JIGSAW, 22-26 February 2010
Tata Institute of Fundamental Research, Mumbai

Multiple Spectral Splits of Supernova Neutrinos

:: arXiv: 0904.3542 – with Dighe, Raffelt, Smirnov ::
:: arXiv: 0904.3542 – with Mirizzi, Tamborra, Tomas ::
:: in progress – with Choubey, Dighe, Mirizzi ::

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Outline

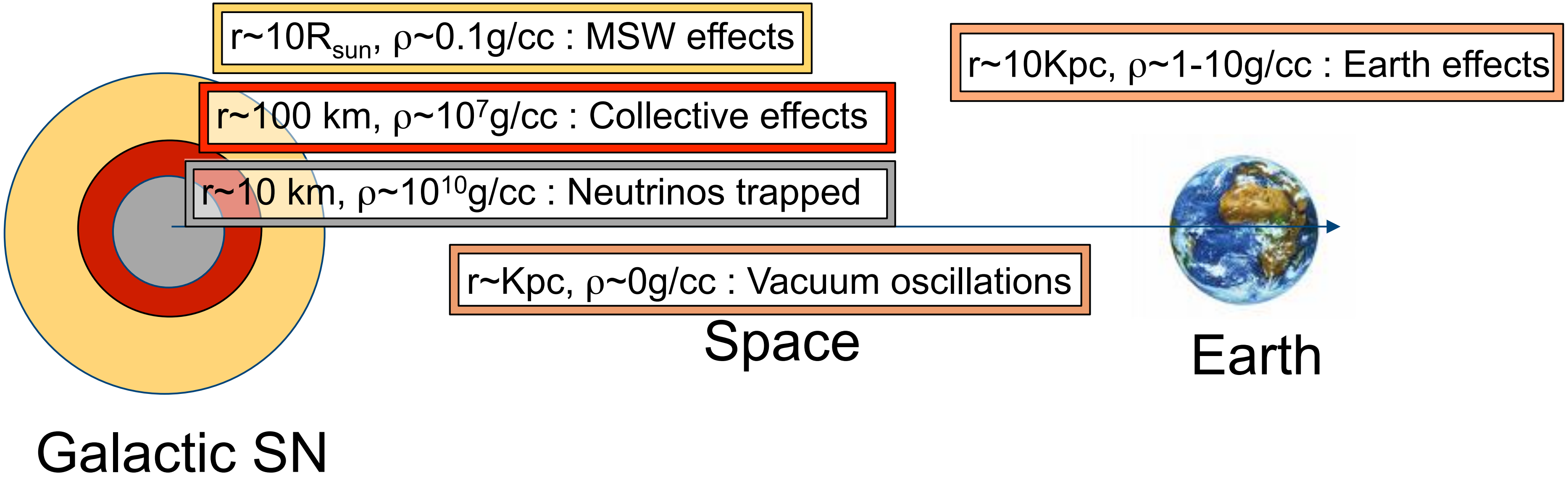
- Inverse SN neutrino problem
- A new layer of difficulty : Collective Oscillations
- A new player : Flux Models
- Rich (Complicated) phenomenology: Spectral Splits
- What should we look for? What could we learn?

Neutrino Oscillations in SN

- Neutrino oscillation usually involves only 2 terms
  - Mass matrix / 2E
  - MSW potential (due to electrons)
- In a SN, neutrinos are very dense and therefore create a similar MSW-like potential.
  - Flavor non-trivial.
  - Coupled neutrino oscillations a.k.a "Collective Effects".
- Neutrino flavor spectra swap in some energy ranges.
- These are called "Collective Effects".

The SN neutrino program

- Calculate an initial neutrino spectrum
- Calculate the changed spectrum due to oscillation effects
- Calculate flux at detector
- Construct variables that distinguish different physics/astro scenarios
- Wait for a SN…



SN collective effects: Summary (old)

- **For IH:**
  - **Exchange ν_e and ν_y above E_c .**
  - **Exchange anti-ν_e and anti- ν_y .**
- **For NH:**
  - **No collective effects.**

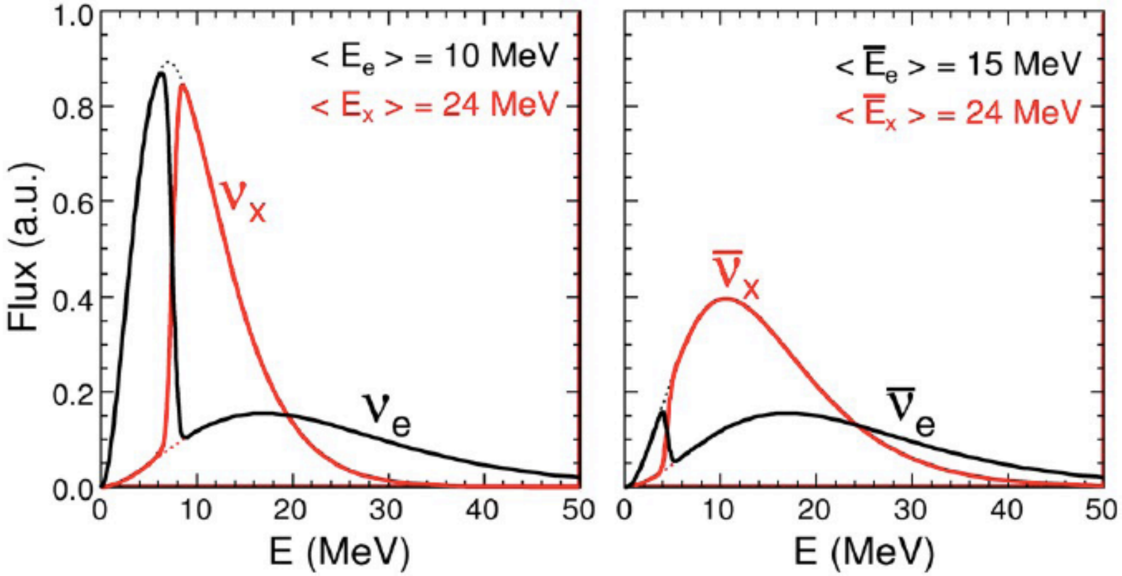
Seminal papers by: Duan, Fuller, Carlson, and Qian (2005, 2006)

Almost 100 papers on collective effects by:

Abazajian, Balantekin, Beacom, Bell, Blennow, Carlson, Dasgupta, Dighe, Dolgov, Duan, Esteban-Pretel, Fogli, Friedland, Fuller, Gava, Goswami, Hannestad, Hansen, Kneller, Kostelecky, Lisi, Lunardini, Marrone, McLaughlin, Mirizzi, Pantaleone, Pastor, Pehlivan, Qian, Raffelt, Samuel, Serpico, Semikoz, Sigl, Smirnov, Stodolsky, Tomas, Volpe, Wong …

SN neutrinos and the spectral split

- **We get a spectral split in neutrinos, and the antineutrinos swap their energy spectra between flavors.**

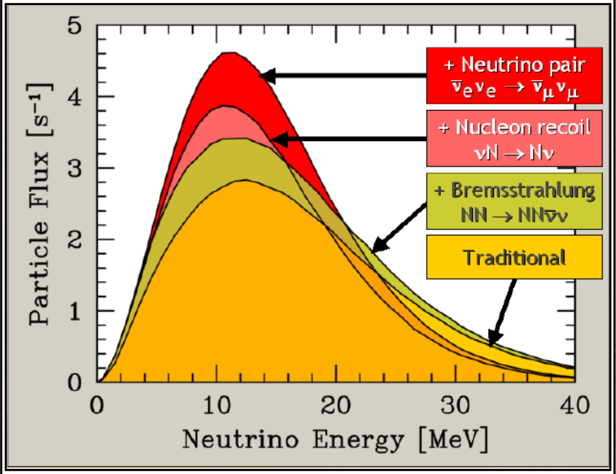


Fogli, Lisi, Marrone, Mirizzi (2007)

