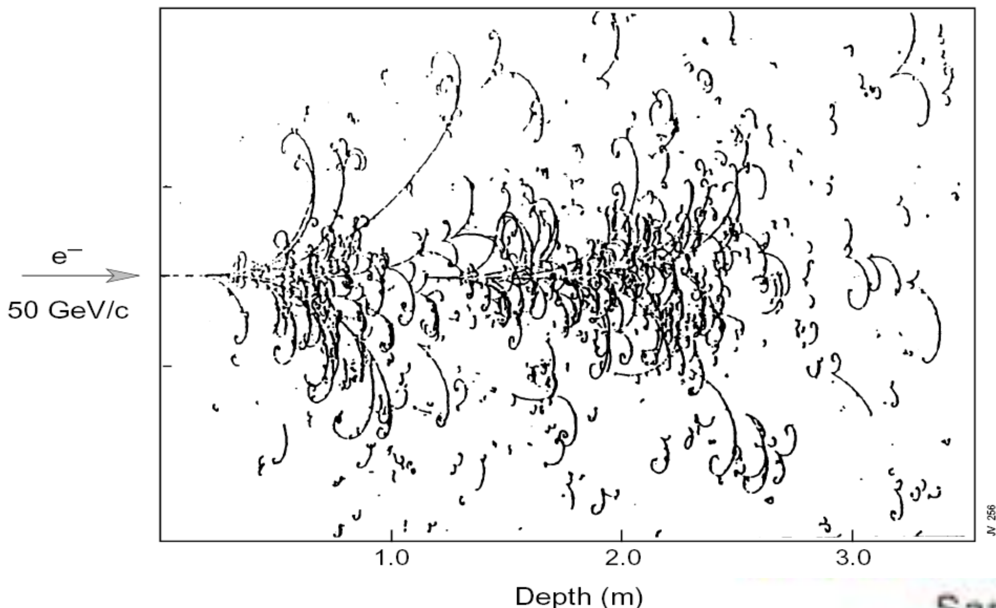
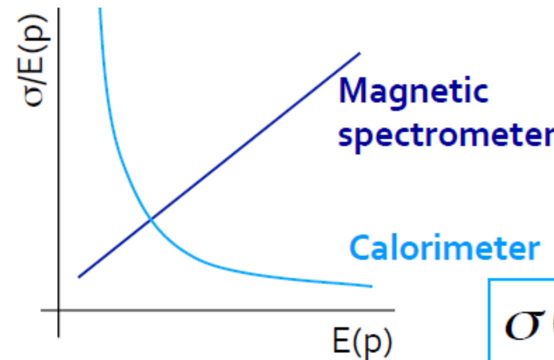
Calorimeter in High Energy Physics

- **Calorimetry : Energy Measurement in Particle physics** by Richard Wigmans
- **Experimental Techniques in High Energy Physics** edited by Thomas Ferbel

Measure energy of electrons, photons and hadrons (including neutral hadrons)

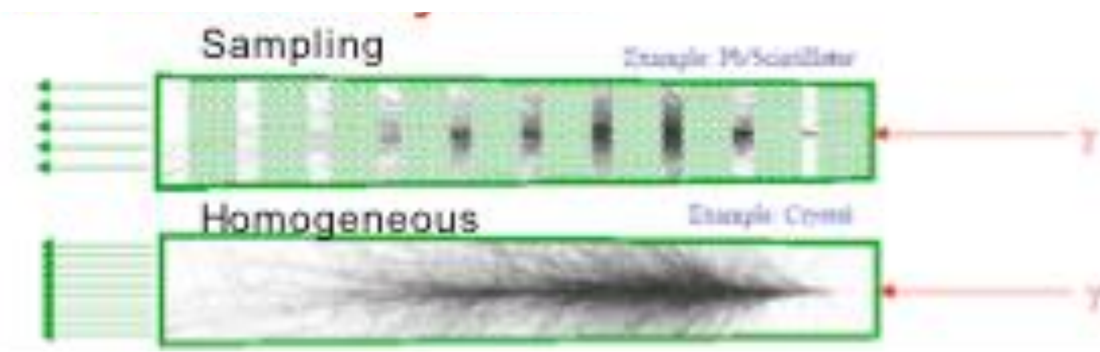
Big European Bubble Chamber filled with Ne:H_2 = 70%:30%,
3T Field, L=3.5 m, $X_0 \approx 34$ cm, 50 GeV incident electron





$$\frac{\sigma(p)}{p} \approx ap$$

$$\frac{\sigma(E)}{E} \approx \frac{a}{\sqrt{E}}$$



Destructive detection method (Calorimeter)

Particle in detecting medium ⇨ Secondary particles (+ medium)

↓

……………. ⇦ Tertiary particles (+ medium)

Gradual degradation in energy → All (mostly) all particles absorbed → Energy converted into heat ⇒ Calorimeters

But, do not measure change in energy (too tiny, 1 GeV = 1.6×10^{-10}J, bolometry, very low temp, very low specific heat, change in temp) → Measure through characteristics interaction with matter (excitation/inonisation)

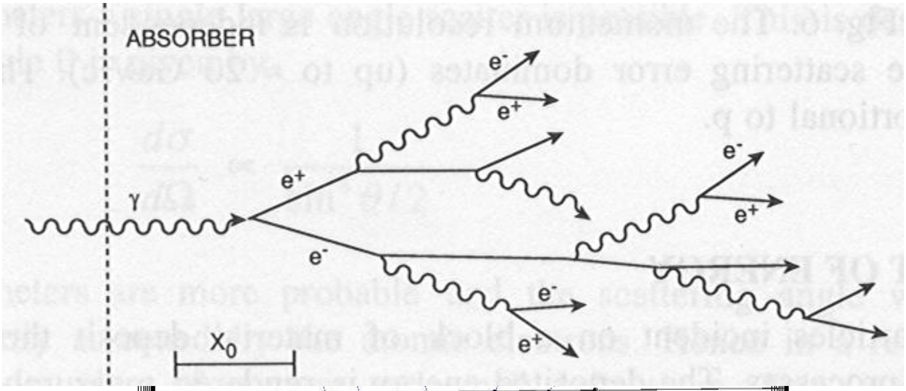
Implementation in Ooty Cosmic Ray air-shower expt in seventies : Complete absorption detector

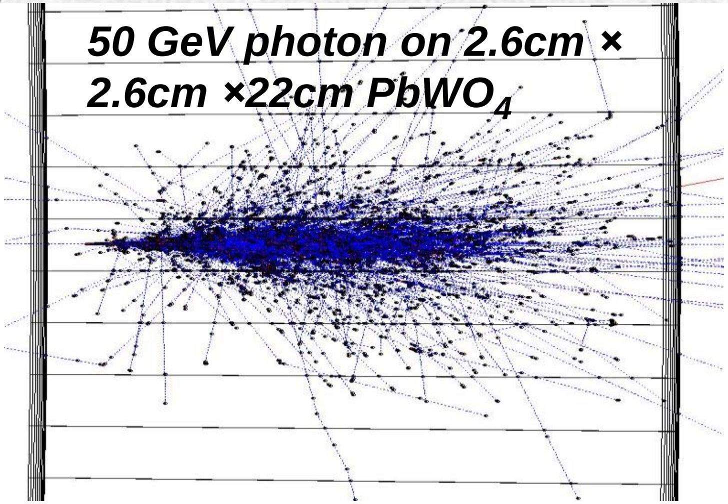
- Only way to detect neutral particles and measure its energy
- Absorption process statistical in nature, $\sigma_E/E \propto E^{-1/2}$ (resolution improve with high E) , whereas in tracker, $\sigma_{p_T}/p_T \propto p_T$.
- Longitudinal depth (containment of shower) ∝ log(E), whereas in tracker size $\propto\sqrt{L}$ for constant σ_p/p. For three point measurement, $\sigma_{p_T}/p_T = \sqrt{3/2}\,\sigma_s \frac{8p_T}{0.3BL^2}$
- Only devices to measure energy of jets and Missing Energy using ~4π coverage
- Fast signal collection (50ns - 1μs) →Trigger

- Electron/photon : interact electromagnetically → Electromagnetic calorimeter (scale with radiation length, X_0)
- Hadron : interact through strong interaction → Hadron calorimeter (scale with interaction length, λ)

Electromagnetic shower

- Electromagnetic cascade propagate via bremsstrahlung and pair production
- Energy gets degraded at each step and number of shower particles increase till, $\varepsilon = \varepsilon_C$ (critical energy) when ionisation/excitation takes over
- Total energy loss in the cascade ≅ energy of incident $e^{\pm}/\gamma$
- Total signal from all track elements in the shower ∞ incident energy





1 X_0 : Remaining energy of electron in 1/e of initial energy and a photon has a probability of 7/9 of pair conversion, rough estimation shower, after t(X_0) generation,

$$\varepsilon(t) = E/2^t \;\; and \;\; n(t) = 2^t$$

$$n(t_{\max}) = E/\varepsilon_C \;\; and \;\; t_{\max} = \ln(E/\varepsilon_C)/\ln 2$$

$$\frac{1}{X_0} = w_1 \left(\frac{1}{X_0}\right)_1 + w_1 \left(\frac{1}{X_0}\right)_2 + \cdots .$$

$$\left.\frac{\Delta\varepsilon}{\Delta x}\right|_{coll} = \left.\frac{\Delta\varepsilon}{\Delta x}\right|_{rad} = -\frac{\varepsilon_C}{X_0}$$

Where X_0 =radiation length =(7/9)λ_{pair}

$$X_0(gm/cm^2) \approx 180\frac{A}{Z^2}$$

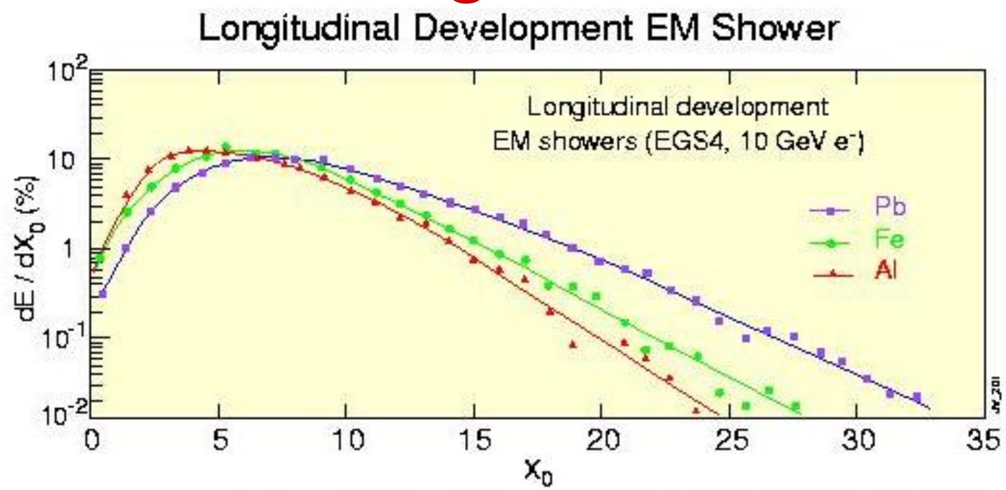
Better than 20% for Z >13

$$\varepsilon_C \approx \frac{700MeV}{Z} \propto \left(\frac{m}{m_e}\right)^2$$

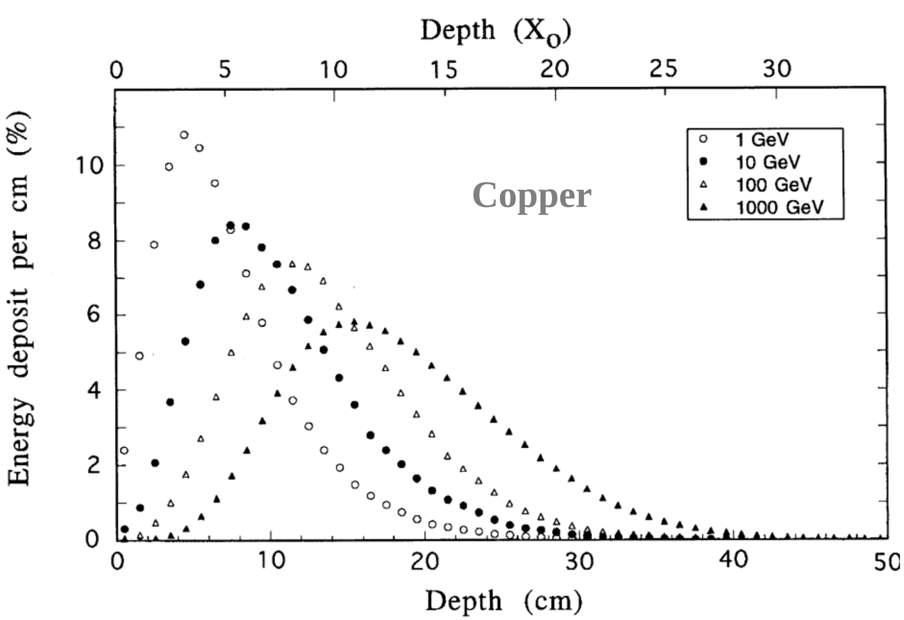
Accurate to 10% for Z >13

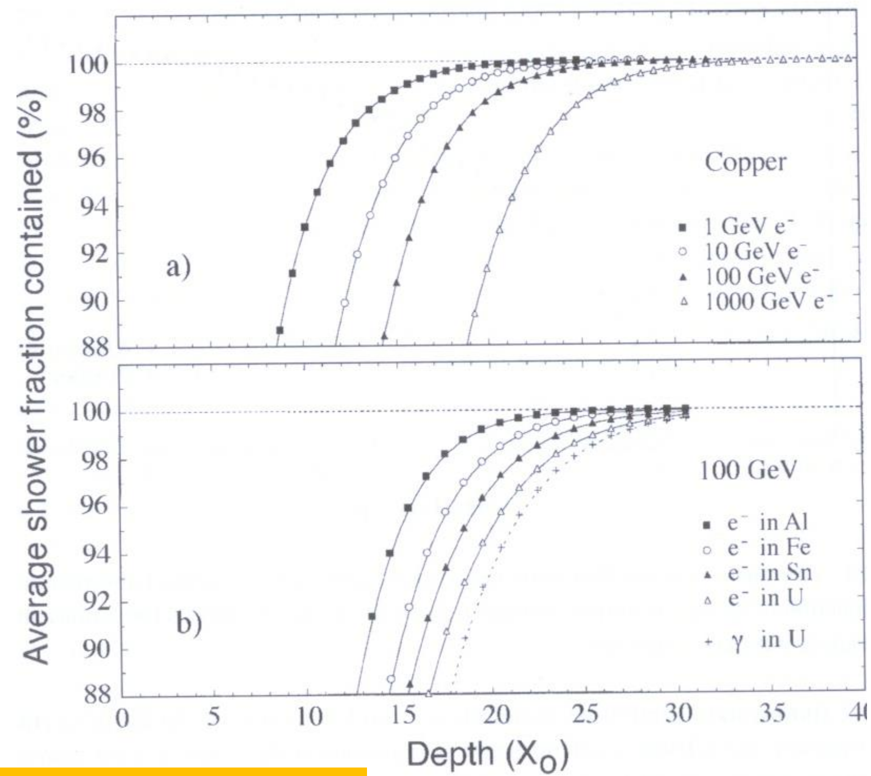
Electromagnetic shower : Longitudinal

Deeper shower profile in Pb :
multiplication continue down to lower energy (low ε_C). After shower maximum, typical exponential falloff of energy deposition caused by the attenuation of photon through Compton interaction.
λ_{att} (= 3.4 ± 0.6 X_0) characterises the slow $\exp(-X_0/\lambda_{att})$ decay of the shower maximum



Radiation length ∝ $1/Z^2$
Compton interaction length ∝ $1/Z$





Typical length of crystal ~16X_0(BELLE) to 25.8 X_0 (CMS)

Longitudinal Electromagnetic shower : $e^{\pm}$ vs γ

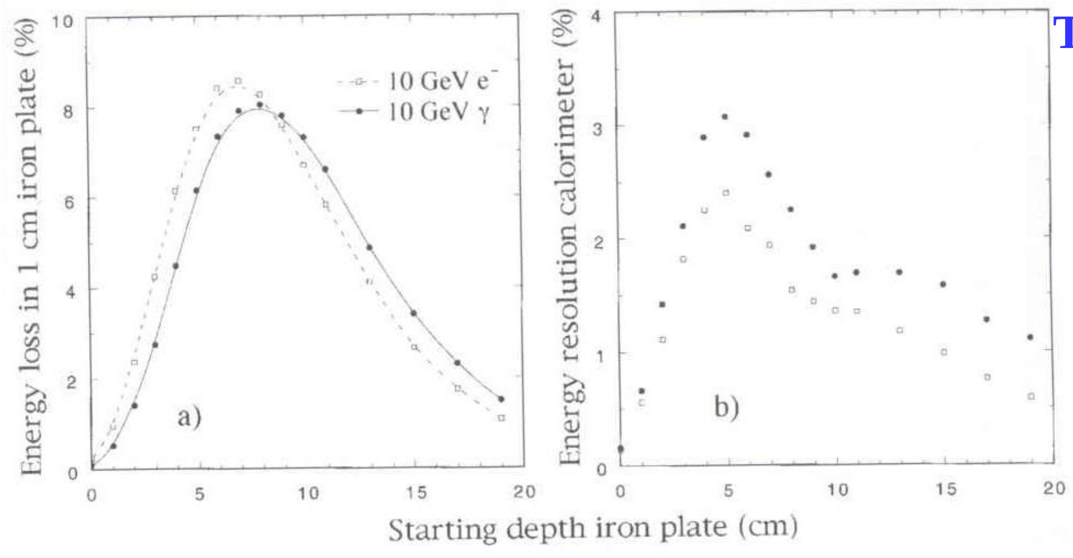
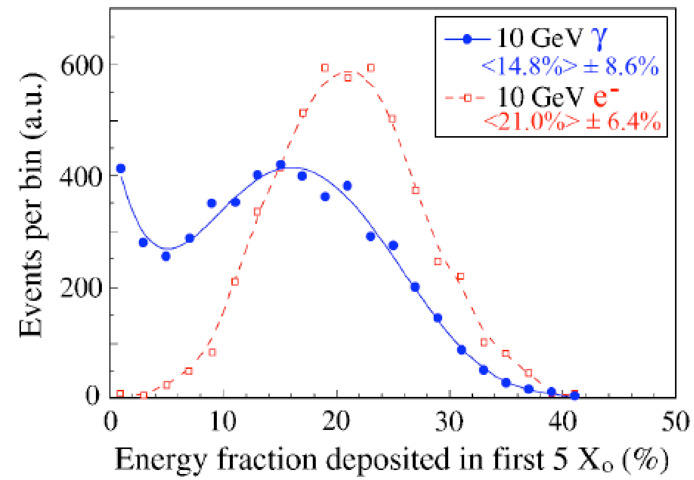


FIG. 5.10. The average calorimeter signal (*a*) and the energy resolution (*b*) of the $Z = 50$ calorimeter for 10 GeV electrons and γs, as a function of the depth at which a given amount of dead material is installed inside the calorimeter. The dead material is represented by a 1 cm thick iron plate, placed perpendicular to the direction of the incident particle. Results from EGS4 simulations.

