



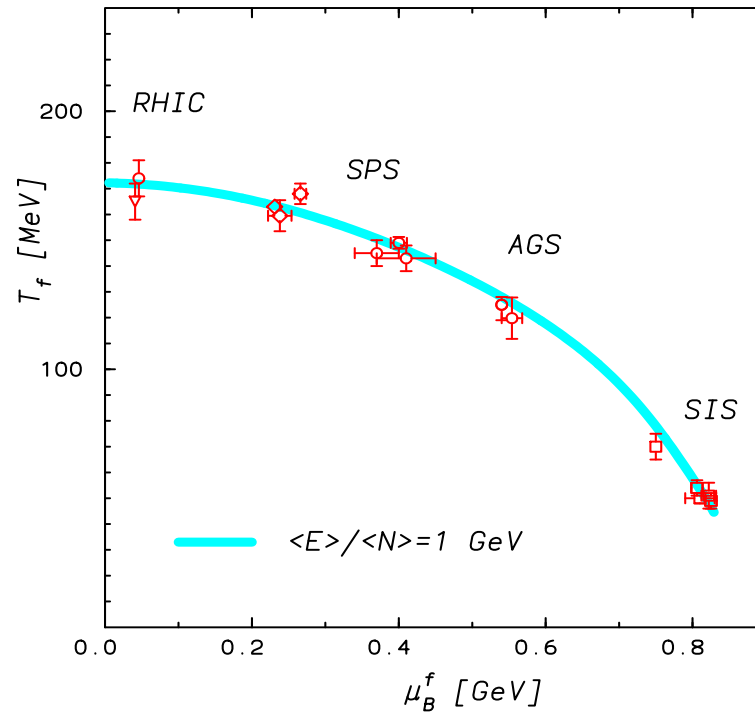
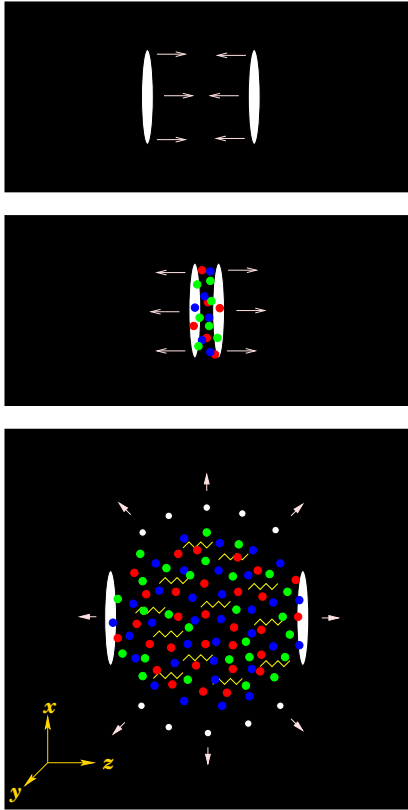
A model approach to thermodynamics of Strong Interactions

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Workshop on QCD in the Nonperturbative Regime, TIFR, 18–20 Nov 2019

Thermalization in HIC



Thermal model:

$$Z(T, \mu_B, \mu_I, \mu_S) = \sum_i Z_i(T, \mu_B, \mu_I, \mu_S) \Rightarrow n_i(T, \mu_B, \mu_I, \mu_S)$$

J. Cleymans and K. Redlich: *Phys. Rev. Lett* 81 (1998) 5284

P. Braun-Munzinger, K. Redlich and J. Stachel:

Quark Gluon Plasma 3, R.C. Hwa and X.-N. Wang, ed., (World Scientific) (nucl-th/0304013)



$$\mathcal{L}_{QCD}^E = \frac{1}{4} F_{\mu\nu}^a F_{\mu\nu}^a + \sum_{f=1}^{N_f} \bar{q}_f \left(\gamma_\mu^E D_\mu + m_f - \mu_f \gamma_0 \right) q_f$$

where,

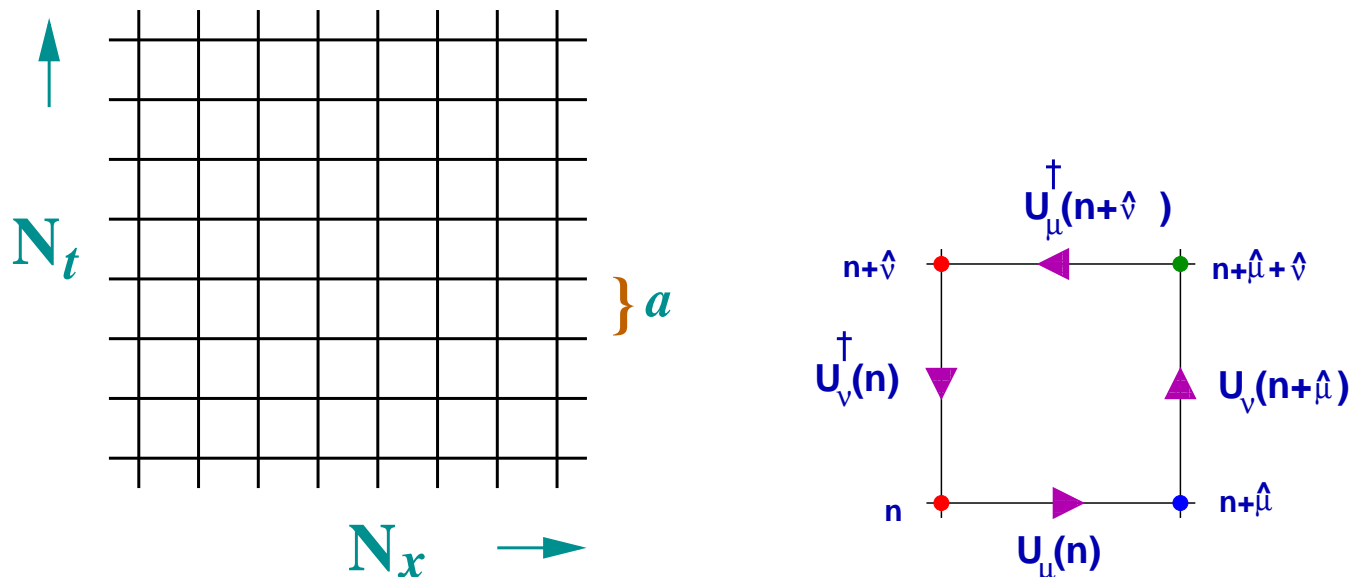
$$\begin{aligned} F_{\mu\nu}^a &= \partial_\mu G_\nu^a - \partial_\nu G_\mu^a + g f^{abc} G_\mu^b G_\nu^c \\ D_\mu &= \partial_\mu - ig T^a G_\mu^a \end{aligned} \quad a = 1, 2, \dots, 8.$$

T^a are SU(3) group generators and f^{abc} are SU(3) structure constants
The QCD partition function is,

$$Z = \int DG_\nu^a Dq_f D\bar{q}_f e^{-\int_0^\beta d\tau \int_{-\infty}^{\infty} d^3x \mathcal{L}_{QCD}^E}$$