- **Actually there is a split in antineutrinos too…**

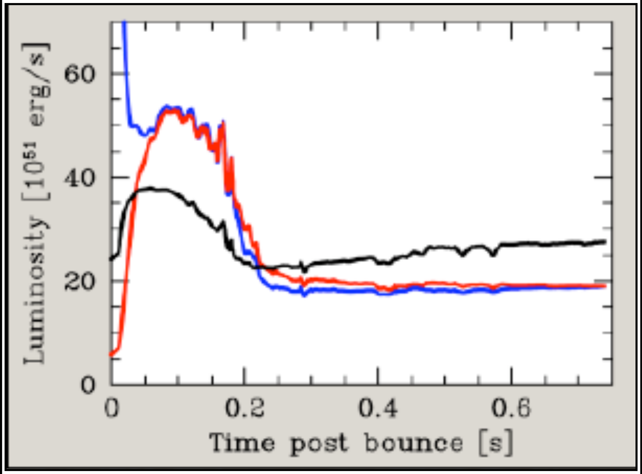
SN Simulations: Garching 2003

Spectra



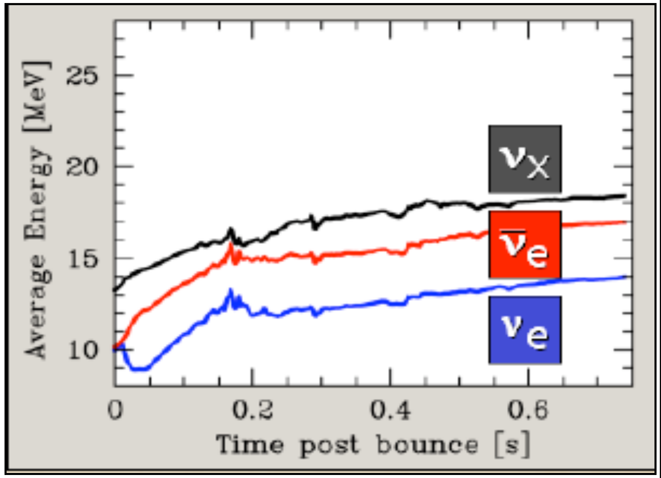
- Almost thermal
- Pinching

Luminosity



- Burst of ν_e
- Crossover
- Cooling

Average Energies



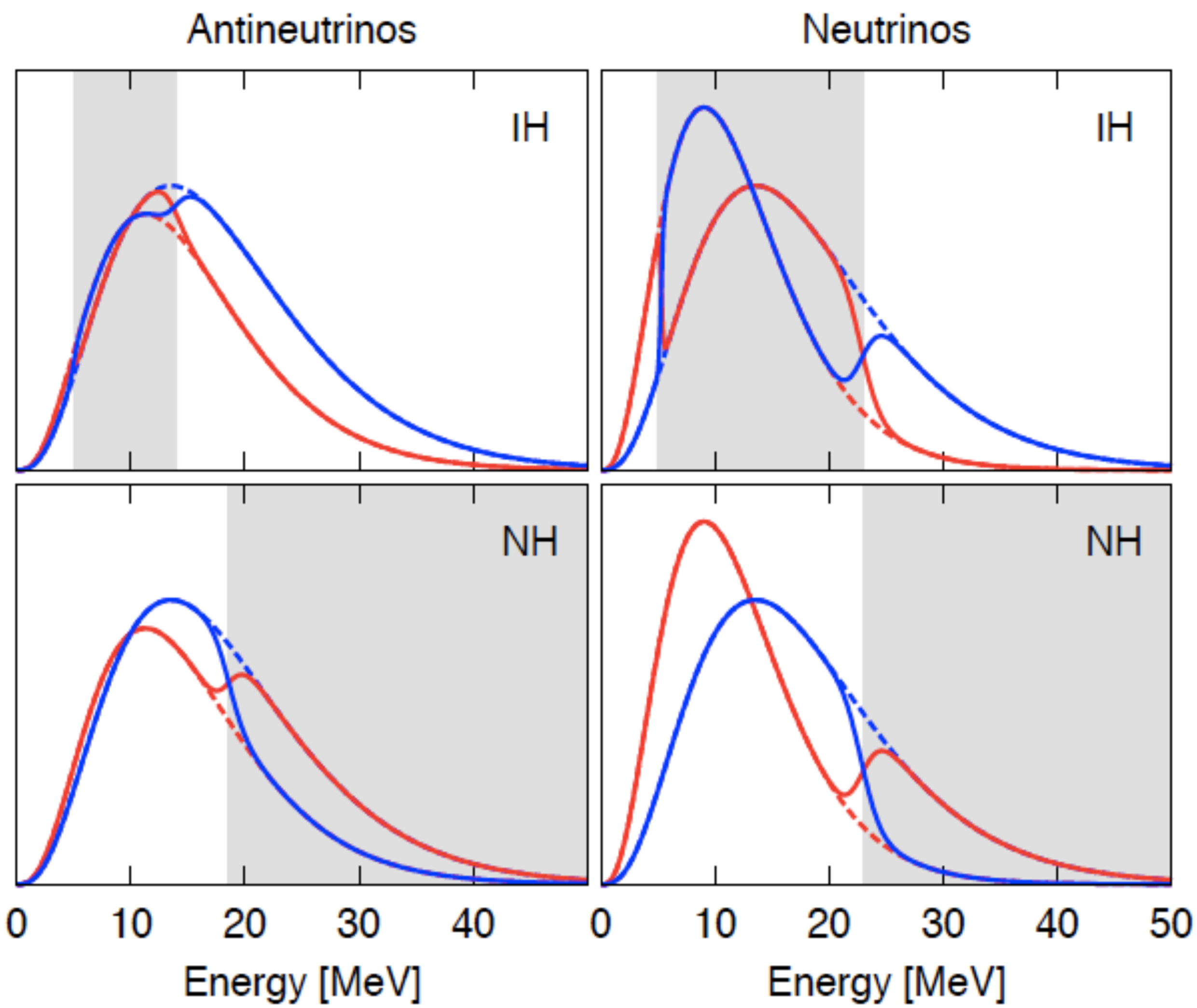
- $E_e < E_{ebar} < E_x$

Garching group, astro-ph/0303226

Slightly different fluxes

- What happens if initial fluxes are changed a bit?
- Let's check out the case of the spectra predicted by Garching group for the cooling phase.
- The essential change : neutrino number fluxes are taken to be $\nu_e : \nu_{ebar} : \nu_x = 0.85 : 0.75 : 1.00$ and not equi-partitioned, as was commonly assumed.

Many spectral splits



- **4 splits in IH.**
- **2 splits in NH.**
- **Why?**

Clearly there is something missing in our understanding.

This could be observationally important…

Part I:
How to predict the final spectra, given the initial spectra.

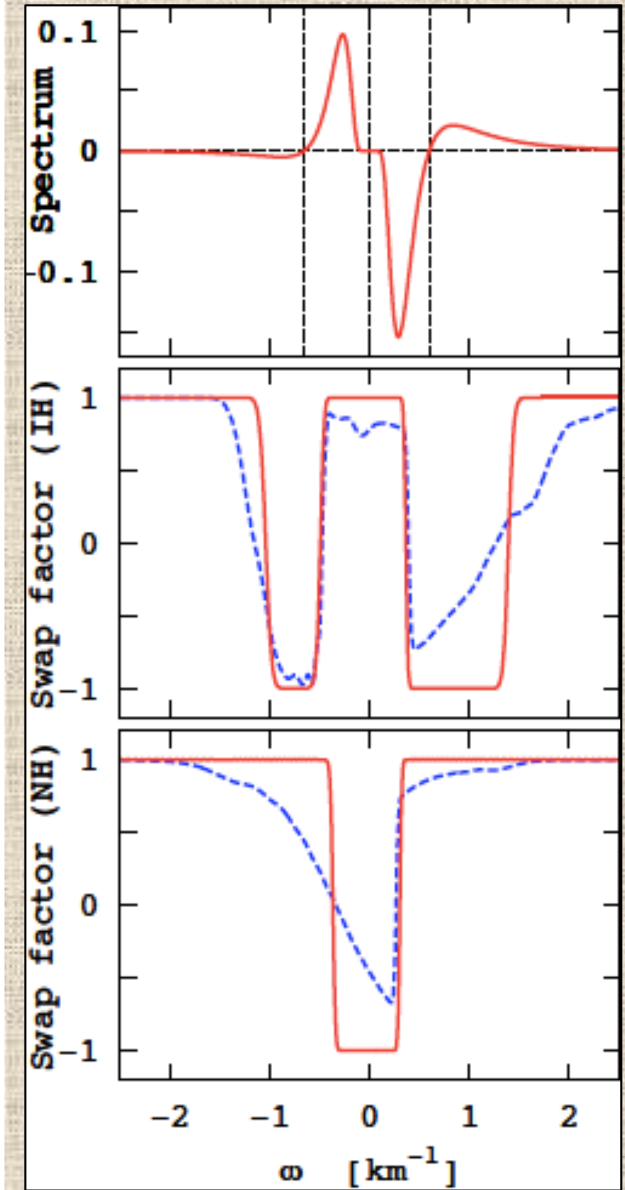
Notation

- We have the flux spectrum f(E) for each flavor.
- However, let's use $\omega = \Delta m^2/2E$ as the x-axis variable.
- Moreover, let's label antineutrinos with $-\omega$.
- Define

$$g(\omega) = \begin{cases} f_e(E) - f_x(E) \text{ for neutrinos} \\ f_x(E) - f_e(E) \text{ for antineutrinos} \end{cases}$$

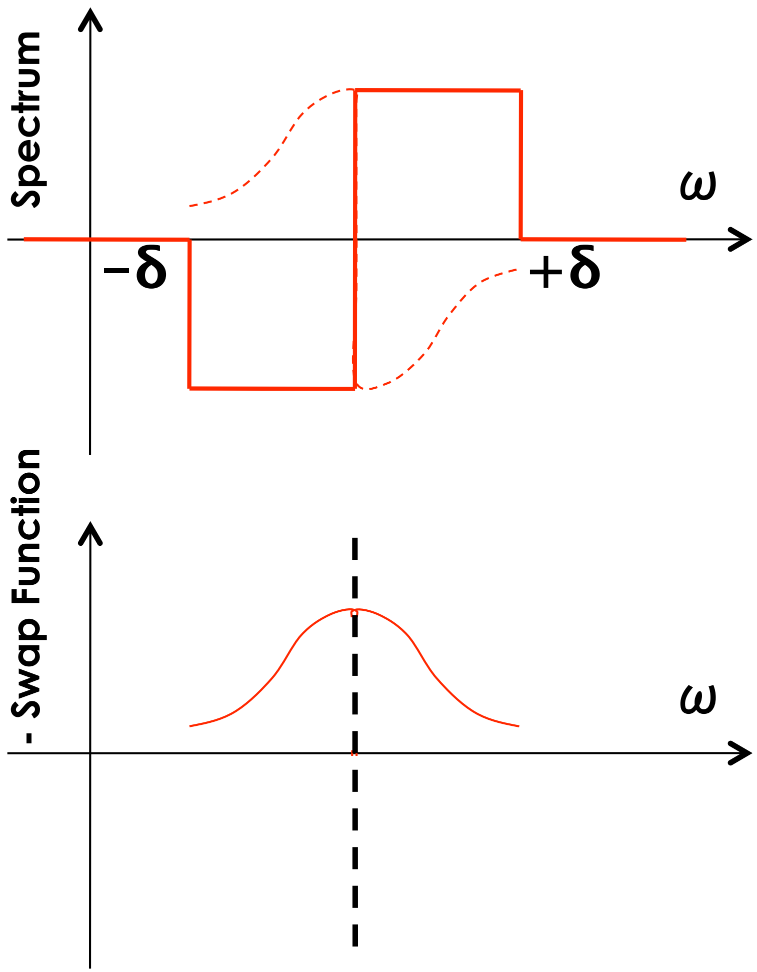
- Now we have put the all the relevant spectral information in a single function $g(\omega)$.
- How does this function look? Let's see…

In the g(ω) variable…



- $g(\omega)=0$ where fluxes equal
- "Swaps" around every "± crossing"
- Each swap flanked by two "splits"
- Splits not always washed out completely by multi-angle effects
- Let's answer some questions now…
  - Why are there swaps around a crossing?
  - Why the ± for IH/NH?
  - What is the width of the swap?

