Study of discrete symmetries at $\Upsilon(5S)$

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B(s) meson system

- Flavour eigenstates
  - $B^0 \equiv b\bar{d}$,$\bar{B}^0 \equiv \bar{b}d$; Mass $\backsim$ 5279 MeV , Lifetime $\backsim 1.5ps$
  - $B_s^0 \equiv b\bar{s}$,$\bar{B}_s^0 \equiv \bar{b}s$; Mass $\backsim$ 5366 MeV , Lifetime $\backsim 1.5ps$
- CP eigenstates
  - CP transformation of Flavoured mesons: $\mathcal{CP}\left|B^0\right\rangle = \left|\bar{B}^0\right\rangle ; \mathcal{CP}\left|\bar{B}^0\right\rangle = \left|B^0\right\rangle$
  - CP eigenstates are thus superposition of flavoured mesons

$$B_+ \equiv \frac{1}{\sqrt{2}}(B^0 + \bar{B}^0) \qquad B_- \equiv \frac{1}{\sqrt{2}}(B^0 - \bar{B}^0)$$

CP even   CP odd

- Effective Hamiltonian
  - Dynamics of $B^0(\bar{B}^0)$ meson is given by time dependent Schrödinger equation

$$i\frac{\partial}{\partial t}\begin{pmatrix} B^0 \\ \bar{B}^0 \end{pmatrix} = \mathcal{H}\begin{pmatrix} B^0 \\ \bar{B}^0 \end{pmatrix} = \begin{pmatrix} M_{11} - i\frac{\Gamma_{11}}{2} & M_{12} - i\frac{\Gamma_{12}}{2} \\ M_{21} - i\frac{\Gamma_{21}}{2} & M_{22} - i\frac{\Gamma_{22}}{2} \end{pmatrix}\begin{pmatrix} B^0 \\ \bar{B}^0 \end{pmatrix}$$

  - M_{11} and M_{22} are masses of B^0 and $\bar{B}^0$ in absence of weak interactions.
  - Eigenvectors corresponding to $\mathcal{H} \Rightarrow \begin{pmatrix} B_H \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ B_L \end{pmatrix}$ $\rightarrow$ Physical particles
  - Eigenvalues of $\mathcal{H} \Rightarrow \mu_{H,L} = m_{H,L} - \frac{i}{2}\Gamma_{H,L}$ $\rightarrow$masses and decay widths

Discrete Symmetries

- CP violation

$$A_{CP} = \frac{\Gamma(B^0 \to l^+\nu_l D^-) - \Gamma(\bar{B}^0 \to l^-\bar{\nu}_l D^+)}{\Gamma(B^0 \to l^+\nu_l D^-) + \Gamma(\bar{B}^0 \to l^-\bar{\nu}_l D^+)}$$

- T violation

$$A_T = \frac{\Gamma(B^0 \to l^+\nu_l D^-) - \Gamma(l^+\nu_l D^- \to B^0)}{\Gamma(B^0 \to l^+\nu_l D^-) + \Gamma(l^+\nu_l D^- \to B^0)}$$

- CPT violation

$$A_{CPT} = \frac{\Gamma(B^0 \to l^+\nu_l D^-) - \Gamma(l^-\bar{\nu}_l D^+ \to \bar{B}^0)}{\Gamma(B^0 \to l^+\nu_l D^-) + \Gamma(l^-\bar{\nu}_l D^+ \to \bar{B}^0)}$$