Non-perturbative QCD



- Quarks sit on the lattice points $q(n)$
- Gluons are on the links $U_\mu(n) = \mathcal{P} \exp \left[ig \int_n^{n+\hat{\mu}a} dy^\sigma G_\sigma^a(y) T^a \right]$
- $V = a^3 (N_x \times N_y \times N_z)$ $\beta = aN_t$
- momentum cutoff $\simeq \frac{1}{a}$ $a \rightarrow 0 \Rightarrow$ Continuum physics



Polyakov Nambu Jona-Lasinio Model :

Ratti et.al. PRD 73 014019 '06.

$$\mathcal{L}_{PNJL} = \bar{q} (i\gamma^\mu D_\mu - m_0 + \mu\gamma^0) q + \frac{\mathcal{G}}{2} \left[(\bar{q}q)^2 + (\bar{q}i\gamma^5 \vec{\tau}q)^2 \right] - \mathcal{U}(\bar{\Phi}, \Phi, T)$$

where $D_\mu = \partial_\mu - igG_\mu$, and $G_\mu = \delta_{\mu 0}G_0$

$$\frac{\mathcal{U}(\Phi, \bar{\Phi}, T)}{T^4} = -\frac{b_2(T)}{2} \bar{\Phi}\Phi - \frac{b_3}{6} (\Phi^3 + \bar{\Phi}^3) + \frac{b_4}{4} (\bar{\Phi}\Phi)^2$$



(De)Confinement at $T \neq 0$

- Finite Temperature

$$\beta = 1/T$$

- Free energy of vacuum

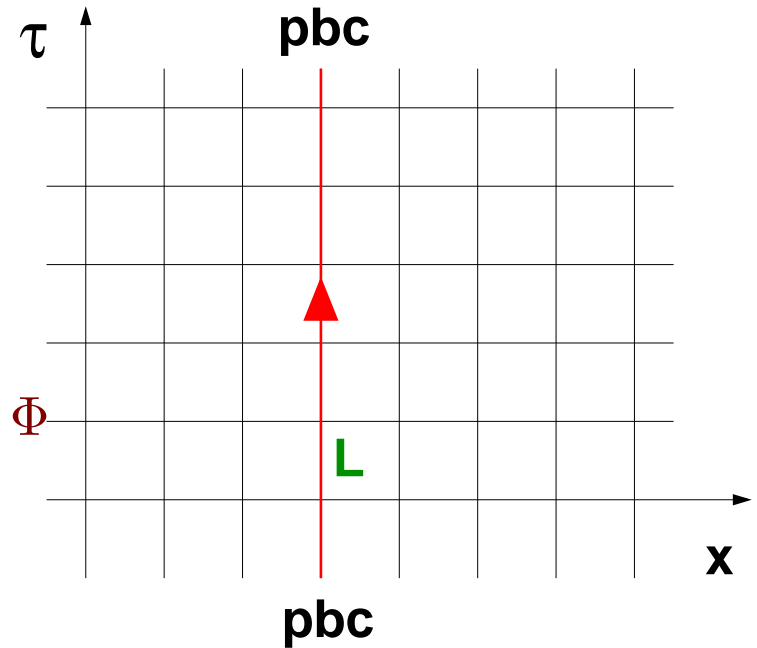
$$F_0 = -T \log Z$$

- Free energy of a single quark

$$F_q - F_0 = -T \log \langle \text{Tr} L(\vec{x}) / 3 \rangle = -T \log \Phi$$

$$L(\vec{x}) = \mathcal{P} \exp \left[- \int_0^{1/T} d\tau G_0(\vec{x}, \tau) \right]$$

Wilson Line/Polyakov Loop



- $\Phi(\vec{x}, \tau) = \begin{cases} \neq 0 \Rightarrow F_q \text{ finite} \Rightarrow \text{deconfined} \\ = 0 \Rightarrow F_q \text{ infinite} \Rightarrow \text{confined} \end{cases}$

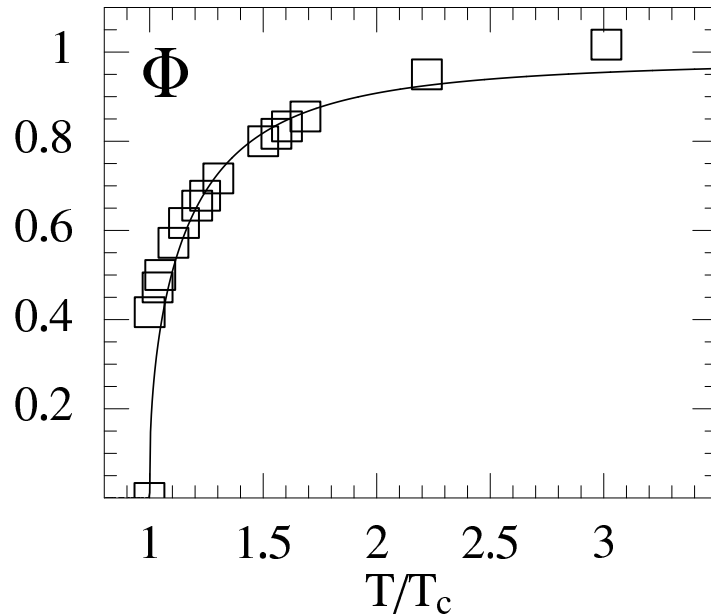
- Use Φ as OP for finite temperature phase transition.

Global $Z(3)$ symmetry

- The gluons G_μ are bosons and satisfy periodic boundary conditions in the Euclidian time and keep the QCD action invariant
 $\Rightarrow G_\mu(\vec{x}, \tau = 0) = G_\mu(\vec{x}, \tau = 1/T)$
- The QCD action is however invariant under a somewhat more general condition
 $\Rightarrow G_\mu(\vec{x}, \tau = 0) = z G_\mu(\vec{x}, \tau = 1/T)$
with $z \in Z(3) = \exp(i2\pi n/3)$, $n = 0, 1, 2$, the center group of $SU(3)$.
- But the Polyakov loop $\Phi(\vec{x}, \tau)$ is not invariant
 \Rightarrow There seems to be a connection between deconfinement and spontaneous $Z(3)$ symmetry breaking !!!



Polyakov Loop Model : (De)Confinement



- Choose some $V(\Phi)$ as a polynomial, parametrized using Lattice:
Lattice EOS: Scavenius et.al. PRC 66 034903 '02.

$$\frac{U(\bar{\Phi}, \Phi, T)}{T^4} = -\frac{b_2(T)}{2} \bar{\Phi} \Phi - \frac{b_3}{6} (\Phi^3 + \bar{\Phi}^3) + \frac{b_4}{4} (\bar{\Phi} \Phi)^2$$

where, $\Phi = \langle \text{Tr} L \rangle$; $\bar{\Phi} = \langle \text{Tr} L^\dagger \rangle$

Global Chiral Symmetry

$$\mathcal{L}_{QCD} = -\frac{1}{4}F_{\mu\nu}^a F_a^{\mu\nu} + \sum_{f=u,d} [i\bar{q}_f \gamma^\mu D_\mu q_f - m_f \bar{q}_f q_f]$$

Symmetries: $SU(3)_c \otimes \underbrace{SU(2)_V \otimes SU(2)_A \otimes U(1)_B \otimes U(1)_A}_{\text{Fermionic}}$

