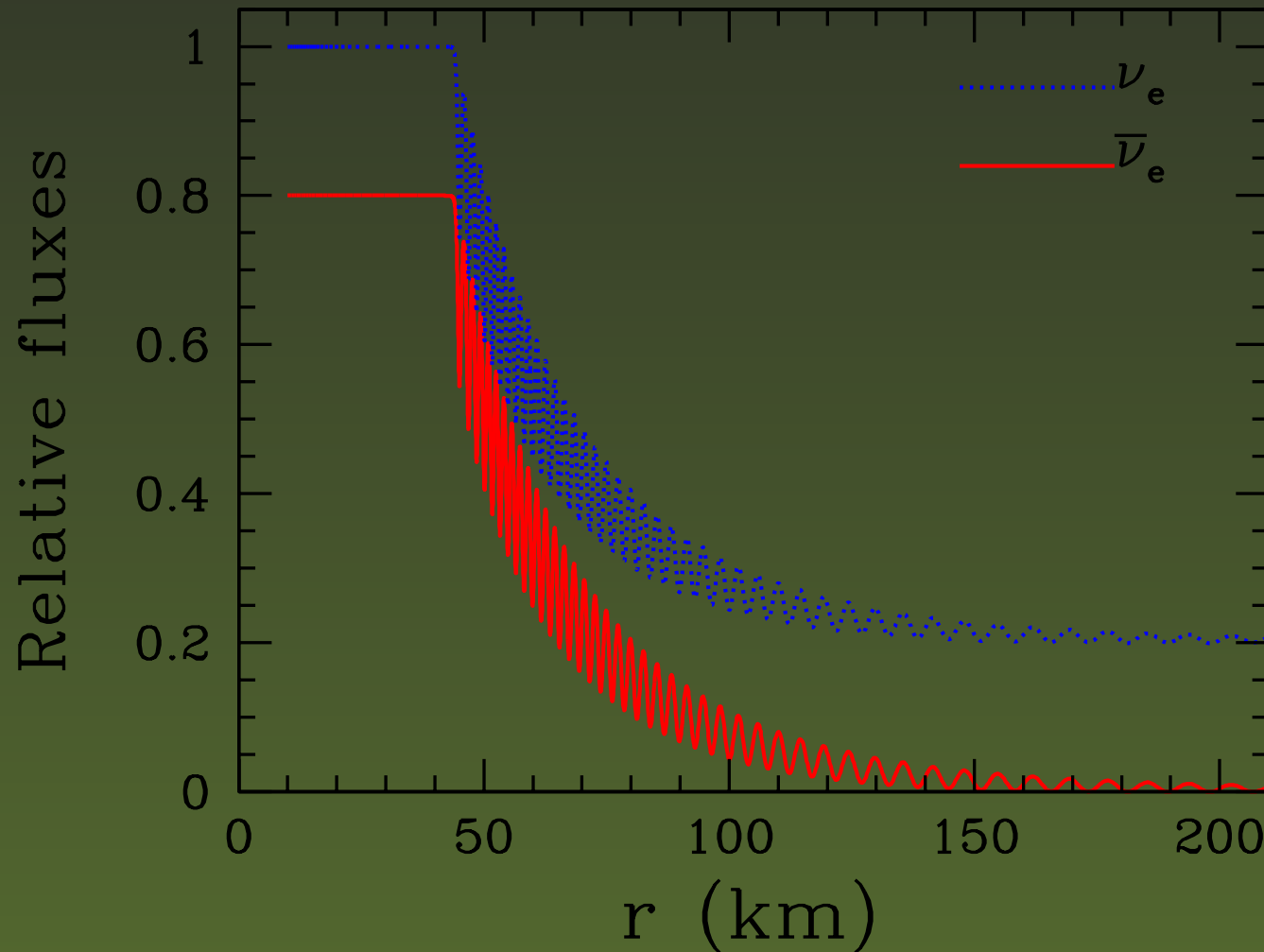
- Collective Effect and Luminosity Variation

SN and Neutrino (Raffelt's slide JIGSAW'07)



Collective effect

Hannestad et al 2006



Evolution Equation :

Ensemble of relativistic neutrinos and Antineutrinos.

Evolution equations are

$$\partial_t \rho = -i[H, \rho] \quad , \quad \partial_t \bar{\rho} = -i[H, \bar{\rho}]$$

Hamiltonian of a neutrino/antineutrino in the ensemble.

$$H = H_{vacuum} + H_{MSW} + H_{\nu\nu}$$

Bloch vector and E.O.M:

For a 2×2 Hermetian Matrix M

$$M = \frac{1}{2}(1 + \mathbf{m} \cdot \bar{\sigma})$$

Where σ Pauli spin Matrices and $\mathbf{m} (m_x, m_y, m_z)$ is called the Bloch Vector.

Bloch vectors corresponding to ρ , $\bar{\rho}$, H_{vacuum} , H_{MSW} , $H_{\nu\nu}$ are \mathbf{P} , \mathbf{P}' , \mathbf{B} , \mathbf{L} , \mathbf{D} respectively.

E.O.M:

E.O.M for neutrino and antineutrino polarization vector \mathbf{P} and \mathbf{P}'

$$\dot{\mathbf{P}} = (\omega\mathbf{B} + \lambda\hat{\mathbf{L}} + \mu\mathbf{D}) \times \mathbf{P}$$

$$\dot{\mathbf{P}}' = (-\omega\mathbf{B} + \lambda\hat{\mathbf{L}} + \mu\mathbf{D}) \times \mathbf{P}'$$

E.O.M:

E.O.M for neutrino and antineutrino polarization vector \mathbf{P} and \mathbf{P}'

$$\dot{\mathbf{P}} = (\omega\mathbf{B} + \lambda\hat{\mathbf{L}} + \mu\mathbf{D}) \times \mathbf{P}$$

$$\dot{\mathbf{P}}' = (-\omega\mathbf{B} + \lambda\hat{\mathbf{L}} + \mu\mathbf{D}) \times \mathbf{P}'$$

$$\mathbf{B} = (\sin 2\theta, 0, -\cos 2\theta)^T \quad ; \quad \omega = \frac{\Delta^2}{2E}$$

E.O.M:

E.O.M for neutrino and antineutrino polarization vector \mathbf{P} and \mathbf{P}'

$$\dot{\mathbf{P}} = (\omega\mathbf{B} + \lambda\hat{\mathbf{L}} + \mu\mathbf{D}) \times \mathbf{P}$$

$$\dot{\mathbf{P}}' = (-\omega\mathbf{B} + \lambda\hat{\mathbf{L}} + \mu\mathbf{D}) \times \mathbf{P}'$$

$$\hat{\mathbf{L}} = (0, 0, 1)^T \quad ; \quad \lambda = \sqrt{2}G_F N_e$$

E.O.M:

E.O.M for neutrino and antineutrino polarization vector \mathbf{P} and \mathbf{P}'

$$\dot{\mathbf{P}} = (\omega\mathbf{B} + \lambda\hat{\mathbf{L}} + \mu\mathbf{D}) \times \mathbf{P}$$

$$\dot{\mathbf{P}}' = (-\omega\mathbf{B} + \lambda\hat{\mathbf{L}} + \mu\mathbf{D}) \times \mathbf{P}'$$

$$\mathbf{D} = \frac{1}{(N_{\nu_e} + N_{\nu_x} + N_{\bar{\nu}_e} + N_{\bar{\nu}_x})} \int dE (n\mathbf{P} - \bar{n}\mathbf{P}')$$

$$\mu = \sqrt{2}G_F(N_{\nu_e} + N_{\nu_x} + N_{\bar{\nu}_e} + N_{\bar{\nu}_x})$$

E.O.M:

$$n = n_{\nu_e} + n_{\nu_x} \quad ; \quad \bar{n} = n_{\bar{\nu}_e} + n_{\nu_x}$$

where n_α 's are the effective number per unit volume per unit energy and given by

$$n_\alpha(r, E) = \frac{D(r)}{2\pi R_\alpha^2} \frac{L_\alpha}{\langle E_\alpha \rangle} \Psi(E)_\alpha$$

N_α 's represent the total effective number density of the α th species.

$$N_\alpha = \int dE n_\alpha$$

Initial Energy Distribution :

Effective number density for the α th species per unit energy

$$n_{\alpha}(r, E) = \frac{D(r)}{2\pi R_{\alpha}^2} \frac{L_{\alpha}}{\langle E_{\alpha} \rangle} \psi(E)_{\alpha}$$

Fermi-Dirac

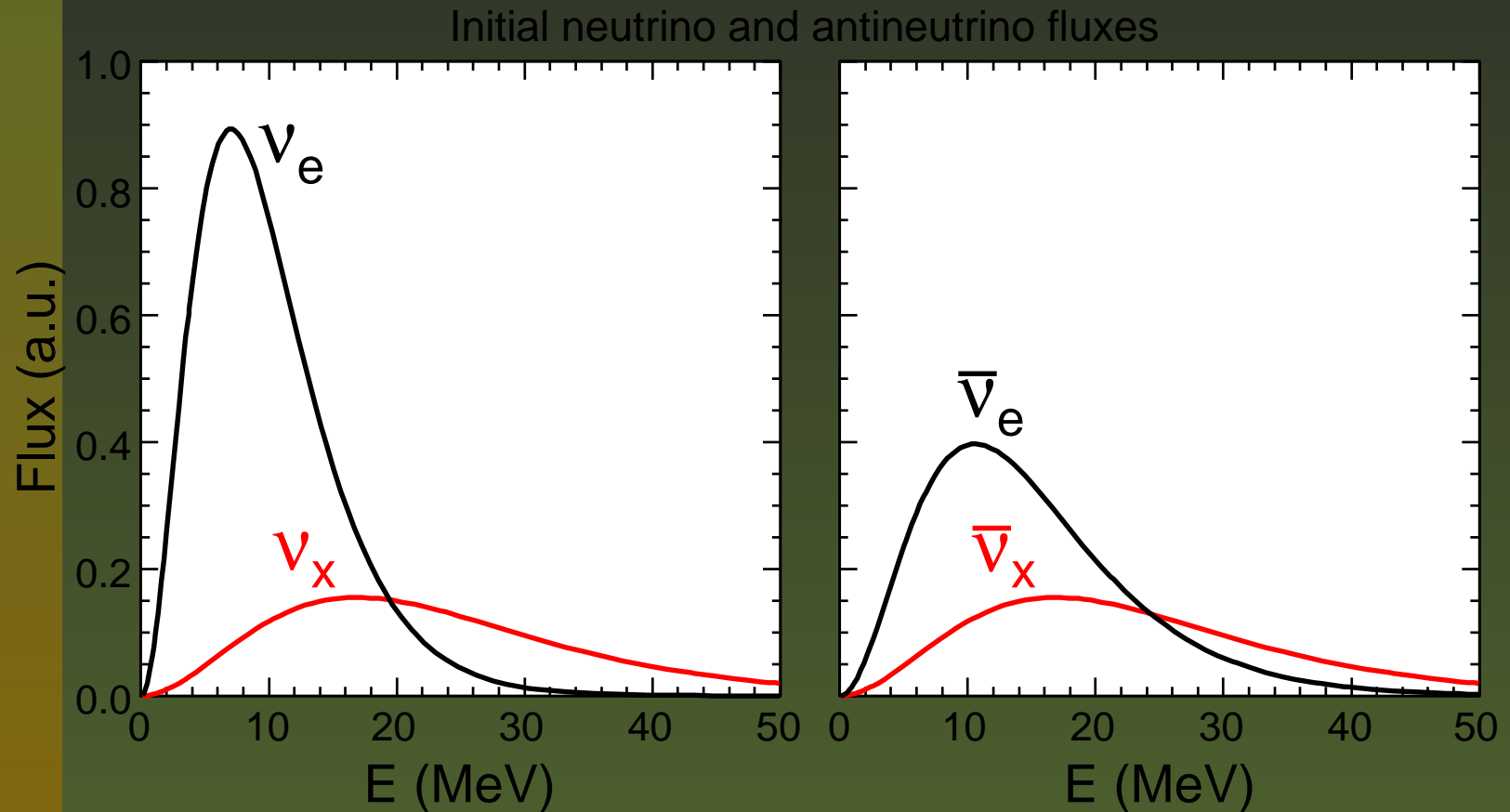
$$\Psi_{\alpha}^{FD}(E) \propto \frac{\beta_{\alpha} (\beta_{\alpha} E)^2}{e^{\beta_{\alpha} E} + 1}$$

where the inverse temperature parameters are

$$\beta_{\nu_e} = 0.315 \text{ MeV}^{-1}; \quad \beta_{\bar{\nu}_e} = 0.210 \text{ MeV}^{-1}$$

