# Aspects of B-Decays

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# My CKM Talk – (en?)Lightning Summary

$R_D$ and $R_{D^*}$ anomalies : Experiments don't match data @ $4\sigma$ (Both taken together)

No deviation from the SM results if only $e$ and $\mu$ are considered

Based on:
arXiv:1610.03038

Model-independent analysis of the $R_D$ and $R_{D^*}$ anomalies

Took six-dimensional operators and their corresponding Wilson Coefficients

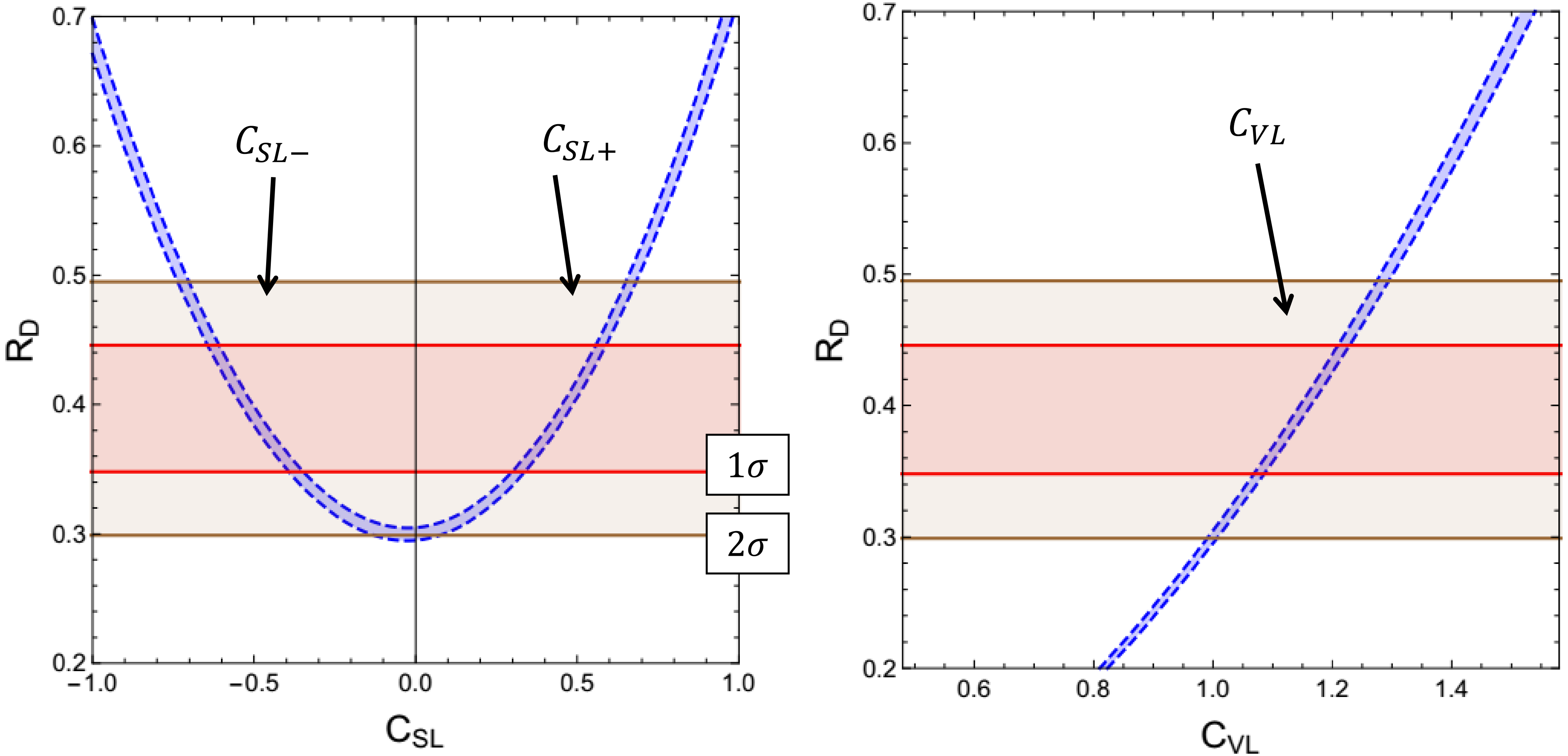
Found ranges for these Wilson Coefficients

Used different observables like Tau polarization, FB Asymmetry and binned $R_{D^*}$ prediction to differentiate between different Wilson Coefficients