- $U(1)_A$ broken by quantum anomalies.
 - $SU(2)_V$ broken explicitly when flavour degeneracy is lifted
e.g. proton and neutron mass splitting.
 - $SU(2)_A$ broken explicitly for non-zero quark mass
where are the chiral partners !!
 - $SU(2)_A$ broken spontaneously; pions are the Goldstone Bosons
→ Measure is the chiral condensate $\langle \bar{q}q \rangle$.
- $$\langle \bar{q}q \rangle = \begin{cases} \neq 0 \Rightarrow \text{symmetry broken} \\ = 0 \Rightarrow \text{symmetry restored} \end{cases}$$

Nambu Jona-Lasinio Model : Chiral aspect

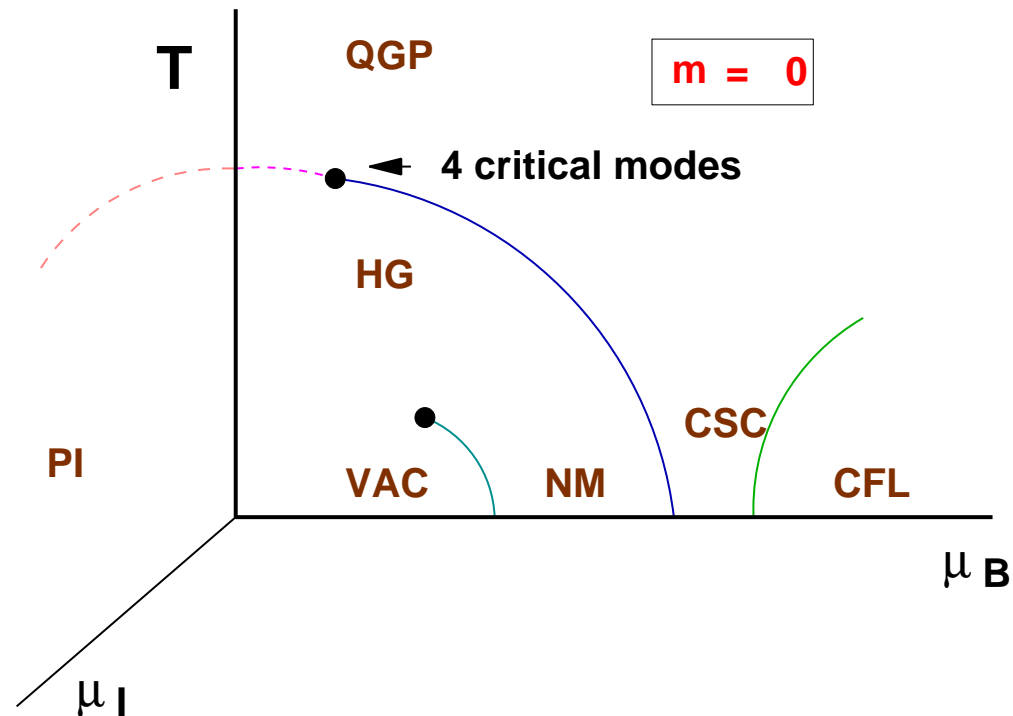
Lagrangian: (1961)

$$\mathcal{L}_{NJL} = \bar{q} (i\gamma^\mu \partial_\mu - m_0 + \mu\gamma^0) q + \frac{\mathcal{G}}{2} \left[(\bar{q}q)^2 + (\bar{q}i\gamma^5 \vec{\tau}q)^2 \right]$$

- Symmetries: $SU(2)_V \otimes SU(2)_A \otimes U(1)_B$.
- Introducing auxillary field variables σ and $\vec{\pi}$ an \mathcal{L}_{eff} is obtained.
- The mean fields $\langle \sigma \rangle = \mathcal{G} \langle \bar{q}q \rangle$ and $\langle \vec{\pi} \rangle = 0$ for $\mu_I < m_\pi$.
- Fit emperical values of m_π , f_π and $g_{\pi NN}$ (RMP [64 649 '92](#)).
Obtain $m_0 = 5.5 \text{ MeV}$, $\mathcal{G} = 10.08 \text{ GeV}^{-2}$, cutoff $\Lambda = 0.651 \text{ GeV}$.
- Thermodynamic properties studied with the thermodynamic potential $\Omega[\sigma, T, \mu_q, \mu_I]$,
where $\mu_q = \frac{\mu_u + \mu_d}{2}$; $\mu_I = \frac{\mu_u - \mu_d}{2}$



Phase Diagram



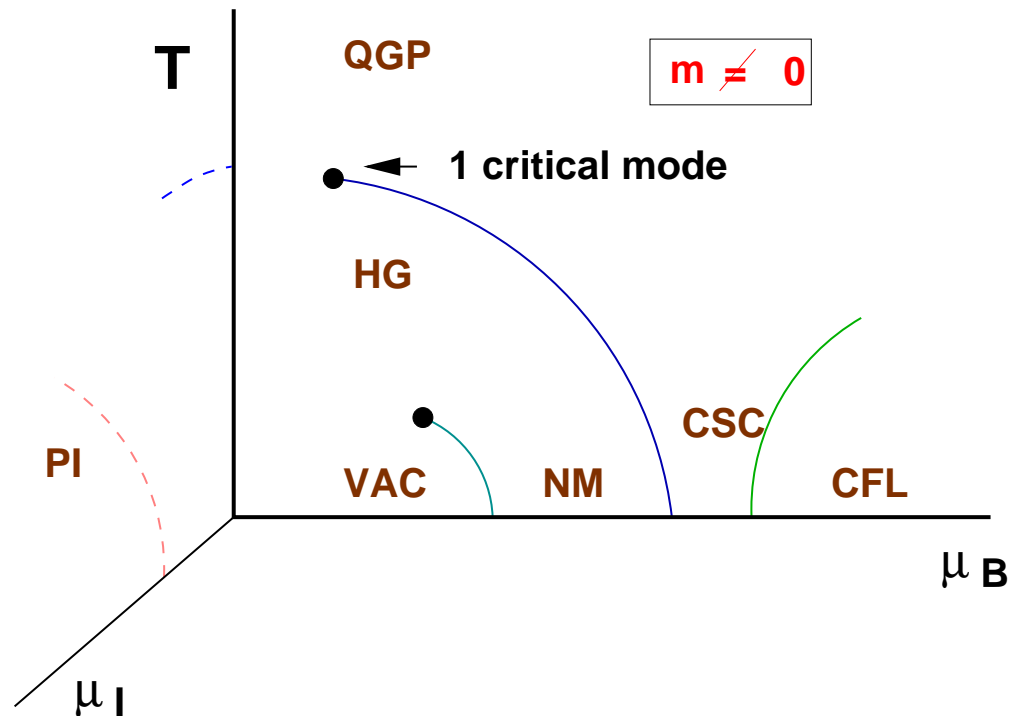
$$\mu_B = \frac{1}{3} \left(\frac{\mu_u + \mu_d}{2} \right) \quad ; \quad \mu_I = \left(\frac{\mu_u - \mu_d}{2} \right)$$

Rajagopal, Wilczek : The Condensed Matter Physics of QCD

Ch. 35, 'Handbook of QCD', M. Shifman, ed., (World Scientific) (hep-ph/0011333).



Phase Diagram



$$\mu_B = \frac{1}{3} \left(\frac{\mu_u + \mu_d}{2} \right) \quad ; \quad \mu_I = \left(\frac{\mu_u - \mu_d}{2} \right)$$

Rajagopal, Wilczek : The Condensed Matter Physics of QCD

Ch. 35, 'Handbook of QCD', M. Shifman, ed., (World Scientific) (hep-ph/0011333).



