Fast flavor conversions: supernova neutrinos

Manibrata Sen

Department of Theoretical Physics, Tata Institute of Fundamental Research, Mumbai, India.

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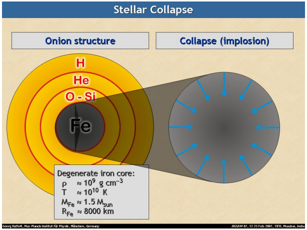
In collaboration with Basudeb Dasgupta(TIFR) and Alessandro Mirizzi(INFN, Bari).

Outline of Talk

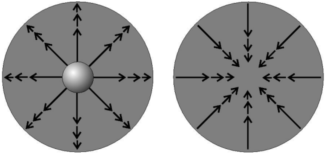
- A Core-Collapse Supernova : Neutrino conversions
- Collective effects in a dense neutrino gas
- Flavor Conversions NEAR the core
- Results

Outline of the talk

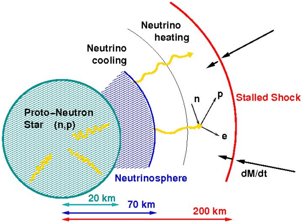
Supernova explosion



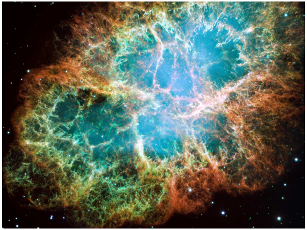
Explosion of a massive $6 - 8\ M_\odot$ star



Collapse of degenerate core.
Bounce and Shock.



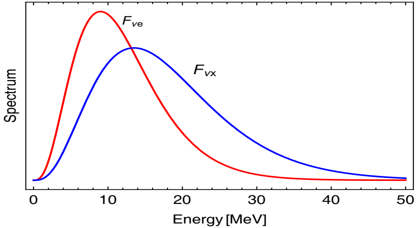
Stalled shock and accretion



Explosion!

Flavor Oscillations in dense media: Why do we care?

- Flavor evolution in a dense media $\rightarrow$ non-linear complicated problem $\rightarrow$ can lead to collective effects.
- Neutrino spectrum, as a result of oscillations?

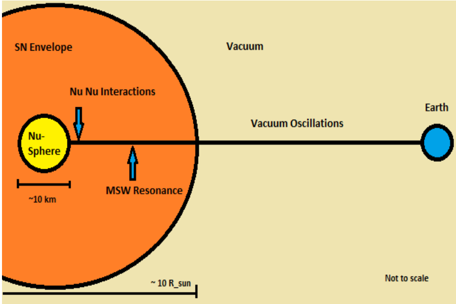


$\Longrightarrow$ **Final Spectra ??**

- Can confirm our idea of SN dynamics.
- Neutrino oscillations can have important impact on explosion dynamics as well as nucleosynthesis.

Prelude : Facts and Trivia

- Flavor conversions of supernova(SN) neutrinos - neutrino flavor conversions during the gravitational collapse of a massive star.



Illustrative of different length scales involved.

$$R_{\nu\text{-sphere}} \simeq 10 \text{ km} \ , \ R_{\text{coll}} \simeq 100 \text{ km} \ , \ R_{\text{MSW}} \simeq 1000 \text{ km} \tag{1}$$

- $\langle E_{\nu_e} \rangle < \langle E_{\bar{\nu}_e} \rangle < \langle E_{\nu_x} \rangle$ where $x = \mu, \tau$.

Ways to describe flavor oscillations

- Schrodinger's equation for flavor states:

$$i\partial_t \begin{bmatrix} \nu_e \\ \nu_\mu \end{bmatrix} = \frac{\Delta m^2}{2E} \begin{bmatrix} \cos 2\vartheta & \sin 2\vartheta \\ \sin 2\vartheta & -\cos 2\vartheta \end{bmatrix} \begin{bmatrix} \nu_e \\ \nu_\mu \end{bmatrix}$$

- Neutrino flavor density matrix

$$\rho = \begin{bmatrix} \langle \nu_e | \nu_e \rangle & \langle \nu_e | \nu_\mu \rangle \\ \langle \nu_\mu | \nu_e \rangle & \langle \nu_\mu | \nu_\mu \rangle \end{bmatrix}$$