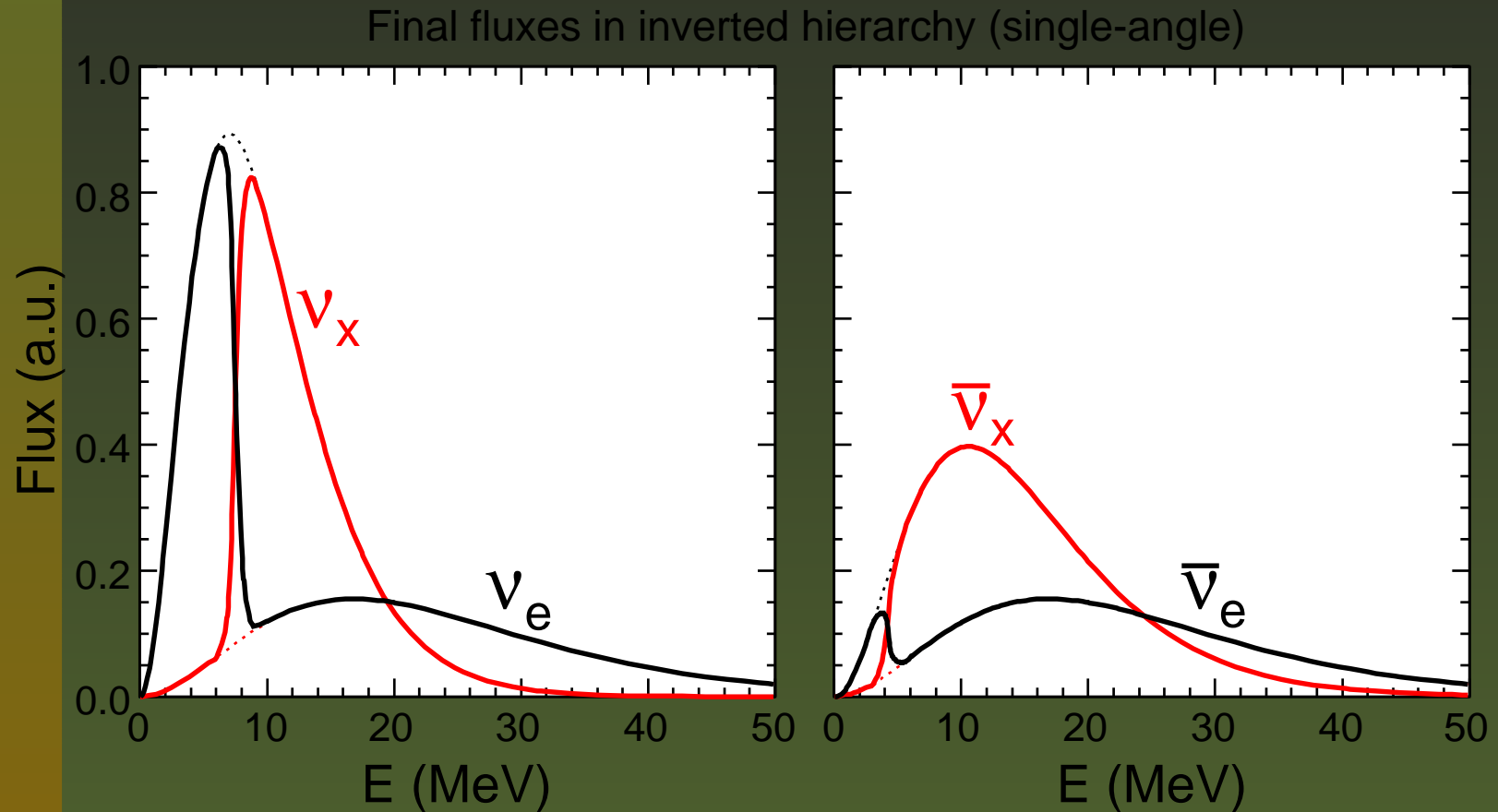
$$\beta_{\nu_x} = \beta_{\bar{\nu}_x} = 0.131 \text{ MeV}^{-1}.$$

Initial Flux (FD)



Fogli et al. JCAP(2007)

Flux Collective Effect



Fogli et al. JCAP(2007)

Pinched Energy Distribution :

Effective number density for the α th species per unit energy

$$n_{\alpha}(r, E) = \frac{D(r)}{2\pi R_{\alpha}^2} \frac{L_{\alpha}}{\langle E_{\alpha} \rangle} \psi(E)_{\alpha}$$

Pinched spectra for different simulations are parameterized as

$$\Psi_{\alpha}(E) = \frac{(1 + \zeta_{\alpha})^{1+\zeta_{\alpha}}}{\Gamma(1 + \zeta_{\alpha})} \left(\frac{E_{\alpha}}{\langle E_{\alpha} \rangle} \right)^{\zeta_{\alpha}} \frac{\exp \left(- (1 + \zeta_{\alpha}) \frac{E_{\alpha}}{\langle E_{\alpha} \rangle} \right)}{\langle E_{\alpha} \rangle},$$

Keil et al. ApJ(2003)

where ζ_{α} is the pinching parameter

Different simulation models have different ζ_{α} and $\langle E_{\alpha} \rangle$

Initial Neutrino Flux:

The initial flux ($\phi_{\nu_\alpha}^0 = \frac{L_\alpha}{\langle E_\alpha \rangle}$) is another important input parameter.

Total emitted SN energy ($E_B = 3 \times 10^{53}$ erg) puts a constraint on flavor luminosities.

$$L_{\nu_e} + L_{\bar{\nu}_e} + 4L_{\nu_x} = \frac{E_B}{\tau}$$

where τ is the luminosity decay timescale. We took $\tau = 10$ seconds.

Thus the initial fluxes of different flavors are also constrained

$$\phi_{\nu_e}^0 \langle E_{\nu_e} \rangle + \phi_{\bar{\nu}_e}^0 \langle E_{\bar{\nu}_e} \rangle + 4\phi_{\nu_x}^0 \langle E_{\nu_x} \rangle = 3 \times 10^{52}$$

Initial Neutrino Fluxes

If the ratio between the initial fluxes of different flavors are

$$\phi_{\nu_e}^0 : \phi_{\bar{\nu}_e}^0 : \phi_{\nu_x}^0 = \phi_{\nu_e}^r : \phi_{\bar{\nu}_e}^r : 1 ,$$

where $\phi_{\nu_e}^r, \phi_{\bar{\nu}_e}^r$ are positive numbers.

$$\phi_{\nu_x}^0 (\phi_{\nu_e}^r \langle E_{\nu_e} \rangle + \phi_{\bar{\nu}_e}^r \langle E_{\bar{\nu}_e} \rangle + 4 \langle E_{\nu_x} \rangle) = 3 \times 10^{52} .$$

The parameters are initial relative fluxes

$$\phi_{\nu_e}^r = \frac{\phi_{\nu_e}^0}{\phi_{\nu_x}^0} , \quad \phi_{\bar{\nu}_e}^r = \frac{\phi_{\bar{\nu}_e}^0}{\phi_{\nu_x}^0}$$

SN Neutrino Parameters

Three representative models motivated by SN simulations.
2 Garching simulations (G1, G2) and 1 Lawrence Livermore (LL).