Fixed initial neutrino density μ



- "Box" spectrum at finite μ.
- Spectrum oscillates to the dotted lines and back.
- Swap function looks like a Lorentzian centered at the crossing at any instant!
  - Collective motion.
  - May be we can solve this analytically?
  - Let's try…

"Deriving" the Lorentzian

- The system has EOM

$$\dot{\mathbf{P}}_\omega = (\omega \mathbf{B} + \lambda \mathbf{L} + \mu \int d\omega_1 \, \mathbf{P}_{\omega_1}) \times \mathbf{P}_\omega$$

- Ansatz:

$$\mathbf{P}_\omega(t) = \begin{pmatrix} -\sin\varphi \, L(\omega) \\ -\frac{\omega}{\Gamma} \, 2\sqrt{1-\cos\varphi} \, L(\omega) \\ 1-(1-\cos\varphi) \, L(\omega) \end{pmatrix} g(\omega)$$

$$L(\omega) = \frac{\Gamma^2}{\Gamma^2 + \omega^2}$$

- This is a merely a parametrization, and putting it back in EOMs we get

$$\dot{\varphi} = \Gamma \sqrt{2(1-\cos\varphi)}$$

$$\Gamma = \frac{\delta}{\sqrt{-1 + e^{2\delta/\mu}}}$$

- EOM of a pendulum.
- Width is exponential in μ.

Changing neutrino density μ

- We know that as we decrease μ (mimicking decreasing neutrino density away from the core) the pendulum damps and relaxes to lowest energy configuration.
- This system involves an adiabatic invariant that *roughly* relates the width of split ω_s to width of Lorentzian Γ.

$$\omega_s = \frac{\pi}{4}\Gamma \frac{2}{1+\sqrt{1+\frac{\pi^2}{4}\frac{\mu}{2\delta}}} = \frac{\delta}{\sqrt{-1+e^{2\delta/\mu}}} \frac{\pi/2}{1+\sqrt{1+\frac{\pi^2}{4}\frac{\mu}{2\delta}}}$$

Some comments

- We showed that there is a pendulum like oscillation about a crossing.
- As μ decreases, this pendulum eventually tips over if it is inverted, i.e. +ive crossings for IH, -ive crossings for NH.
- Thus there are swaps around a crossing (B.P conserved).
- Width of the swap is related to Γ and depends on initial μ: wider swaps for larger initial μ. Exponentially thin swaps for small. Also depends on δ, i.e. box width.

Analogy to Spin Magnetic Resonance

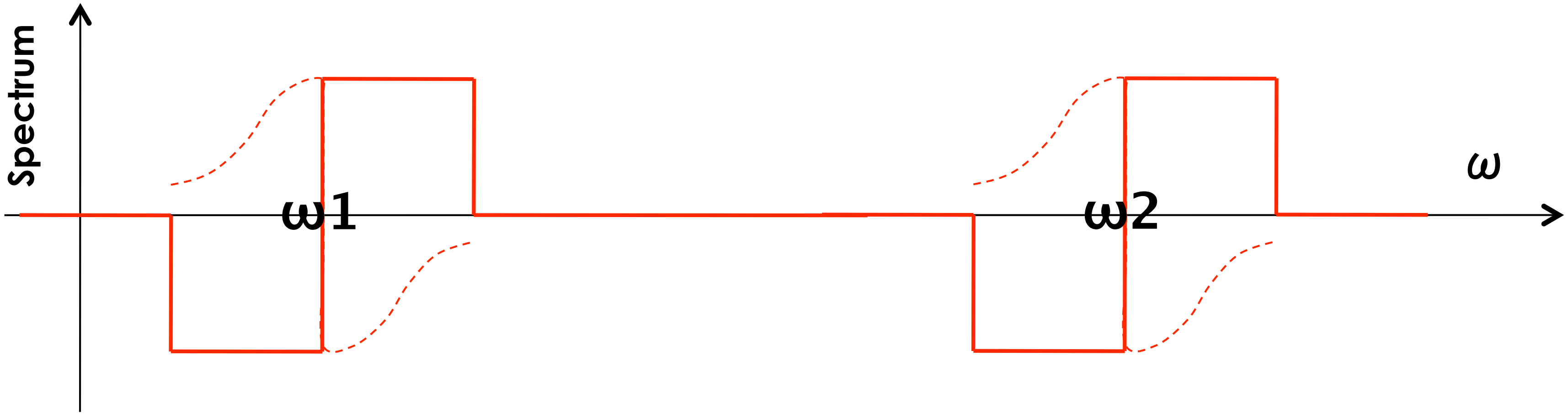
- We break the collective magnetic field into a parallel and perpendicular component, drop the former.

$$\dot{\mathbf{P}}_{\omega} = (\omega\mathbf{B} + \mu\int d\omega_1\, \mathbf{P}^{\parallel}_{\omega_1} + \mu\int d\omega_1\, \mathbf{P}^{\perp}_{\omega_1}) \times \mathbf{P}_{\omega}$$

- For ω=0, P is on-resonance (the mode has the same frequency as the transverse magnetic field)!
- Others are slightly off-resonance by ω, and their amplitude falls off as a Lorentzian, as in SMR.

But that's still only two splits …

- What happens if two copies of the box are put far apart in ω-space?

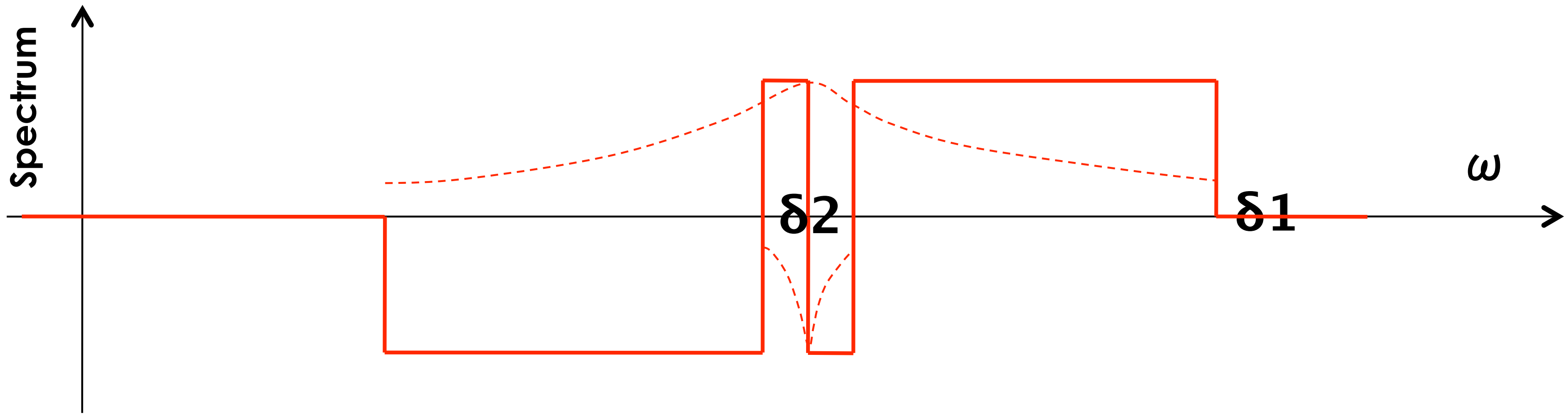


- Each box acts like an independent pendulum; the transverse field due to the other is averaged to zero.

$$\dot{\mathbf{P}}_{\omega} = (\omega \mathbf{B} + \mu \int d\omega_1 \, \mathbf{P}_{\omega_1} + \mu \int d\omega_2 \, \mathbf{P}_{\omega_2}) \times \mathbf{P}_{\omega}$$

What happens when they are close

- What happens if two "boxes" are put close together in the ω-space



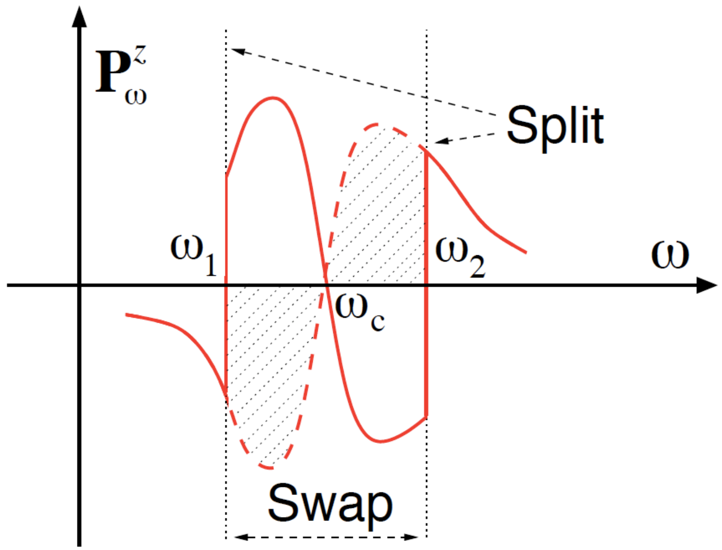
- The inner block acts like a superimposed oscillator on the bigger one. The inner swap-width is exponentially small.