  Use EoM $i d_t \rho = [H, \rho]$

- Expand density matrices in Pauli basis:

$$\rho = 1/2 \left[\mathrm{Tr}(\rho) + \mathbf{P} \cdot \sigma \right] \text{ and } H = \omega \mathbf{B} \cdot \sigma$$

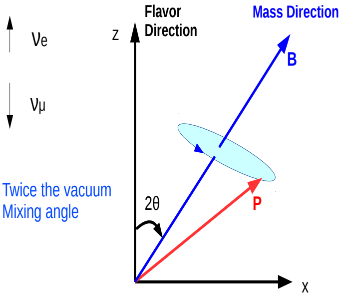
  where $\omega = \dfrac{\Delta m^2}{2E}$ and $\mathbf{B} = \{\sin 2\vartheta, 0, \cos 2\vartheta\}$.

- Gives a spin-precession equation

$$\dot{\mathbf{P}} = \omega \mathbf{B} \times \mathbf{P}$$

  where $\mathbf{P}$ is the polarisation vector.

Flavor oscillation as spin precession



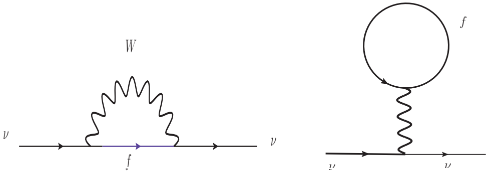
$$\dot{\mathbf{P}} = \omega \mathbf{B} \times \mathbf{P}$$

Flavor polarization vector precesses around the mass direction with frequency $\omega = \dfrac{\Delta m^2}{2E}$

C.W Kim *et al.*(2006)

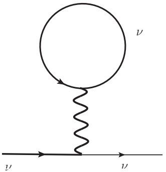
Interacting Hamiltonian

- Vacuum oscillation.
- Matter effect : forward scattering with electrons.



L. Wolfenstein (1977), S. Mikheyev, A. Smirnov (1985)

- Nu-Nu interaction : scattering with same/different flavors.



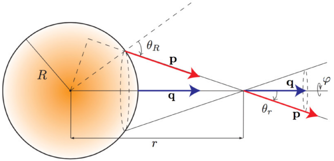
J. Pantaleone (1992)

Non-linearity from neutrino-neutrino interactions

- Effective Hamiltonian $H = H_{vac} + H_{MSW} + H_{\nu\nu}$ where

$$
\begin{aligned}
H_{\rm vac} &= \omega = \frac{M^2}{2E_p} \\
H_{\rm MSW} &= \lambda = \sqrt{2} G_F N_e \ \ {\rm diag}\{1,0,0\} \\
H_{\nu\nu} &= \sqrt{2} G_F \int \frac{d^3q}{(2\pi)^3} \ (1 - \vec{v_p}.\vec{v_q})(\rho_q - \bar{\rho_q})
\end{aligned}
$$

Define $\mu = \sqrt{2} G_F N_\nu$.



H. Duan *et al.*(2006)

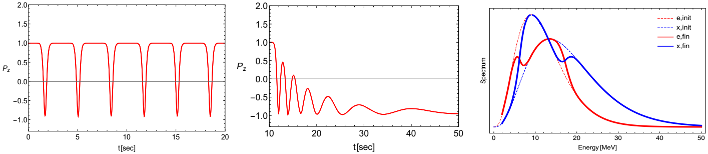
- $H_{\nu\nu} \sim \mu(\mathbf{P} - \bar{\mathbf{P}}) \Rightarrow$ non-linear term $\Rightarrow$ collective effects.

Outline of the talk

Collective effects : new phenomena

- Synchronized oscillations: ν and $\bar{\nu}$ of all energies oscillate with the same frequency.

$$\dot{\mathbf{P}} = \omega \mathbf{B} \times \mathbf{P} + \mu(\mathbf{P} - \bar{\mathbf{P}}) \times \mathbf{P} \longrightarrow \mu >> \omega$$



- Coherent $\nu_e \bar{\nu}_e \leftrightarrow \nu_x \bar{\nu}_x$ oscillations. Intermediate μ.
- Realistic declining μ can cause complete conversion.
- ν_e and ν_x spectra swap completely, but only within certain energy ranges. Occurs in both hierarchies.