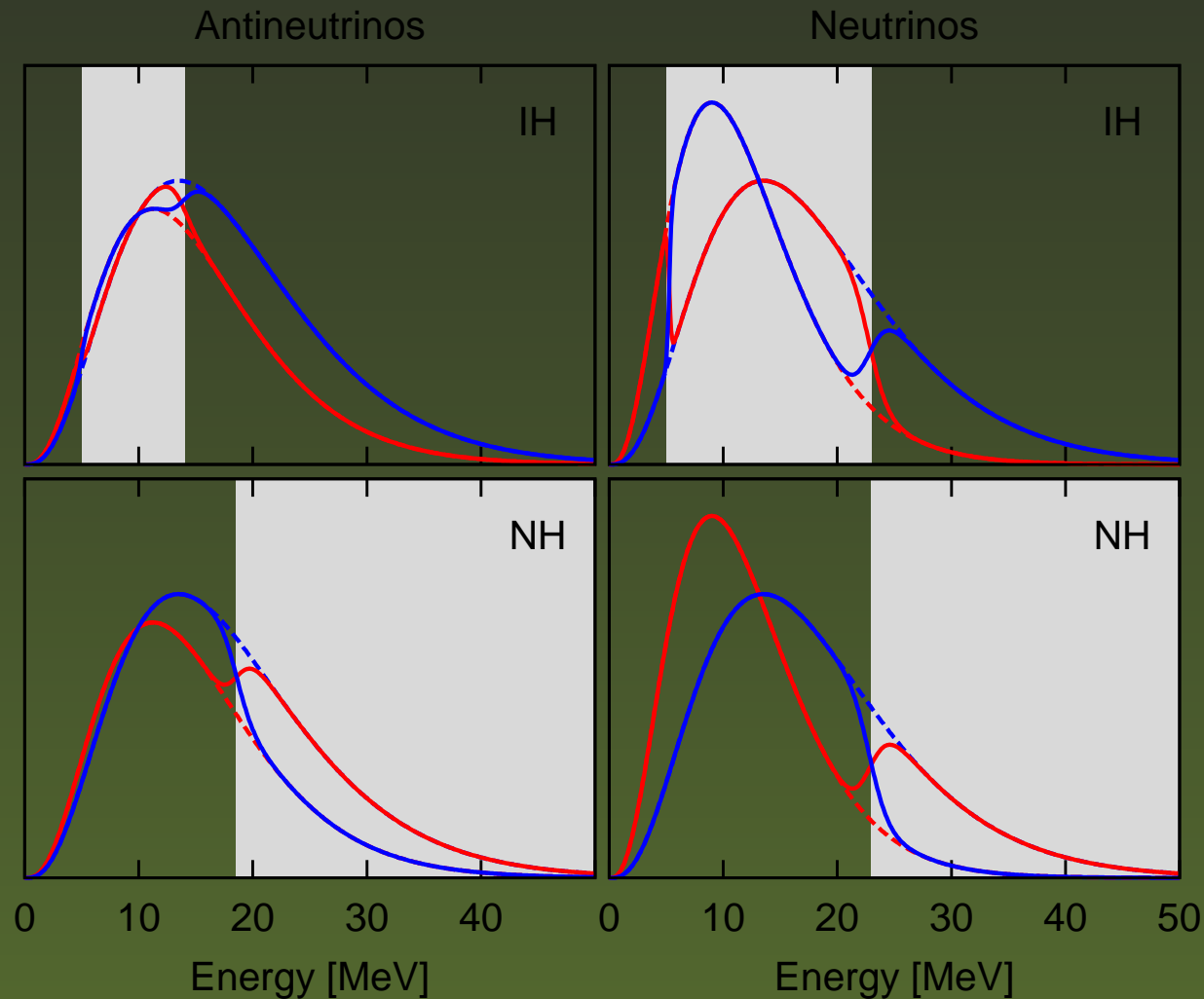
Another ‘plausible’ choice, say ‘G3’. Dasgupta et al. PRL (2009)

Fitting Parameters

Model	$\langle E_{\nu_e} \rangle$ (MeV)	$\langle E_{\bar{\nu}_e} \rangle$ (MeV)	$\langle E_{\nu_x} \rangle$ (MeV)	ζ_{ν_e} $= \zeta_{\bar{\nu}_e}$	$\zeta_{\bar{\nu}_x}$	$\phi_{\nu_e}^r = \frac{\phi_{\nu_e}^0}{\phi_{\nu_x}^0}$	$\phi_{\bar{\nu}_e}^r = \frac{\phi_{\bar{\nu}_e}^0}{\phi_{\nu_x}^0}$
LL	12	15	24	3	4	2.00	1.60
G1	12	15	18	3	4	0.80	0.80
G2	12	15	15	3	4	0.50	0.50
G3	12	15	18	3	3	0.85	0.75

Multiple Split Spectra 'G3'

Dasgupta et al. PRL (2009)



Luminosity Variation

$$\frac{1}{2} \leq \frac{L_{\nu_e}}{L_{\nu_x}} \leq 2 ; \quad \frac{1}{2} \leq \frac{L_{\bar{\nu}_e}}{L_{\nu_x}} \leq 2$$

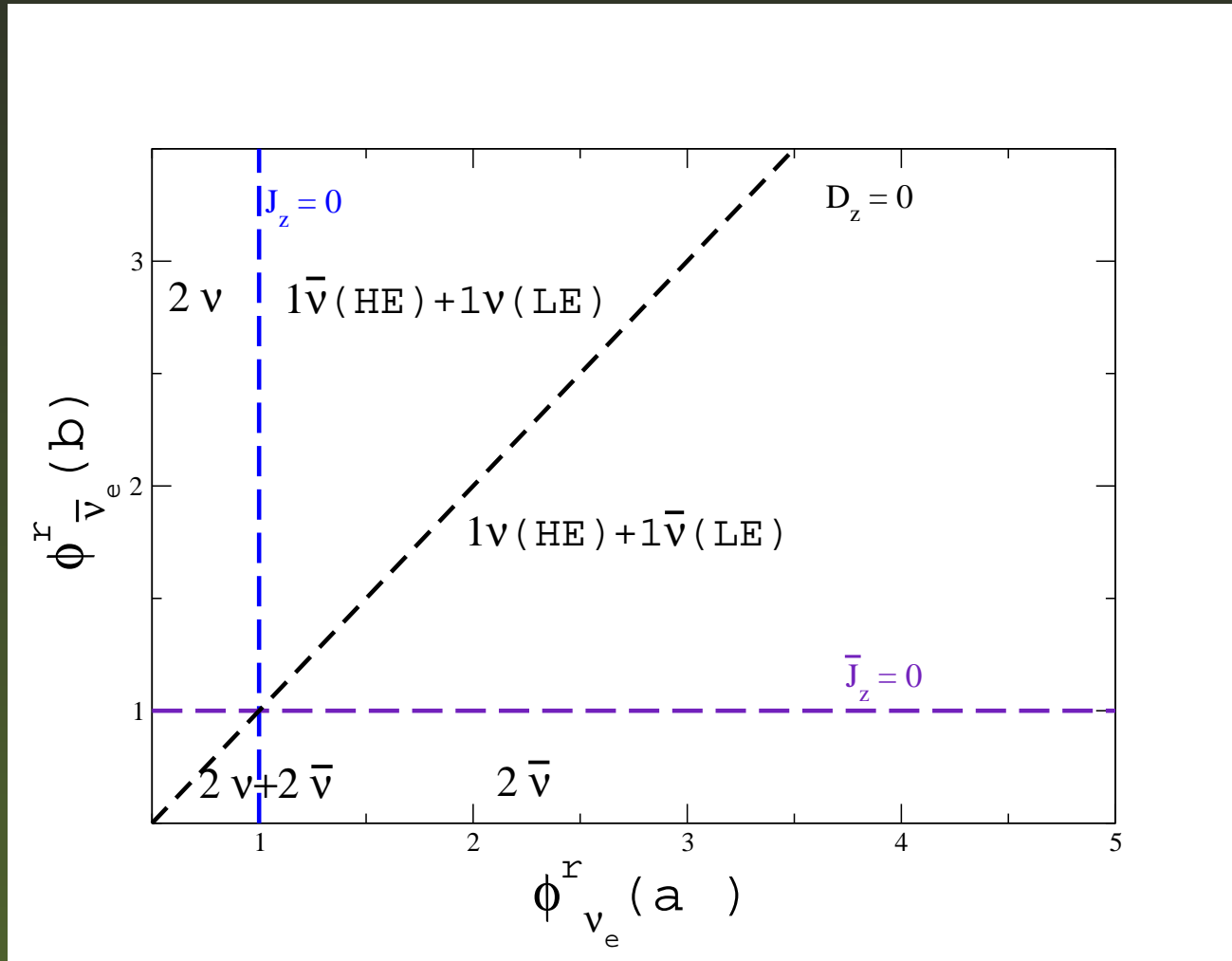
$$\frac{1}{2} \frac{\langle E_{\nu_x} \rangle}{\langle E_{\nu_e} \rangle} \leq \phi_{\nu_e}^r \leq 2 \frac{\langle E_{\nu_x} \rangle}{\langle E_{\nu_e} \rangle} ; \quad \frac{1}{2} \frac{\langle E_{\nu_x} \rangle}{\langle E_{\bar{\nu}_e} \rangle} \leq \phi_{\bar{\nu}_e}^r \leq 2 \frac{\langle E_{\nu_x} \rangle}{\langle E_{\bar{\nu}_e} \rangle} .$$