PNJL Model : Confinement + Chiral aspects

Lagrangian: Ratti et.al. PRD 73 014019 '06.

$$\mathcal{L}_{PNJL} = \bar{q} (i\gamma^\mu D_\mu - m_0 + \mu\gamma^0) q + \frac{\mathcal{G}}{2} \left[(\bar{q}q)^2 + (\bar{q}i\gamma^5\vec{\tau}q)^2 \right] - U(\bar{\Phi}, \Phi)$$

where $D_\mu = \partial_\mu - igG_\mu$,
and $G_\mu = \delta_{\mu 0}G_0$

- Introducing auxillary field variables σ and $\vec{\pi}$ an \mathcal{L}_{eff} is obtained, with the replacement $\exp[-G_0/T] \rightarrow \Phi$
- The mean fields $\langle\sigma\rangle = \mathcal{G}\langle\bar{q}q\rangle$ and $\langle\vec{\pi}\rangle = 0$ for $\mu_I < m_\pi$.
- Thermodynamic properties studied with $\Phi(T)$ and σ from the thermodynamic potential $\Omega[\bar{\Phi}, \Phi, \sigma, T, \mu_q, \mu_I]$, where $\mu_q = \frac{\mu_u + \mu_d}{2}$; $\mu_I = \frac{\mu_u - \mu_d}{2}$



PNJL Model: 2 flavors

The thermodynamic potential: [Ratti et.al. PRD 73 014019 '06.](#)

$$\begin{aligned}\Omega &= \mathcal{U}(\Phi, \bar{\Phi}, T) + 2G_1(\sigma_u^2 + \sigma_d^2) + 4G_2\sigma_u\sigma_d \\ &- \sum_{f=u,d} 2T \int \frac{d^3p}{(2\pi)^3} \left\{ \ln \left[1 + 3 \left(\Phi + \bar{\Phi} e^{-(E_f - \mu_f)/T} \right) e^{-(E_f - \mu_f)/T} + e^{-3(E_f - \mu_f)/T} \right] \right. \\ &+ \left. \ln \left[1 + 3 \left(\bar{\Phi} + \Phi e^{-(E_f + \mu_f)/T} \right) e^{-(E_f + \mu_f)/T} + e^{-3(E_f + \mu_f)/T} \right] \right\} \\ &- \sum_{f=u,d} 6 \int \frac{d^3p}{(2\pi)^3} E_f \theta(\Lambda^2 - \vec{p}^2)\end{aligned}$$

where,

$$\frac{\mathcal{U}(\Phi, \bar{\Phi}, T)}{T^4} = -\frac{b_2(T)}{2} \bar{\Phi}\Phi - \frac{b_3}{6} (\Phi^3 + \bar{\Phi}^3) + \frac{b_4}{4} (\bar{\Phi}\Phi)^2$$

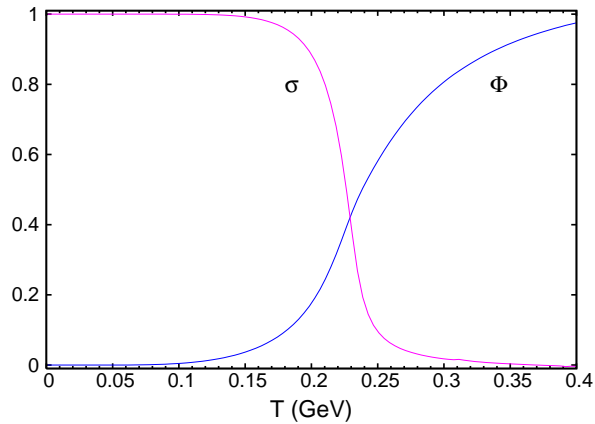


Models

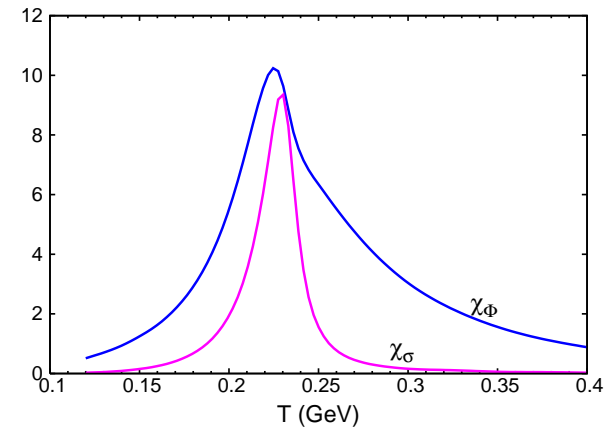
- Introduce models incorporating the global symmetries of QCD
 - Aim is to develop physical insights
 - May be too simplified ...
 - Find out problems and keep improving



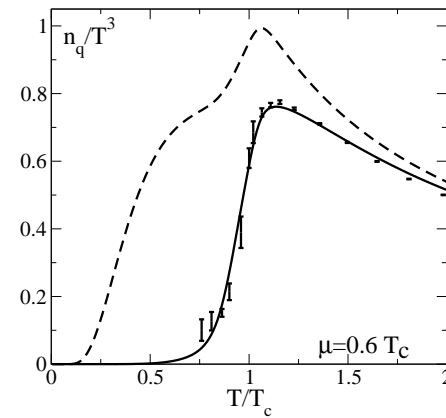
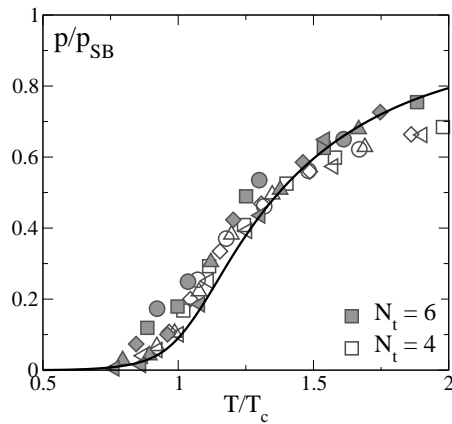
Thermodynamics



Variation of the OPs



$T_c \sim 230$ MeV (Peak separation ~ 5 MeV)

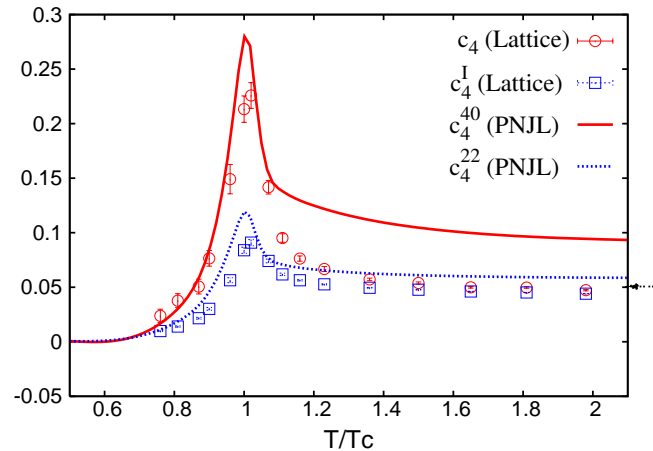
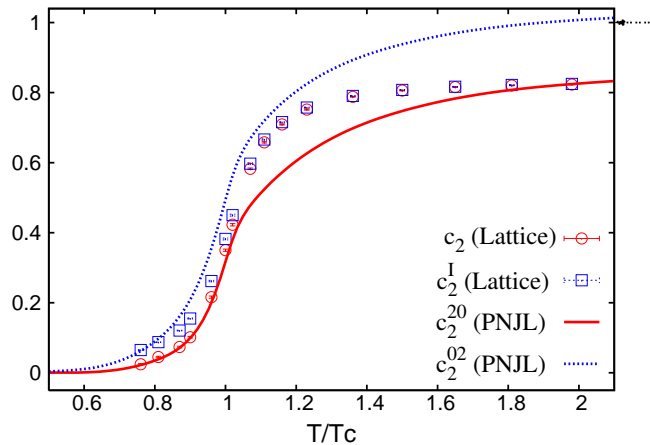


Lattice: CP-PACS PRD 64 074510 '01.