What's special about the box?

- Short answer: Nothing!
- Long answer: Although any function around the crossing works fine, doing the integrals is harder/impossible. Also, the uniqueness and stability of the solution is not guaranteed.

Great expectations

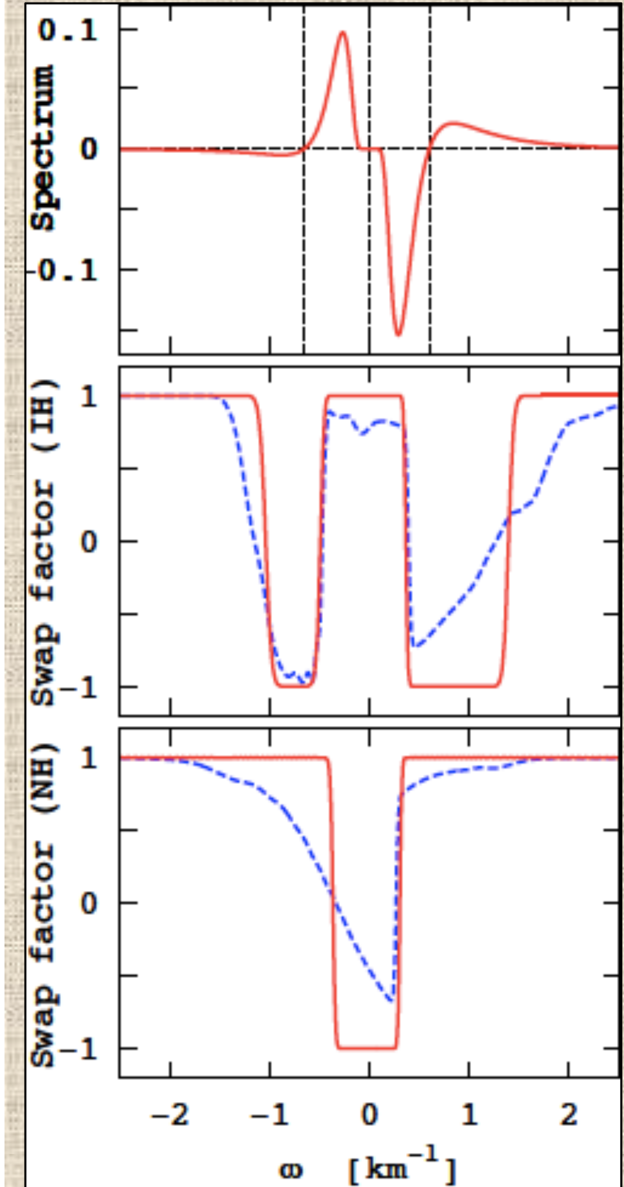
- The basic picture …



  - One swap for every ± crossing for IH/NH.
  - Width of each swap depends exponentially on μ and also on the δ for the block around that crossing.
  - Each swap approx. preserves lepton number B.P locally.

- When blocks are close more complicated things can happen, and it would be interesting to study…

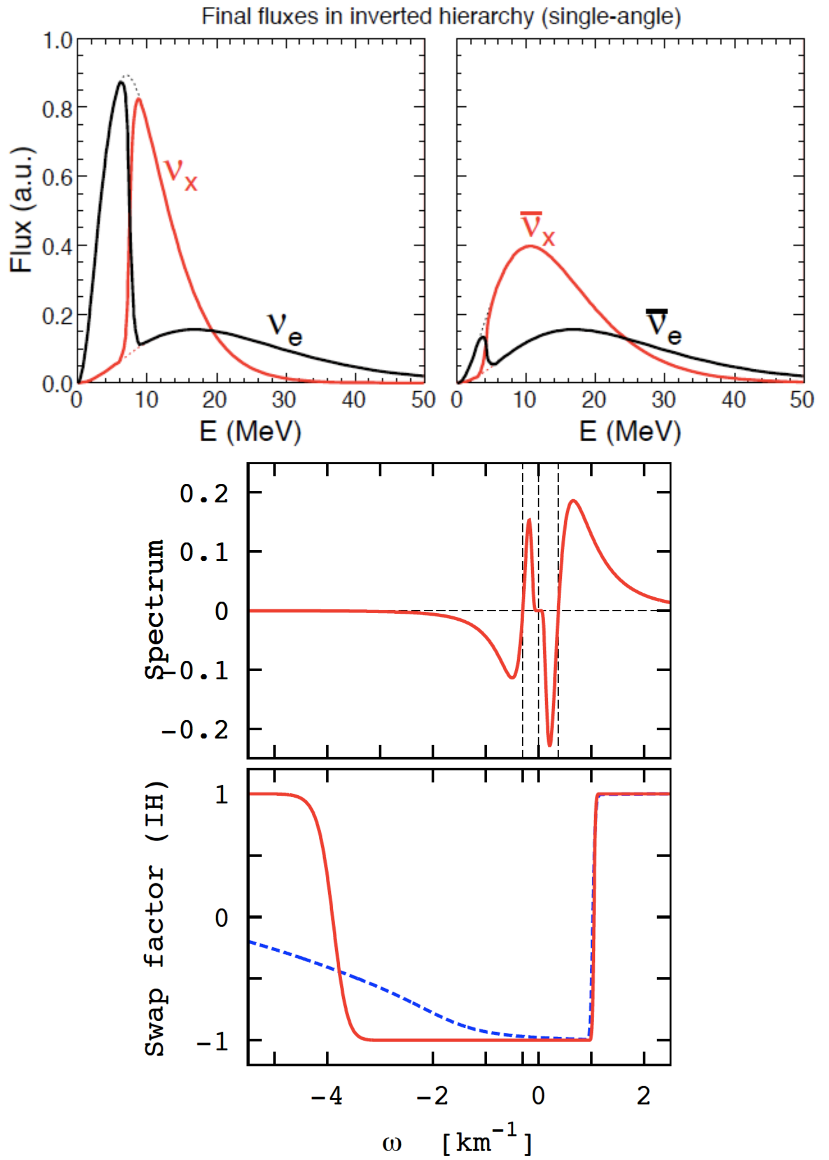
Cooling phase fluxes : Recap



- Swaps around every "± crossing"
- Each swap flanked by two splits
- Splits not always washed out completely by multi-angle effects

- We have answered the questions…
  - Why are there swaps around a crossing?
  - Why the ± for IH/NH?
  - What is the width of the swap?

Accretion phase example: Recap



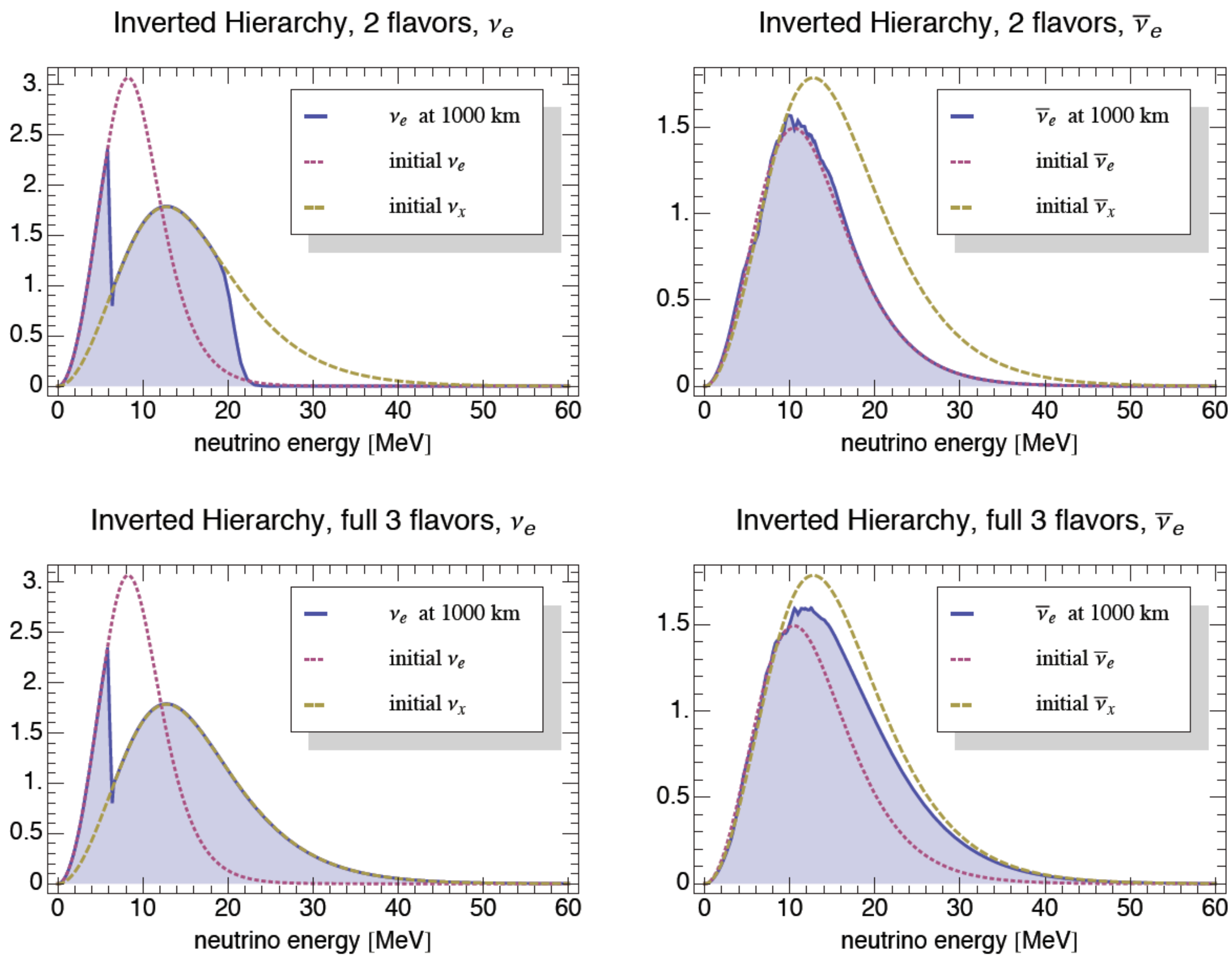
- We should have seen 4 splits, but we see 2 only, because the inner swap is exponentially narrower.
- In fact even in NH we should get two splits (but again they are narrow and the flux is low at low-|ω| to see anything).