B Factories

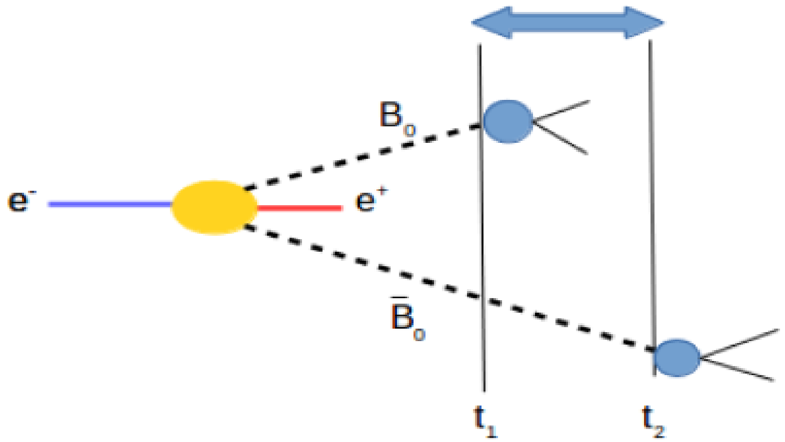
- Antisymmetric B factories are needed where e^- and e^+ beams collide at a centre of mass energy of 10.58 GeV $\backsim$ mass of $\Upsilon(4s)$.
  - PEP-II e^+e^- collider at SLAC

    e^+beam energy= 3.1 GeV

    e^- beam energy= 9.0 GeV

  - This gives $\Upsilon(4s)$ a boost $(\beta\gamma) \backsim 0.56$
- $\Upsilon(4s)$ then decays to $B\bar{B}$ $(B^0\bar{B}^0,\ B^+B^-)$ through strong processes.
- Purpose of an antisymmetric factory is to physically separate the decays of two B mesons $\mathcal{O}(100\mu m)$

Using $\Delta t \backsim \Delta z/\beta\gamma c$, time difference between two decays can be measured(1.5ps)



Initial state

- $J^{PC}(\Upsilon(4s)) = 1^{--}$.
- Bose-Einstein symmetry $\implies \mathcal{CP} = +$.
- S=0, L=1. $\implies \mathcal{C} = -; \mathcal{P} = -$ $[\mathcal{C} = (-1)^{L+S}; \mathcal{P} = (-1)^{L}]$
- Wavefunction of $B^0 \bar{B}^0$ pair is antisymmetric

$$|i\rangle = \frac{1}{\sqrt{2}}[\left|B^0(\vec{k})\bar{B}^0(-\vec{k}\right\rangle - \left|\bar{B}^0(\vec{k})B^0(-\vec{k})\right\rangle]$$

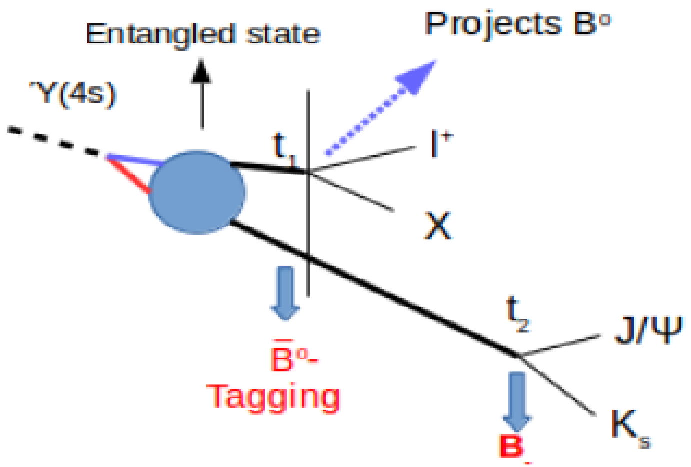
In terms of CP eigenstates,

$$|i\rangle = \frac{1}{\sqrt{2}}[\left|B_+(\vec{k})B_-(-\vec{k})\right\rangle - \left|B_-(\vec{k})B_+(-\vec{k})\right\rangle]$$

EPR Entanglement

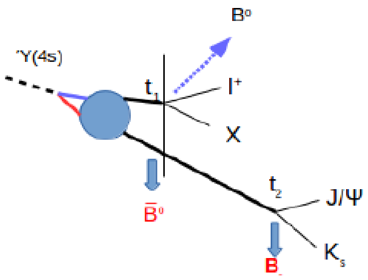
- EPR Entanglement states, For the entangled state of the two mesons, the individual state of each neutral meson is not defined before its collapse as a filter imposed by the observation of the decay.
- The state of the first B to decay at t_1 dictates the state of other B.
  - Using $\Delta b = \Delta q$ rule, we know that only $B^0 \rightarrow l^+$ and $\bar{B}^0 \rightarrow l^-$
  - $\bar{B}^0$ can now evolve independently and will undergo mixing.
  - **Tagging Mechanism**