Typical longitudinal shower profile

$$\frac{dE}{dt} = Eb\frac{(bt)^{a-1}e^{-bt}}{\Gamma(a)};$$

$$t_{max} = (a-1)/b \;\; and \;\; b \approx 0.5$$



Incident particle	$e^{\pm}$	γ
Shower maximum, $t_{max}(X_0)$	$\ln(\varepsilon/\varepsilon_C) - 1$	$\ln(\varepsilon/\varepsilon_C) - 0.5$
Center of gravity $t_{med}(X_0)$ (half energy absorbed)	$t_{MAX} + 1.4$	$t_{MAX} + 1.7$
98% shower containment $t_{98}(X_0)$	t_{MAX} + 4 λatt	t_{MAX} + 4 λatt
Number of $e^{\pm}$ at the peak	$0.3(\varepsilon/\varepsilon_C)[\ln((\varepsilon/\varepsilon_C)-0.37]^{-1/2}$	$0.3(\varepsilon/\varepsilon_C)[\ln((\varepsilon/\varepsilon_C)-0.31]^{-1/2}$

Electromagnetic shower : Lateral

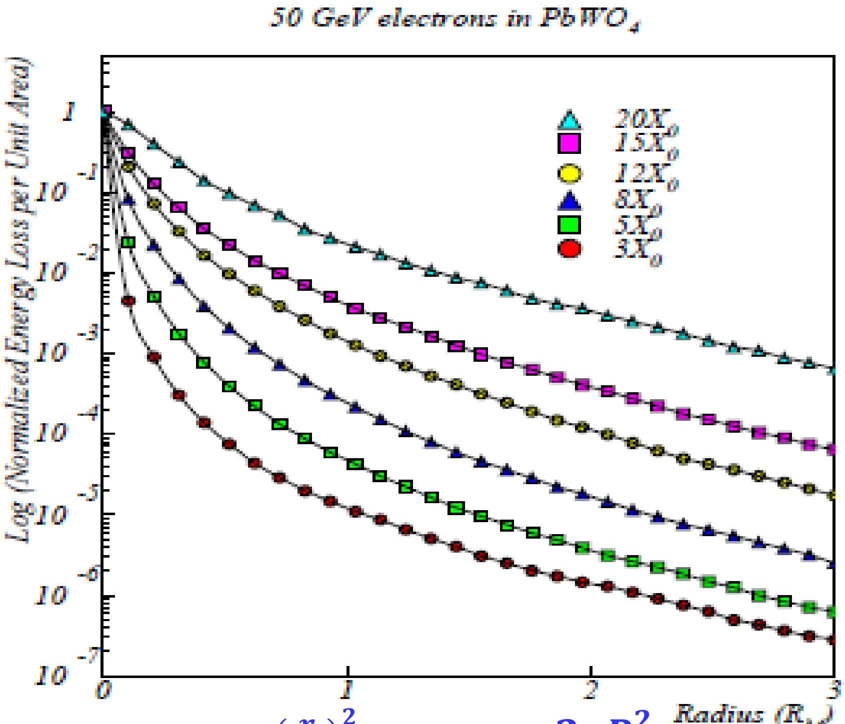
Transverse shower size is governed by

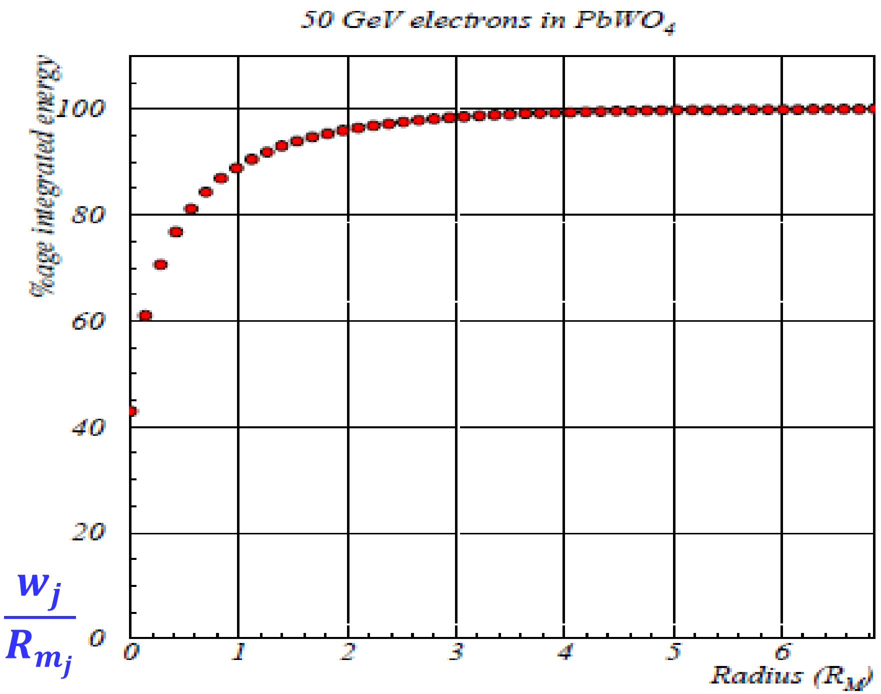
- Typical angle of bremsstrahlung emission at high energies
- Multiple scattering at low energies
- Propagation of photon

For measuring total energy of the cascade, measure energy deposit inside a cylinder of radius ρ. ρ_M (Moliere radius) contains 90% of shower energy = $E_S \times X_0/\varepsilon_C$ = 7 × A / Z ($gm\text{-}cm^{-2}$), where $E_S = m_e c^2 (4\pi/\alpha)^{1/2}$ = 21 MeV

$\rho_M \equiv$ Average lateral deflection of electrons of energy ε_C after traversing one X_0

Lateral size of crystal ~ ρ_M . Optimisation of noise, cost/ Separation of two particles





$$F_{E(r)} = (A_C)e^{-\left(\frac{r}{R_C}\right)^2} + (A_T)\frac{2rR_T^2}{\left(r^2 + R_T^2\right)^2}$$

$$0.9 = \int_0^{2\pi} d\phi \int_0^{Rm} F_{E(r)} r dr$$

$$\frac{1}{R_m} = \sum \frac{w_j}{R_{m_j}}$$

Choice of electromagnetic calorimetric material

Crystal		NaI(Tl)	CsI(Tl)	CsI	BaF_2	BGO	CeF_3	$PbWO_4$	LAr	Plastic	Pb	Cu	Fe	U
Density	gm/ cm^3	3.67	4.51	4.51	4.89	7.13	6.16	8.28	1.4	1.03	11.4	8.96	7.87	19.0
Rad. Lenght	cm	2.59	1.85	1.85	2.06	1.12	1.68	0.89	13.5	42.4	0.56	1.43	1.7	0.32
Moliére rad	cm	4.5	3.8	3.8	3.4	2.4	2.6	2.2						
Inter. length	cm	41.4	36.5	36.5	29.9	22.0	25.9	22.4	65.0	78.9	17.6	15.1	16.7	11.0
Decay time	ns	250	1000	35	630	300	10	15		1 - 5				
				6	0.9		30	5						
Peak emission	nm	410	565	420	300	480	310	420		370-430				
				310	220		340	440						
Rel. light yield	%	100	45	5.6	21	9	10	0.7		28-34				
				2.3	2.7									
D(LY)/dT	%/°C	≈0	0.3	−0.6	−2.0	−1.6	0.15	−1.9	–					
r.i. (n)		1.85	1.80	1.80	1.56	2.20	1.68	2.16	1.6	1.58				

NaI(Tl) : Light output =*7%*

LAr : dE/dx=2.2 MeV/cm, mobility ~ 5 mm/μs at 1 KV/mm

Radiation hardness

Homogeneous and Sampling calorimeter

Deterioration of energy resolution due to shower leakage

$$\frac{\sigma}{E} = \left[\frac{\sigma}{E}\right]_{f=0}\left[1+2\sqrt{E}\,f\right]$$

f **= fraction of energy loss through leakage**

Radiation length of plastic is 42.4cm. Required crystal length ~1050cm to confine 98% of shower energy, which is not feasible due to

- **Growth in industry (crystal)**
- **Cost**
- **Nonlinearity along crystal**

Energy resolution : (a) fluctuation in cascading and (b) fluctuation due to sampling (depends on both active/passive material) → Worse energy resolution

Homogeneous calorimeter

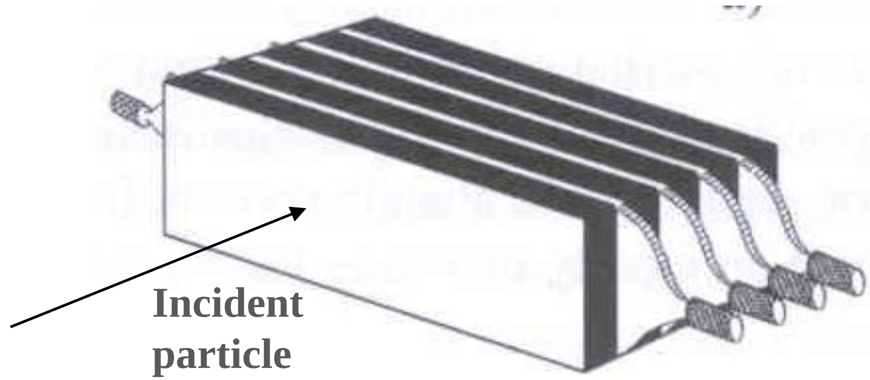
Same material used for (1) degrading (absorbing) the energy and (2) generating measurable signal

Signal generated through

Scintillation	Cherenkov radiation
High light yield	Low light yield
Low threshold	High threshold
Good resolution	Resolution worse by factor ~2-10
Radiation damage	Radiation hard

Sampling calorimeter

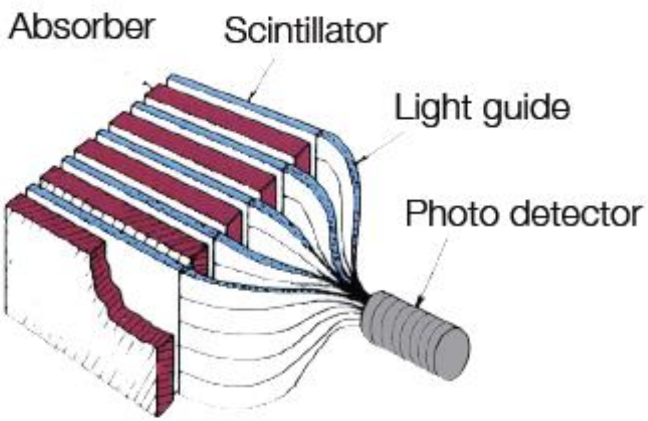
- **Energy degraded by a passive material**
- **Signal seen in active element through excitation/ionisation**



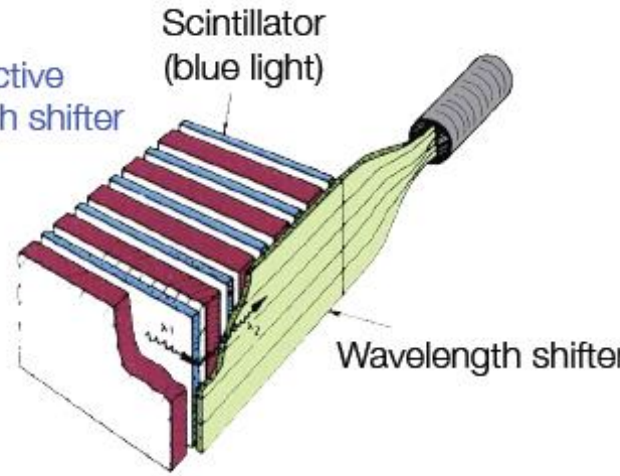
Design of sampling calorimeters

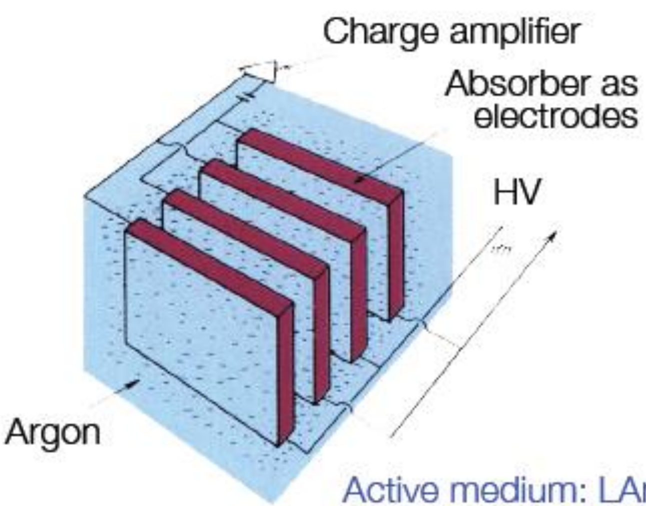
Possible setups

Scintillators as active layer;
signal readout via photo multipliers



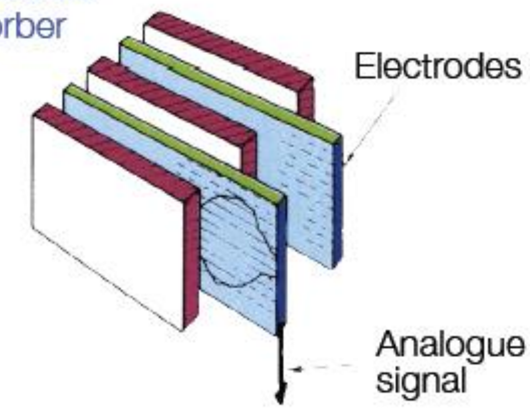
Scintillators as active layer; wave length shifter to convert light



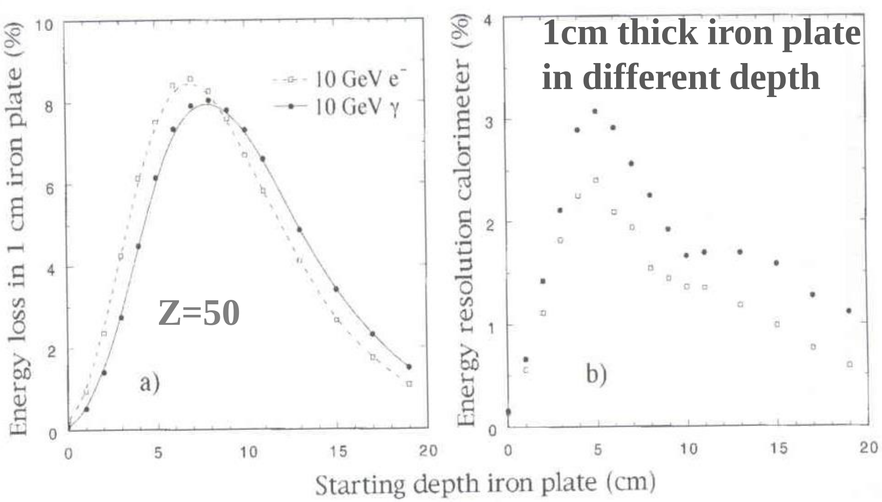


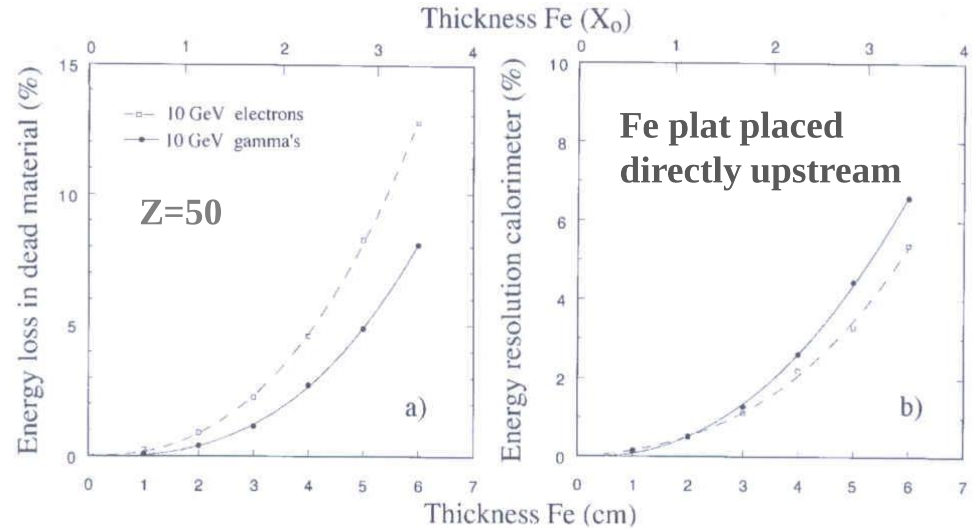
Active medium: LAr; absorber embedded in liquid serve as electrods

Ionization chambers between absorber plates

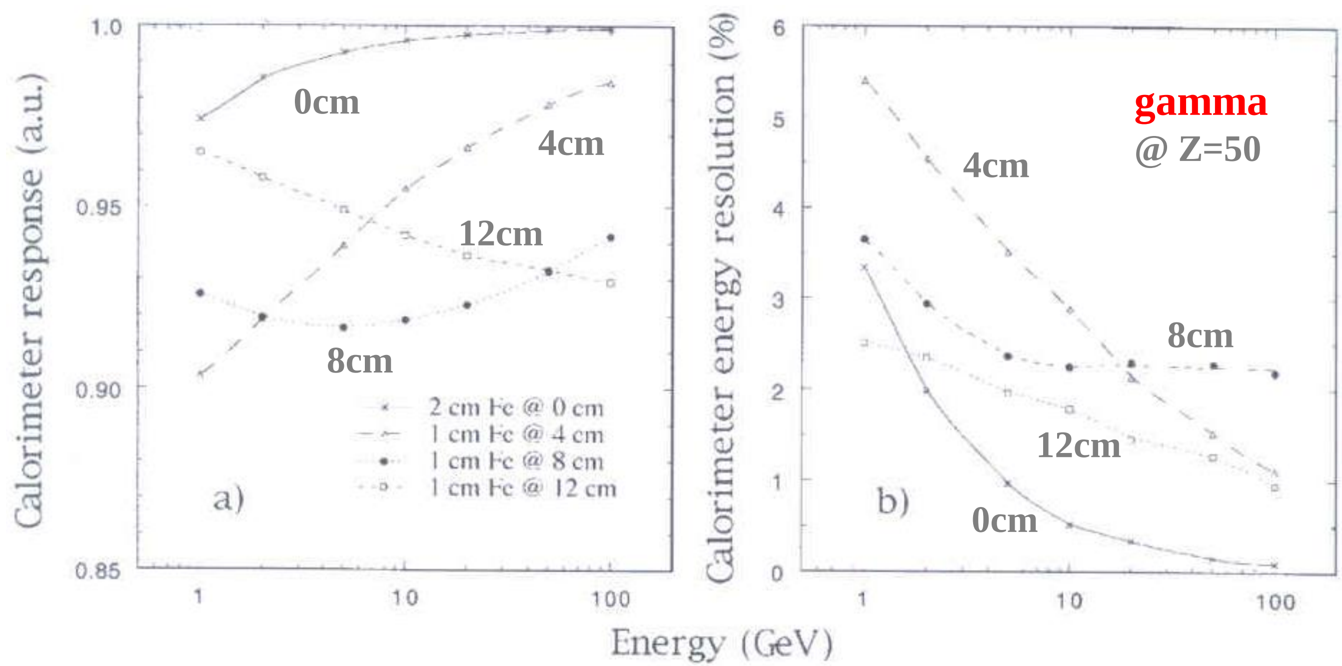


Construction principle





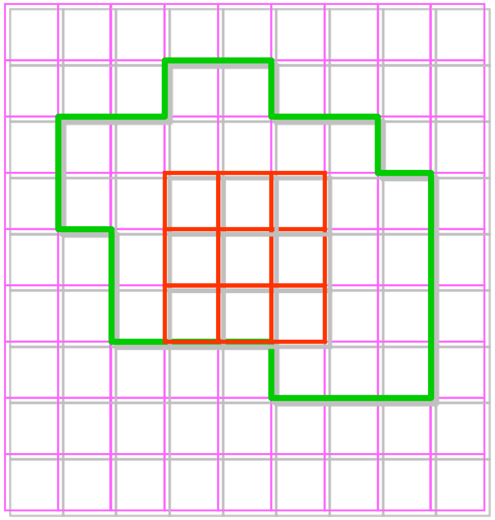
- In case of electron, energy deposit started at the entrance point, whereas for photon, on the average it is after $(9/7)X_0$. Thus shower depth is more in gamma
- Energy deposit in 1[st] X_0 : For photon it varies from 0 (no pair production) or twice the energy deposit due to electron (pair production at early stage, energy loss due to electron and positron)