# My CKM Talk – Lightning Summary

## Explaining $R_D$

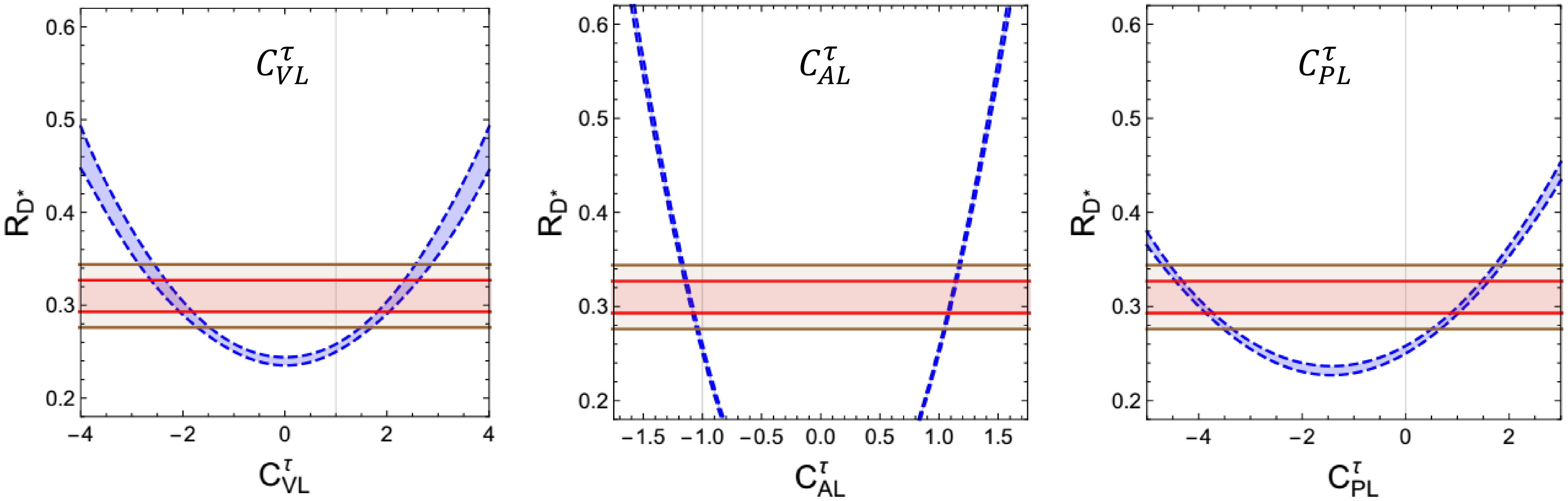
- $R_D$ dependent on : $C_{VL}^{\tau}$ and $C_{SL}^{\tau}$