Odds and Ends

- Three-flavor effects?
- Do a survey of various SN flux models and check what kind of split patterns one gets.
- Is there a simple picture to this?
- Can one show that this will/won't have experimental relevance?

Part II:
Three Flavor Effects

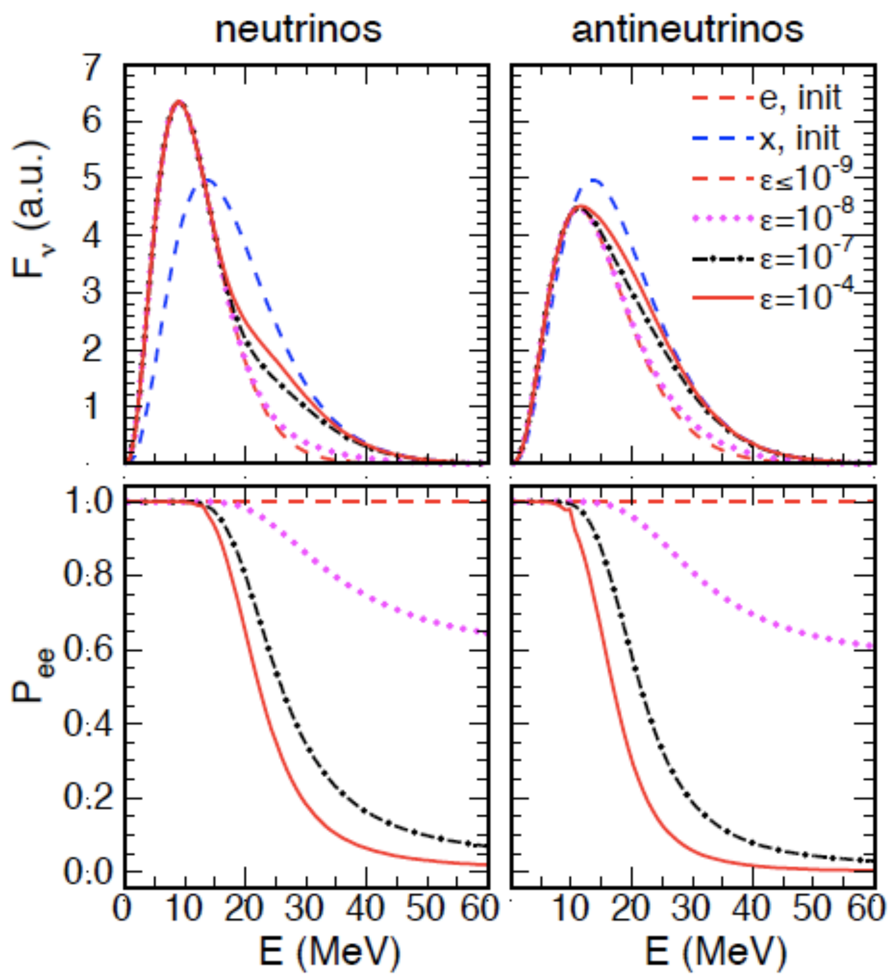
Three-flavor effects



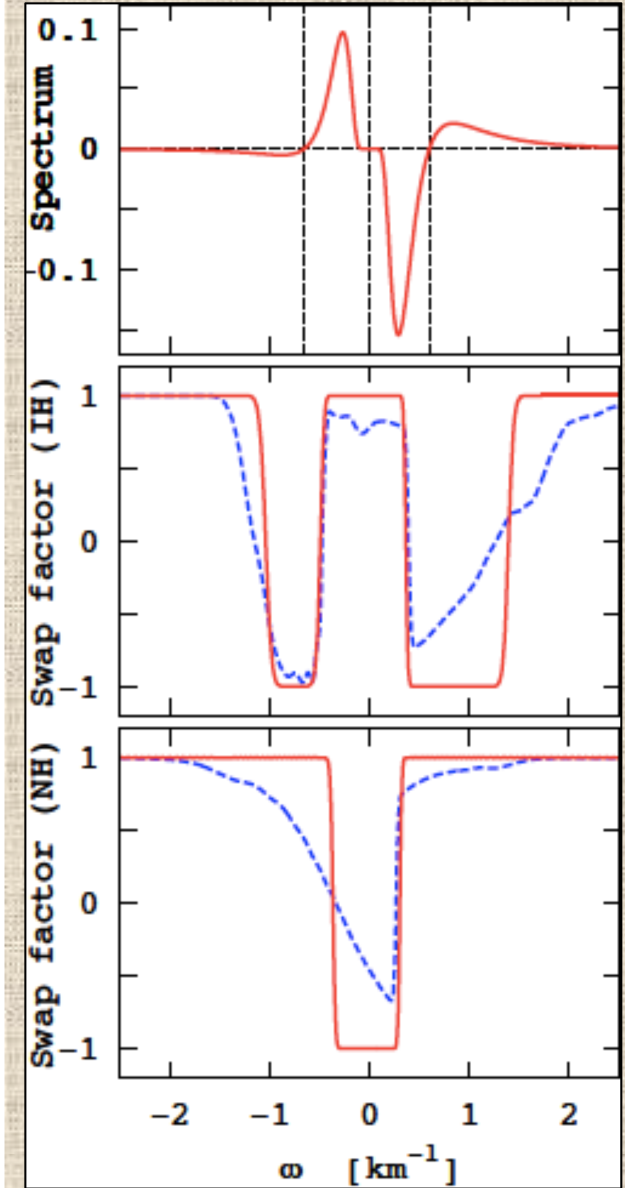
Alexander Friedland, arXiv:1001:0996

Solar Δm^2 driven effects

- **Usually not adiabatic, i.e.**
  - **$\omega = \Delta m^2/E \approx \Gamma$ (pendulum frequency) less than rate at which μ is decreasing.**
- **Some initial disturbance helps to kick-start swaps.**



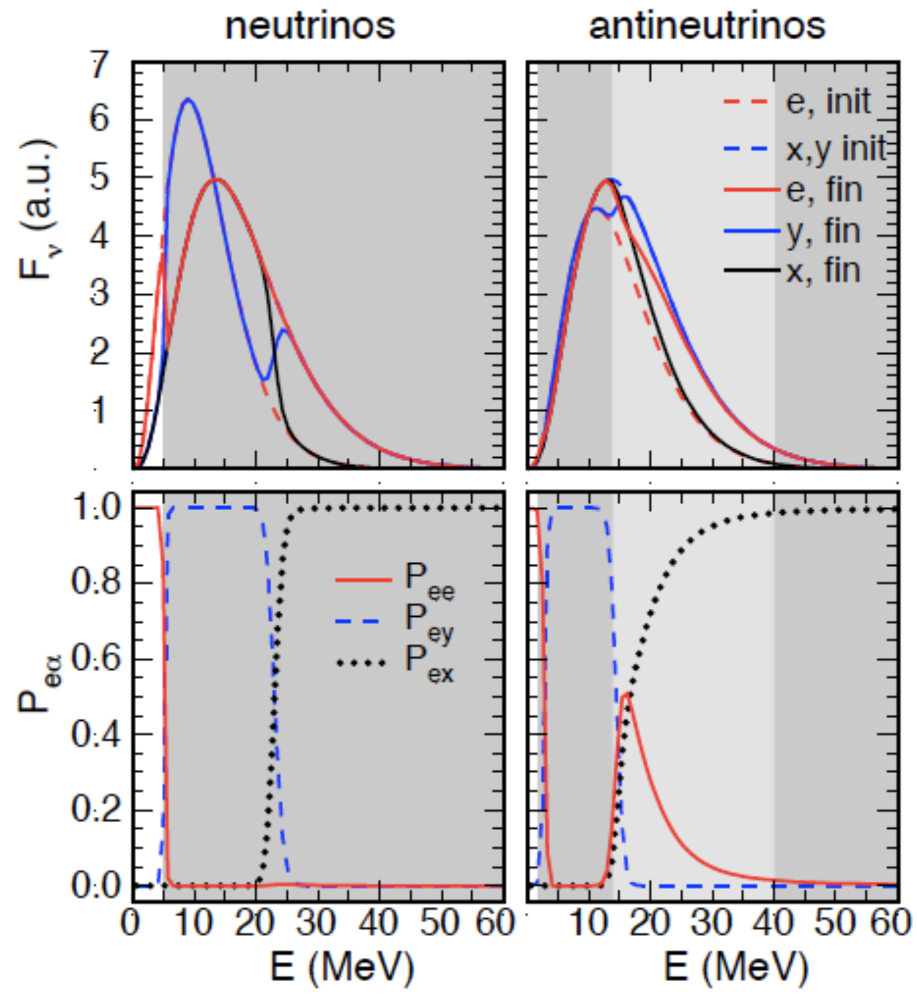
Try anything twice



- g(ω) is processed twice
- Step 1: by atmospheric Δm^2 (NH/IH)
- Step 2: by solar Δm^2
- Interplay of these two steps
  - NH: cooperate
  - IH: compete with each other
- Step 1 gives required disturbance.