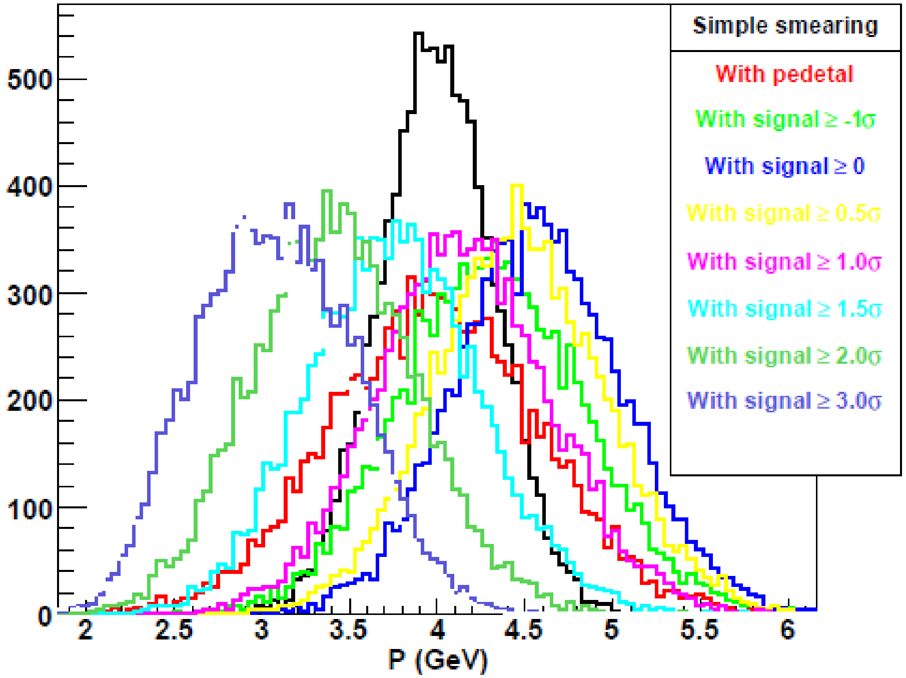


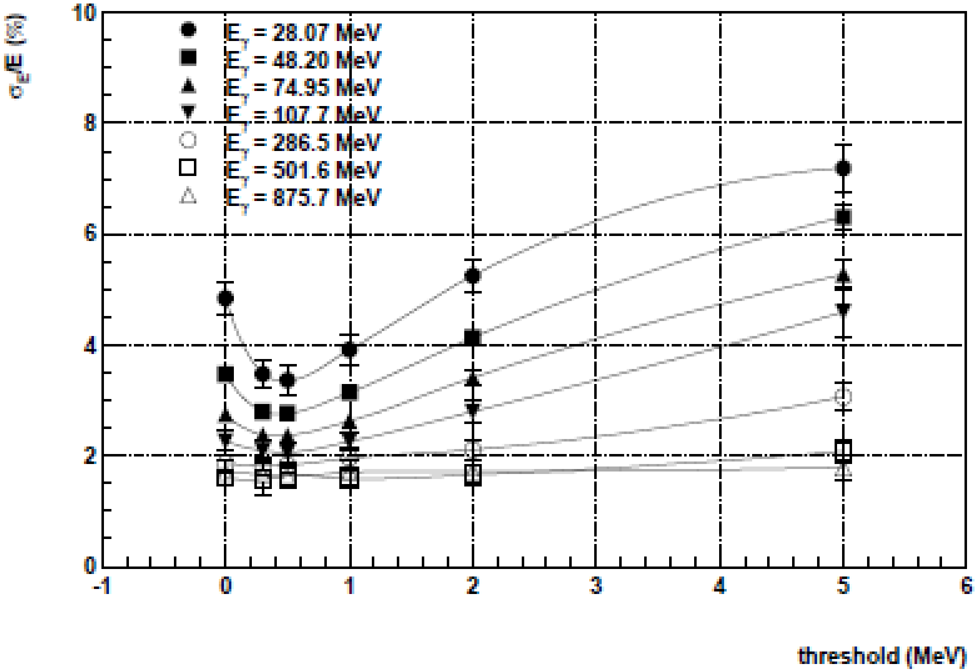
Effect of inactive medium in different place

Energy resolution in EM calorimeter

- Size of crystal ~ Moliere radius
- Sort out crystals, which have energy greater than a certain threshold value to reduce the effect of noise
- Looks for seed crystal with the energy greater than certain value and ad add nearby by eight crystal with it (3×3), (5×5) or (7×7)
- Optimisation based on noise level and signal height







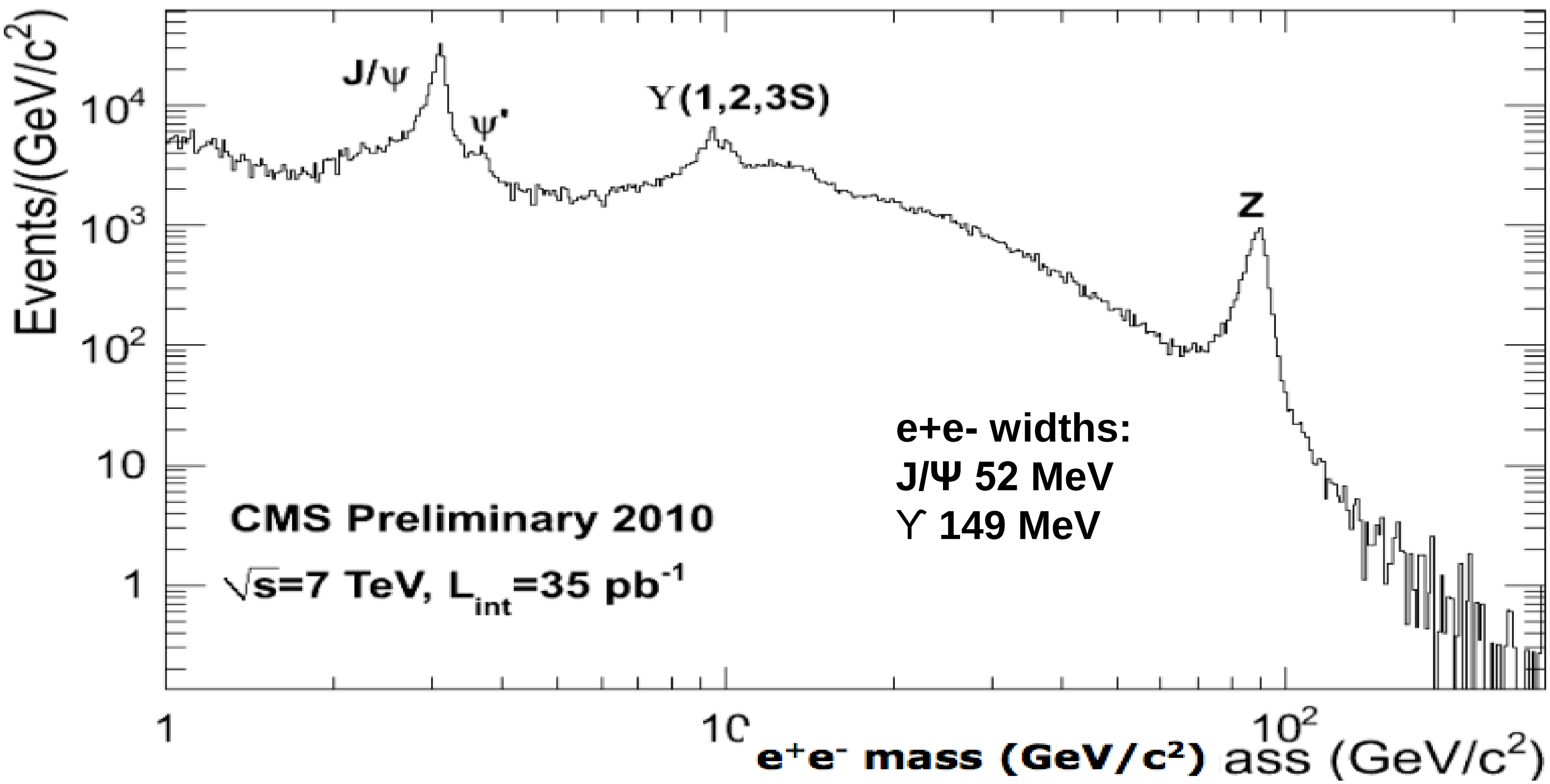
Resolution as well as absolute calibration depends on threshold value

Performance of an Electromagnetic calorimeter

$$\left(\frac{\sigma}{E}\right)^2 = \left(\frac{S}{\sqrt{E}}\right)^2 + \left(\frac{N}{E}\right)^2 + C^2$$

Calibration :
$\pi/\eta \to \gamma\gamma$
$J/\psi \to e^+e^-$ / $\mu^+\mu^-$
$Z \to e^+e^-/\mu^+\mu^-/qq$

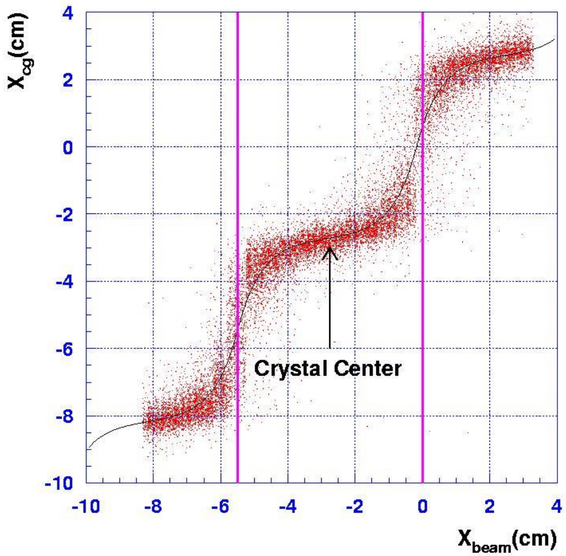
E (Calorimeter) /P(tracker)

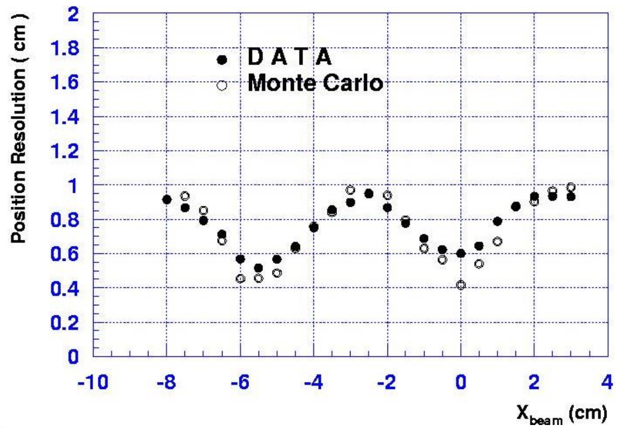


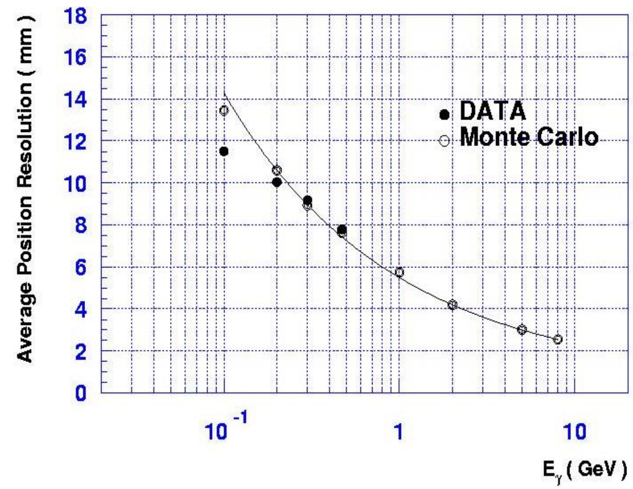
Performance of an Electromagnetic calorimeter

Localisation of shower done by centre of gravity

Spatial resolution ⇒ Fluctuation in lateral profile, $E^{-1/2}$







S-shape correction of position resolution

- **EM calorimeter : R_{eff} = size of crystal in Moliere radius**
- **Hadron calorimeter : Linear transverse dimension (in unit of λ_{int})**
- **In general $\sigma_{x,y} = c_1 \oplus c_2/\sqrt{E} \oplus c_3/E$**

$$x_{CG} = \frac{\sum E_i x_i}{\sum E_i}$$

$$x_{true} = P_0 \tanh(P_1 x_{COG})$$

$$x'_{corr} = A \tan^{-1}(Bx')$$

$$\sigma_{x,y} \approx \frac{0.1 R_{eff}}{\sqrt{0.1E}}$$

$$\sigma_x^h = \exp(2d)$$

Energy resolution of EM calorimeter

How do we see energy ? Not directly from shower, but mainly from the ionisation energy loss of charged particles only, e.g., in Compton scattering visible energy is the energy from electron, not directly from incoming/outgoing photon.

Maximum number of track segment $\eta_{max} = E/\eta$, where η=threshold for observing an element (or average detectable track length, $\langle T_d \rangle \propto E$)

Intrinsic resolution due to fluctuation in shower propagation

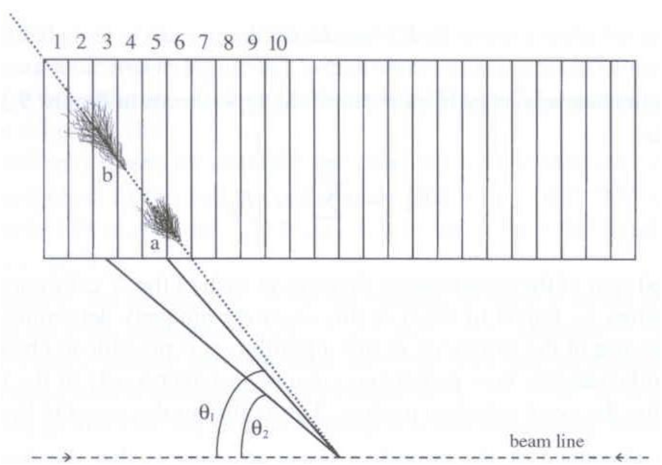
- **(c1) Electronic Noise**
- **(c2) Particle other the one in interest, e.g., pile-up**
- **(c3) Analog to digital : loss of information**
- **(c4) Shift in pedestal level**
- **(b1) Fluctuation in cascading, charges/neutral ratio, sampling etc.**
- **(b2) Photon/p.e. statistics**

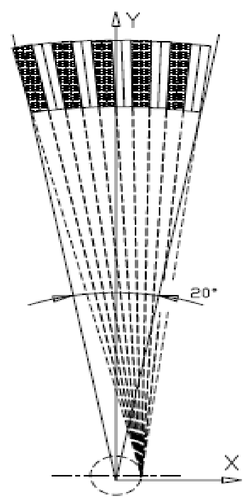
Overall resolution : $$\frac{\sigma_E}{E} = a \oplus \frac{b}{\sqrt{E}} \oplus \frac{c}{E}$$

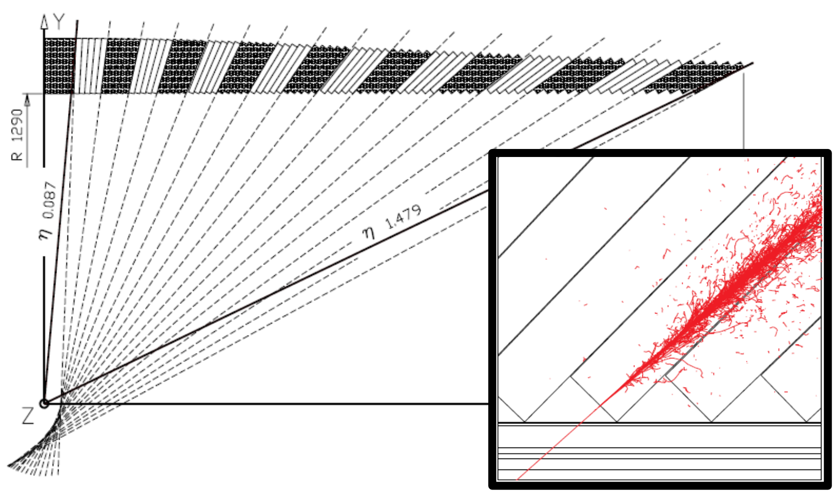
- **(a1) Non-uniformity in signal generation, e.g., thickness of scintillator, uniformity of scintillator properties, position of shower**
- **(a2) Collection of photon : crystal shape, fraction of crystal surface covered by the PMT, reflectivity at surface, self attenuation**
- **(a3) Propagation of photon, attenuation, surface loss, bending of fibre**
- **(a4) Loss in splice, connectors**
- **(a5) Cell-to-cell Inter calibration error**
- **(a6) Non containment of shower, energy leakage in rear/side ($E^{-(1/4)}$), albedo**
- **(a7) Energy deposit in dead areas in front or inside the calorimeter**
- **(a8) Fluctuation in timing measurement(TDC)**
- **(a9) Position dependent QE of photon-transducer (and/or cell-to-cell variation), e.g. PMT/SiPM**
- **(a10) Gain of PMT/APD/SiPM + HV stability**
- **(a11) dL/dT, variation with temperature**
- **(a12) Gas composition, contamination of electronegative substance(in particular, oxygen), temperature, pressure**

Design of EM calorimeter : Goal 4-vector of γ

- **Concept : Longitudinal segmentation of EM crystal and direction from the measurement in front and back side and may be few more intermediate points (not possible because of large signal in front photo detector, while particle passed though it, e.g., in $PbWO_4$ p.e./MeV~4, energy loss of heavily ionised particle in 100μm ~100 MeV/g-cm^2 ×100 μm × 3 g/cm^3 = 3MeV ~10^6 p.e.~ 1TeV energy of particle). Same problem with back too, but in reduced form (e.g. ECAL spike, large noise in CMS HF due to passage of particle in PMT window/fibre bundle).**
- **Need precise measurement of vertex position to measure 4-vector**
- **Calorimeter tower should be pointing towards vertex positions, otherwise depending on shower depth, position measurement will have large uncertainty.**
- **But, crystal points to vertex, particle may pass through gap without any interaction, thus a small inclination, e.g., 3^o is used for CMS ECAL (both in η and ϕ).**
- **Use preshower detector to measure the position precisely and/or better γ/π^0 separation**







Variety of hadronic interaction

Hadronic interaction:

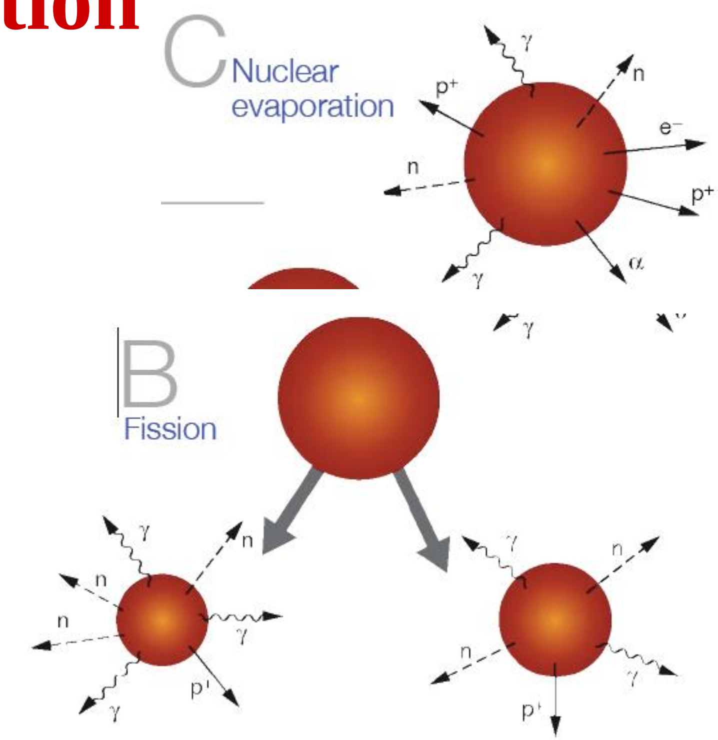
Elastic:

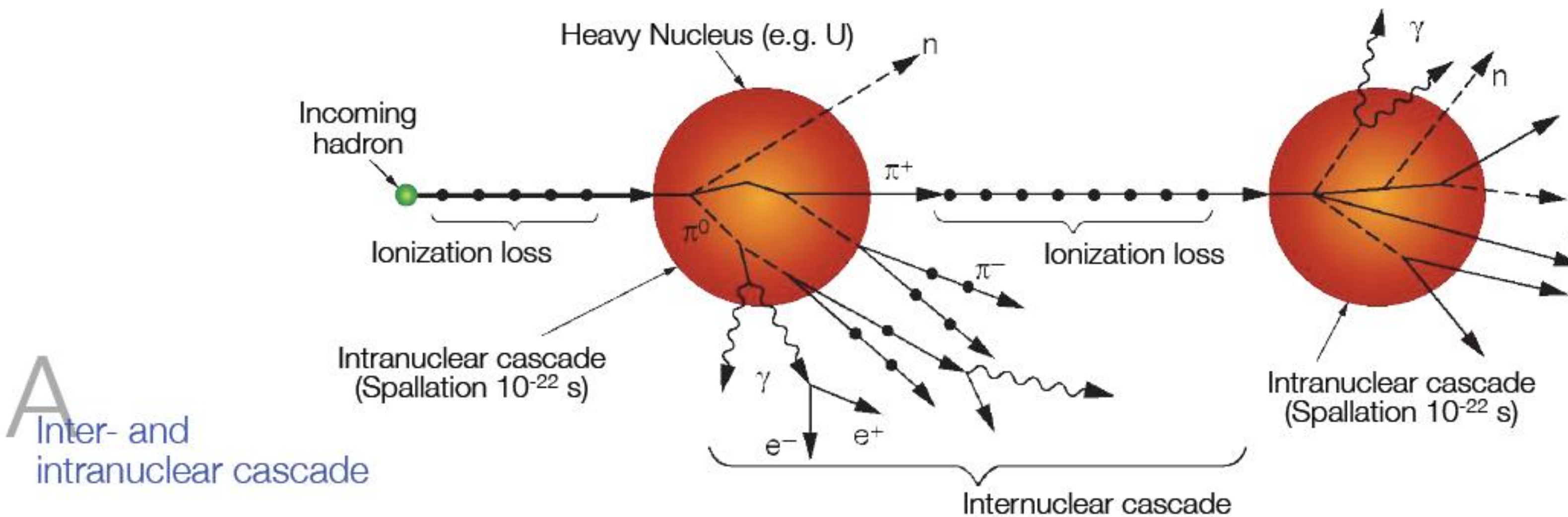
$$p + \text{Nucleus} \rightarrow p + \text{Nucleus}$$

Inelastic:

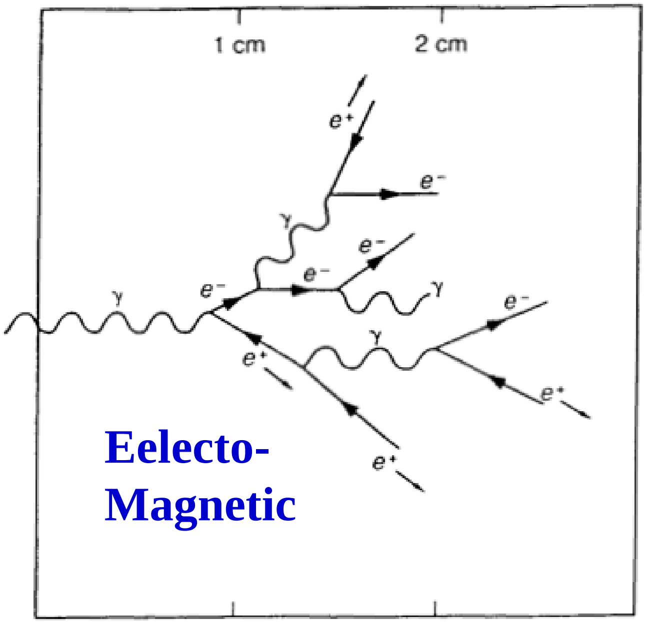
$$p + \text{Nucleus} \rightarrow$$
$$\pi^+ + \pi^- + \pi^0 + \ldots + \text{Nucleus}^*$$

$$\begin{bmatrix} \text{Nucleus}^* & \rightarrow \text{Nucleus A} + n, p, \alpha, \ldots \\ & \rightarrow \text{Nucleus B} + 5p, n, \pi, \ldots \\ & \rightarrow \text{Nuclear fission} \end{bmatrix}$$



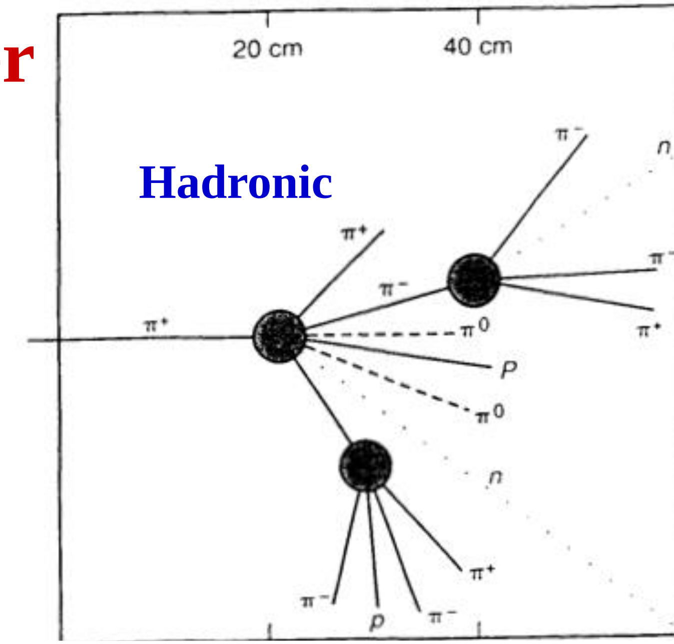


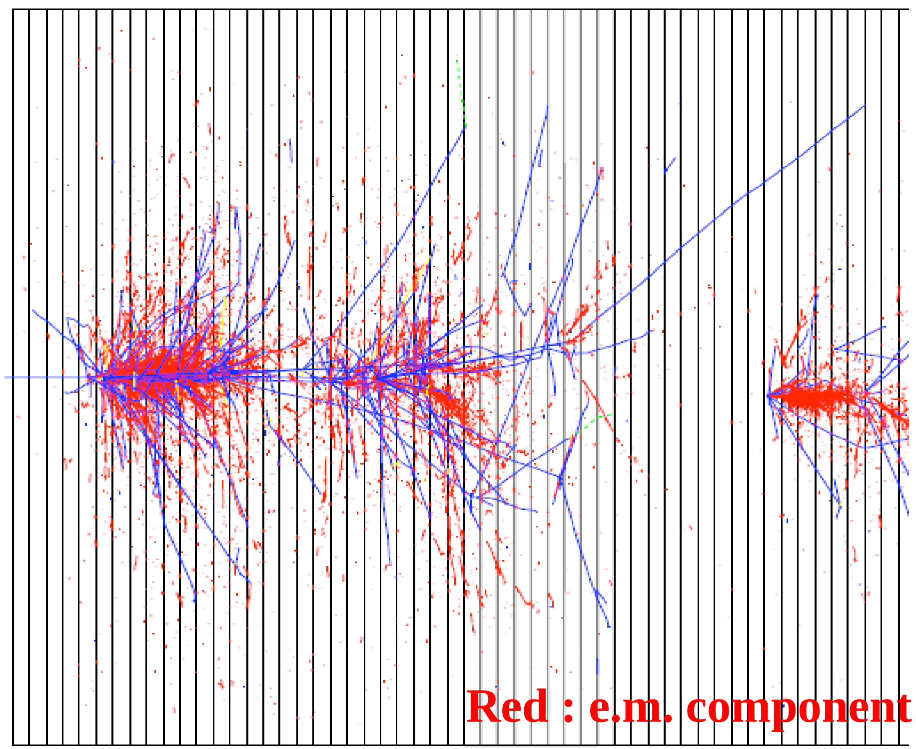
Hadron shower

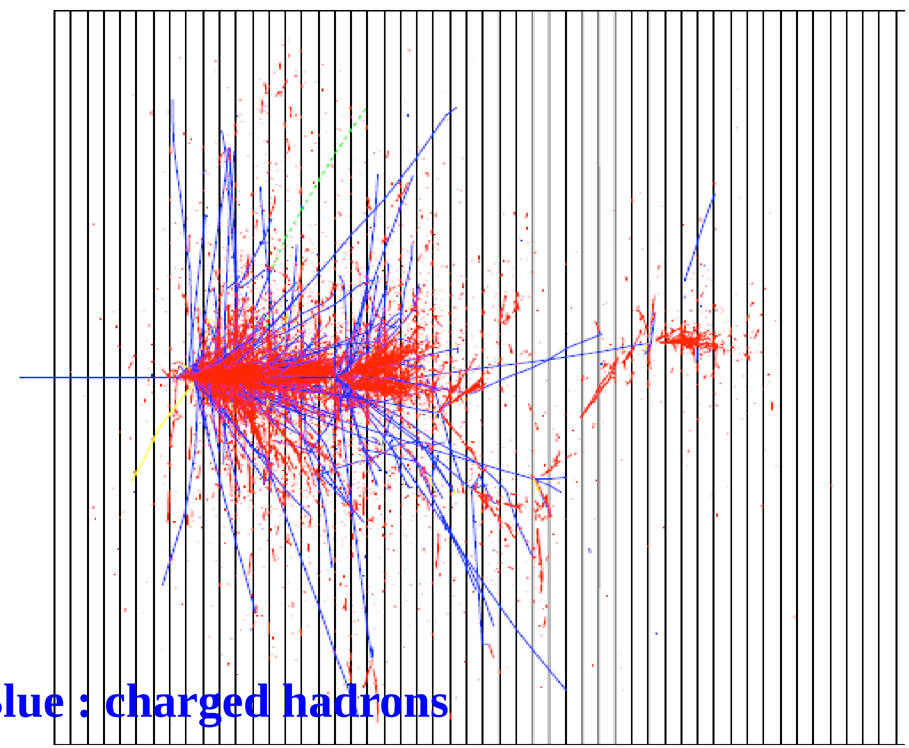


As compared to EM showers, hadron showers are :

- Broader and more penetrating
- Subject to larger fluctuations – more erratic and varied





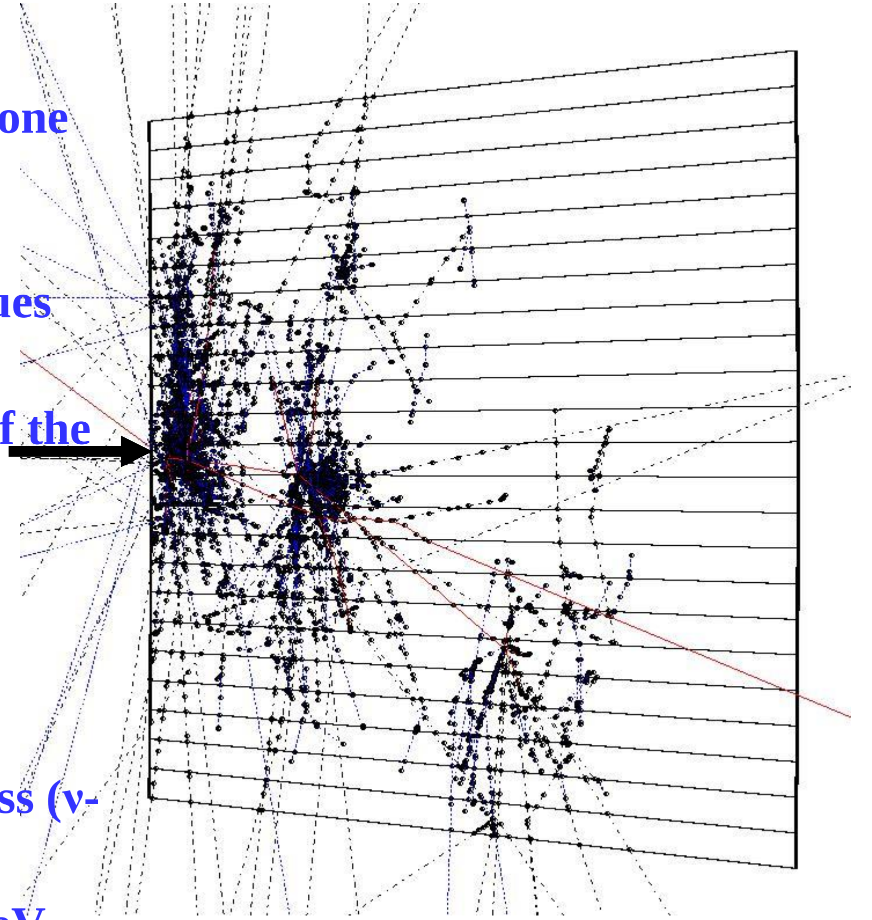


Red : e.m. component Blue : charged hadrons

Hadron calorimeter

Hadronic cascade is like electromagnetic cascade, but of greater variety and complexity due to hadronic processes

- **Multiple production in inelastic interactions, average energy requires for the production of one pion, E_0 ~0.7 (1.3) GeV for Fe(Pb)**
- **Transverse momentum distribution of the produced particles sharp peaking at small values <P_T> ~350 MeV**
- **Leading particle takes large (~50%) fraction of the available energy, $D(z)=(\alpha+1)(1-z)^{\alpha}/z$, α ~ 4 (LEP) - 6 (Tevatron)**
- **Invisible nuclear de-excitation energy (~20% energy lost in the form of binding/evaporation energy)**
- **Semi-leptonic / leptonic decay cause energy loss (ν-energy completely and μ-energy partially)**
- **12% energy carried by neutron with k.e.~1 MeV and 3% by photons of energy ~1 MeV**
- **Cross section is smaller than in EM process (scale of interaction length, λ larger than in EM process, X_0)**

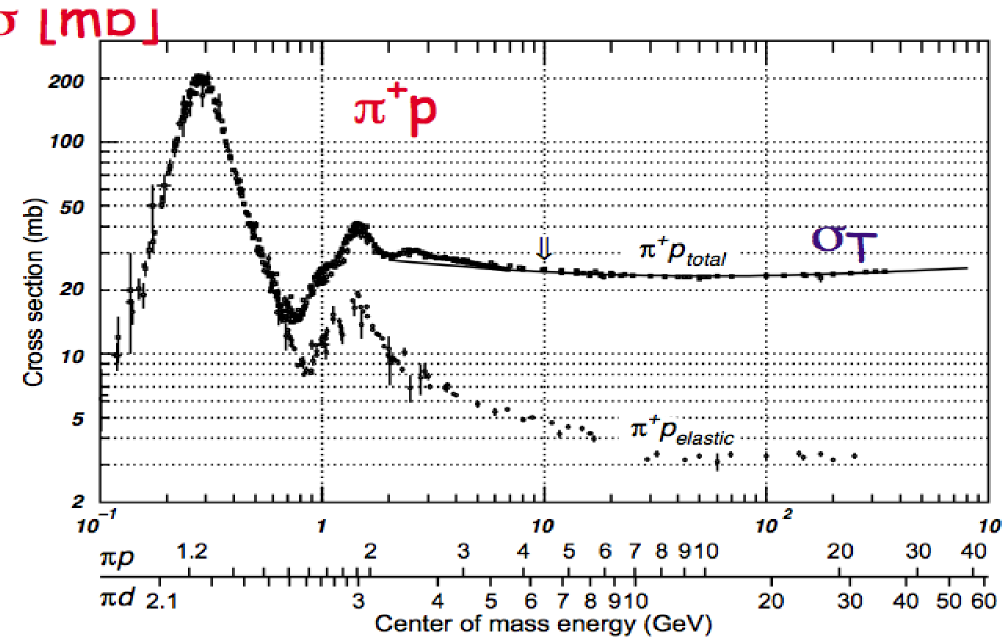


Shower due to 150 GeV K^+ beam on 2.6 cm × 2.6 cm × 71 cm $PbWO_4$ crystal (odd example) . Back-scattered particles (mainly neutron and photon)

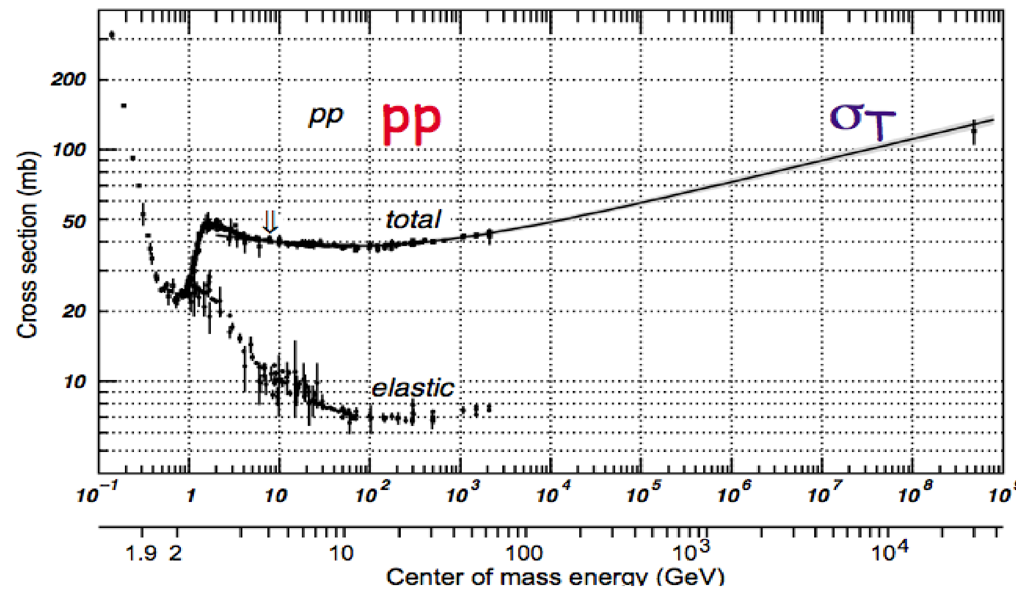
Hadronic interaction

$$P = \exp(-l/\lambda_{\text{int}});$$

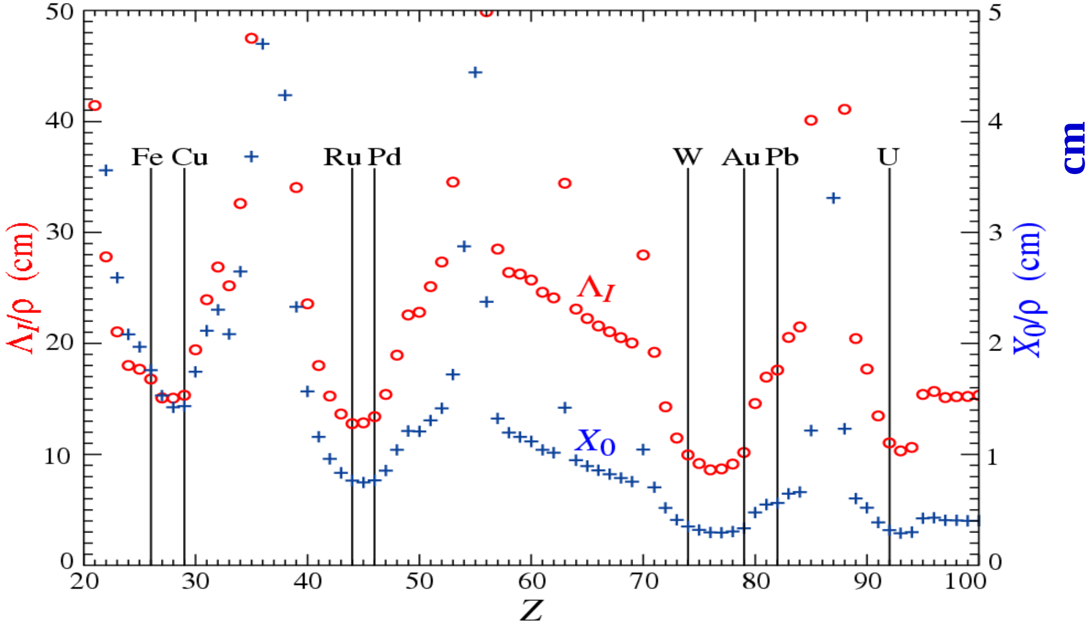
$$\lambda_{\text{int}} = \frac{A}{N_A \sigma_{tot}} \approx 35A^{1/3}(gm/cm^2)$$