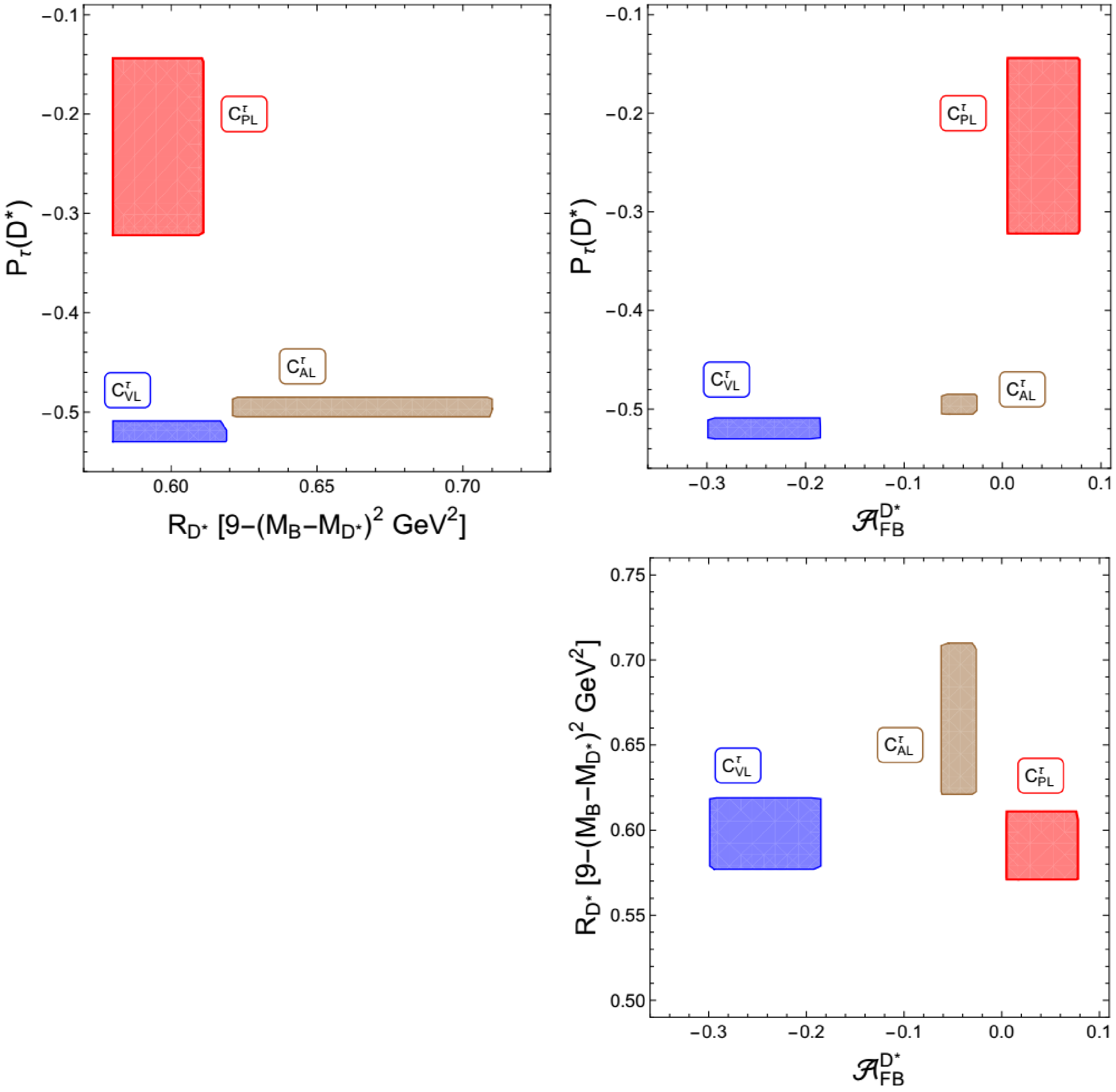


# My CKM Talk – Lightning Summary

# Explaining $R_{D^*}$

- $R_{D^*}$ dependent on : $C^{\tau}_{VL}, C^{\tau}_{AL}$ and $C^{\tau}_{PL}$



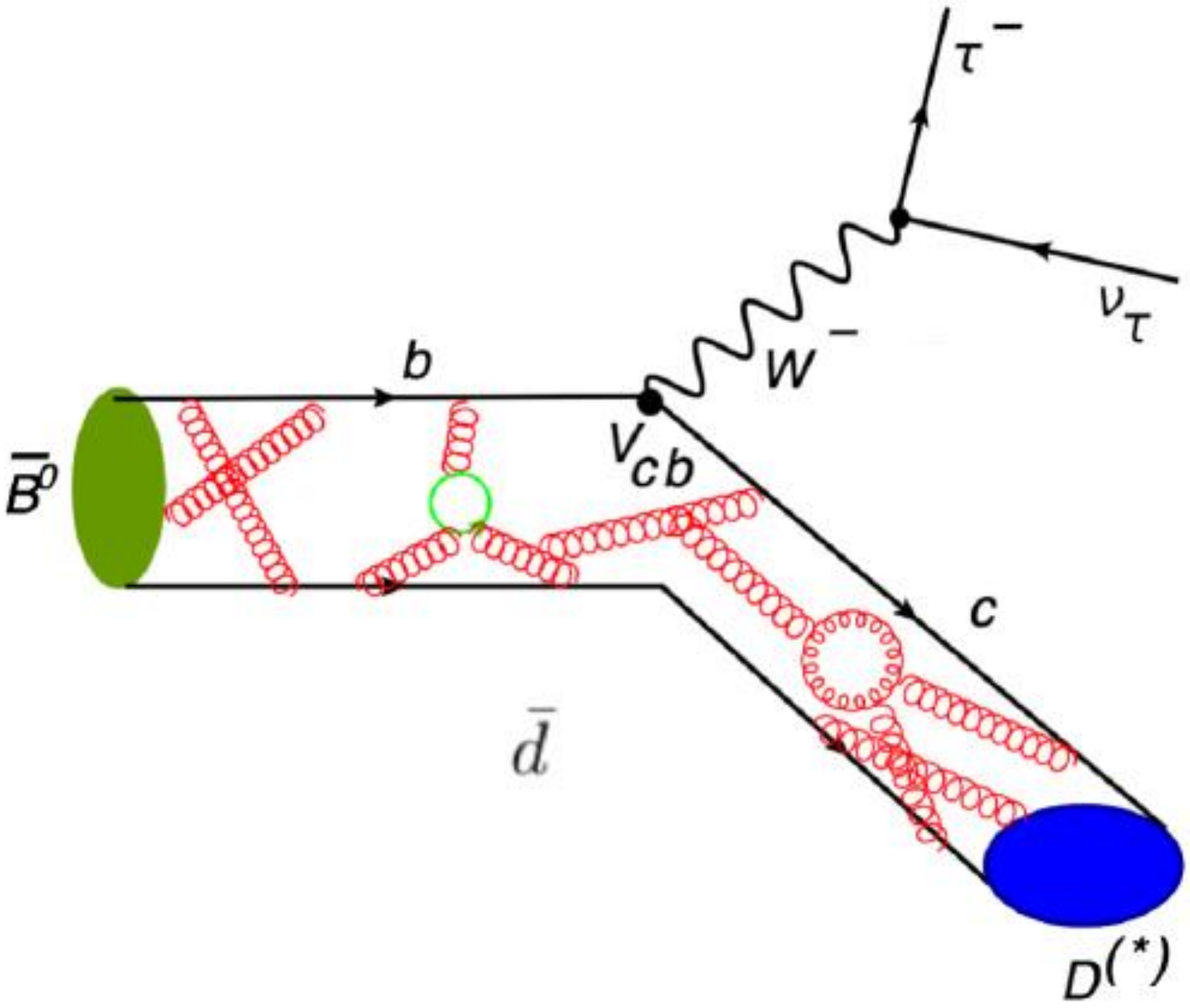



# My CKM Talk – Lightning Summary

## Differentiating Between the Scenarios

- Can differentiate between the different Wilson coefficients
- Urged experimentalists to make this measurement

# The Hadronic Problem!




Hadronic elements

Non-perturbative effects

Cannot be calculated in perturbation theory

Put it into form factors

# Separating Long and Short Distance Physics

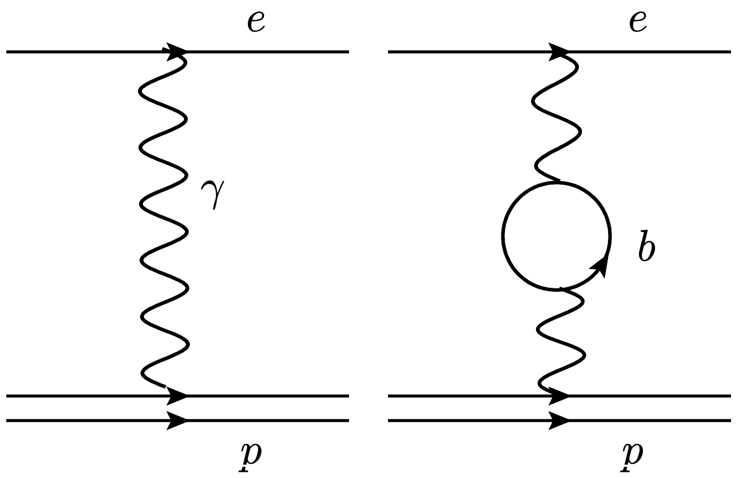
**Hydrogen Atom**

How does the presence of the bottom quark in the proton affect the electronic energy levels?