PNJL model: Ratti et.al. PRD 73 014019 '06.



Fluctuations



● **Lattice:** $c_2 \sim c_2^I \sim 80\%$ SB limit ; $c_4 \sim c_4^I \rightarrow$ SB limit

● **PNJL:** $c_2 \neq c_2^I, c_4 \neq c_4^I$

c_2 and c_4 away from SB limit ; c_2^I and $c_4^I \rightarrow$ SB limit

Lattice: [Bielefeld PRD 71 054508 '05](#)

PNJL model: [RR et.al. PRD 73 114007 '06](#); [PRD 77 094015 '07](#).



PNJL + Van der Monde

$$\begin{aligned}
 \Omega &= \mathcal{U}(\Phi, \bar{\Phi}, T) + 2G_1(\sigma_u^2 + \sigma_d^2) + 4G_2\sigma_u\sigma_d \\
 &- \sum_{f=u,d} 2T \int \frac{d^3p}{(2\pi)^3} \left\{ \ln \left[1 + 3 \left(\Phi + \bar{\Phi} e^{-(E_f - \mu_f)/T} \right) e^{-(E_f - \mu_f)/T} + e^{-3(E_f - \mu_f)/T} \right] \right. \\
 &+ \left. \ln \left[1 + 3 \left(\bar{\Phi} + \Phi e^{-(E_f + \mu_f)/T} \right) e^{-(E_f + \mu_f)/T} + e^{-3(E_f + \mu_f)/T} \right] \right\} \\
 &- \sum_{f=u,d} 6 \int \frac{d^3p}{(2\pi)^3} E_f \theta(\Lambda^2 - \vec{p}^2)
 \end{aligned}$$

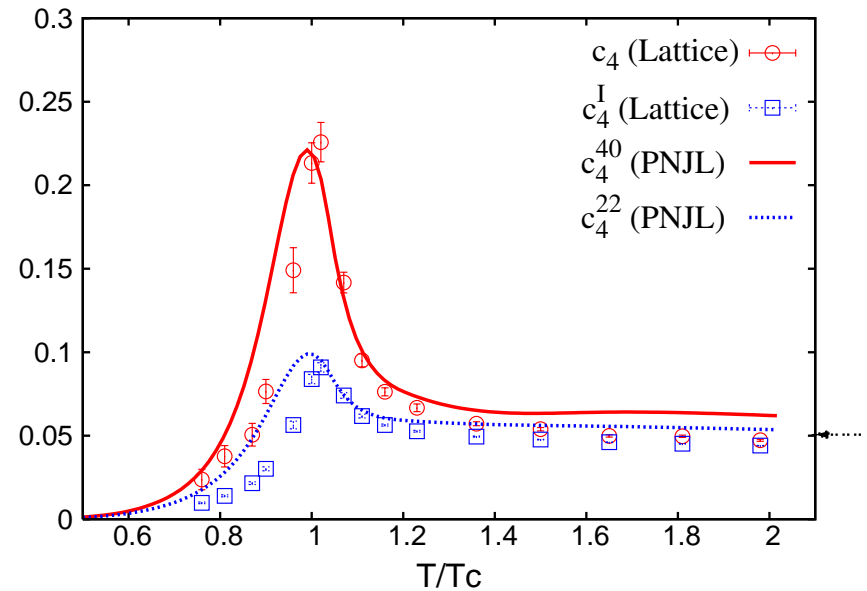
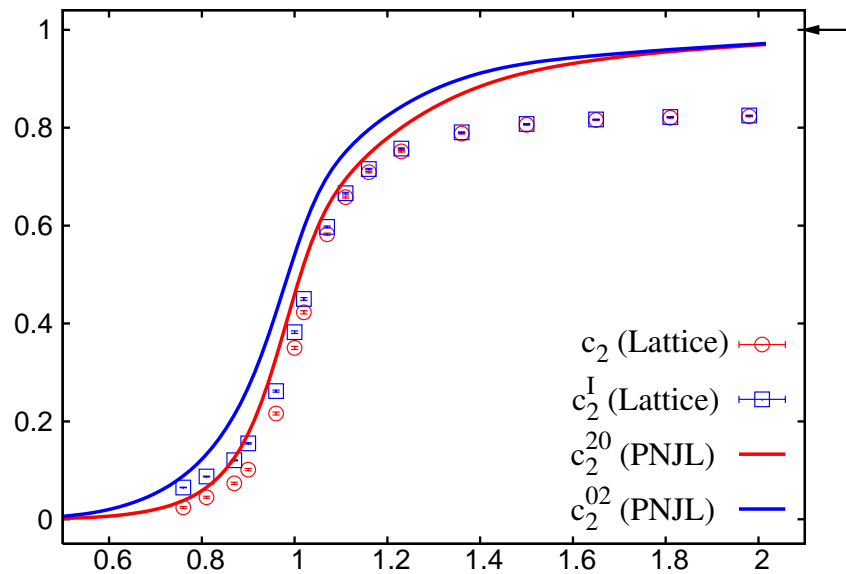
where,

$$\begin{aligned}
 \frac{\mathcal{U}(\Phi, \bar{\Phi}, T)}{T^4} &= -\frac{b_2(T)}{2} \bar{\Phi}\Phi - \frac{b_3}{6} (\Phi^3 + \bar{\Phi}^3) + \frac{b_4}{4} (\bar{\Phi}\Phi)^2 \\
 &+ \kappa \ln[1 - 6\bar{\Phi}\Phi + 4(\Phi^3 + \bar{\Phi}^3) - 3(\bar{\Phi}\Phi)^2]
 \end{aligned}$$

PNJL model: [RR et. al. PRD 77 094024 '08.](#)