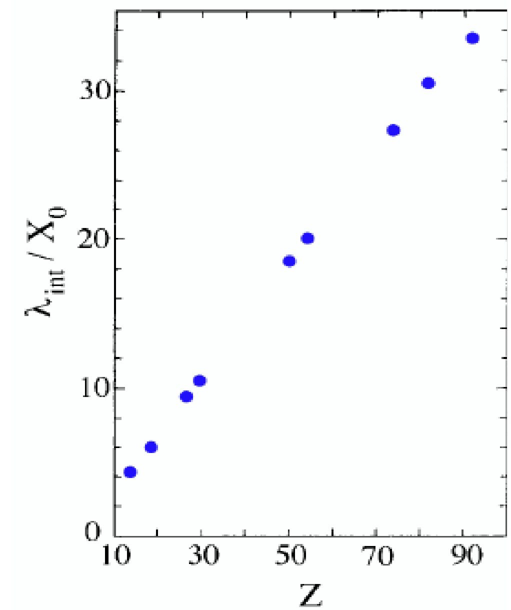


- **Total cross section for pp (πp) in fixed target experiment at 100 GeV is 38mb (24mb).**
- **In general, λ_{int} is quoted for proton, thus 10 λ_{int} detector is in fact only ~7 λ_{int} for pion , thus sail through probability of proton ($\exp(-10) \approx 5\times10^{-5}$) is very much different from pion ($\exp(-7) \approx 10^{-3}$).**

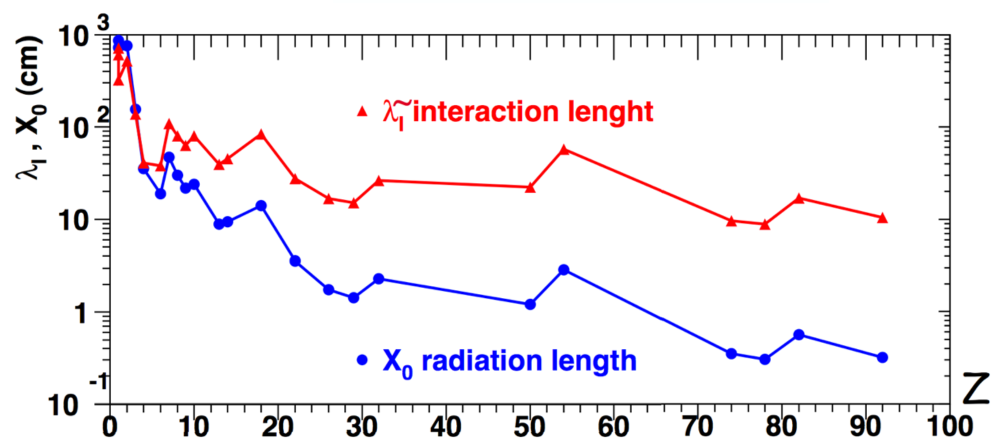


Interaction length (λ_{int}) vs radiation length (X_0)





Material	Density ($g\text{-}cm^{-3}$)	X_0 (cm)	λ_{int} (cm)
H_2(liquid)	0.0708	890	734
He (liquid)	0.125	755	568
Li	0.534	155	134
Be	1.85	35.3	42.1



- **Interaction length is always not larger than radiation length !!!!!**

High Z material for EM calorimeter : Minimum λ_{int} for same length of X_0, reduce the probability of hadronic shower inside ECAL.

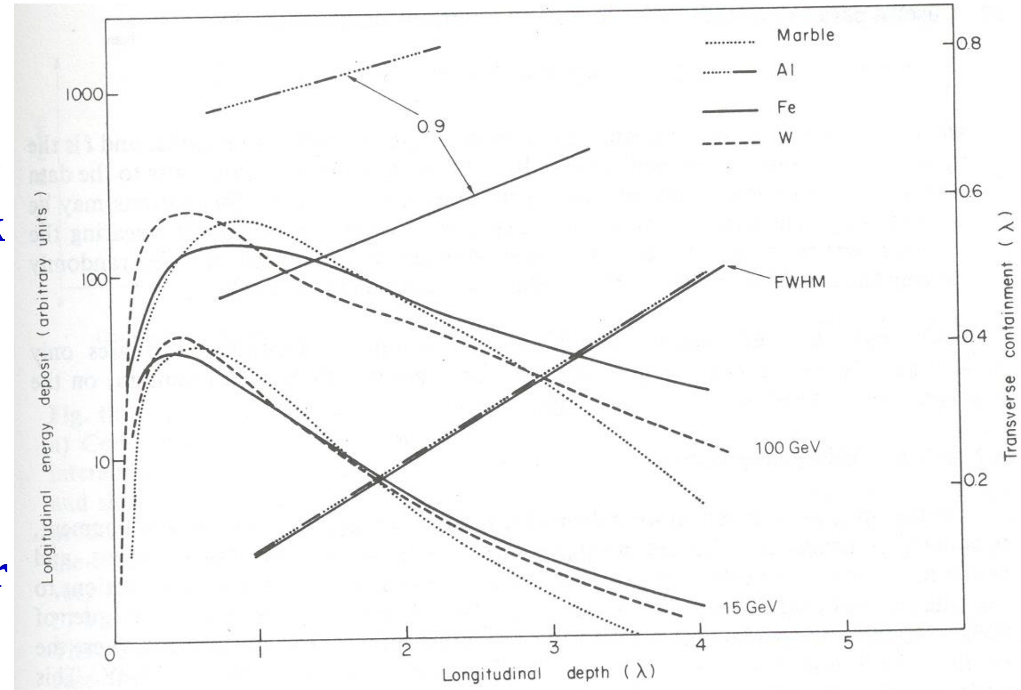
Hadronic shower development

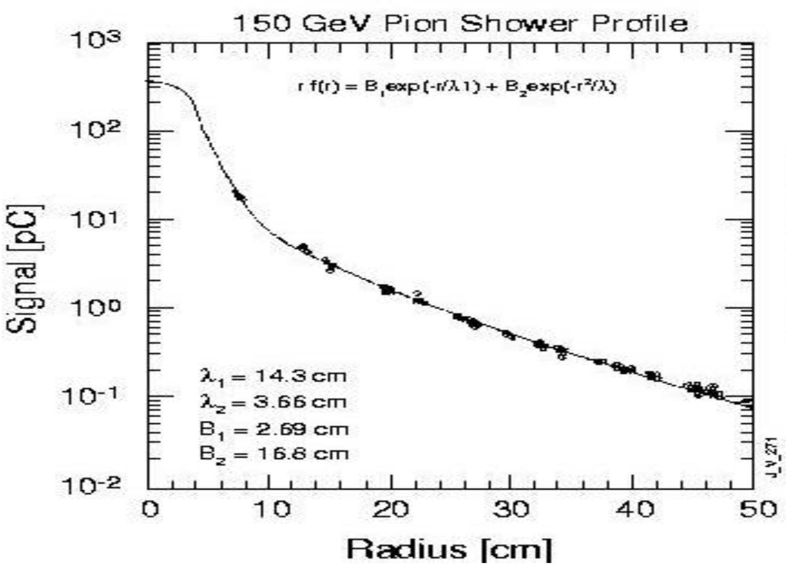
Shower maximum $t_{MAX}(\lambda) \approx 0.2 \ln E + 0.7$, **smaller depth in high Z material due to the smaller ratio of X_0/λ**

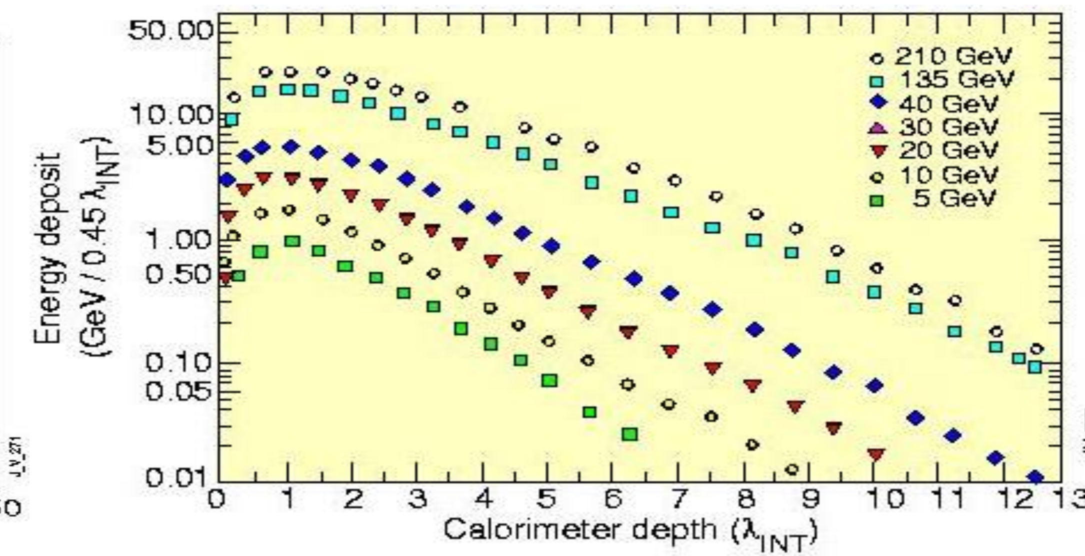
95% energy containment $t_{95}(\lambda) \approx t_{Max} + 2.5\, \lambda_{att}$, where $\lambda_{att} = \lambda E^{0.13}$, **with an weak energy dependence for high Z**

Transverse dimension $R_{95} \leq \lambda$; **does not scale with λ and is smaller in high Z substances**

Peak in lateral and longitudinal shower profile is due to π^0/η (mostly in first interaction length, ***quartz fibre output)***

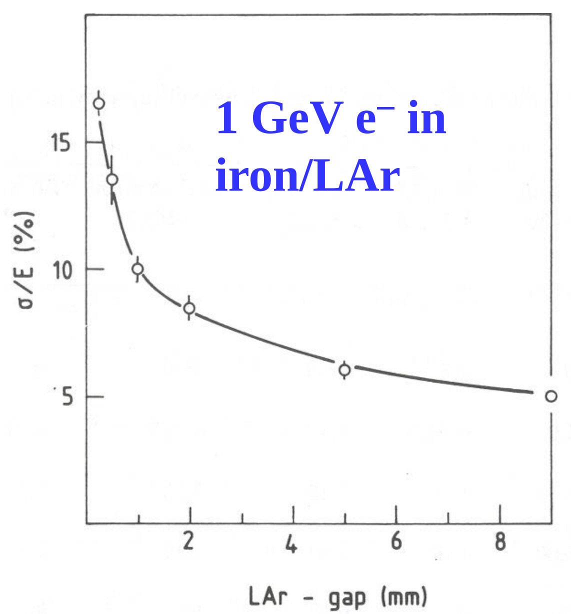


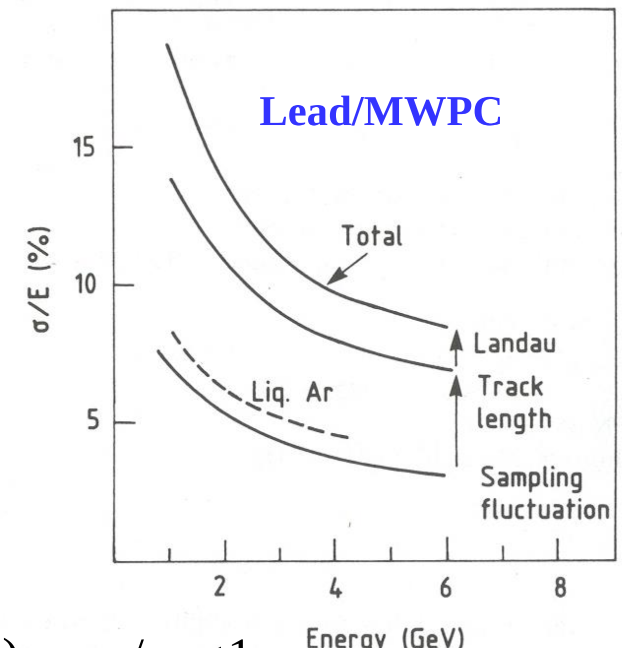




Energy resolution in sampling calorimeter

- **Intrinsic sampling fluctuation :** Total number of track crossing $N_x = T/d = E/\varepsilon_C d = E/\Delta E$, where T=total track length = E/ε_C, d=distance between active plates and ΔE=energy loss per unit cell, $\sigma(E)/E_{sampling} = \sigma(N_x)/N_x = 1/N_x^{-1/2}$
- **Landau fluctuation** of the energy deposit in the active material, $[\sigma(e)/E]_{Landau} \approx 3/[\sqrt{N_x} \times \ln(1.3 \times 10^4 \delta)]$, δ(MeV) is the energy loss per active detector plane
- **Path length fluctuation** in the active and passive material





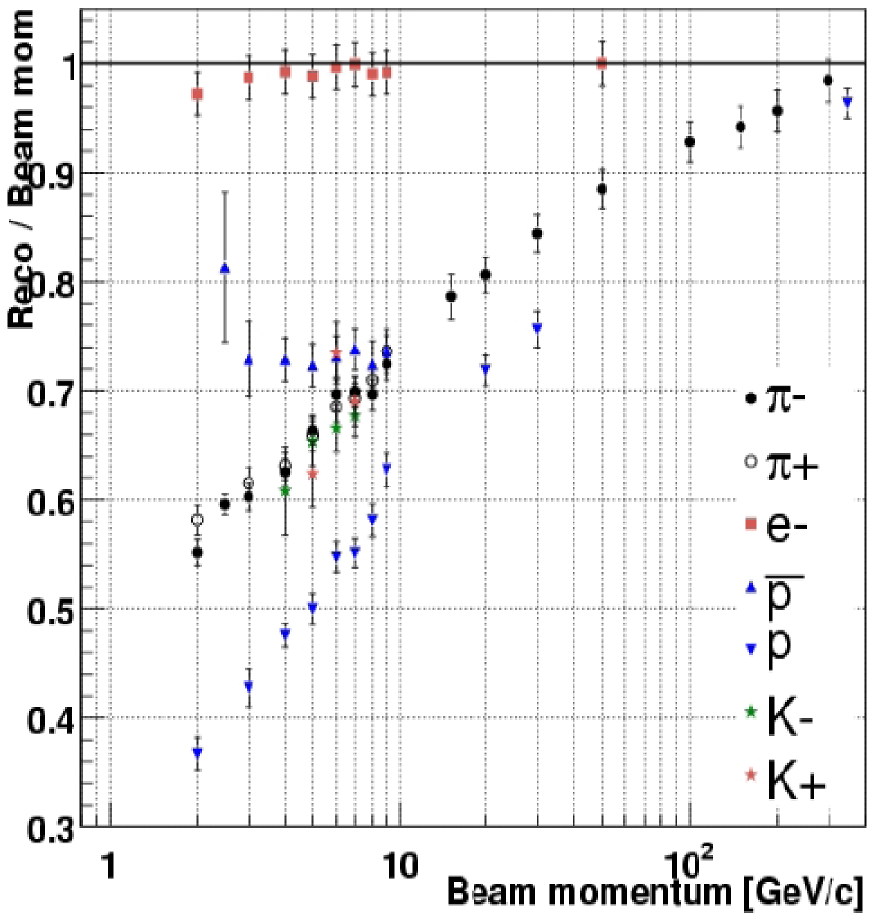
Transition effect : $n_{ep}^{el}(visible) < n_{ep}^{el}(expected)$ or $e/\mu < 1$

Multiple scattering try to increase the effective path length in high-Z material (absorber) relative to the low-Z active material

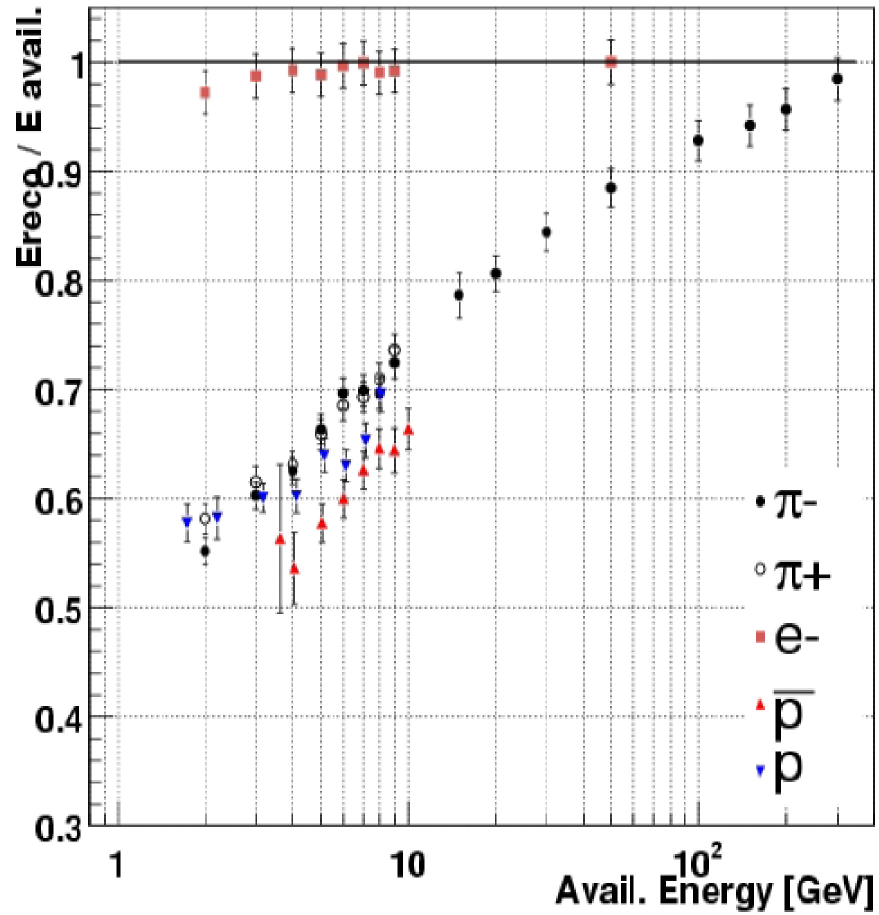
In the last generation of cascade consisting of low-energy particles has saturation effect (in scintillator/liquid Ar)