**Not much!**
But why?

Correction to the ground level:

$$E_0 = \frac{1}{2} m_e \alpha^2 \left[ 1 + \underbrace{\mathcal{O}\left( \frac{m_e^2}{m_b^2} \right)}_{10^{-8}} \right]$$

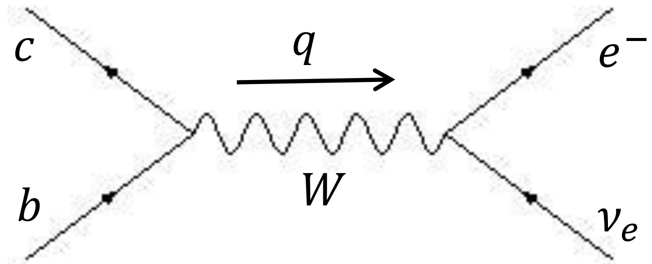



But… Separation of Scales allows one to forget about the high energy physics

# Separating Long and Short Distance Physics

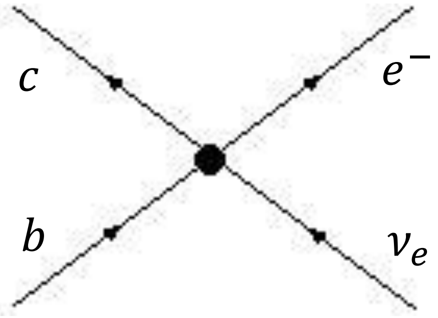
**Four Fermi Theory -** $b \to c + e^- + \bar{\nu}_e$

Consider the process in the b-rest frame




Full theory

Integrate out the W - boson




Effective Theory

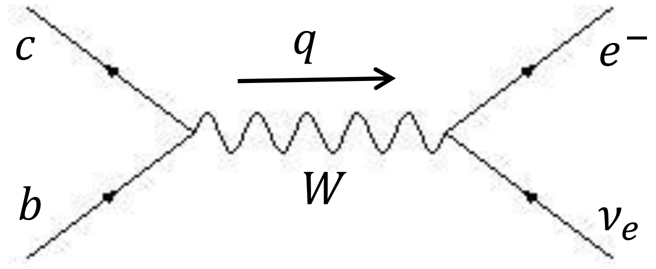
W-boson is much heavier than the CM energy – doesn't affect physics at low scales

Energy-momentum transfer is limited by b-quark mass; b-quark is far less massive than $W$-boson

$$\boxed{|q| < m_b \ll M_W}$$

# Separating Long and Short Distance Physics

**Scale of separation -** $\mu$




Choose a scale $\mu$ above which the **short distance physics** is given by the Wilson coefficients
Below the scale, there is **long distance physics** contained in the effective operators

$$\mathcal{H}_{eff} = \frac{4G_F}{\sqrt{2}} V_{cb} \sum_i \boldsymbol{C_i(\mu)} \boldsymbol{\mathcal{O}_i(\mu)}$$

The whole combination is independent of $\mu$

Good choice : $\mu \sim m_B$

Scale dependence of WCs $\Rightarrow$ Given by the anomalous dimensions (Won't talk about that!)

# Our Observables

Consider the differential branching ratio:

$$\frac{d^2\mathcal{B}}{dq^2 d\cos\theta} = \mathcal{N}\,|p_{D^{(*)}}|\,\left[\boldsymbol{a_\ell(q^2)} + \boldsymbol{b_\ell(q^2)}\cos\theta + \boldsymbol{c_\ell(q^2)}\cos^2\theta\right]$$

$$\mathcal{B} = \int \mathcal{N}\,|p_{D^{(*)}}|\,\left[2\boldsymbol{a_\ell(q^2)} + \frac{2}{3}\boldsymbol{c_\ell(q^2)}\right]$$

$R_{D^*}$(binned and unbinned) & $P_\tau$ = functions of $a_\ell$ and $c_\ell$

$\boxed{\textbf{Not } \boldsymbol{b_\ell}}$

FB Asymmetry = function of $b_\ell$

$\boxed{\textbf{TADA!}}$

# Operator Basis

$$\mathcal{O}^{cb\ell}_{\rm VL} = [\bar{c}\,\gamma^\mu\, b][\bar{\ell}\,\gamma_\mu\, P_L\,\nu] \qquad \mathcal{O}^{cb\ell}_{\rm VR} = [\bar{c}\,\gamma^\mu\, b][\bar{\ell}\,\gamma_\mu\, P_R\,\nu]$$

$$\mathcal{O}^{cb\ell}_{\rm AL} = [\bar{c}\,\gamma^\mu\,\gamma_5\, b][\bar{\ell}\,\gamma_\mu\, P_L\,\nu] \qquad \mathcal{O}^{cb\ell}_{\rm AR} = [\bar{c}\,\gamma^\mu\,\gamma_5\, b][\bar{\ell}\,\gamma_\mu\, P_R\,\nu]$$

$$\mathcal{O}^{cb\ell}_{\rm SL} = [\bar{c}\, b][\bar{\ell}\, P_L\,\nu] \qquad \mathcal{O}^{cb\ell}_{\rm SR} = [\bar{c}\, b][\bar{\ell}\, P_R\,\nu]$$

$$\mathcal{O}^{cb\ell}_{\rm PL} = [\bar{c}\,\gamma_5\, b][[\bar{\ell}\, P_L\,\nu] \qquad \mathcal{O}^{cb\ell}_{\rm PR} = [\bar{c}\,\gamma_5\, b][[\bar{\ell}\, P_R\,\nu]$$

$$\mathcal{O}^{cb\ell}_{\rm TL} = [\bar{c}\,\sigma^{\mu\nu}\, b][\bar{\ell}\,\sigma_{\mu\nu}\, P_L\,\nu] \qquad \mathcal{O}^{cb\ell}_{\rm TR} = [\bar{c}\,\sigma^{\mu\nu}\, b][\bar{\ell}\,\sigma_{\mu\nu}\, P_R\,\nu]$$