G. Raffelt *et al.*(2007), B. Dasgupta *et al.*(2009)

Bipolar Oscillations : Linear stability analysis

- Deep inside → high density → flavor and mass states almost equal.
- Consider 2 flavors ν_e and ν_x . The flavor density matrices

$$\rho = \begin{bmatrix} \rho_{ee} & \rho_{ex} \\ \rho_{xe} & \rho_{xx} \end{bmatrix}$$

- The EoM is given by

$$id_t\rho_p = i(\partial_t + \vec{v_p}.\vec{\nabla})\rho_p = [H_p, \rho_p]$$

where,

$$H_p = \underbrace{\frac{M^2}{2E_p}}_{\omega_p} + \underbrace{\sqrt{2}G_F N_e}_{\lambda} + \sqrt{2}G_F N_\nu \int \frac{d^3q}{(2\pi)^3}\,(1 - \vec{v_p}.\vec{v_q})(\rho_q - \bar{\rho_q})$$

- $\omega < 0$ for antineutrino.
- Neglect collisions.
- Can linearise in small off-diagonal elements.

$$\rho = \frac{\mathrm{Tr}\rho}{2} + \frac{g_{\omega v \phi}}{2}\begin{bmatrix} s & S \\ S^* & -s \end{bmatrix}$$

A. Dighe *et al.*(2011)

Linear stability analysis

- Here $s^2 + S^2 = 1$. Assume $S << 1$ and linearise in S.
- Look for solutions far from neutrinosphere $(r >> R_{\rm ns})$.
- Take $S \sim Q_{\omega v z} e^{-i\Omega_t t - i\vec{\Omega_r}.\vec{r}}$. This gives us an eigenvalue equation.

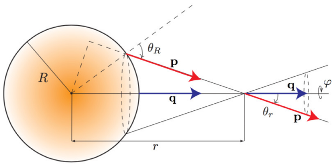
$$\begin{aligned} i(\Omega_t + \vec{v}\cdot\vec{\Omega_r})Q_{\omega v z} &= \left(\omega + \lambda + \mu \int \frac{d\Gamma'}{(2\pi)} \left(1 - v_z v'_z - \vec{v_T}.\vec{v_T}'\right) g_{\omega' v' \phi'}\right) Q_{\omega v z} \\ &\quad - \mu \int \frac{d\Gamma'}{(2\pi)} \left(1 - v_z v'_z - \vec{v_T}.\vec{v_T}'\right) g_{\omega' v' \phi'} Q_{\omega' v' z'} \end{aligned}$$

- Complex values of $\Omega = \gamma + i\kappa$ with $\kappa > 0$ signals an instability.
- Evolution in space : Put $\partial_t \longrightarrow 0$.
- Evolution in time : Put $\vec{v}\cdot\vec{\nabla} = 0$.

Linear stability analysis

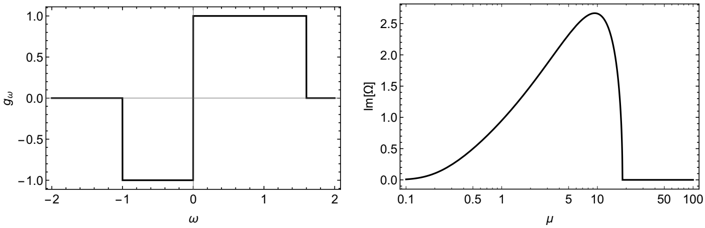
- Simplifications:
  1. Single angle emission
  2. Spherical symmetry. Large distance approximation.

H. Duan *et al.*(2006)



- Symmetries not sacrosanct. Breaking of symmetries leads to interesting results $\rightarrow$ Multi angle matter suppression, instability in NH due to breaking of symmetries.

LSA Example: Box Spectrum



A. Dighe *et al.*(2011)

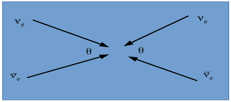
- Spectrum g_ω=box spectra.
- Identical angular distributions for all neutrinos.
- Growth rate of instability $\mathrm{Im}\ \Omega_r \propto \sqrt{\omega\mu} > \omega$. Significant flavor conversions at $r \sim O(10^2)$ km from neutrinosphere.
- $\mathrm{Im}\ \Omega_r$ non-zero for a certain range of μ.