Available energy of shower (TestBeam)

Using beam energy



Using available energy

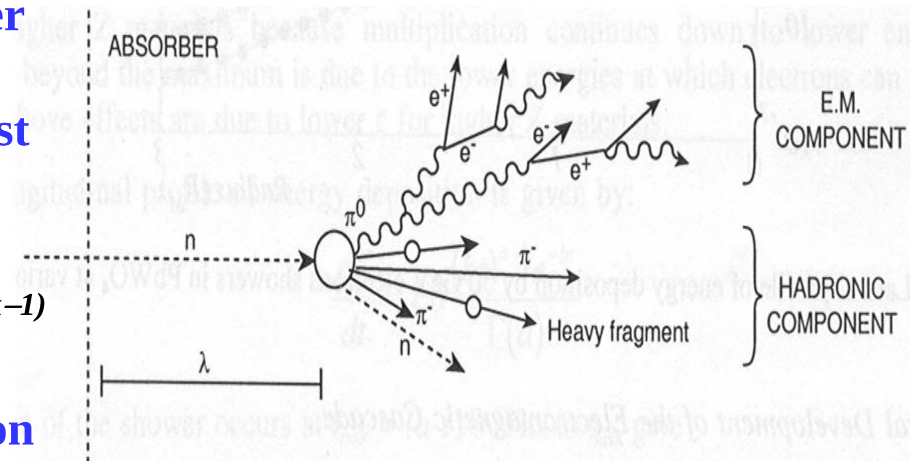


Available energy:
Proton : $E_{kinetic}$
Anti-proton: $E_{kinetic} + 2 \times M_{proton}$

Hadronic shower in physics event contains many particles in comparison to single particle in testbeam/ calibration

Nonlinearity and EM fraction

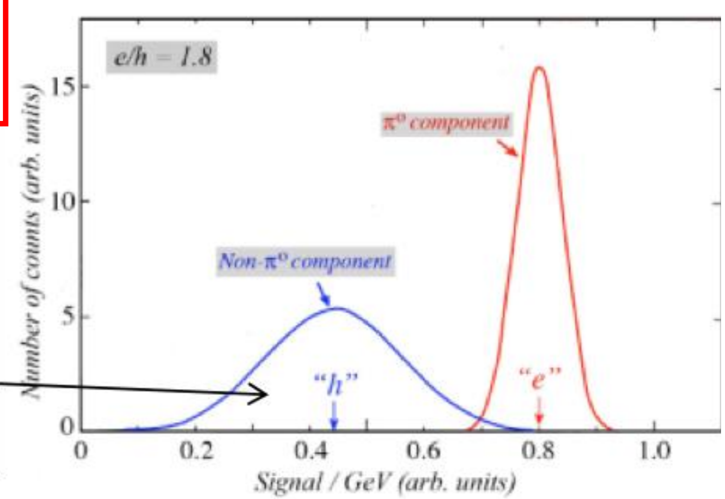
- **The fraction of EM component in the shower makes wider and non-Gaussian energy distribution (known as *e/h ratio ~1.4* for most of the EM material)**
- $\langle f_{EM} \rangle$ **increase logarithmically with energy,** $f_{em} \approx 0.11 \ln(E)$, $\approx 1-(1-1/3)^n$, $\approx 1-(E/E0)^{(k-1)}$ **leads to an tail in upper side of the distribution due to event-by-event fluctuation of π^0 /η components**
- f_{em} *is ~15% less for proton induced shower than pion (Mainly due to baryon number conservation)*



- **Considerable energy goes to π^0/η, which decay electromagnetically and give rise to electromagnetic cascade. They differ both in longitudinal and lateral size as well as visible energy in detector**

Charged hadrons	**20%**
Nuclear fragments, p	**25%**
Neutrons, soft γ's	**15%**
Breakup of nuclei	**40%**

Either not detected or often to slow to be within detector time window
= **Invisible energy**
e/h > 1

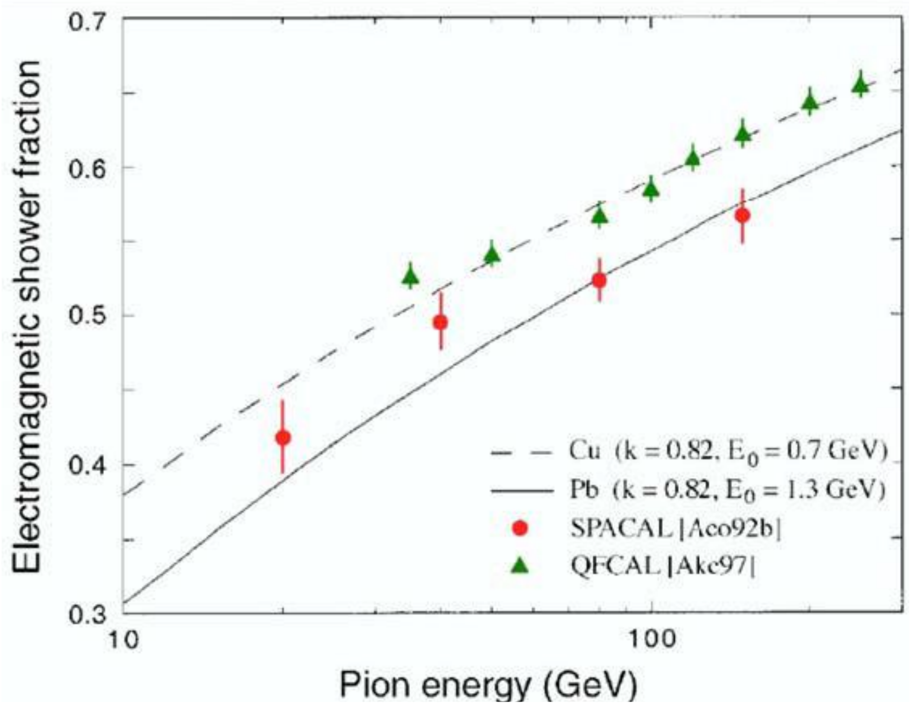


Fluctuations in the em shower component (f_{em})

- Why are these so important
  - EM calorimeter response ≠ non-EM response ($e/h \neq 1$)
  - Event-to-Event fluctuations are large and non-Gaussian
  - $\langle f_{em} \rangle$ depends on shower energy and age
- Cause of all common problems in hadron calorimeters
  - Energy scale different from electrons, in energy-dependent way
  - Hadronic non-linearity
  - Non-Gaussian response function
  - Poor energy resolution
  - Calibration of the sections of a longitudinally segmented detector

Energy resolution due to fluctuation of f_{em}

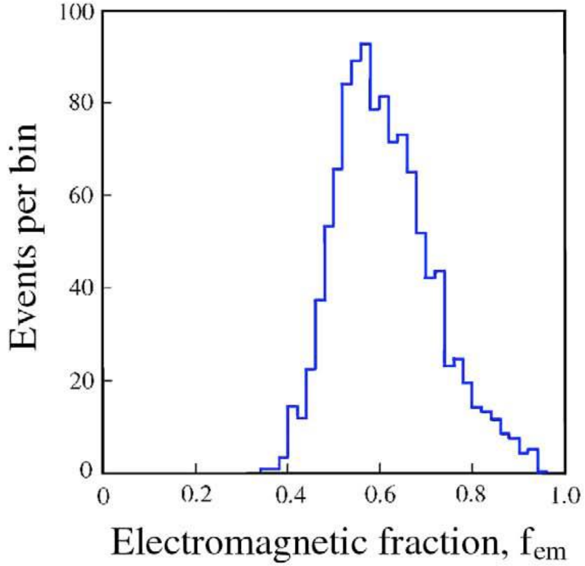
- On the average 1/3 of the mesons produced at each interaction will be π^0's (π^+, π^0, and π^- are equally produced, isospin symmetry)
- Assume that a fraction of EM energy, f_{em} is produced at each step :
  - After 1st step : f_{em}
  - After 2nd step : $f_{em} + f_{em}(1-f_{em})$
- F_{em} , the fraction of EM energy in the shower :
  - $F_{em} = f_{em} \sum (1-f_{em})^{n-1}$, after n generation
  - $F_{em} = 1 - (1-f_{em})^n$,
- Thus,
  - At low energy $F_{em} = f_{em}$
  - At very high energy $F_{em} \to 1$



$$< f_{em} > = 1 - (E/E_0)^{k-1} \Rightarrow$$

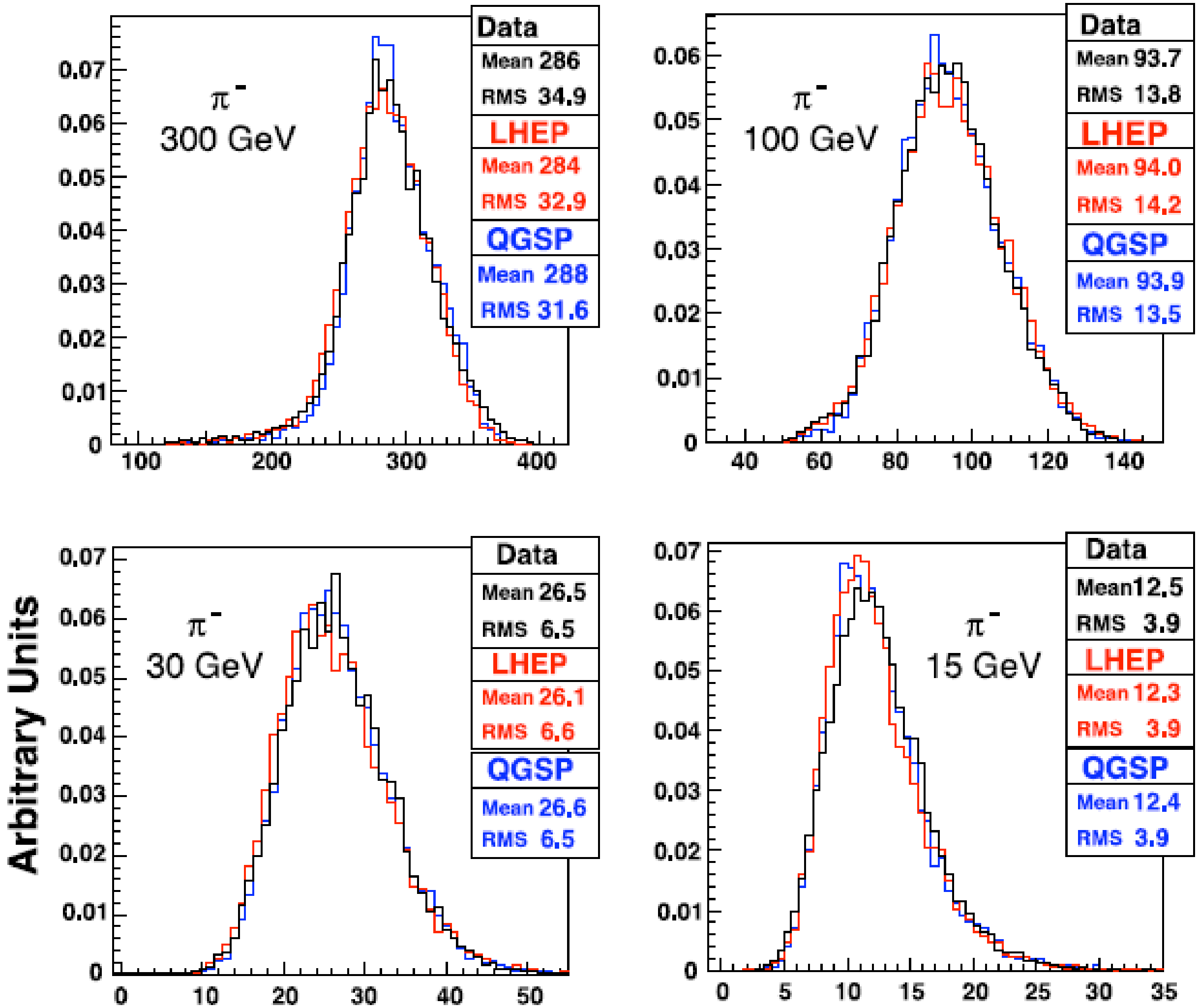
$$\sigma(f_{em}) / < f_{em} > = (E/E_0)^{l-1}$$

Fluctuation in fem are large and non-Poissonian



Energy in Hadronic shower

- **Due to shower leakage, high energy shower has a tail in lower side**
- **Tail in upper side is due to shower fluctuation, mainly f_{em}**



Energy of secondary particles in generation ν

$$e(\nu) = \frac{E}{< n >^{\nu}}; \qquad e(\nu_{\max}) = E_{th} = \frac{E}{< n >^{\nu_{\max}}}$$

<n> =secondaries/primary in each generation

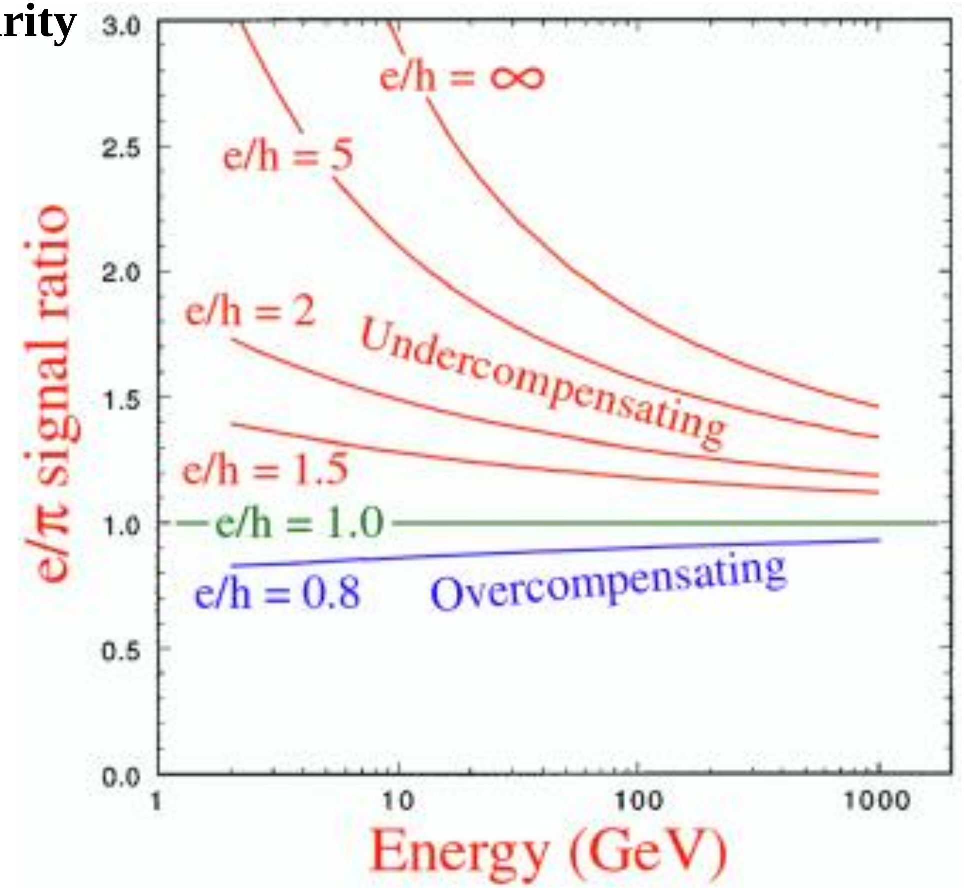
$$n^{\nu_{\max}} = \frac{E}{E_{th}} \Rightarrow \nu_{\max} = \ln(E/E_{th})/\ln < n >$$

e/h and e/π

- e/h: not directly measurable
- e/π: ratio of response between electron-induced and pion-induced shower

$$\frac{e}{\pi} = \frac{e}{f_{em}e + (1 - f_{em})h} = \frac{e}{h} \cdot \frac{1}{1 + f_{em}(e/h - 1)}$$

- e/h is energy independent
- e/π depends on E via $f_{em}(E)$ ⇒ non-linearity
- Approaches to achieve compensation:
  - e/h ⇒ 1 right choice of materials or
  - f_{em} ⇒ 1 (high energy limit)



Experimentally e/h ratio can not be directly measured, but can be done by measuring e/π for large energy range and use empirical formula for f_{em}, e.g., $f_{em} = 1 - (E/E_0)^{(k-1)}$, where E_0 and k are free parameters

Nonlinearity and EM fraction

- **Response of EM component is e**
- **Response of non-EM component is h**
- **E is the energy in the incident energy and the measured energy to electrons (E_e) and charged pions (E_π) related like,**

e/h > 1 : Under compensating
e/h = 1 : compensating
e/h < 1 : Over compensating

$$E_e = eE, \quad E_\pi = [ef_{em} + h(1 - f_{em})]E$$

$$\frac{E_e}{E_\pi} \equiv \frac{e}{\pi} = \frac{(e/h)}{[(e/h)f_{em} + (1 - f_{em})]}$$

E_e/E_π gives the degree of non-compensation

$$Consider \quad dE_\pi = [(e-h)df_{em}]\ E$$

$$\frac{dE}{E} = \frac{df_{em}\,|(e/h) - 1|}{[(e/h)f_{em} + (1 - f_{em})]}$$

$$\left.\frac{dE}{E}\right|_{comp} \sim \frac{1}{\sqrt{f_0(n)}} \sim \frac{1}{\sqrt{\ln E}}$$

Hadron non-linearity and e/h

- Non-linearity determined by e/h value of the calorimeter
- Measurement of non-linearity is one of the methods to determine e/h
- Assuming linearity for EM showers, $e(E_1)=e(E_2)$:
- Difference between the responses to different types of hadrons

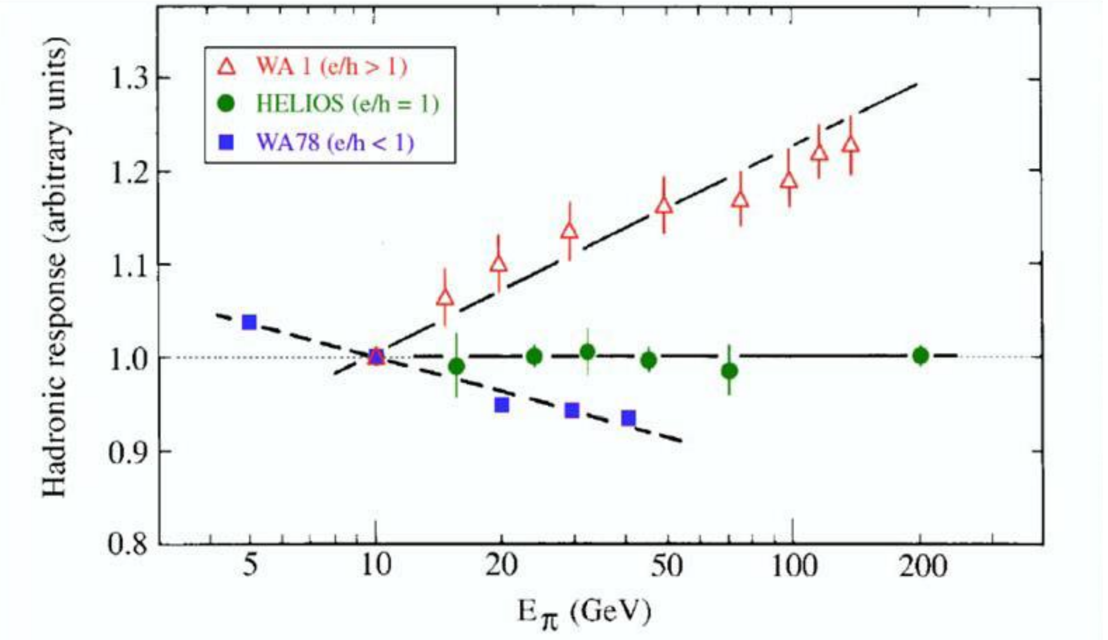


FIG. 3.14. The response to pions as a function of energy for three calorimeters with different e/h values: the WA1 calorimeter ($e/h > 1$, [Abr 81]), the HELIOS calorimeter ($e/h \approx 1$, [Ake 87]) and the WA78 calorimeter ($e/h < 1$, [Dev 86, Cat 87]). All data are normalized to the results for 10 GeV.