Model	$\langle E_{\nu_e} \rangle$	$\langle E_{\bar{\nu}_e} \rangle$	$\langle E_{\nu_x, \bar{\nu}_x} \rangle$	$\phi_{\nu_e; ll}^r$	$\phi_{\nu_e; ul}^r$	$\phi_{\bar{\nu}_e; ll}^r$	$\phi_{\bar{\nu}_e; ul}^r$
LL	10	15	24	1.20	4.80	0.80	3.2
G1/G3	12	15	18	0.75	3.00	0.60	2.4

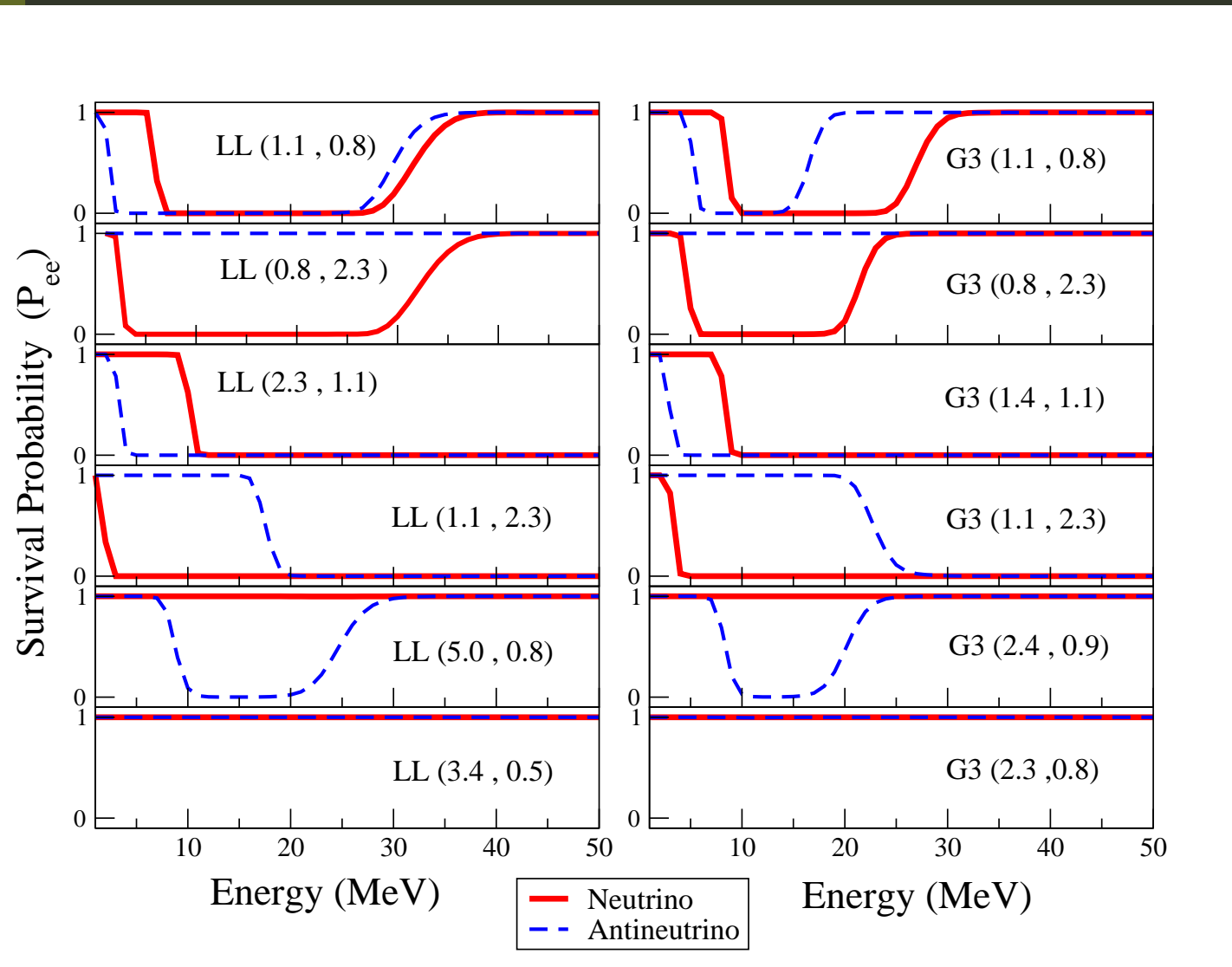
We Study in the range

$$0.5 \leq \phi_{\nu_e}^r \leq 5.0 ; \quad 0.5 \leq \phi_{\bar{\nu}_e}^r \leq 3.5 .$$