And their corresponding Wilson coefficients $C^{\tau}_{VL}, C^{\tau}_{AL}$ etc

Not all contribute to the two decays

**Related to the "popular" basis**

| | |
|---|---|
| $\mathcal{O}^{cb\ell}_{9} = [\bar{c}\,\gamma^\mu\,\mathrm{P}_L\, b][\bar{\ell}\,\gamma_\mu\,\nu]$ | $\mathcal{O}^{cb\ell\,\prime}_{9} = [\bar{c}\,\gamma^\mu\,\mathrm{P}_R\, b][\bar{\ell}\,\gamma_\mu\,\nu]$ |
| $\mathcal{O}^{cb\ell}_{10} = [\bar{c}\,\gamma^\mu\,\mathrm{P}_L\, b][\bar{\ell}\,\gamma_\mu\gamma_5\,\nu]$ | $\mathcal{O}^{cb\ell\,\prime}_{10} = [\bar{c}\,\gamma^\mu\,\mathrm{P}_R\, b][\bar{\ell}\,\gamma_\mu\gamma_5\,\nu]$ |
| $\mathcal{O}^{cb\ell}_{s} = [\bar{c}\,\mathrm{P}_L\, b][\bar{\ell}\,\nu]$ | $\mathcal{O}^{cb\ell\,\prime}_{s} = [\bar{c}\,\mathrm{P}_R\, b][\bar{\ell}\,\nu]$ |
| $\mathcal{O}^{cb\ell}_{p} = [\bar{c}\,\mathrm{P}_L\, b][[\bar{\ell}\,\gamma_5\,\nu]$ | $\mathcal{O}^{cb\ell\,\prime}_{p} = [\bar{c}\,\mathrm{P}_R\, b][[\bar{\ell}\,\gamma_5\,\nu]$ |
| $\mathcal{O}^{cb\ell}_{T} = [\bar{c}\,\sigma^{\mu\nu}\, b][\bar{\ell}\,\sigma_{\mu\nu}\,\nu]$ | $\mathcal{O}^{cb\ell}_{T5} = [\bar{c}\,\sigma^{\mu\nu}\, b][\bar{\ell}\,\sigma_{\mu\nu}\gamma_5\,\nu]$ |

# Form Factors & Symmetries

**Consider**

$$\langle D(p_D, M_D)|\mathcal{O}|\bar{B}(p_B, M_B)\rangle$$

Pseudoscalar $(0^-)$ Pseudoscalar $(0^-)$

$$\langle D^*(p_{D^*}, M_{D^*})|\mathcal{O}|\bar{B}(p_B, M_B)\rangle$$

Vector $(1^+)$ Pseudoscalar $(0^-)$

**Since the matrix element transforms in a certain way, only those operators transforming properly w.r.t the mesonic fields contribute to the transition**

**Symmetry Principle**

✓ Vector
Scalar
Tensor

✗ Axialvector
Pseudoscalar

✓ Vector
Pseudoscalar
Axialvector
Tensor

✗ Scalar

# Form Factors: $B \to D$ Decays

$$\langle D(p_D, M_D)|\bar{c}\gamma^\mu b|\bar{B}(p_B, M_B)\rangle = \boldsymbol{A}\,(p_B + p_D)^\mu + \boldsymbol{B}\,(p_B - p_D)^\mu = \boldsymbol{A}\,(p_B + p_D)^\mu + \boldsymbol{B}\,q^\mu$$

Etc.

$q_\mu$

$$\begin{aligned}
\langle D(p_D, M_D)|\bar{c}\gamma^\mu b|\bar{B}(p_B, M_B)\rangle &= F_+(q^2)\left[(p_B + p_D)^\mu - \frac{M_B^2 - M_D^2}{q^2} q^\mu\right] \\
&\quad + F_0(q^2)\frac{M_B^2 - M_D^2}{q^2} q^\mu \\
\langle D(p_D, M_D)|\bar{c}\gamma^\mu\gamma_5 b|\bar{B}(p_B, M_B)\rangle &= 0 \\
\langle D(p_D, M_D)|\bar{c}b|\bar{B}(p_B, M_B)\rangle &= F_0(q^2)\frac{M_B^2 - M_D^2}{m_b - m_c} \\
\langle D(p_D, M_D)|\bar{c}\gamma_5 b|\bar{B}(p_B, M_B)\rangle &= 0
\end{aligned}$$

# Form Factors: $B \to D^*$ Decays

$\langle D^*(p_{D^*}, M_{D^*})|\bar{c}\gamma_\mu b|\bar{B}(p_B, M_B)\rangle$ ⇒ Transforms as an axial vector

Three vectors in the system: $p_B^\mu$, $p_{D^*}^\mu$ & $\epsilon^\mu$

**Challenge:** Use these three vectors to form an object which transforms like an axial vector -

Only one possibility: $\boldsymbol{A}\, \varepsilon_{\mu\nu\rho\sigma}\epsilon^{*\nu} p_B^\rho p_{D^*}^\sigma$