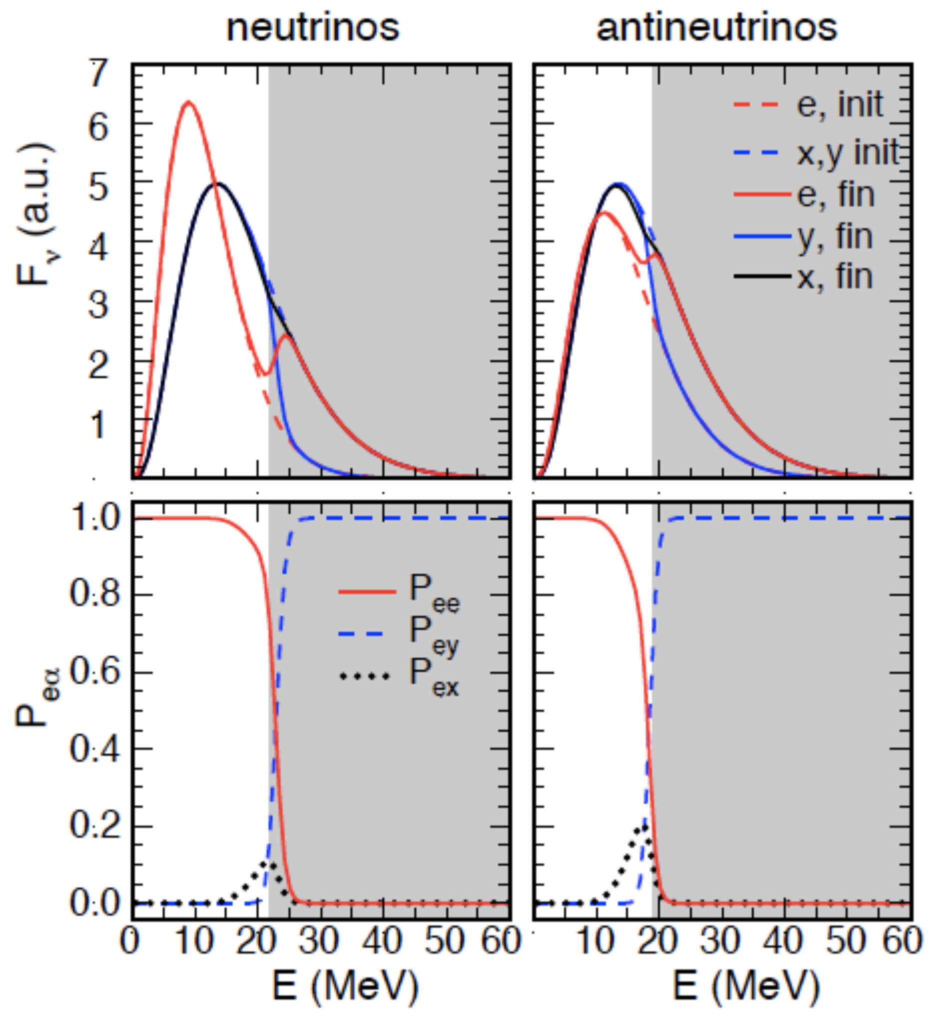
Inverted Hierarchy

- Atmospheric swaps (e,y)
- Solar swaps (e,x)
- Higher energy split is transferred from e to x
- Non-adiabatic effects
- In short: It's a mess! But a mess that we understand!



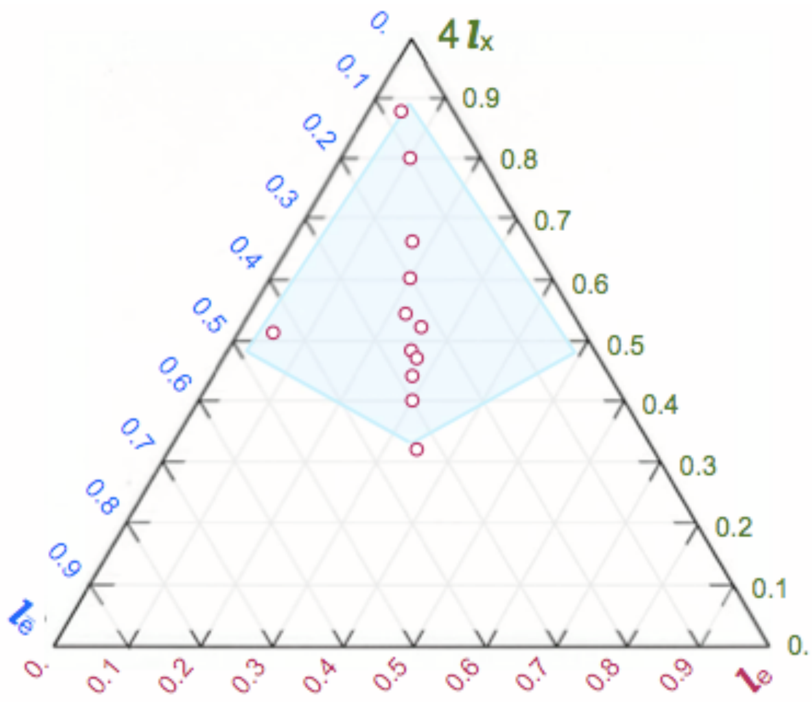
Normal Hierarchy

- **Almost same as 2-flavors.**
- **Solar driven conversions are too slow to compete.**
- **Simple prediction**
  - **High energy spectrum of e and y flavors are swapped.**
- **Let's get a bit more ambitious…**