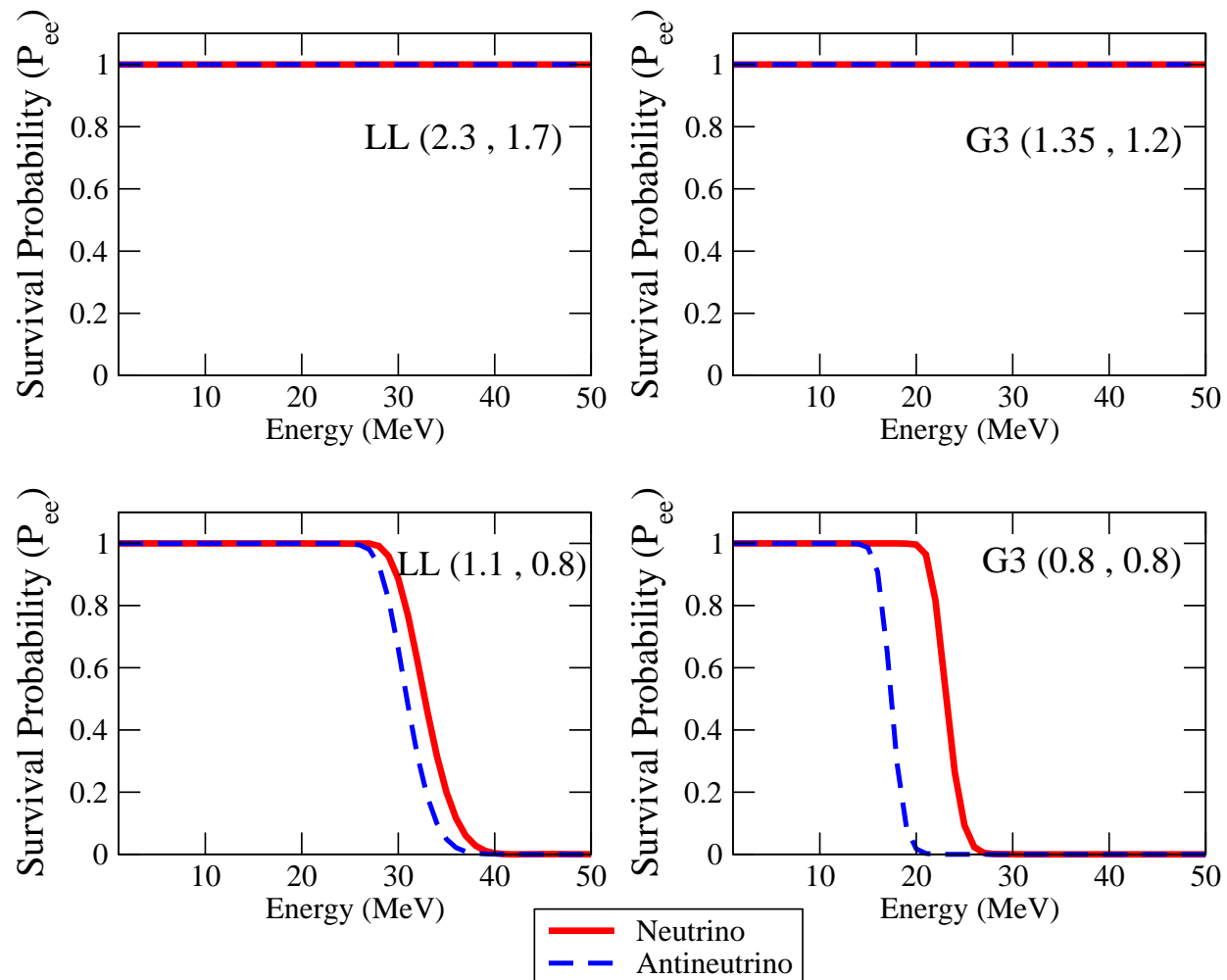
Spectral Split Regions



Spectral Split Patterns (IH)



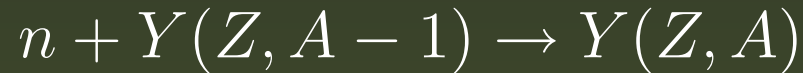
Spectral Split Patterns (NH)



- r-Process and Collective Effect :

r-Process Nucleosynthesis

- Heavy Neutron (n) rich nuclei (beyond Iron group) are synthesized by rapid neutron capture,



- r-Process \Rightarrow Rapid process of neutron capture $t(n, \gamma) \ll$ beta decay life time (t_β).
- r-process requires high neutron(n) number density ($> 10^{20} \text{ cm}^{-3}$)
- Possible site \Rightarrow “ ν driven wind” ahead of “hot bubble” in SN.

r-Process Nucleosynthesis

- Heating by neutrino driven wind coming from neutrino-sphere



- Important quantity whose evolution should be studied is

$$\text{Electron fraction } (Y_e) = \frac{\text{No of electrons}}{\text{No of Baryons}}$$

- For Neutron rich conditions $Y_e < 0.5$ (Preferably < 0.45).
- This is minimal requirement, other constraint on Entropy, Temperature.

r-Process Nucleosynthesis

The expression for Y_e involves reaction rates

$$Y_e \simeq \frac{\lambda_{\nu en}}{\lambda_{\nu en} + \lambda_{\bar{\nu} ep}} = \frac{1}{1 + \frac{\lambda_{\bar{\nu} ep}}{\lambda_{\nu en}}}$$

Qian et al PRL(1993)

where,

$$\lambda_{\nu N} \simeq \frac{L_\nu}{4\pi R_\nu^2} \frac{\int dE \sigma_{\nu N} f_\nu(E)}{\int dE E f_\nu(E)}$$

N can be either p or n.

r-Process Nucleosynthesis

Electronfraction

$$Y_e \simeq \frac{1}{1 + \frac{\lambda_{\bar{\nu}_e p}}{\lambda_{\nu_e n}}}$$

In terms of the initial relative flux $\phi_{\nu_e}^r$ and $\phi_{\bar{\nu}_e}^r$

$$\frac{\lambda_{\bar{\nu}_e p}}{\lambda_{\nu_e n}}(r) = \frac{\int_0^\infty \sigma_{\bar{\nu}_e p}(E) (P_{\bar{\nu}_e}^c(r, E) \phi_{\bar{\nu}_e}^r \Psi_{\bar{\nu}_e}(E) + (1 - P_{\bar{\nu}_e}^c(r, E)) \Psi_{\nu_x}(E)) dE}{\int_0^\infty \sigma_{\nu_e n}(E) (P_{\nu_e}^c(r, E) \phi_{\nu_e}^r \Psi_{\nu_e}(E) + (1 - P_{\nu_e}^c(r, E)) \Psi_{\nu_x}(E)) dE}$$

r-Process Nucleosynthesis

$$\frac{\lambda_{\bar{\nu}_{ep}}}{\lambda_{\nu_{en}}}(r) = \frac{\int_0^\infty \sigma_{\bar{\nu}_{ep}}(E) (P_{\bar{\nu}_e}^c(r, E) \phi_{\bar{\nu}_e}^r \Psi_{\bar{\nu}_e}(E) + (1 - P_{\bar{\nu}_e}^c(r, E)) \Psi_{\nu_x}(E)) dE}{\int_0^\infty \sigma_{\nu_{en}}(E) (P_{\nu_e}^c(r, E) \phi_{\nu_e}^r \Psi_{\nu_e}(E) + (1 - P_{\nu_e}^c(r, E)) \Psi_{\nu_x}(E)) dE}$$

The cross section used are

$$\sigma_{\nu_{en}}(E_{\nu_e}) \approx 9.6 \times 10^{-44} \left(\frac{E_{\nu_e} + \Delta_{np}}{\text{MeV}} \right)^2 \text{ cm}^2 ,$$

$$\sigma_{\bar{\nu}_{ep}}(E_{\bar{\nu}_e}) \approx 9.6 \times 10^{-44} \left(\frac{E_{\bar{\nu}_e} - \Delta_{np}}{\text{MeV}} \right)^2 \text{ cm}^2 ,$$

Δ_{np} (1.293 MeV) is mass difference between neutron and proton.