Fluctuations – 2 flavors



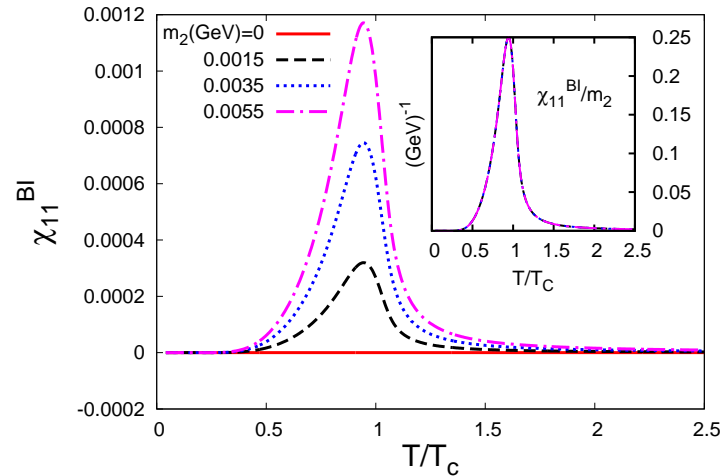
- **Lattice:** $c_2 \sim c_2^I \sim 80\%$ SB limit ; $c_4 \sim c_4^I \rightarrow$ SB limit
- **PNJL:** $c_2 \sim c_2^I \rightarrow$ SB limit ; $c_4 \sim c_4^I \rightarrow$ SB limit

Lattice: [Bielefeld PRD 71 054508 '05](#)

PNJL model: [RR et. al. PRD 77 094024 '08.](#)



Baryon-Isospin correlation

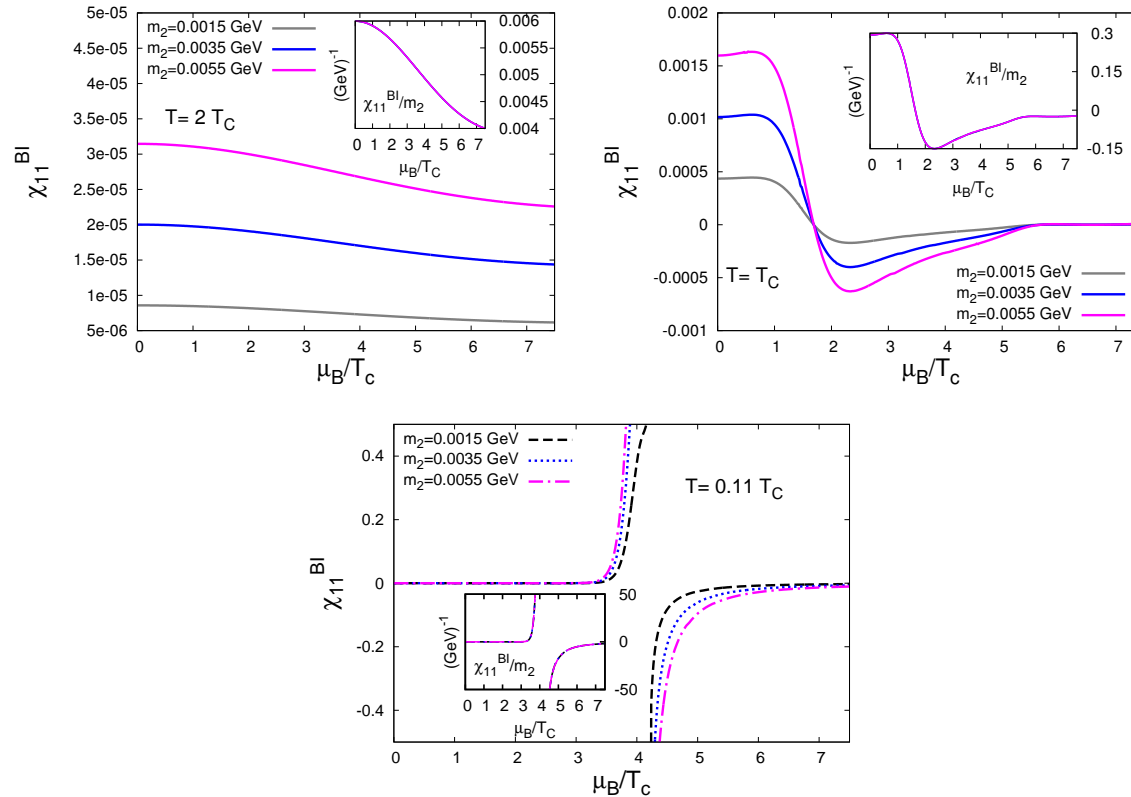


- We consider the baryon-isospin correlator $\chi^{BI} = \frac{1}{3}(\chi^{uu} - \chi^{dd})$
- Conventional choice is baryon-charge correlator $\chi^{BQ} = \frac{1}{9}(2\chi^{uu} - \chi^{dd} + \chi^{ud})$
 - For isospin symmetric matter, $m_u = m_d$, and $\chi^{BI} = 0 \neq \chi^{BQ}$
 - Thus χ^{BI} would be an excellent signal for $m_u \neq m_d$
 - Introduce $m_1 = (m_u + m_d)/2$ and $m_2 = (m_u - m_d)/2$; $m_2 \ll m_1$.
- At low T , $m_2 \ll m_1$ and at high T , $m_2 \ll T$
 - Surprise!! The correlation scales with m_2 !!

PNJL model: RR et. al. PRC 89 064905 '14.



Isospin breaking $T \neq 0, \mu_B \neq 0$

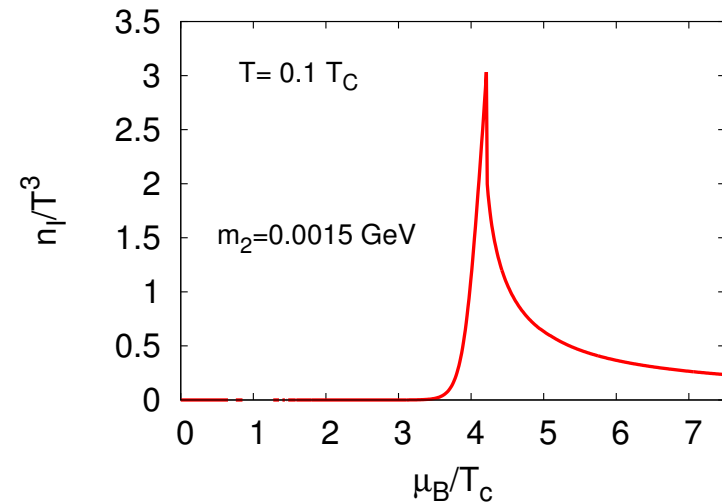
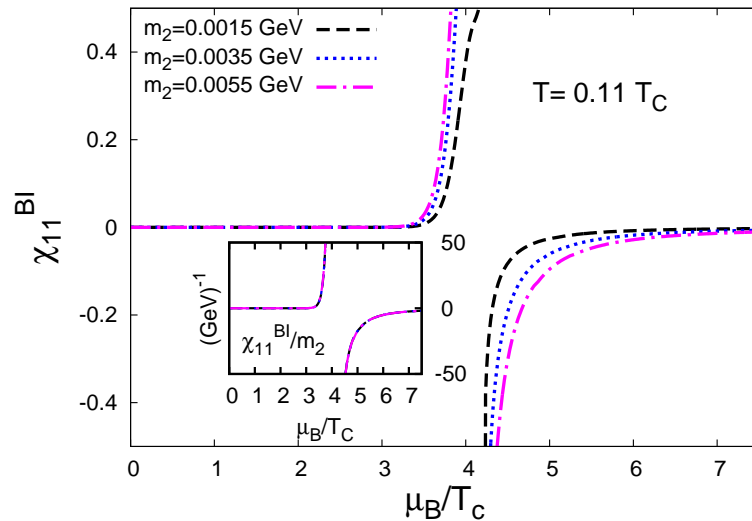


- We find m_2 scaling is valid over the whole $T - \mu_B$ plane.
- The scaling is useful to identify m_2 from experiments
- Sign of χ_{11}^{BI} is useful for identifying a first order transition.