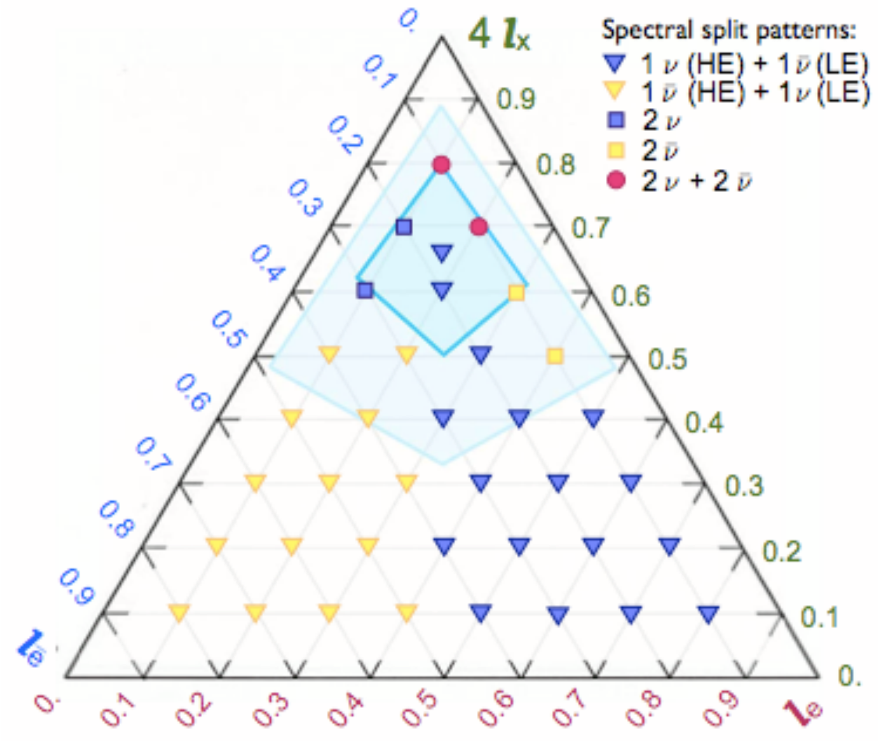


Part III:
Survey of Flux Models and Pattern Hunting

Ternary Diagrams



- **Luminosity of 3 species**
- **Typically L_e=L_ebar**

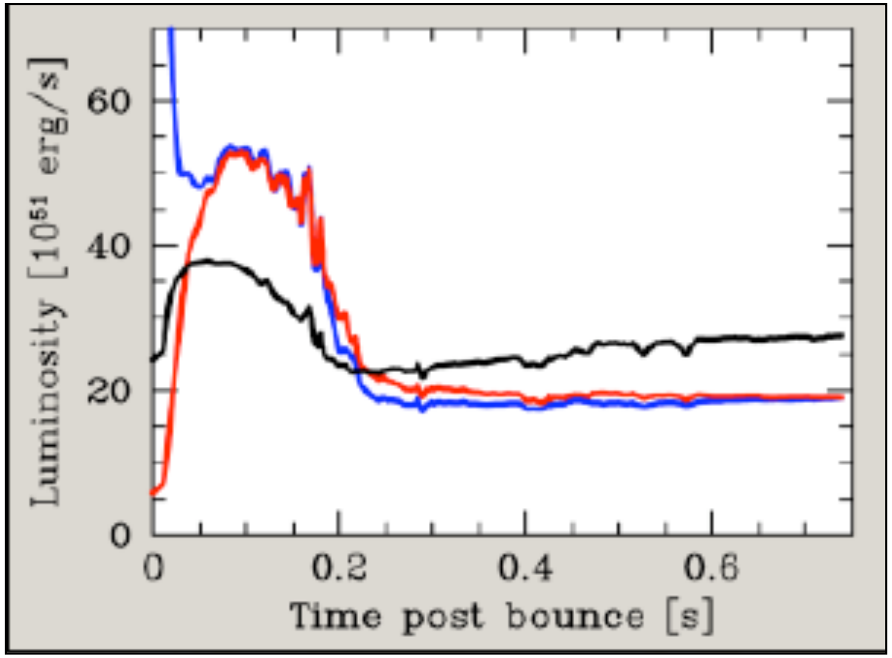


- **A pattern of splits**

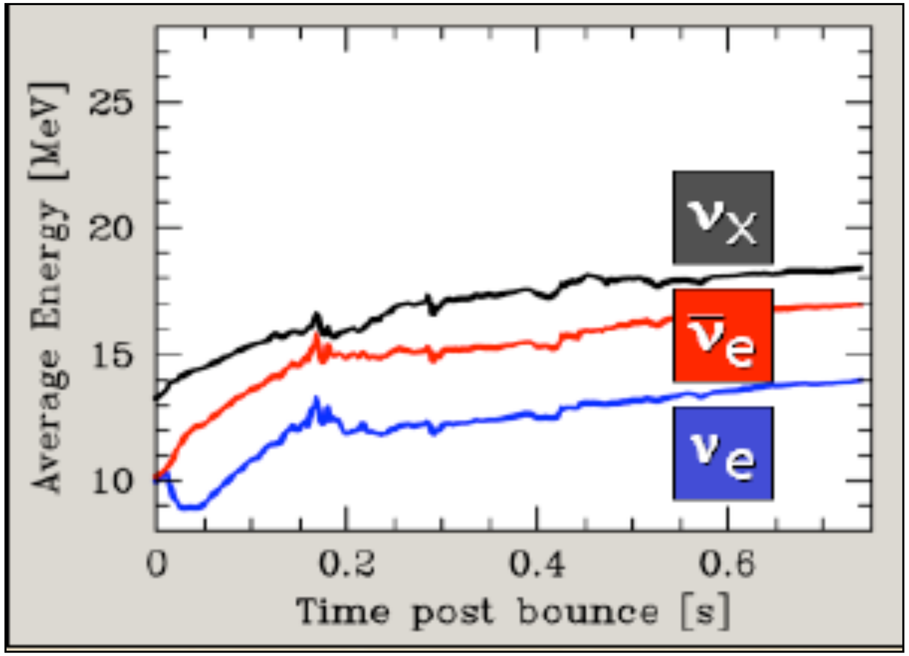
Fogli, Lisi, Marrone, Tamborra, arXiv:0907.5115