r-Process Nucleosynthesis

$$\frac{\lambda_{\bar{\nu}_{ep}}}{\lambda_{\nu_{en}}}(r) = \frac{\int_0^\infty \sigma_{\bar{\nu}_{ep}}(E) (P_{\bar{\nu}_e}^c(r, E) \phi_{\bar{\nu}_e}^r \Psi_{\bar{\nu}_e}(E) + (1 - P_{\bar{\nu}_e}^c(r, E)) \Psi_{\nu_x}(E)) dE}{\int_0^\infty \sigma_{\nu_{en}}(E) (P_{\nu_e}^c(r, E) \phi_{\nu_e}^r \Psi_{\nu_e}(E) + (1 - P_{\nu_e}^c(r, E)) \Psi_{\nu_x}(E)) dE}$$

$$P_{\bar{\nu}_e/\nu_e}^c(r_s, E) = 0$$

$$\frac{\lambda_{\bar{\nu}_{ep}}}{\lambda_{\nu_{en}}}(r) = \frac{\int_0^\infty \sigma_{\bar{\nu}_{ep}}(E) \Psi_{\nu_x}(E) dE}{\int_0^\infty \sigma_{\nu_{en}}(E) \Psi_{\nu_x}(E) dE},$$

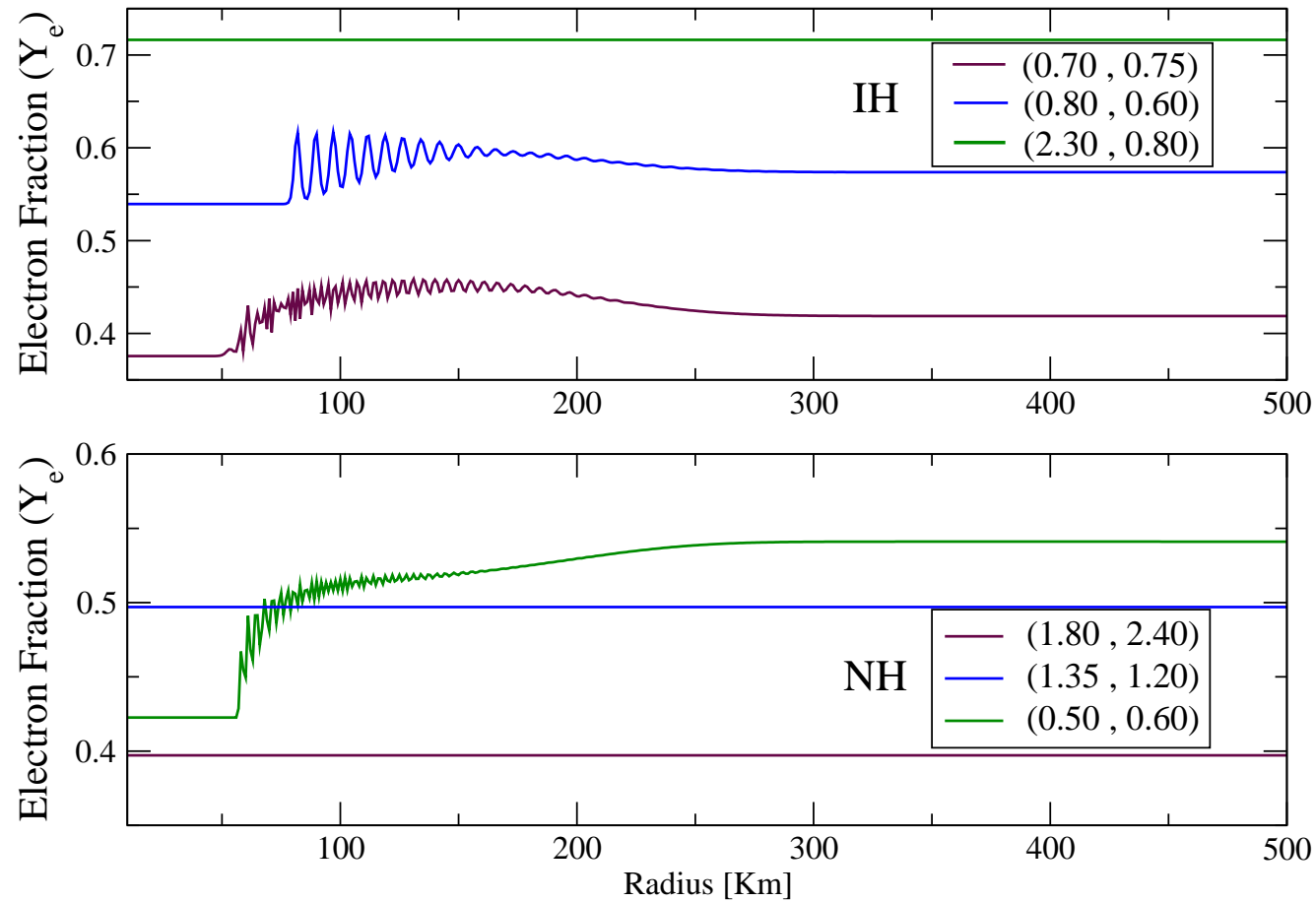
r-Process Nucleosynthesis

$$\frac{\lambda_{\bar{\nu}_{ep}}}{\lambda_{\nu_{en}}}(r) = \frac{\int_0^\infty \sigma_{\bar{\nu}_{ep}}(E) (P_{\bar{\nu}_e}^c(r, E) \phi_{\bar{\nu}_e}^r \Psi_{\bar{\nu}_e}(E) + (1 - P_{\bar{\nu}_e}^c(r, E)) \Psi_{\nu_x}(E)) dE}{\int_0^\infty \sigma_{\nu_{en}}(E) (P_{\nu_e}^c(r, E) \phi_{\nu_e}^r \Psi_{\nu_e}(E) + (1 - P_{\nu_e}^c(r, E)) \Psi_{\nu_x}(E)) dE}$$