Observed particle	decaying particle
l^+	B^0
l^-	$\bar{B}^0$
$J/\Psi K_S$	B_-
$J/\Psi K_L$	B_+



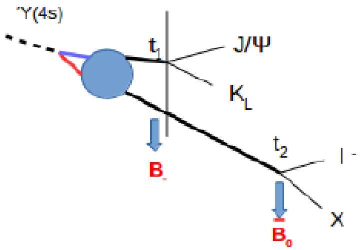
T-conjugate process

Process



$\bar{B}^0 \to B_-$

T-Conjugated Process



$B_- \to \bar{B}^0$

All possible T conjugated processes:

Reference		*T*-conjugate	
Transition	Final state	Transition	Final state
$\bar{B}^0 \to B_-$	$(\ell^+ X, J/\psi K_S)$	$B_- \to \bar{B}^0$	$(J/\psi K_L, \ell^- X)$
$B_+ \to B^0$	$(J/\psi K_S, \ell^+ X)$	$B^0 \to B_+$	$(\ell^- X, J/\psi K_L)$
$\bar{B}^0 \to B_+$	$(\ell^+ X, J/\psi K_L)$	$B_+ \to \bar{B}^0$	$(J/\psi K_S, \ell^- X)$
$B_- \to B^0$	$(J/\psi K_L, \ell^+ X)$	$B^0 \to B_-$	$(\ell^- X, J/\psi K_S)$

CP and CPT asymmetries

The same methodology can be applied to test CP and CPT invariance.

Reference		*CP*-conjugate	
Transition	Final state	Transition	Final state
$\overline{B}^0 \to B_-$	$(\ell^+ X, J/\psi K_S)$	$B^0 \to B_-$	$(\ell^- X, J/\psi K_S)$
$B_+ \to B^0$	$(J/\psi K_S, \ell^+ X)$	$B_+ \to \overline{B}^0$	$(J/\psi K_S, \ell^- X)$
$\overline{B}^0 \to B_+$	$(\ell^+ X, J/\psi K_L)$	$B^0 \to B_+$	$(\ell^- X, J/\psi K_L)$
$B_- \to B^0$	$(J/\psi K_L, \ell^+ X)$	$B_- \to \overline{B}^0$	$(J/\psi K_L, \ell^- X)$

Reference		*CPT*-conjugate	
Transition	Final state	Transition	Final state
$\overline{B}^0 \to B_-$	$(\ell^+ X, J/\psi K_S)$	$B_- \to B^0$	$(J/\psi K_L, \ell^+ X)$
$B_+ \to B^0$	$(J/\psi K_S, \ell^+ X)$	$\overline{B}^0 \to B_+$	$(\ell^+ X, J/\psi K_L)$
$B^0 \to B_-$	$(\ell^- X, J/\psi K_S)$	$B_- \to \overline{B}^0$	$(J/\psi K_L, \ell^- X)$
$B_+ \to \overline{B}^0$	$(J/\psi K_S, \ell^- X)$	$B^0 \to B_+$	$(\ell^- X, J/\psi K_L)$