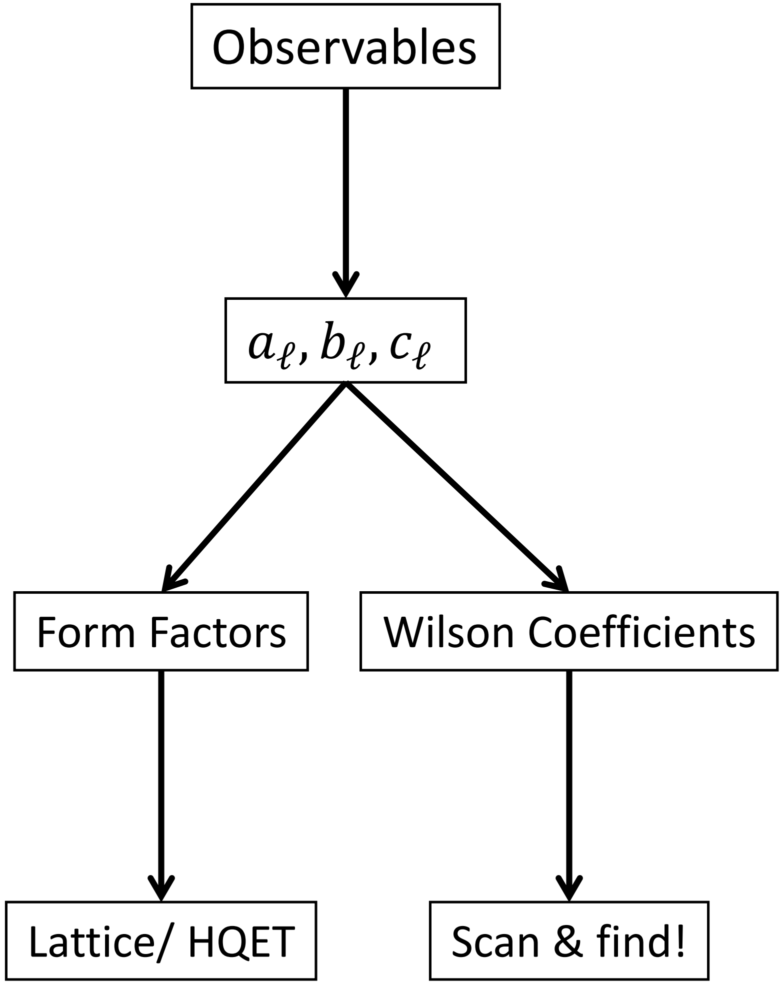
$$\langle D^*(p_{D^*}, M_{D^*})|\bar{c}\gamma_\mu b|\bar{B}(p_B, M_B)\rangle = i\varepsilon_{\mu\nu\rho\sigma}\epsilon^{\nu *} p_B^\rho p_{D^*}^\sigma \frac{2V(q^2)}{M_B + M_{D^*}}$$

$$\begin{aligned}
\langle D^*(p_{D^*}, M_{D^*})|\bar{c}\gamma_\mu\gamma_5 b|\bar{B}(p_B, M_B)\rangle &= 2M_{D^*}\frac{\epsilon^*.q}{q^2} q_\mu A_0(q^2) + (M_B + M_{D^*})\left[\epsilon_\mu^* - \frac{\epsilon^*.q}{q^2} q_\mu\right] A_1(q^2) \\
&\quad - \frac{\epsilon^*.q}{M_B + M_{D^*}}\left[(p_B + p_{D^*})_\mu - \frac{M_B^2 - M_{D^*}^2}{q^2} q_\mu\right] A_2(q^2) \\
\langle D^*(p_{D^*}, M_{D^*})|\bar{c}b|\bar{B}(p_B, M_B)\rangle &= 0 \\
\langle D^*(p_{D^*}, M_{D^*})|\bar{c}\gamma_5 b|\bar{B}(p_B, M_B)\rangle &= -\epsilon^*.q\,\frac{2M_{D^*}}{m_b + m_c} A_0(q^2)
\end{aligned}$$

# Eye to Eye with the Observables!



$$
\begin{aligned}
a_\ell^D(+) &= \frac{2\left(M_B^2 - M_D^2\right)^2}{\left(m_b - m_c\right)^2} |\mathbf{C}_{\mathbf{SL}}^\ell|^2 \mathbf{F}_\mathbf{0}^\mathbf{2} \\
&+ m_\ell \left[ \frac{4(M_B^2 - M_D^2)^2}{q^2 \left(m_b - m_c\right)} \mathcal{R}\left(\mathbf{C}_{\mathbf{VL}}^\ell \mathbf{C}_{\mathbf{SL}}^{\ell *}\right) \mathbf{F}_\mathbf{0}^\mathbf{2} \right] \\
&+ m_\ell^2 \left[ \frac{2\left(M_B^2 - M_D^2\right)^2}{q^4} |\mathbf{C}_{\mathbf{VL}}^\ell|^2 \mathbf{F}_\mathbf{0}^\mathbf{2} \right] \\
b_\ell^D(+) &= -m_\ell \left[ \frac{8|p_D| M_B \left(M_B^2 - M_D^2\right)}{q^2 \left(m_b - m_c\right)} \mathcal{R}\left(\mathbf{C}_{\mathbf{SL}}^\ell \mathbf{C}_{\mathbf{VL}}^{\ell *}\right) \mathbf{F_0 F_+} \right] \\
&- m_\ell^2 \left[ \frac{8|p_D| M_B \left(M_B^2 - M_D^2\right)}{q^4} |\mathbf{C}_{\mathbf{VL}}^\ell|^2 \mathbf{F_0 F_+} \right] \\
c_\ell^D(+) &= m_\ell^2 \left[ \frac{8|p_D|^2 M_B^2}{q^4} |\mathbf{C}_{\mathbf{VL}}^\ell|^2 \mathbf{F}_+^\mathbf{2} \right]
\end{aligned}
$$