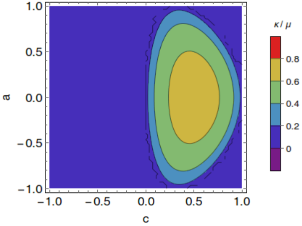
Faster Conversions

- Pre-2006 : Flavor conversions mainly in MSW regions $r \sim O(10^3)$ km. MSW conversions $\propto \omega$
- Post-2006 : Collective effects. Significant flavor conversions at $r \sim O(10^2)$ km from neutrinosphere. Rates $\text{Im } \Omega \propto \sqrt{\omega\mu}$.
- Faster conversions: $\text{Im } \Omega \propto \mathcal{O}(\mu) \sim 10^5\omega$? Can occur for massless neutrinos. Non-trivial angular distributions? Near the source.

R.F Sawyer(2015)

- Simplest model. Shows fast conversion. Still far from source.



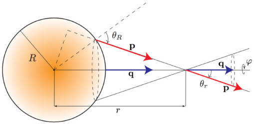


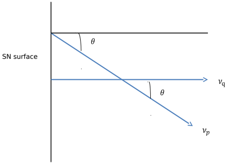
G. Raffelt *et al.*(2016)

Outline of the talk

Aim of our work

- Closer look at conditions for fast flavor conversions.
- Do a linear stability analysis, and check for fast conversions near the source of emission $r \sim \mathcal{O}(1)$ m.
- Discard the "bulb model", and because of the near field effect, model the source as an infinitely long plane.

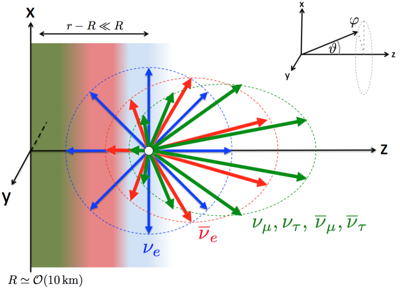


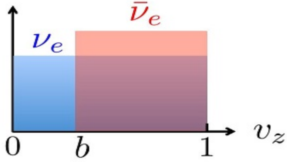


- Use flavor dependent angular spectrum. Realistic approximation.
- Include backward going modes also.
- Consider evolution in time (stationary soln) as well as in space (homogeneous soln).

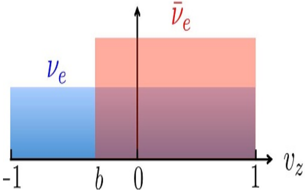
Angular spectrum of emission

- Consider different cones of emission for ν and $\bar{\nu}$. Can consider inward going rays also.





(a) Without inward rays



(b) With inward rays

Performing the LSA

- The eigenvalue eqn from LSA is

$$
\begin{aligned}
i(\Omega_t + \vec{v}\cdot\vec{\Omega_r})Q_{\omega v z} &= \left(\omega + \lambda + \mu \int \frac{d\Gamma'}{(2\pi)} \left(1 - v_z v_z' - \vec{v_T}.\vec{v_T}'\right) g_{\omega' v' \phi'}\right) Q_{\omega v z} \\
&\quad -\mu \int \frac{d\Gamma'}{(2\pi)} \left(1 - v_z v_z' - \vec{v_T}.\vec{v_T}'\right) g_{\omega' v' \phi'} Q_{\omega' v' z'}
\end{aligned}
$$

- Define

$$
\begin{aligned}
\epsilon &= \int \frac{d\Gamma}{2\pi}\; g_{\omega v_z \phi}, \quad \epsilon_v = \int \frac{d\Gamma}{2\pi} v_z\; g_{\omega v_z \phi} \\
\epsilon_{vs(c)} &= \int \frac{d\Gamma}{2\pi}\; \sqrt{1 - v_z^2}\; \mathrm{s}_\phi(\mathrm{c}_\phi) g_{\omega v_z \phi}
\end{aligned}
$$