Results by Babar

Coefficient	Assumed value	Fit value
$\Delta S_{\mathrm{T}}^{+} = S_{\ell^-,K_L}^{-} - S_{\ell^+,K_S}^{+}$	-1.4	-1.57 ± 0.15
$\Delta S_{\mathrm{T}}^{-} = S_{\ell^-,K_L}^{+} - S_{\ell^+,K_S}^{-}$	1.4	1.25 ± 0.19
$\Delta C_{\mathrm{T}}^{+} = C_{\ell^-,K_L}^{-} - C_{\ell^+,K_S}^{+}$	0.0	-0.07 ± 0.14
$\Delta C_{\mathrm{T}}^{-} = C_{\ell^-,K_L}^{+} - C_{\ell^+,K_S}^{-}$	0.0	-0.09 ± 0.14
$\Delta S_{\mathrm{CP}}^{+} = S_{\ell^-,K_S}^{+} - S_{\ell^+,K_S}^{+}$	-1.4	-1.65 ± 0.11
$\Delta S_{\mathrm{CP}}^{-} = S_{\ell^-,K_S}^{-} - S_{\ell^+,K_S}^{-}$	1.4	1.54 ± 0.13
$\Delta C_{\mathrm{CP}}^{+} = C_{\ell^-,K_S}^{+} - C_{\ell^+,K_S}^{+}$	0.0	0.03 ± 0.10
$\Delta C_{\mathrm{CP}}^{-} = C_{\ell^-,K_S}^{-} - C_{\ell^+,K_S}^{-}$	0.0	-0.09 ± 0.10
$\Delta S_{\mathrm{CPT}}^{+} = S_{\ell^+,K_L}^{-} - S_{\ell^+,K_S}^{+}$	0.0	-0.25 ± 0.22
$\Delta S_{\mathrm{CPT}}^{-} = S_{\ell^+,K_L}^{+} - S_{\ell^+,K_S}^{-}$	0.0	0.04 ± 0.13
$\Delta C_{\mathrm{CPT}}^{+} = C_{\ell^+,K_L}^{-} - C_{\ell^+,K_S}^{+}$	0.0	-0.04 ± 0.15
$\Delta C_{\mathrm{CPT}}^{-} = C_{\ell^+,K_L}^{+} - C_{\ell^+,K_S}^{-}$	0.0	-0.06 ± 0.13

- $\Delta S = \frac{2\Im(\lambda)}{1+|\lambda|^2}$; $\Delta C = \frac{1-|\lambda|^2}{1+|\lambda|^2}$; $\lambda \propto \frac{<f|H|B^0>}{<f|H|\bar{B}^0>}$.
- Results are in accordance with Standard Model within C.L. of 6σ.

$\Upsilon(5s)$

- $\Upsilon(5s)$ is a bound state of $b\bar{b}$ at resonance 10.860 GeV.

Υ(10860) DECAY MODES

	Mode	Fraction (Γ_i/Γ)	Confidence level
Γ_1	$B\overline{B}X$	($76.2\ ^{+2.7}_{-4.0}$) %	
Γ_2	$B\overline{B}$	(5.5 $\pm$1.0) %	
Γ_3	$B\overline{B}^*$ + c.c.	(13.7 $\pm$1.6) %	
Γ_4	$B^*\overline{B}^*$	(38.1 $\pm$3.4) %	
Γ_5	$B\overline{B}^{(*)}\pi$	< 19.7 %	90%
Γ_6	$B\overline{B}\pi$	(0.0 $\pm$1.2) %	
Γ_7	$B^*\overline{B}\pi$ + $B\overline{B}^*\pi$	(7.3 $\pm$2.3) %	
Γ_8	$B^*\overline{B}^*\pi$	(1.0 $\pm$1.4) %	
Γ_9	$B\overline{B}\pi\pi$	< 8.9 %	90%
Γ_{10}	$B_s^{(*)}\overline{B}_s^{(*)}$	(20.1 $\pm$3.1) %	
Γ_{11}	$B_s\overline{B}_s$	(5 $\pm$5) $\times 10^{-3}$	
Γ_{12}	$B_s\overline{B}_s^*$ + c.c.	(1.35$\pm$0.32) %	
Γ_{13}	$B_s^*\overline{B}_s^*$	(17.6 $\pm$2.7) %	

The analysis discussed for $\Upsilon(4S)$ can be carried out at resonance $\Upsilon(5S)$ but that is more tricky because of two reasons.

- It decays to a state of entangled B ($\Delta m = 0.51 ps^{-1}$; $\Delta\Gamma = 0.007 s^{-1}$) mesons as well as B_s ($\Delta m = 17.8 ps^{-1}$; $\Delta\Gamma = 0.08 ps^{-1}$) mesons.

- Different final states have different C-parities.