# Symmetries in the Standard Model

# A Big Basket of SM Symmetries

(Aside)

A lot of global symmetries of the Standard Model -

(not the usual gauge symmetry)

*Introduction to Flavour Physics*
Grossmann
arXiv: 1006.3534

**"Accidental" Symmetries**

Flavour symmetry of the Q-doublets

$$\mathcal{G}_F^q = U(3)_Q \times U(3)_U \times U(3)_D$$

Flavour symmetry of the U-type singlets

Flavour symmetry of the D-type singlets

No. of generators : $\mathbf{9 \times 3 = 27}$

Write down Yukawa fields as spurions; in this $\mathcal{G}_F^q$ space:

$$\mathcal{L}_Y = \bar{Q}_L\, \boldsymbol{Y_U}\, \tilde{\phi}\, U_R + \bar{Q}_L\, \boldsymbol{Y_D}\, \phi\, D_R$$

**Spurions**

$Q_L = (3, 1, 1)$

$U_R = (1, 3, 1)$

$D_R = (1, 1, 3)$

$\boldsymbol{Y_U = (3, \bar{3}, 1)}$

$\boldsymbol{Y_D = (3, 1, \bar{3})}$

# A Big Basket of SM Symmetries

(Aside)

Yukawas → $3 \times 3$ matrices; Total of **36 independent parameters**

**Broken Symmetry:**

$$U(3)_Q \times U(3)_U \times U(3)_D \qquad \textbf{27 generators}$$

$$\Big\downarrow \langle \boldsymbol{Y_U} \rangle, \langle \boldsymbol{Y_D} \rangle \qquad \textbf{26 broken generators}$$

$$U(1)_B \qquad \textbf{1 generator}$$

Use $N_{broken} = 26$ to rotate away most of the Yukawa

$$\boldsymbol{N_{physical} = N_{total} - N_{broken} = 36 - 26 = 10}$$

Interpret $N_{physical} = 10$ as **6 quark masses, 3 mixing angles and 1 CP violating phase**

Homework

Try this with 2 generations of quarks, instead of 3. Show that there is no CP violation.

# Homework Solution

(CheatSheet)

For two generations, the symmetry is: $U(2)_Q \times U(2)_U \times U(2)_D$ $\downarrow$ $U(1)_B$

12 generators

11 broken generators

1 generator

No. of Yukawa elements = $8 \times 2 = 16$

Thus, $N_{physical} = 16 - 11 = 5$

Interpret them as the masses of the 4 quarks and the rotation angle in the $2 \times 2$ mixing matrix

**No phase appears → No CP violation**

# Heavy Quark Effective Theory

**Effective Field Theory Course :**
By Prof. Iain Stewart
MIT OpenCourseWare (OCW)

**References for this section:**

1. *Review of Heavy Quark Effective Theory*
   Thomas Mannel
   hep-ph/9611411
2. *Heavy Quark Expansion*
   Thorsten Feldmann
   Talk in 2010: Feldmann.pdf
3. *Heavy Quark Physics*
   Aneesh Manohar and Mark Wise
   CUP published book

# Setting It Up

Have a heavy quark in the process – take $m_Q \to \infty$ ; gives a model-independent starting point

The mass of the quark is typically $m_Q \gg \Lambda_{QCD}$

Expand in powers of $1/m_Q$ or $\Lambda_{QCD}/m_Q$

Non-Relativistic form of the Dirac equation

On-shell momentum

Momentum: $p_Q = m_Q v + k$

"Tickle"

$v^\mu \Rightarrow$ Some velocity parameter
$v^2 = 1$

Quark field: $Q(x) = e^{-iM_Q v.x}[Q_v(x) + B_v(x)]$

Why the exponential?

Components: $e^{iM_Q v.x}\frac{1+\not{v}}{2}Q = Q_v$ $\quad e^{iM_Q v.x}\frac{1-\not{v}}{2}Q = B_v$

Propagator: $\frac{\not{p} + M_Q}{p^2 - M_Q^2 + i\varepsilon} = \left(\frac{1+\not{v}}{2}\right)\frac{1}{v.k + i\,\varepsilon}$

# HQET Lagrangian

The (matter part) HQET Lagrangian can be derived from the QCD Lagrangian

$$\mathcal{L}_{HQET} = (\bar{Q}_v + \bar{B}_v)\, e^{iM_Q v.x} \left[\not{v}\, iv.D + i\,\not{D}_T\; - M_Q\right] e^{-iM_Q v.x} (Q_v + B_v)$$

$$i\not{D} = \not{v}\, iv.D + i\,\not{D}_T$$

Simplify

Hint: Use –
$\not{v} Q_v = Q_v$
$\not{v} B_v = -B_v$

$$\mathcal{L}_{HQET} = \bar{Q}_v iv.D Q_v \;-\; \bar{B}_v\left(iv.D + 2\,M_Q\right)B_v + \bar{Q}_v i D_T B_v + \bar{B}_v i\, D_T Q_v$$