Flux Models: Garching 2003

- **Luminosity:**

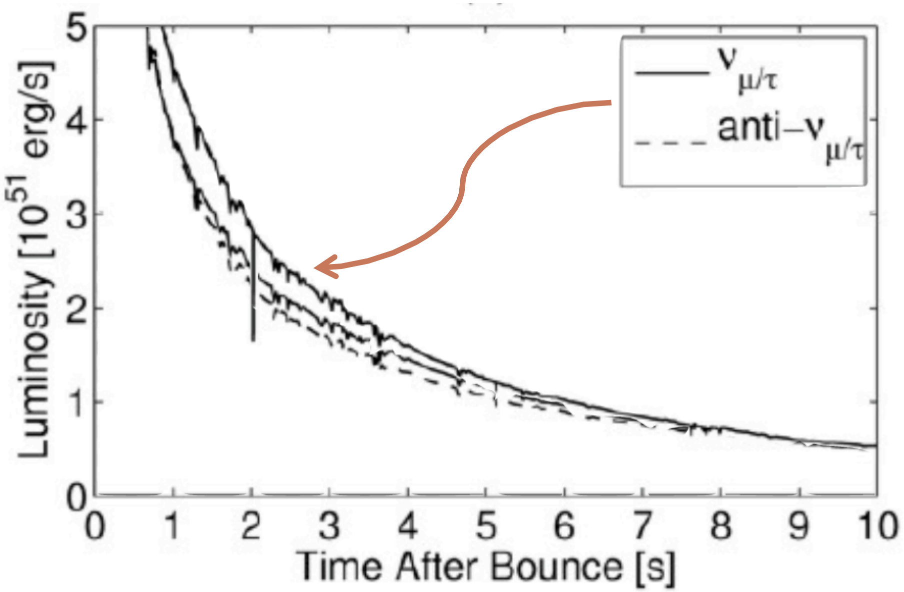


- **Average Energy:**

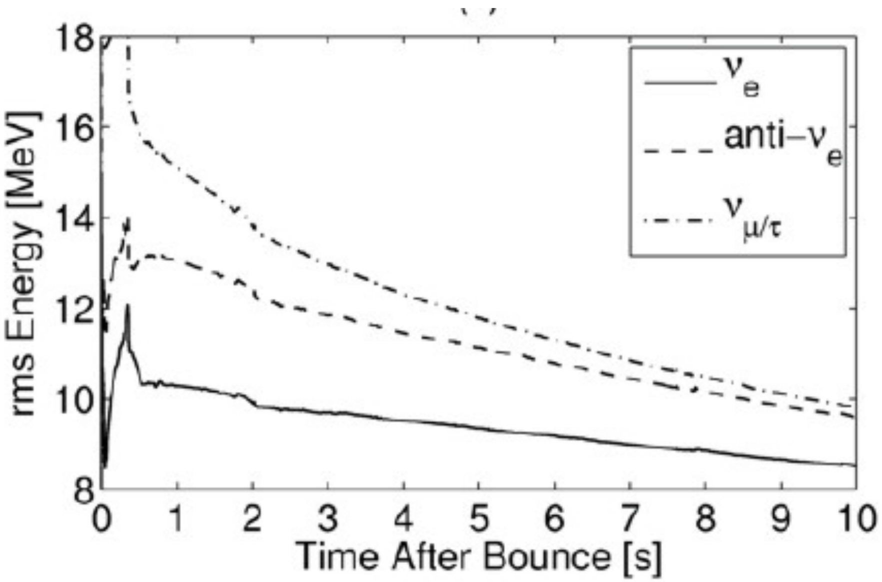


Flux Models: Basel 2009

- **Luminosity:**

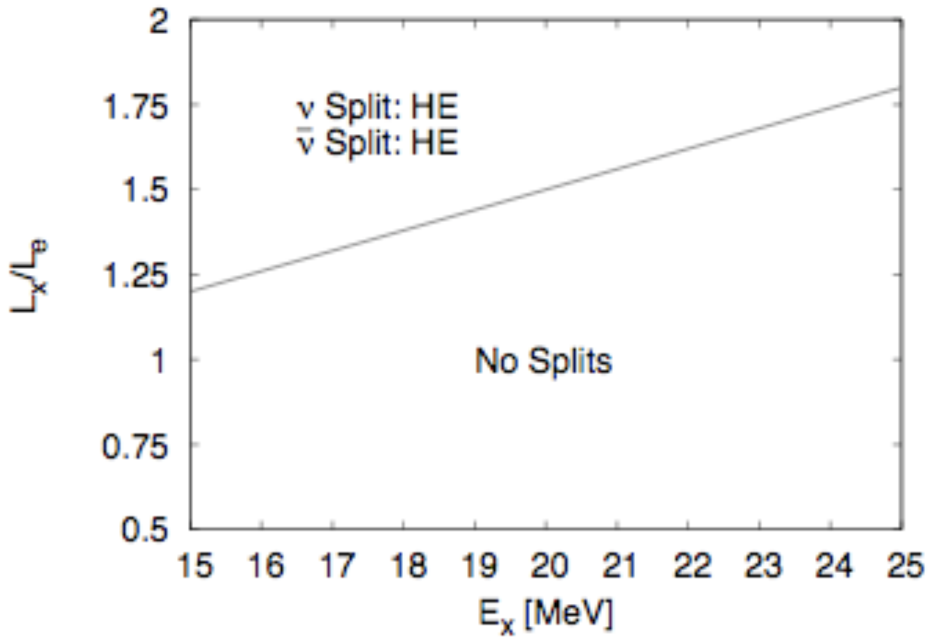


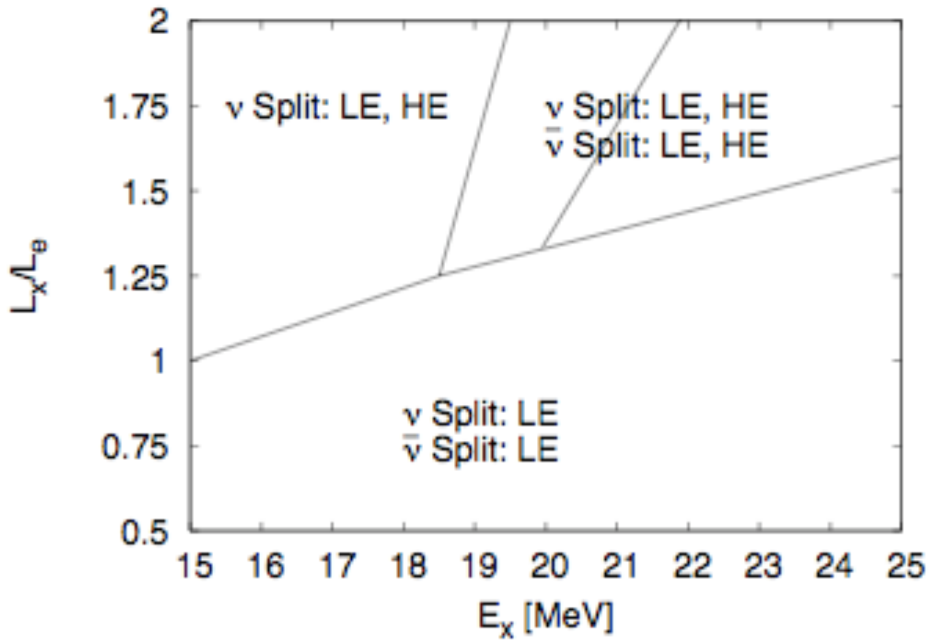
- **Average Energy:**



Basel group, arXiv:0908.1871

Split Patterns in NH and IH

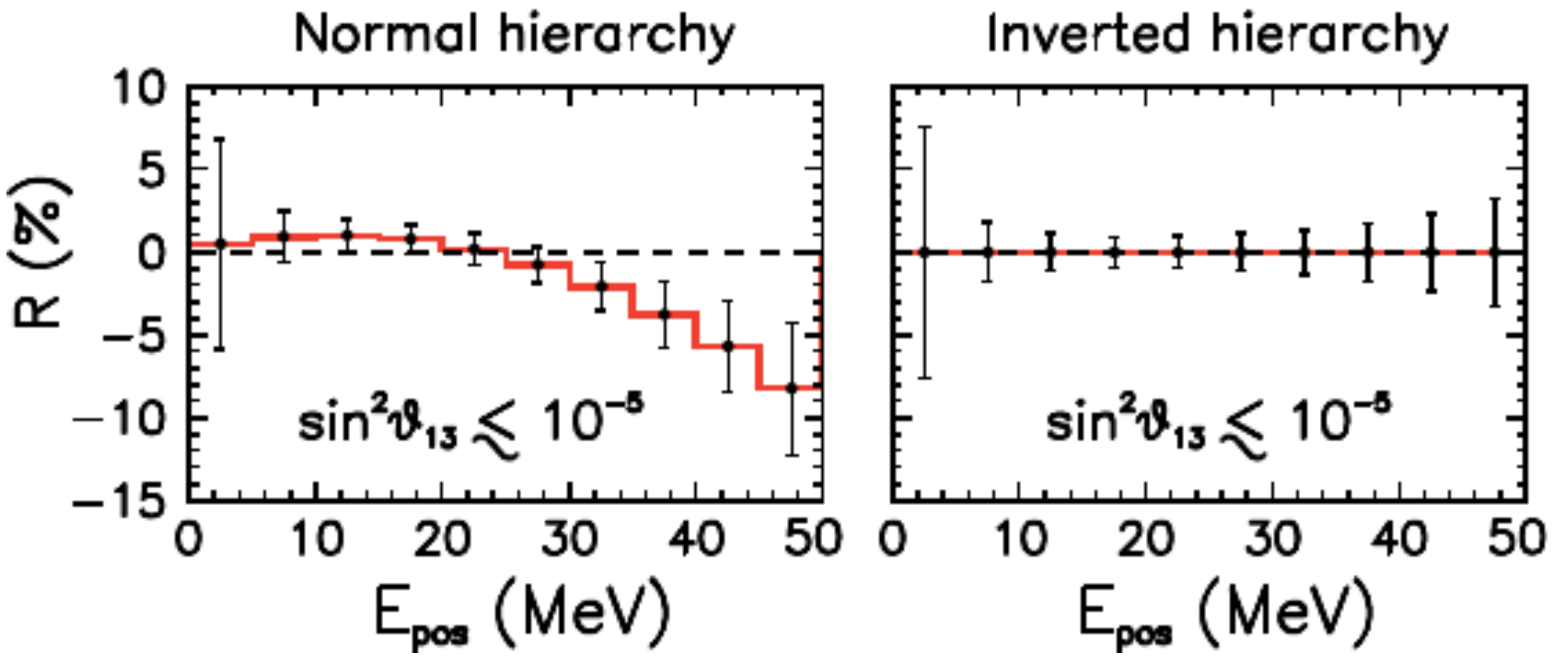




- **A given model at time, is a point on this plane**
  - **Include MSW effects, vacuum mixing, Earth matter effects…**
  - **Look at ν_e and anti-ν_e spectra at various detectors…**
  - **See if there are some simple ways to extract NH/IH or SN physics…**

NH/IH determination

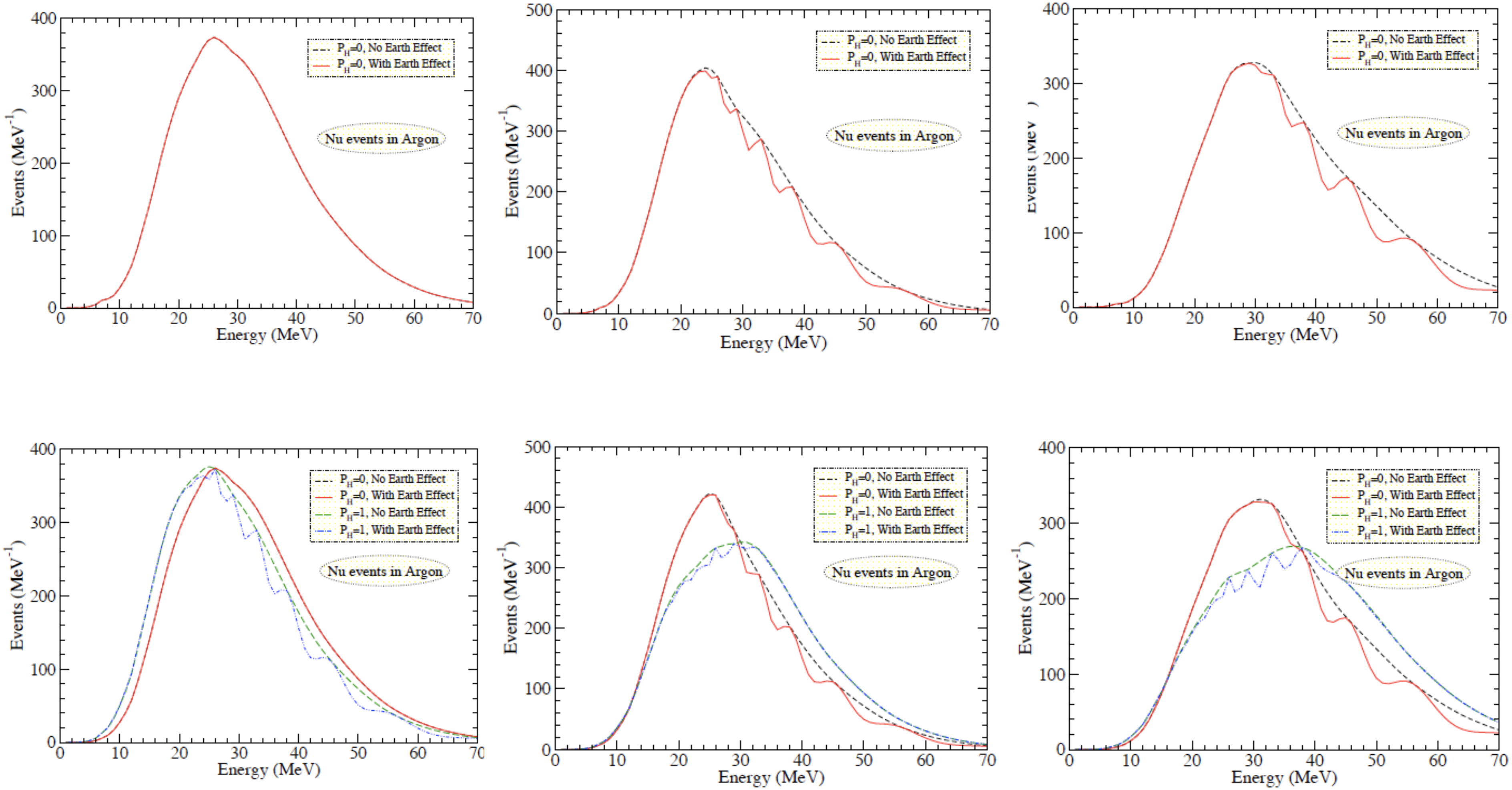
- **Look at the early signal (< 1 sec) in antineutrinos using ratio of events at two WC detectors.**



Dasgupta, Dighe, Mirizzi, arXiv:0802.1481

Hierarchy+Shock-wave effects

- **More complicated time-dependent signatures**



Conclusions

- Rich phenomenology…important to remember that we will have a time-dependent signal.
- Collective effects can be very different over these times.
- Theory still not complete…but in good shape.
- Lots to do…