$$\frac{\pi(E_1)}{\pi(E_2)} = \frac{f_{em}(E_1).e/h + [1 - f_{em}(E_1)]}{f_{em}(E_2).e/h + [1 - f_{em}(E_2)]}$$

$$e/h = 1 \Rightarrow \frac{\pi(E_1)}{\pi(E_2)} = 1$$

Hadronic response

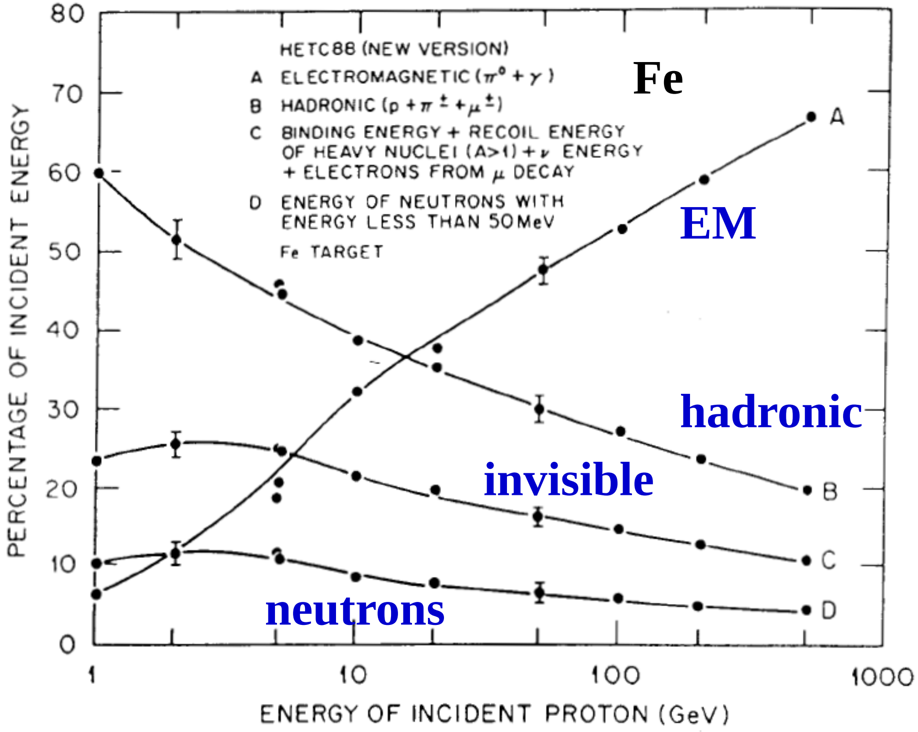
- **Energy deposition mechanisms relevant for the absorption of the non-EM shower energy:**
- **Ionization by charged pions f_{rel} (Relativistic shower component).**
- **spallation protons f_p (non-relativistic shower component).**
- **Kinetic energy carried by evaporation neutrons f_n**
- **The energy used to release protons and neutrons from calorimeter nuclei, and the kinetic energy carried by recoil nuclei do not lead to a calorimeter signal. This is the invisible fraction f_{inv} of the non-em shower energy**
- **The total hadron response can be expressed as:**
  - $h = f_{rel} . rel + f_p . p + f_n . n + f_{inv} . inv \; ; \quad f_{rel} + f_p + f_n + f_{inv} = 1$
- **Normalizing to mip and ignoring the invisible component**

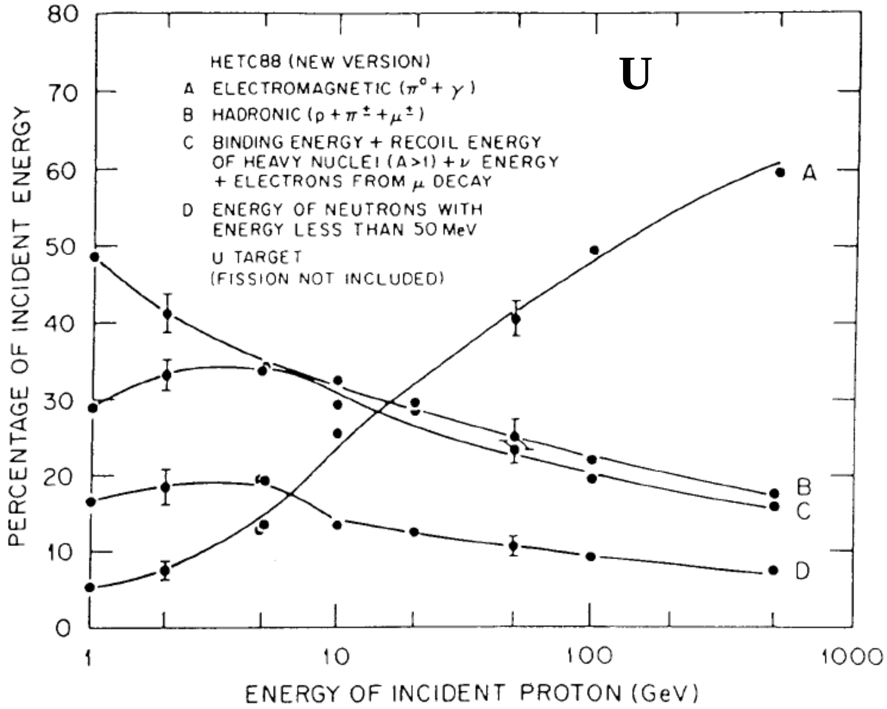
$$\frac{e}{h} = \frac{\frac{e}{mip}}{f_{rel} \cdot \frac{rel}{mip} + f_p \cdot \frac{p}{mip} + f_n \cdot \frac{n}{mip}}$$

- **The e/h value can be determined once we know the calorimeter response to the three components of the non-em shower**

Hadronic shower : energy fractions

$$E_p = f_{em}\, e + (1 - f_{em})h; \quad h = f_{rel} . rel + f_p . p + f_n . n + f_{inv} . inv$$





Compensating Calorimeter and ^{238}U

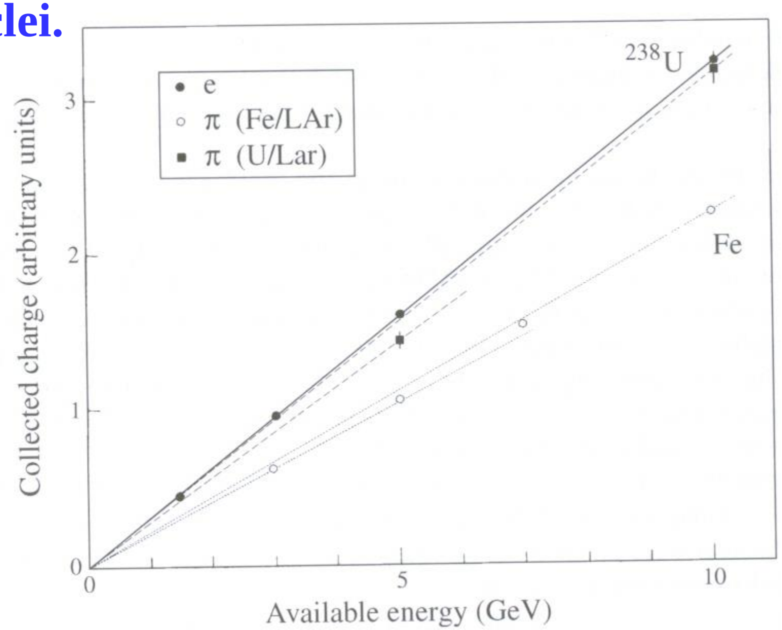
- e/h value can't be measured directly, but can be derived from experimental measurements of e/π signal ratio

$$\frac{e}{\pi} = \frac{e/h}{1 - f_{em}[1 - e/h]}$$

$$\frac{e}{h} = \frac{e/mip}{f_{rel}.rel/mip + f_p.p/mip + f_n.n/mip}$$

- f_{rel}, f_p and f_n : average fraction of energy in the non-em shower component carried by relativistic charged particle, spallation protons (dE/dx, Z-dependency, range(sampling fraction, frequency), saturation), and evaporation neutrons (Nuclear reaction, inelastics, elastic scattering), respectively.
- Homogeneous calorimeter, e/h is always less than one, because of invisible energy, e.g., nuclear binding energy and K.E. of recoil nuclei.

  - Use ^{238}U in passive material (energy release in neutron induced nuclear fission)
  - Construct detector with Fe and U and compare results
  - Looks like result follows the working principle

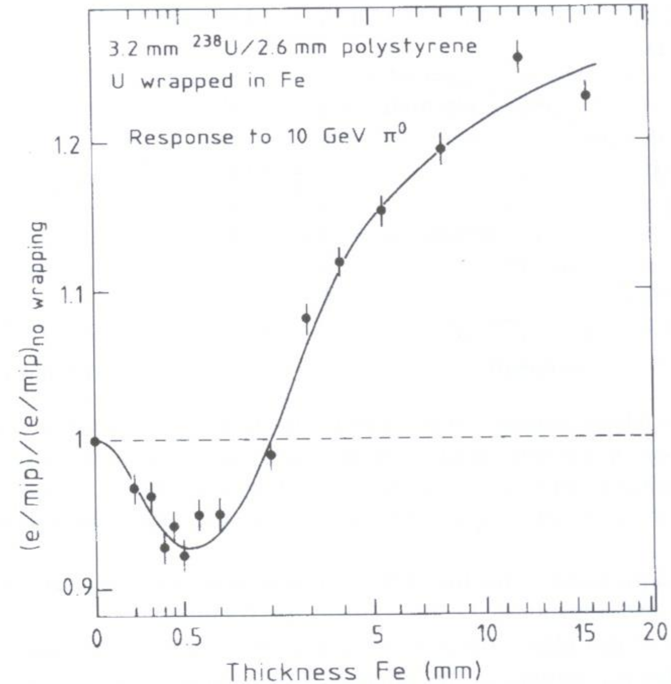


Compensating Calorimeter

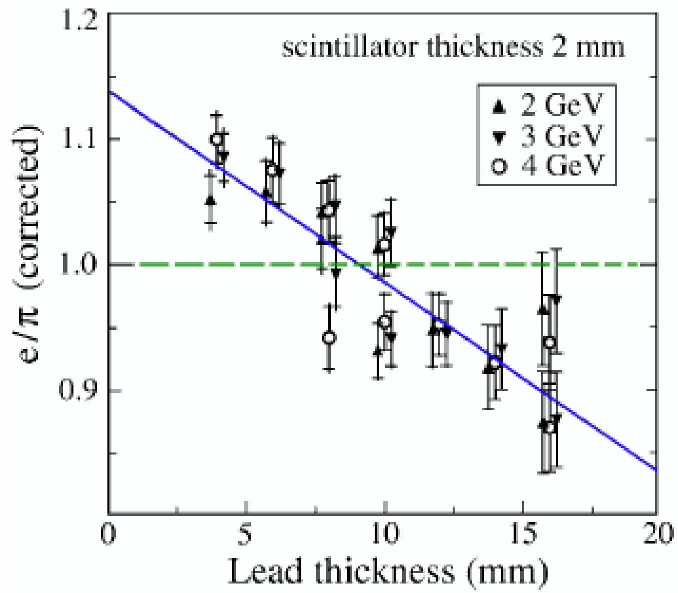
- **2. Reducing the em response by hardware (High Z absorber)**
  - **Photo electric is mainly happen in absorber material (∝ Z^5), e.g., for U/Scint : 3mm:2.5mm, ratio is**

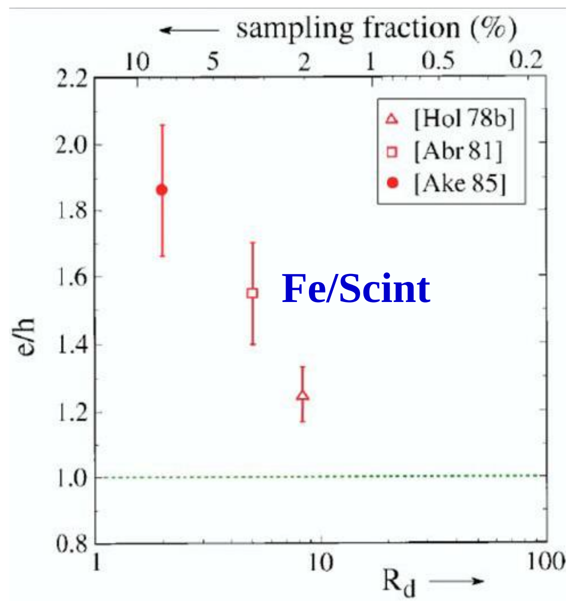
$$\frac{\sigma_a}{\sigma_p}\cdot\frac{A_p}{A_a}\cdot\frac{\rho_a}{\rho_p}\cdot\frac{d_a}{d_p}=\frac{1.7}{76}\cdot\frac{238}{12}\cdot\frac{1.18}{18.95}\cdot\frac{2.5}{3}=0.023$$

  - **thus if the p.e. occurs close to boundary region, electron can escape to active volume**
  - **500 μm of iron ~ range of electron of energy 700KeV, boundary of domination of Compton over p.e.**



- **Compensation adjusting the sampling frequency**
- **Works best with Pb and U**
- **In principle also possible with Fe, but only few n generated**





Compensating Calorimeter

Compensation for ^{238}U/Scintillator and Pb/Scintillator calorimeters requires absorber/scintillator thickness ratio 1:1 and 4:1

hadrons	Pb	$\sigma_{samp} = 41.2 \pm 9.9\%/\sqrt{E}$	$\sigma_{intr} = 13.4 \pm 4.7\%/\sqrt{E}$
	U	$\sigma_{samp} = 31.1 \pm 0.9\%/\sqrt{E}$	$\sigma_{intr} = 20.4 \pm 2.4\%/\sqrt{E}$
Electrons	Pb	$\sigma_{samp} = 23.5 \pm 0.5\%/\sqrt{E}$	$\sigma_{intr} = 0.3 \pm 5.1\%/\sqrt{E}$
	U	$\sigma_{samp} = 16.5 \pm 0.5\%/\sqrt{E}$	$\sigma_{intr} = 2.2 \pm 4.8\%/\sqrt{E}$

The best performance for EM particle deteriorate hadronic performance (incompatible with e/h=1)

In Fe/Scint need ratio > 10:1 $\Rightarrow$
deterioration of longitudinal segmentation

Compensating Calorimeter : capturing slow neutron

What about original concept of uranium and fission fragment ?

In general fission increase non-em response less than 10%. But, for D0, e/h ~1.12, which is by increasing signal integration time from 0.1μs to 2μs.

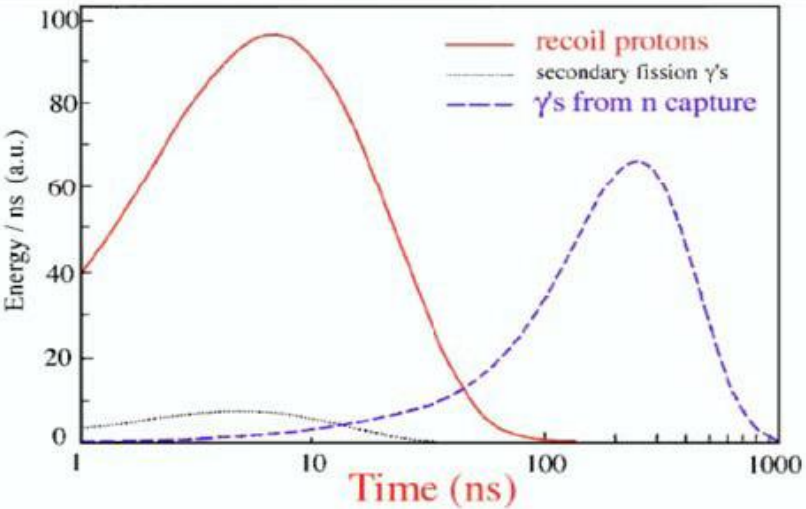


FIG. 3.22. Time structure of various contributions from neutron-induced processes to the hadronic signals of the ZEUS uranium/plastic-scintillator calorimeter [Bru 88].

Large fraction of neutron energy captured and released after >100ns

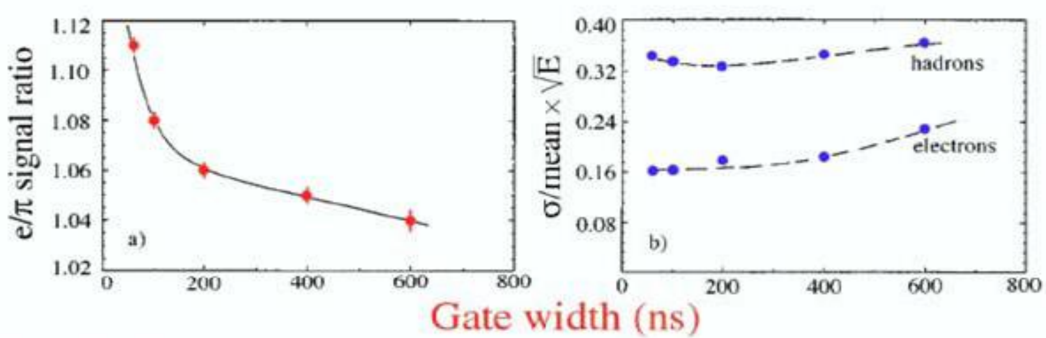


FIG. 3.23. The ratio of the average ZEUS calorimeter signals from 5 GeV/c electrons and pions (a) and the energy resolutions for detecting these particles (b), as a function of the charge integration time [Kru 92].