PNJL model: [RR et. al. PRC 89 064905 '14.](#)



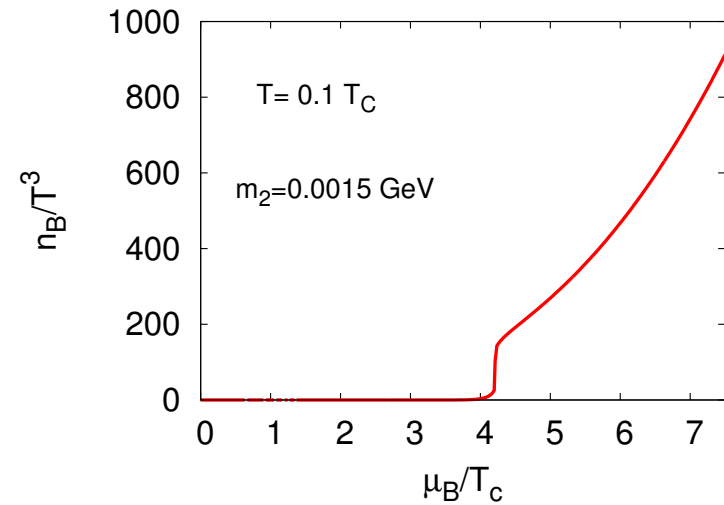
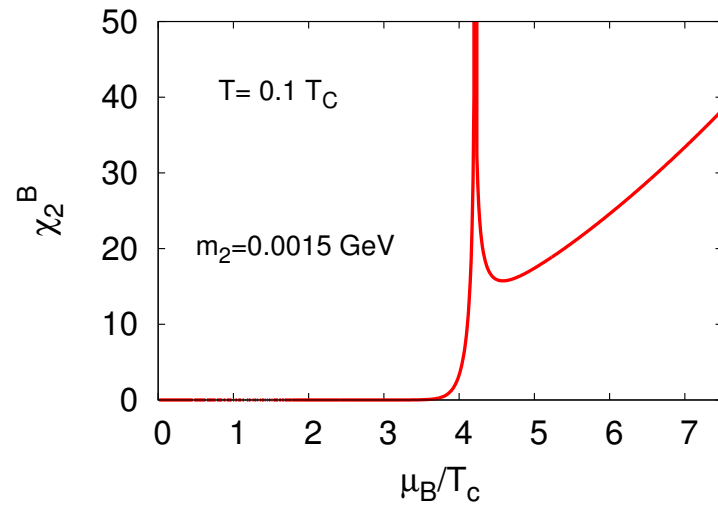
Isospin breaking $T \neq 0, \mu_B \neq 0$



- Large discontinuity in $\chi_{11}^{BI}(\mu_B)$ at a first order transition.
- Jump from positive to negative is understood from the relation $\chi_{11}^{BI}(\mu_B) = \partial n_I / \partial \mu_B$
- There is a non-zero isospin number even though no isospin chemical potential.
- Both the isospin number and baryon-isospin correlations seem to be good signal for the first order transition.



Baryon number and fluctuation



- Large fluctuation occurs in $\chi_{20}^B(\mu_B)$ at a first order transition.
- The baryon number shows a jump near the transition and then keeps on increasing.



PNJL model: 2+1 flavors

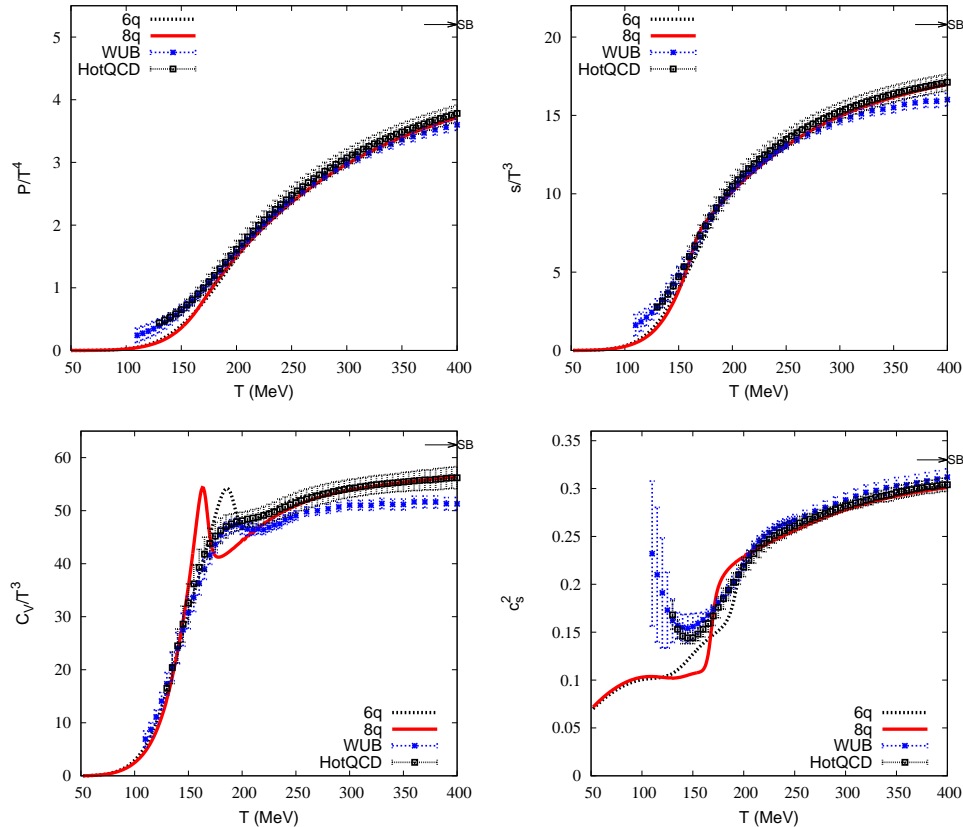
The thermodynamic potential:

$$\begin{aligned}\Omega &= \mathcal{U}'[\Phi, \bar{\Phi}, T] + 2g_S \sum_{f=u,d,s} \sigma_f^2 - \frac{g_D}{2} \sigma_u \sigma_d \sigma_s + 3 \frac{g_1}{2} (\sum_{f=u,d,s} \sigma_f^2)^2 \\ &\quad + 3g_2 \sum_{f=u,d,s} \sigma_f^4 - 6 \sum_{f=u,d,s} \int_0^\Lambda \frac{d^3 p}{(2\pi)^3} E_f \Theta(\Lambda - |\vec{p}|) \\ &\quad - 2T \sum_{f=u,d,s} \int_0^\infty \frac{d^3 p}{(2\pi)^3} \ln \left[1 + 3(\Phi + \bar{\Phi} e^{-\frac{(E_f - \mu)}{T}}) e^{-\frac{(E_f - \mu)}{T}} + e^{-3\frac{(E_f - \mu)}{T}} \right] \\ &\quad - 2T \sum_{f=u,d,s} \int_0^\infty \frac{d^3 p}{(2\pi)^3} \ln \left[1 + 3(\bar{\Phi} + \Phi e^{-\frac{(E_f + \mu)}{T}}) e^{-\frac{(E_f + \mu)}{T}} + e^{-3\frac{(E_f + \mu)}{T}} \right]\end{aligned}$$

where, g_S is the usual four-fermi interaction, g_D is the coupling for the 't Hooft determinant and g_1 and g_2 are the 8q coupling constants needed to remove an infinite potential well close to the classical vacuum.