$B_v$ field propagator is suppressed!

$$\boxed{\mathcal{L}_{HQET} = \bar{Q}_v (iv.D) Q_v}$$

Only one type of field – no massive anti-matter fields

No pair production

No annihilation

# Spin and Flavour Symmetries

$$\boxed{\mathcal{L}_{HQET} = \bar{Q}_v(iv.D)Q_v}$$

The Lagrangian is invariant under rotations in flavour space – no mass appears

Also spin symmetry

Spin of the heavy quark and the light degrees of freedom are separately conserved

$\boldsymbol{SU(2N_h)}$ ⇒ **Spin-flavour symmetry**

Consider spin symmetry : $m_{B^*} = m_B$ in exact HQ symmetry limit → Obviously not true

Hyperfine corrections break degeneracy!

$$m_{H^*} - m_H \propto \frac{1}{m_Q} \Rightarrow \left(m_{H^*}^2 - m_H^2\right) = \text{const}$$

$$m_{B^*}^2 - m_B^2 \approx 0.49\ \text{GeV}^2$$
$$m_{D^*}^2 - m_D^2 \approx 0.55\ \text{GeV}^2$$

For Flavour symmetry, the spectator quark flavour must not matter

$$m_{H_s} - m_H = \text{const}$$

$$m_{B_s} - m_B \approx 100\ \text{MeV}$$
$$m_{D_s} - m_D \approx 100\ \text{MeV}$$

# Calculation of the Form Factors (Technical)

$$\langle D|V^\mu|B\rangle = \boldsymbol{h_+}(v_b + v_c)^\mu + \boldsymbol{h_-}(v_b + v_c)^\mu$$

$$\boldsymbol{w = v_b . v_c}$$

$$\langle D^*|V^\mu|B\rangle = \boldsymbol{h_V}\varepsilon^{\mu\nu\alpha\beta}\epsilon_\nu v_{b\alpha} v_{c\beta}$$

$$\boxed{\text{No } \epsilon^* . v_c \text{ term – why?}}$$

$$\langle D^*|A^\mu|B\rangle = -i\boldsymbol{h_{A1}}(w)(w+1)\epsilon^{*\mu} + i\boldsymbol{h_{A2}}(w)(\epsilon^* . v_b)v_b^\mu + i\boldsymbol{h_{A3}}(w)(\epsilon^* . v_b)v_c^\mu$$

HQET relates all these form factors to each other

$$\left\langle H^c\middle|\bar{c}\Gamma b\middle|H^b\right\rangle = \left\langle H^c\middle|\bar{c}_v\Gamma b_v\middle|H^b\right\rangle$$

- Current is invariant under individual rotations of b and c
- Thus $\Gamma \rightarrow D(R)_c\Gamma\, D(R)_b$ is a symmetry
- Represent currents by operators each containing a heavy field

$$\boxed{\bar{c}_{v'}\Gamma b_v = \mathrm{Tr}\,(X\, \bar{H}^c_{v_c}\Gamma H^b_{v_b})}$$

- Put in the most general form of $X$
- $X = X_0 + X_1 \not{v}_b + X_2 \not{v}_c + X_3 \not{v}_b \not{v}_c$

$$\boxed{\bar{c}_{v'}\Gamma b_v = -\xi(\mathrm{w})\mathrm{Tr}\,(\bar{H}^c_{v_c}\Gamma H^b_{v_b})}$$

# Calculation of the Form Factors (Technical)

$$\boxed{\bar{c}_{v'}\Gamma b_v = -\xi(\mathrm{w})\mathrm{Tr}\,(\bar{H}^c_{v_c}\Gamma H^b_{v_b})}$$

Put in different gamma matrices and compute trace!

$$\langle D|V^\mu|B\rangle = h_+(v_b+v_c)^\mu + h_-(v_b+v_c)^\mu \Rightarrow \boldsymbol{\langle D|V^\mu|B\rangle = \xi(w)[v_b^\mu + v_c^\mu]}$$

$$\langle D^*|V^\mu|B\rangle = h_V\varepsilon^{\mu\nu\alpha\beta}\epsilon_\nu v_{b\alpha}v_{c\beta} = \boldsymbol{\xi(w)\varepsilon^{\mu\nu\alpha\beta}\epsilon_\nu v_{b\alpha}v_{c\beta}}$$

$$\begin{aligned}\langle D^*|A^\mu|B\rangle &= -ih_{A1}(w)(w+1)\epsilon^{*\mu} + ih_{A2}(w)(\epsilon^*.v_b)v_b^\mu + ih_{A3}(w)(\epsilon^*.v_b)v_c^\mu \\ &= \boldsymbol{-i\,\xi(w)[(w+1)\epsilon^{*\mu} - (\epsilon^*.v_b)v_c^\mu]}\end{aligned}$$

$$\boxed{\begin{gathered}h_+(w) = h_v(w) = h_{A1}(w) = h_{A3}(w) = \xi(w)\\ h_-(w) = h_{A2}(w) = 0\end{gathered}}$$

*Did not show, but true:* Decay widths are related to these HQET parameters. They are measured experimentally.

# Summary

Introduction to Effective Theory using Four Fermi interaction

- Separation of Scales
- The issue of the factorization scale $\mu$

Form Factors and Symmetries

- Symmetry transformations constrain the possibilities
- Use Formfactors to hide our ignorance

Global Symmetries of the Standard Model

Heavy Quark Effective Theory

- Heavy Quark Symmetry – simplified
- Calculation of form factors (sketch)