Long integration time:
- collect more hadron E ⇒ closer to compensation
- integrate additional noise ⇒ worse resolution

Compensating Calorimeter

3. **Hydrogen (scintillator) in the active material (inelastic scattering of neutron)**

Structure ↓/ Particle→	1KeV n	10KeV n	100KeV n	1MeV n	mip
Fe/LH_2 (1/1vol)	93.6%	95.9%	95.6%	92.6%	2.4%
Fe/LAr (1/1vol)	2.0%	2.8%	11.4%	21.5%	15.5%
Pb/LH_2(1/1vol)	99.2%	99.2%	98.8%	98.3%	2.2%

H_2 fraction (%)	1MeV neutron	mips	n/mip ratio
1%	36.9%	0.0227%	1630
2%	54.1%	0.0458%	1180
5%	75.3%	0.118%	640
10%	86.6%	0.249%	350
20%	93.5%	0.558%	170
30%	96.1%	0.953%	100
40%	97.5%	1.47%	66

H_2 fraction (%)	1MeV neutron	mips	n/mip ratio
50%	98.3%	2.20%	45
60%	98.9%	3.26%	30
70%	99.3%	4.98%	20
80%	99.6%	8.24%	12
90%	99.8%	16.8%	5.9
95%	99,9%	29.9%	3.3
99%	99.95%	66.6%	1.5

By changing scintillator fraction n/mip can changed from 1.5 to 1630 (saturation effect reduces this ratio) and choose appropriate ratio to have e/h=1

Compensation by increasing fraction of hydrogen

- **Compensation with hydrogenous active detector**
- **Elastic scattering of soft neurons on protons**

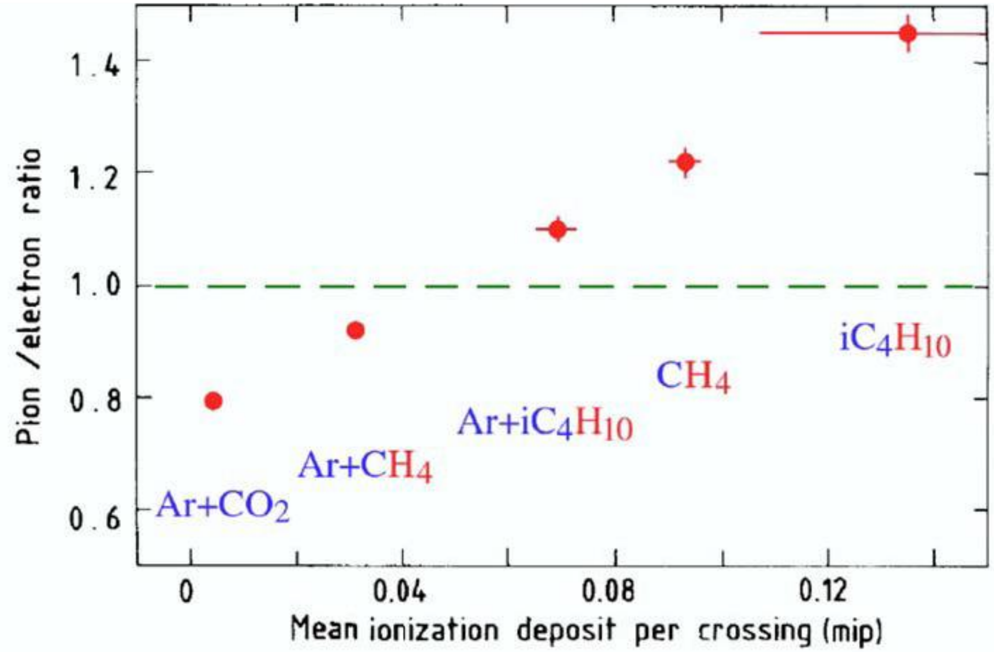


FIG. 3.32. The pion/electron signal ratio, averaged over the energy range 1 5 GeV, measured for different gas mixtures with the uranium/gas calorimeter of the L3 Collaboration. The horizontal scale gives the (calculated) average energy deposit in a chamber gap by slow neutrons [Gal 86].

Plastic scintillator : source of hydrogen as active medium

Compensation is an average effect, but uncorrelated effect in binding energy loss and neutron induced signal may deteriorate calorimeter performance in compensating calorimeter

4. Design of compensating calorimeter

Speciality in Calorimeter: In general, EM response ≠ hadronic response, event-to-event fluctuation is large and non-Gaussian nature of hadronic shower. Measure all main fluctuations, obtain excellent energy resolution. Spatial (fine fibers), EM fraction (Cerenkov and scintillation), binding energy losses/neutrons (time readout, third fiber)

2 m long rods (10 λint) with no longitudinal segmentation

Dual/Triple READout Module (DREAM/TREAM)

- **Achieved resolution <30%/√E (ideal case is 13%)**
- **EM resolution <5%/√E**
- **The entire detector can be calibrated with electrons**

Hadron & Jets :
NIMA537 (2005) 537

Electrons :
NIMA536 (2005) 29

Muons :
NIMA533 (2004) 305

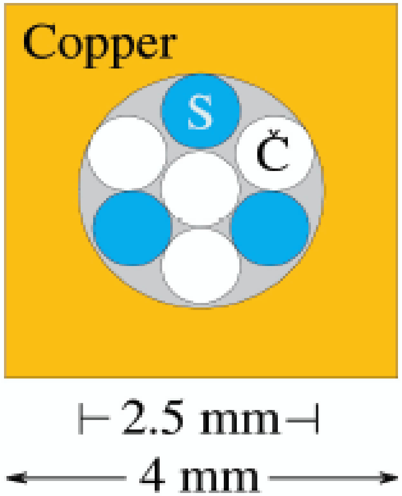
4. Dual readout : Scintillator (hadronic component) and Cherenkov(EM component) Compensating Calorimeter

- Hadronic response (normalized to electrons):

$$R(f_{\rm em}) = f_{\rm em} + \frac{1}{e/h}\left[1 - f_{\rm em}\right], \qquad e/h = 1.3\ (\mathrm{S}),\quad 5\ (\check{\mathrm{C}})$$

- Q/S response *ratio* related to $f_{\rm em}$ value $\rightarrow$ find $f_{\rm em}$ from Q/S:

$$\frac{Q}{S} = \frac{R_Q}{R_S} = \frac{f_{\rm em} + 0.20\ (1 - f_{\rm em})}{f_{\rm em} + 0.77\ (1 - f_{\rm em})}$$



- Correction to measured signals (regardless of energy):

$$S_{\rm corr} = S_{\rm meas}\left[\frac{1 + p_1/p_0}{1 + f_{\rm em}\cdot p_1/p_0}\right], \qquad \text{with} \quad \frac{p_1}{p_0} = (e/h)_S - 1$$

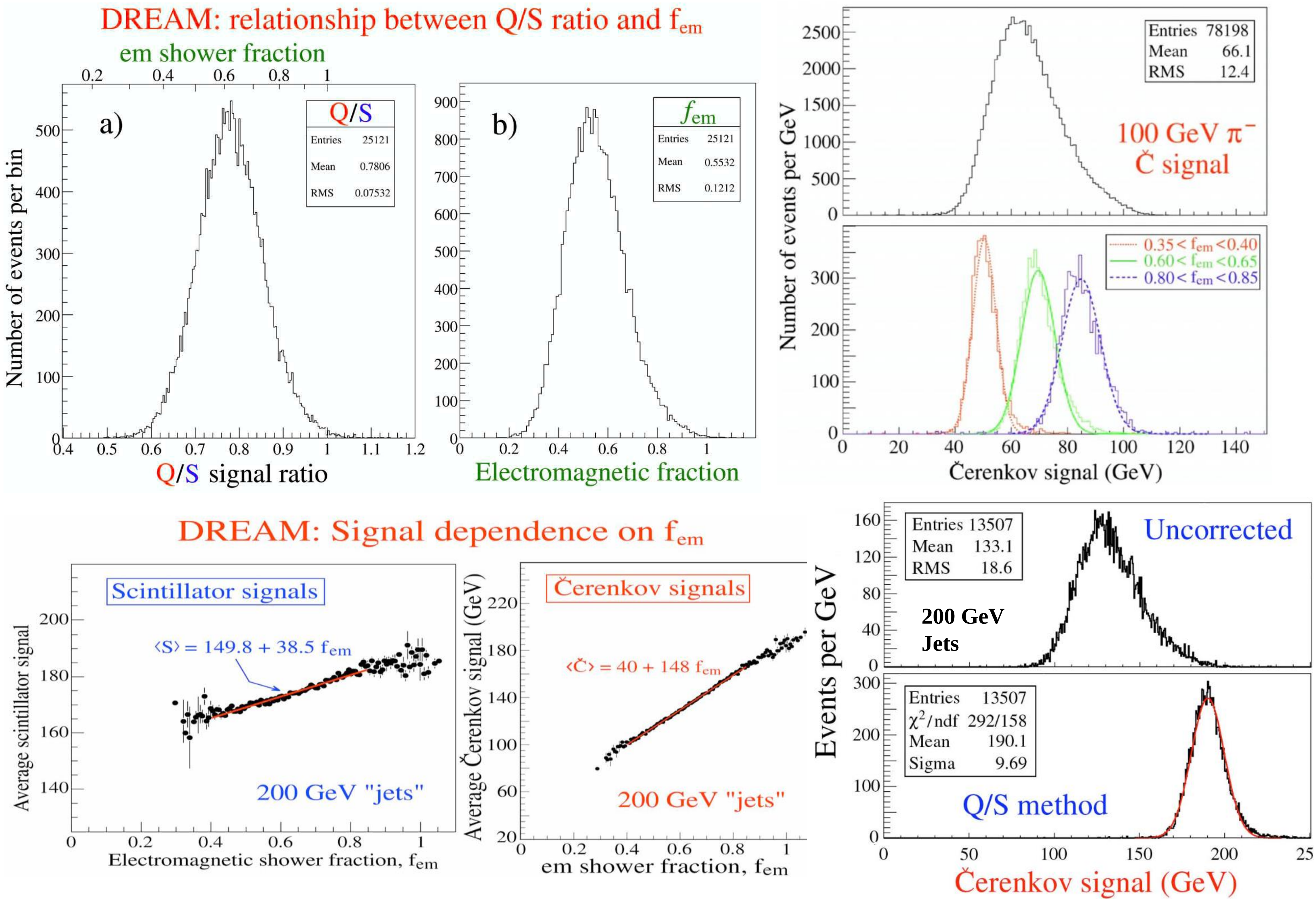
$$Q_{\rm corr} = Q_{\rm meas}\left[\frac{1 + p_1/p_0}{1 + f_{\rm em}\cdot p_1/p_0}\right], \qquad \text{with} \quad \frac{p_1}{p_0} = (e/h)_{\check{C}} - 1$$

$$E_{corr} = E_{meas} \times \frac{e}{\pi}$$

$$\frac{e}{\pi} = \frac{(e/h)}{[(e/h)f_{em} + (1 - f_{em})]}$$

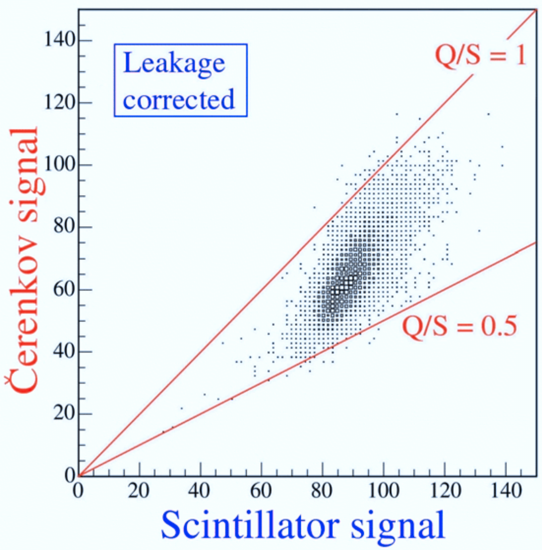
$$R(f_{em}) = \frac{f_{em}[(e/h) - 1] + 1}{e/h} = \frac{1}{e/h} + \frac{e/h - 1}{e/h} f_{em} = p_0 + p_1 \times f_{em}$$

Dream : Effect of event selection based on f_{em}



Determination of E

DREAM: The (energy-independent) Q/S method



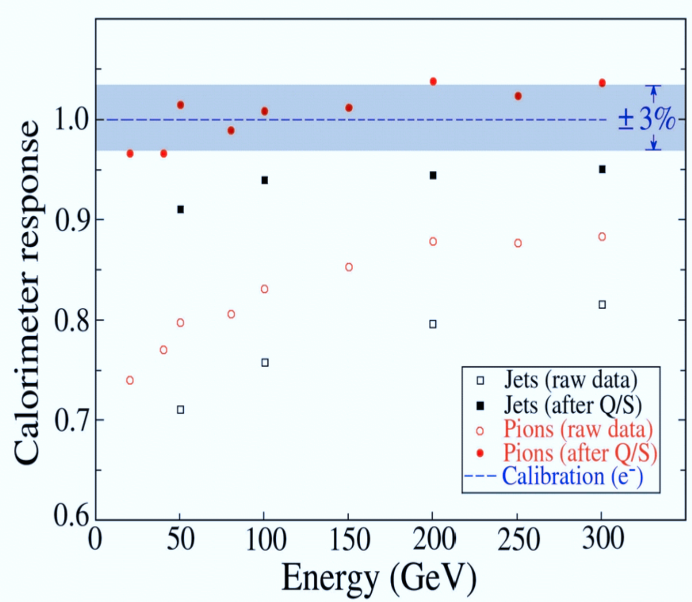
$$S = E\left[f_{em} + \frac{1}{(e/h)_S}(1-f_{em})\right]$$

$$Q = E\left[f_{em} + \frac{1}{(e/h)_Q}(1-f_{em})\right]$$

$e/h = 1.3$ (S), 5 (Q)

$$\frac{Q}{S} = \frac{f_{em} + 0.20\,(1-f_{em})}{f_{em} + 0.77\,(1-f_{em})}$$

Hadronic response: Effect Q/S correction



$$S = E[f + h_S(1-f)], \quad h_S = (h/e)_S$$

$$Q = E[f + h_Q(1-f)], \quad h_Q = (h/e)_Q$$

$$S(1-h_Q) = E\left[(1-h_Q)f + h_S(1-h_Q)(1-f)\right]$$

$$= E\left[f - (h_Q + h_S)f + h_S - h_S h_Q(1-f)\right]$$

Similarly,

$$Q(1-h_S) = E\left[f - (h_Q + h_S)f + h_Q - h_S h_Q(1-f)\right]$$

Thus, $S(1-h_Q) - Q(1-h_S) = E\left[h_S - h_Q\right]$

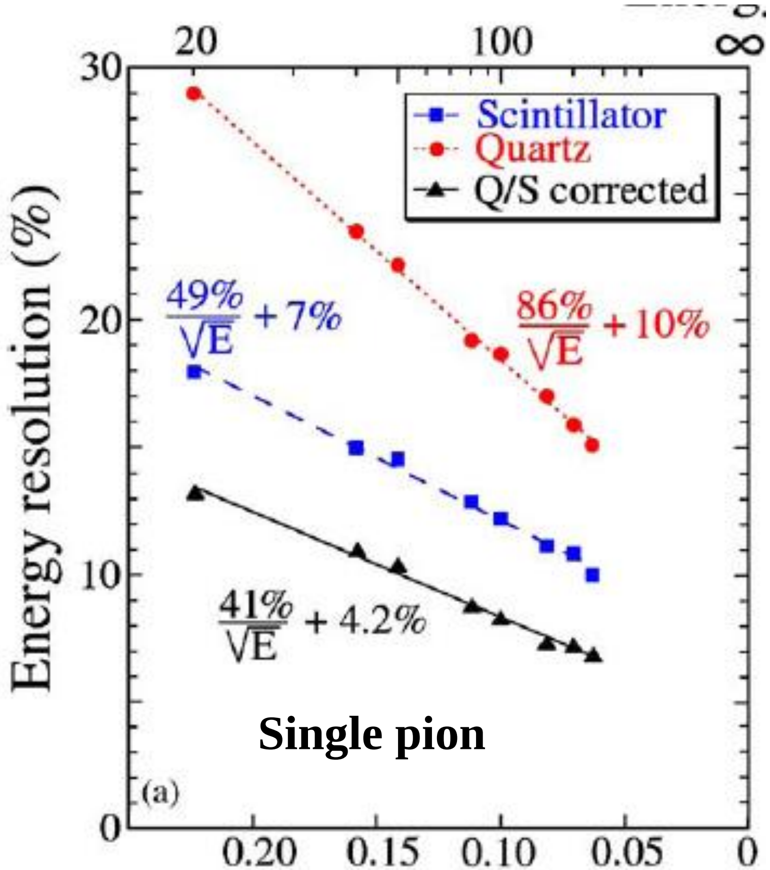
$$\therefore E = \frac{S(1-h_Q) - Q(1-h_S)}{h_S - h_Q} = \frac{S - Q\dfrac{1-h_S}{1-h_Q}}{\dfrac{h_S - h_Q}{1-h_Q}}$$

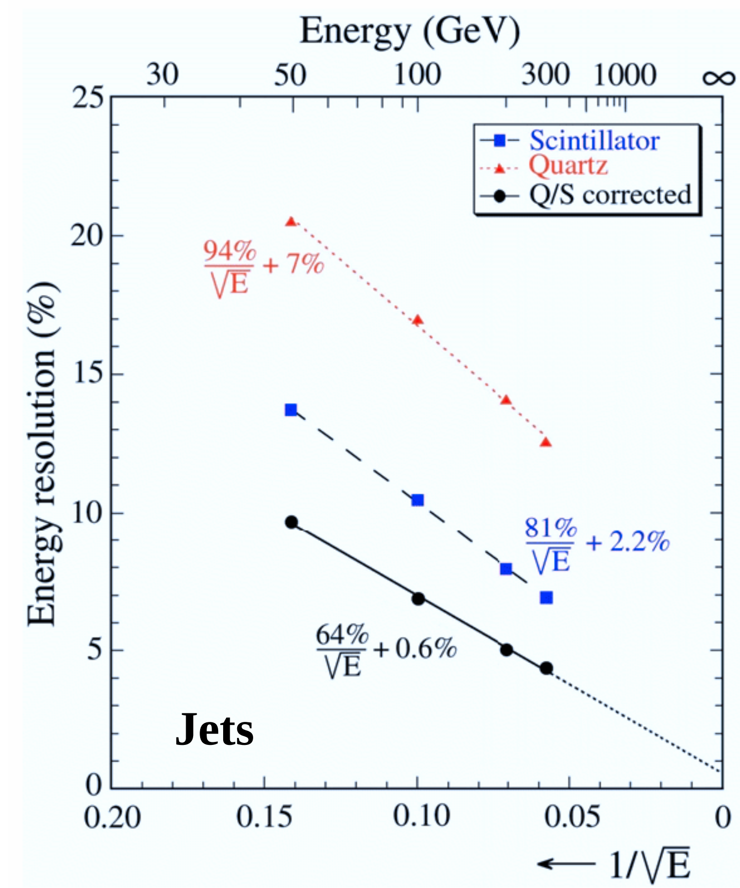
$$= \frac{S - Q\dfrac{1-h_S}{1-h_Q}}{1 - \dfrac{1-h_S}{1-h_Q}} = \frac{S - Q\chi}{1-\chi}$$

where, $\chi = \dfrac{1-h_S}{1-h_Q} \sim 0.3$

Q/S<1 ⇒ ~25% of the scintillator signal from pion showers is caused by nonrelativistic particles, typically protons from spallation or elastic neutron scattering

Compensating Calorimeter : Dual readout





- **D(TREAM) seems capable of meeting/exceeding ILC hadronic calorimeter performance requirement, linearity but not resolution (by removing leakage fluctuation, expecting resolution =20/√E+2.3%)**
- **The entire detector can be calibrated with electrons only**

Indications of Non compensating calorimeter

- Non-linearity in signal
- Non-Gaussian response function
- Difference between the responses to different types of hadrons

Richard Wigmans, CERN Detector Seminar, June 6, 2008

LESSONS FROM 25 YEARS OF R&D

- *LESSON 1:* Energy resolution is determined by *fluctuations*, ***not*** by average values
- *LESSON 2:* Digital calorimetry has been tried *and abandoned,* for good reasons
- *LESSON 3a:* A narrow signal distribution is useless if the mean value is incorrect *Correct energy scale* is at least as as important as good resolution

  LESSON 3b: *Longitudinal segmentation means asking for trouble*
- *LESSON 4:* GEANT based MC simulations of hadronic shower development are *fundamentally flawed* ⟶ useless as design tool
- *LESSON 5:* If you want to improve hadronic calorimeter performance ⟶ *reduce/eliminate the (effects of) fluctuations that dominate the performance* :

  i) Fluctuations in the em shower fraction, f_{em}

  ii) Fluctuations in visible energy (nuclear binding energy losses)