- Equation simplifies

$$
\left[\omega + \lambda - \Omega_t + \mu\epsilon - \mu v_z \epsilon_v - \mu\sqrt{1 - v_z^2}\left(\epsilon_{vc}\mathrm{c}_\phi + \epsilon_{vs}\mathrm{s}_\phi\right)\right] Q =
$$

$$
\mu \int \frac{d\Gamma'}{(2\pi)} \left(1 - v_z v_z' - \sqrt{(1 - v_z^2)(1 - v_z'^2)}\; \mathrm{c}_{(\phi - \phi')}\right) g_{\omega' v_z' \phi'} Q'
$$

Performing the LSA

- Write the functional form of Q to be

$$Q = \frac{a + bv_z + c\ \sqrt{1 - v_z^2}\ \mathrm{c}_\phi + d\ \sqrt{1 - v_z^2}\ \mathrm{s}_\phi}{\left[\omega + \lambda - \Omega_t + \mu\epsilon - \mu v_z \epsilon_v - \mu\sqrt{1 - v_z^2}\left(\epsilon_{vc}\mathrm{c}_\phi + \epsilon_{vs}\mathrm{s}_\phi\right)\right]}$$

- Eigenvalue equation

$$\begin{bmatrix} I_{0,0}^{0,0} - 1 & I_{1,0}^{0,0} & I_{0,1}^{1,0} & I_{0,1}^{0,1} \\ -I_{1,0}^{0,0} & -I_{2,0}^{0,0} - 1 & -I_{1,1}^{1,0} & -I_{1,1}^{0,1} \\ -I_{0,1}^{1,0} & -I_{1,1}^{1,0} & -I_{0,2}^{2,0} - 1 & -I_{0,2}^{1,1} \\ -I_{0,1}^{0,1} & -I_{1,1}^{0,1} & -I_{0,2}^{1,1} & -I_{0,2}^{0,2} - 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = 0$$

where (using $v_z = \mathrm{c}_\theta$)

$$I_{m,n}^{\alpha,\beta} = \mu \int \frac{d\Gamma}{2\pi} \left(\frac{\mathrm{c}_\phi^\alpha\ \mathrm{s}_\phi^\beta\ \mathrm{c}_\theta^m\ \mathrm{s}_\theta^n}{\left[\omega + \lambda - \Omega_t + \mu\epsilon - \mu\mathrm{c}_\theta\epsilon_v - \mu\mathrm{s}_\theta\left(\epsilon_{vc}\mathrm{c}_\phi + \epsilon_{vs}\mathrm{s}_\phi\right)\right]} \right) g_{\omega v_z \phi}$$

Equation of motion

- For simplicity, assume the initial spectrum of emission is independent of ϕ. This simplifies the eigenvalue matrix to a block diagonal form.

$$\begin{bmatrix} I_{0,0}-1 & I_{1,0} & 0 & 0 \\ -I_{1,0} & -I_{2,0}-1 & 0 & 0 \\ 0 & 0 & -I_{0,2}/2-1 & 0 \\ 0 & 0 & 0 & -I_{0,2}/2-1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = 0 \qquad (2)$$

The eigenvalue equations are, for the **axially symmetric case** :

$$\left(I_{0,0}-1\right)\left(I_{2,0}+1\right)-I_{1,0}^{2}=0 \qquad (3)$$

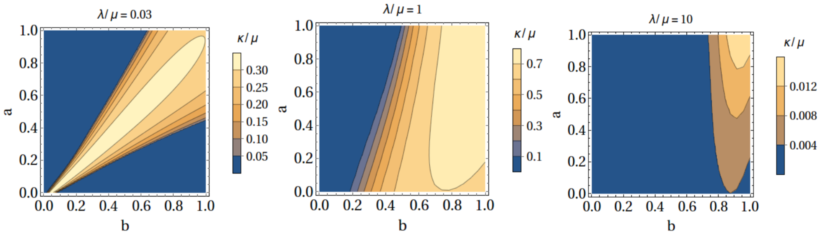
and for the **axial symmetry breaking case**:

$$\left(\frac{I_{0,2}}{2}+1\right)=0 \qquad (4)$$

Outline of the talk

Results: Evolution in space

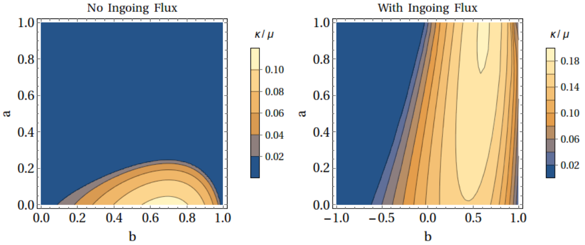
- Growth rates $\kappa = \text{Im}(\Omega_r)$ in units of $\mu \simeq 10^5$ km^{-1} $\Rightarrow$ large growth.