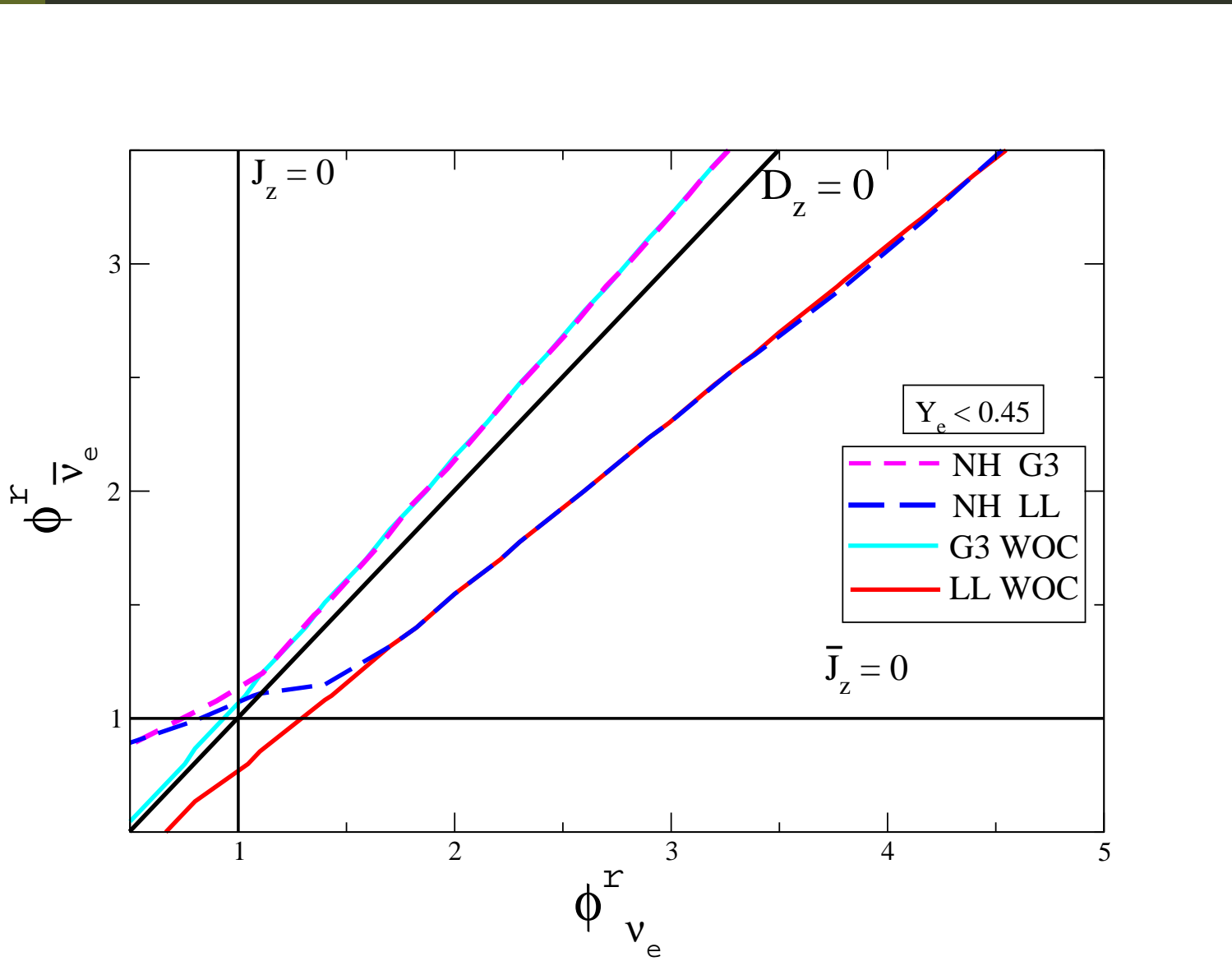
$$P_{\bar{\nu}_e/\nu_e}^c(r_s, E) = 1$$

$$\frac{\lambda_{\bar{\nu}_{ep}}}{\lambda_{\nu_{en}}}(r) = \frac{\int_0^\infty \sigma_{\bar{\nu}_{ep}}(E) \phi_{\bar{\nu}_e}^r \Psi_{\bar{\nu}_e}(E) dE}{\int_0^\infty \sigma_{\nu_{en}}(E) \phi_{\nu_e}^r \Psi_{\nu_e}(E) dE},$$

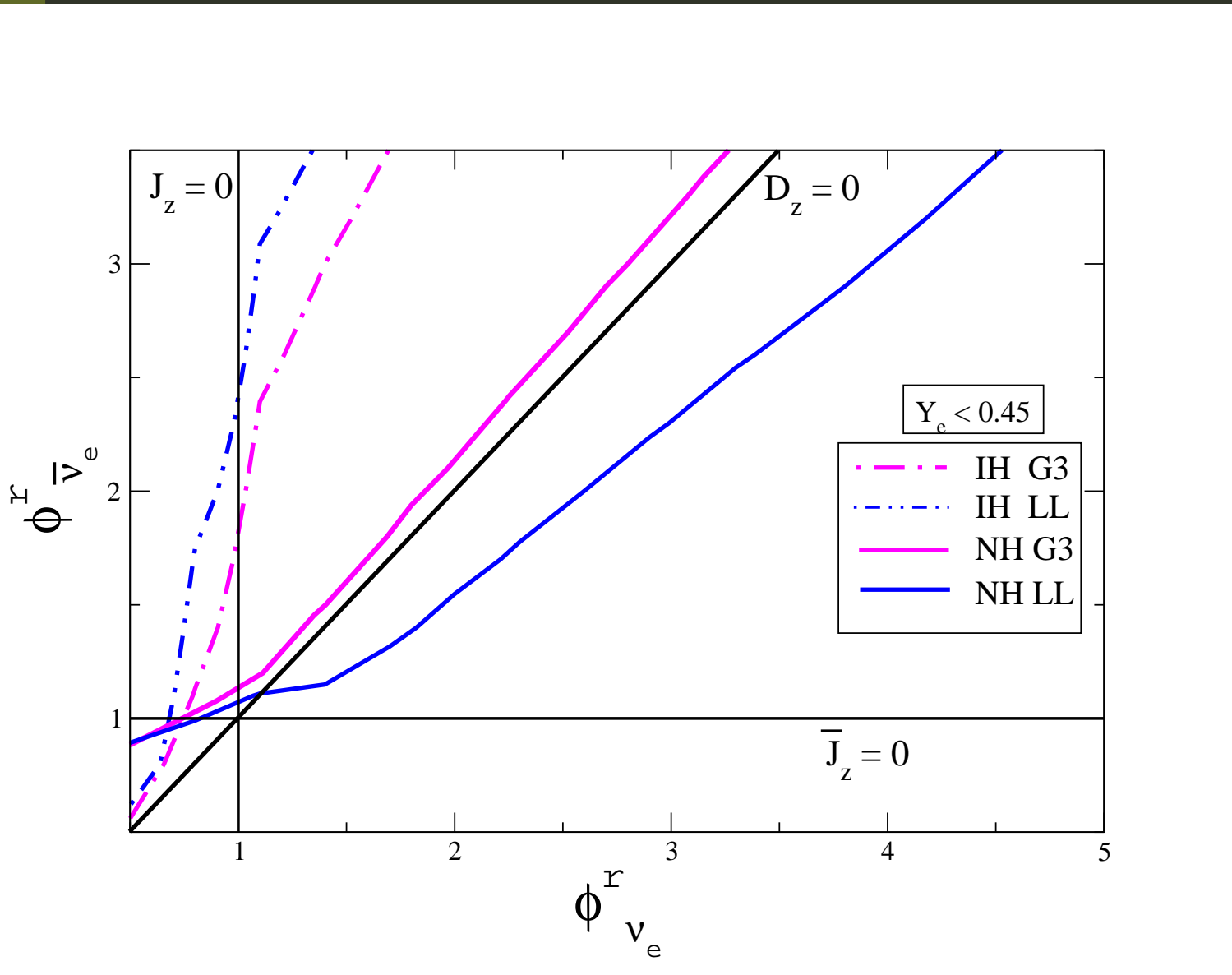
Y_e Vs Radius



Exclusion Plot $Y_e < 0.45$: NH



Exclusion Plot $Y_e < 0.45$



Conclusion

- Different than active-sterile oscillation.
- IH the parameter space is more constrained than NH.
- For successful r-process all initial relative fluxes are not allowed.

THANK YOU !