Parametrizing quantities

- Following parametrization has been used to study violation of discrete symmetries

$$\epsilon \equiv \frac{\epsilon_1 + \epsilon_2}{2} \qquad \delta = \epsilon_2 - \epsilon_1$$

- Consider the conditions imposed by discrete symmetries on elements of effective mass matrix
  - CP conservation $\Longrightarrow \Im(M_{12} CP^*_{12}) = \Im(\Gamma_{12} CP^*_{12}) = 0$ and $H_{11} = H_{22}$.
  - CPT conservation $\Longrightarrow H_{11} = H_{22}$.
  - T invariance $\Longrightarrow \Im(M_{12} CP^*_{12}) = \Im(\Gamma_{12} CP^*_{12}) = 0$
- The conditions in terms of ϵ and δ,
  - $\Re(\epsilon) \neq 0$ signals CP and T violation, if $\Delta\Gamma \neq 0$;
  - $\Im(\epsilon) \neq 0$ indicates CP and T violation;
  - $\Re(\delta) \neq 0$ implies CP and CPT violation;
  - $\Im(\delta) \neq 0$ means CP and CPT violation, if $\Delta\Gamma \neq 0$.
- Objective is to constrain these 4 observables.

$$|B_H\rangle = \frac{1}{\sqrt{1+\epsilon_1^2}}(|B_+\rangle + \epsilon_1 |B_-\rangle)$$

$$|B_L\rangle = \frac{1}{\sqrt{1+\epsilon_2^2}}(|B_-\rangle + \epsilon_2 |B_+\rangle)$$

Asymmetries

- Defining different asymmetries as
  1. **Equal semileptonic:** This is defined as

$$A_1 = \frac{P(l^+, l^+) - P(l^-, l^-)}{P(l^+, l^+) + P(l^-, l^-)}$$

where $P(a, b)$ is the transition probability of observering a at time t_1 and b at time t_2.

$$P(l^+, l^+) \implies \bar{B}^0 \to B^0$$
$$P(l^-, l^-) \implies \bar{B}^0 \to B^0$$

  2. **Unequal semileptonic:** which is defined as

$$A_2 = \frac{P(l^+, l^-) - P(l^-, l^+)}{P(l^+, l^-) + P(l^-, l^+)}$$

$$P(l^+, l^-) \implies \bar{B}^0 \to \bar{B}^0$$
$$P(l^-, l^+) \implies B^0 \to B^0$$

  3. **leptonic-hadronic:** It involves detection of one leptonic flavour specific state and one hadronic eigenstate. It is defined as

$$A_3 = \frac{P(K_s, l^+) - P(K_l, l^-)}{P(K_s, l^+) + P(K_l, l^-)}$$

$$P(K_s, l^+) \implies B_+ \to B^0$$
$$P(K_s, l^-) \implies B_+ \to \bar{B}^0$$

C=-1 state

- In the expressions, x and y are defined as, $x = \frac{\Delta m}{\Gamma}$; $y = \frac{\Delta\Gamma}{\Gamma}$
- tm is the time difference in observation of a and b.
- ϵ and δ have been redefined as $\kappa\epsilon$ and $\kappa\delta$ respectively, where κ is the expansion parameter. Only leading terms are shown here:
  -
$$A_1 = 4\kappa\Re(\epsilon)$$
  -
$$A_2 = -\frac{2\kappa(\Im(\delta)\sin(\Delta\mathrm{mtm}) - \Re(\delta)\sinh(\Delta\Gamma\mathrm{tm}))}{\cosh(\Delta\Gamma\mathrm{tm}) + \cos(\Delta\mathrm{mtm})}$$
  -
$$A_3 = \kappa[\frac{(\Im(\delta) - 2\Im(\epsilon))\sin(\Delta\mathrm{mtm})}{\sinh(\Delta\Gamma\mathrm{tm}) + \cosh(\Delta\Gamma\mathrm{tm})} +$$
$$\frac{(-\Re(\delta) + 2\Re(\epsilon))(\sinh(\Delta\Gamma\mathrm{tm}) + \cosh(\Delta\Gamma\mathrm{tm}) - \cos(\Delta\mathrm{mtm}))}{\sinh(\Delta\Gamma\mathrm{tm}) + \cosh(\Delta\Gamma\mathrm{tm})}]$$