Instability rates for different values of a and b, for three different values of $\lambda/\mu =$ 0.03, 1, and 10.

- $a \Rightarrow$ neutrino-antineutrino asymmetry. $b \Rightarrow$ angular asymmetry of emission.
- No instability for $b = 0$. Need non-trivial angular spectrum.
- κ maximum for $\lambda \sim \mu$.

Results: Evolution in time

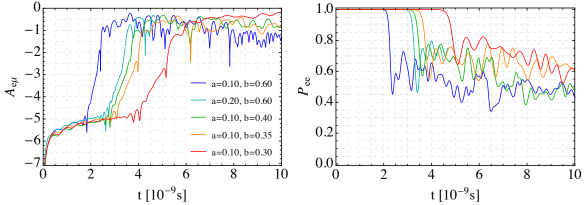


Rates for different values of a and b, for evolution in time, without including inward going modes (left panel) and including inward going modes (right panel).

- No matter suppression.
- Inclusion of inward modes increases the growth rates, making fast conversions stronger.

Growth rates : Full Numerical Solution

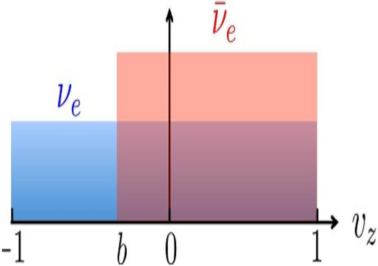
- Numerical solution of the fully nonlinear EoMs(no inward going modes). Matches with linear stability in linear regime.



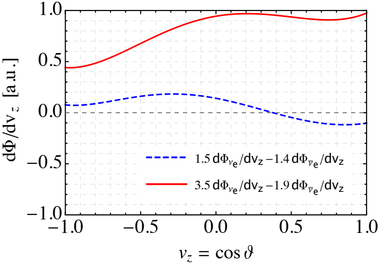
Left : Instability growth rates $A_{ex}(t) = \log[S(t)]$. Right : Electron neutrino survival probability.

- Fast conversions at timescale of $t \sim O(10^{-8}\text{sec}) \Rightarrow$ at distances of $r \sim O(1\text{ m})$ from neutrinosphere.
- $P_{ee} \simeq 0.5 \Rightarrow$ flavor averaging.

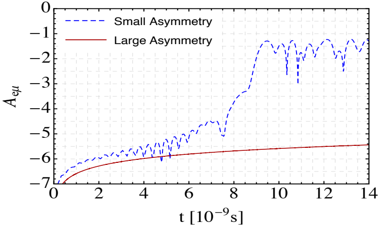
Growth rates: Crossed Angular spectrum

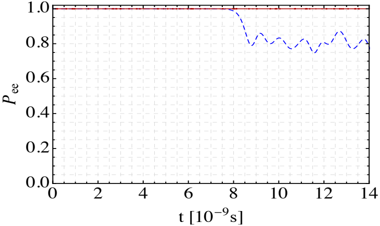


(a)



(b)





Open Questions

- Look at dispersion relations - evolution in space AND time. Study instability in Ω_t and Ω_r plane.
- Why do we need a crossing in the angular spectra?
- Is there true flavor averaging? Need to include collisions to have a clearer answer.
- Spectra formation?

Conclusions

- Find fast conversions at a distance of $\sim O(1 \text{ m})$ from the neutrinosphere.
- Flavor dependent angular spectrum seem to be essential for these fast conversions.
- Fast conversions lead to averaging of flavor information.
- Can be crucial for SN explosion and nucleosynthesis.

THANK YOU