C=+1 state

- $A_1 =$

$$\frac{\begin{aligned}2\kappa\Big[-\Re(\delta)\left(x^3\sinh(\Delta\Gamma tm) + x^3 y\cosh(\Delta\Gamma tm) + 2xy\cos(\Delta mtm) + 8y\sin(\Delta mtm)\right) \\ + \Im(\delta)\left(2x^2\cosh(\Delta\Gamma tm) + x^2\cos(\Delta mtm) + 3x\sin(\Delta mtm) + 8y\sinh(\Delta\Gamma tm)\right) \\ + 2x\Re(\epsilon)\left(-\cos(\Delta mtm) + x^2\cosh(\Delta\Gamma tm) + x^2 y\sinh(\Delta\Gamma tm) + x\sin(\Delta mtm)\right)\Big]\end{aligned}}{x\left(-\cos(\Delta mtm) + x^2\cosh(\Delta\Gamma tm) + x^2 y\sinh(\Delta\Gamma tm) + x\sin(\Delta mtm)\right)}$$

- $A_2 =$

$$\frac{\begin{aligned}-4\kappa\Big[\Im(\delta)\left(-x^2\cosh(\Delta\Gamma tm) + x^2\cos(\Delta mtm) + 2x\sin(\Delta mtm) - 4y\sinh(\Delta\Gamma tm)\right) + \\ \Re(\delta)(-2x\sinh(\Delta\Gamma tm) + xy\cosh(\Delta\Gamma tm) - xy\cos(\Delta mtm) - 4y\sin(\Delta mtm))\Big]\end{aligned}}{x\left(\cos(\Delta mtm) + x^2\cosh(\Delta\Gamma tm) + x^2 y\sinh(\Delta\Gamma tm) - x\sin(\Delta mtm)\right)}$$

C=+1 state

- $A_3 =$

$$\kappa \frac{\begin{pmatrix} \cosh(\Delta\Gamma \mathrm{tm})\left[2\left(x^2+1\right)\left(\Re(\delta)+2\Re(\epsilon)\right)\left(x^2+(y+2)^2\right)\right]- \\ 2\cos(\Delta \mathrm{mtm})(y+1)\left[-x\left(\Im(\delta)\left(3x^2-y(y+4)\right)+2\Im(\epsilon)\left(x^2+(y+2)^2\right)\right)+\right. \\ \left.\Re(\delta)\left(x^2(4y+7)-y^2+4\right)+2\Re(\epsilon)\left(x^2+(y+2)^2\right)\right]+ \\ 2\sin(\Delta \mathrm{mtm})(y+1)\left[\Im(\delta)\left(x^2(4y+7)-y^2+4\right)+2\Im(\epsilon)\left(x^2+(y+2)^2\right)+\right. \\ \left.x\Re(\delta)\left(3x^2-y(y+4)\right)+2x\Re(\epsilon)\left(x^2+(y+2)^2\right)\right]- \\ 2\sinh(\Delta\Gamma \mathrm{tm})\left(x^2+1\right)\left(\Re(\delta)+2\Re(\epsilon)\right)\left(x^2+(y+2)^2\right) \end{pmatrix}}{\left(2\left(x^2+1\right)\left(x^2+(y+2)^2\right)\right)\left(\cosh[\Delta\Gamma tm]-\sinh[\Delta\Gamma tm]\right)}$$

Conclusion

- Final asymmetry is

$$A = \frac{R1[P(l^+,l^+)_- - P(l^-,l^-)_-] + R2[P(l^+,l^+)_+ - P(l^-,l^-)_+]}{R1[P(l^+,l^+)_- + P(l^-,l^-)_-] + R2[P(l^+,l^+)_+ - P(l^-,l^-)_+]}$$

where R1 are R2 are branching ratios for C=-1 and +1 states respectively.

- These asymmetries can be measured in B factories and the parameters; $\Re(\delta), \Im(\delta), \Re(\epsilon), \Im(\epsilon)$ can be constrained.
- This idea can be extended for LHCb as well, where the Bbbar pairs can be written as a linear combination of symmetric and antisymmetric states.