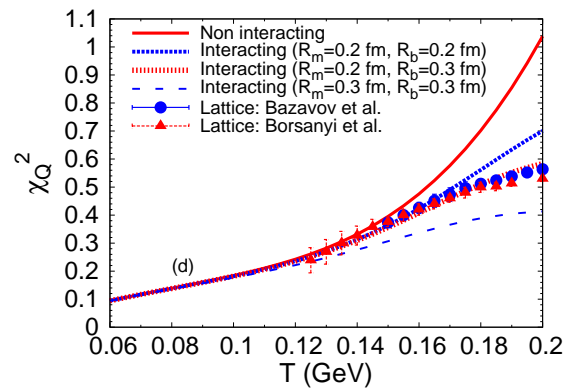
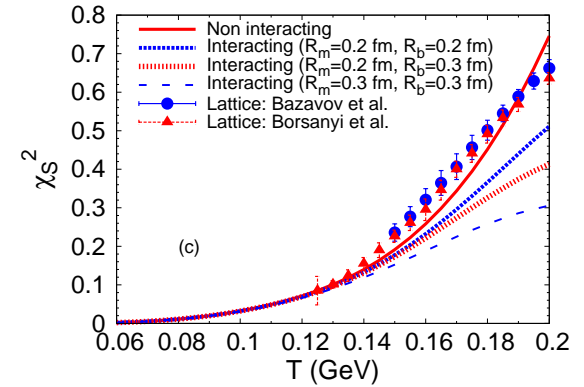
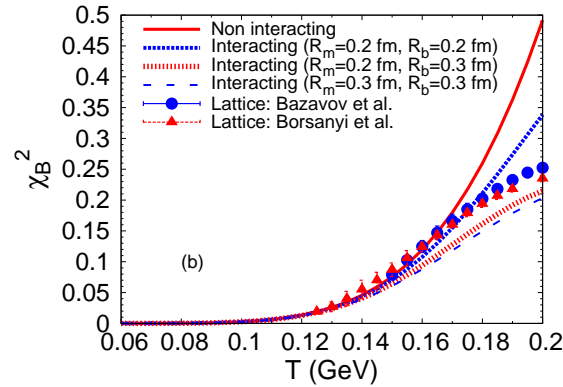
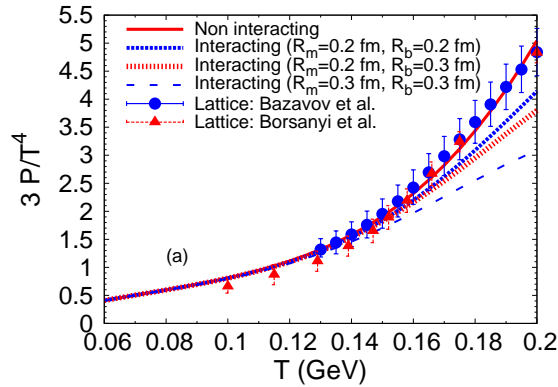
PNJL vs Continuum LQCD



HotQCD: Phys. Rev. D 86, 034509 (2012).
WUB: J. High Energy Phys. 01 138 (2012).
PNJL model: [RR et. al. PRD 95 054005 '17.](#)



HRG vs LQCD I

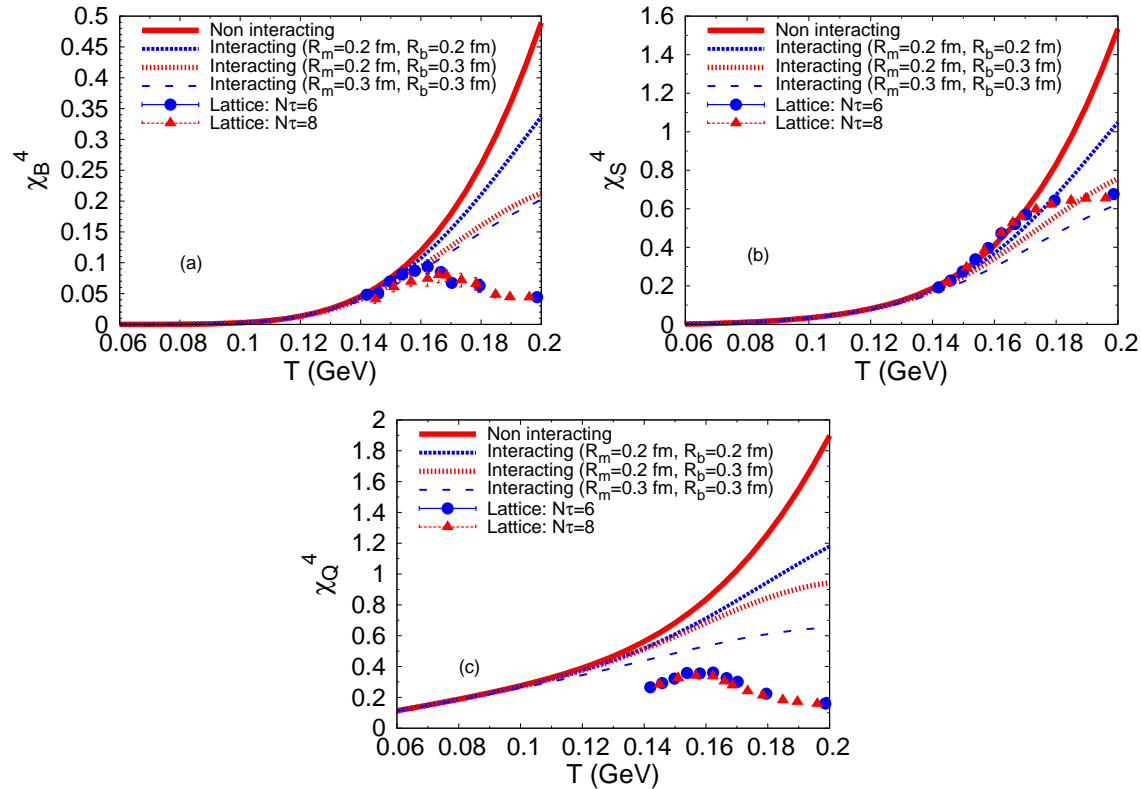


RR *et al.*, PRC 90 034909 '14.

- Included hard core repulsion between hadrons to obtain fluctuations
- Reasonable agreement with Lattice QCD data for intermediate T .



HRG vs LQCD II

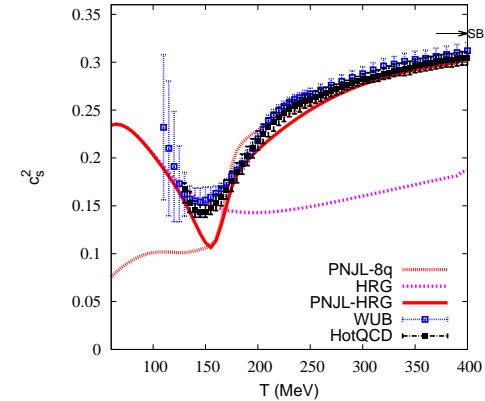
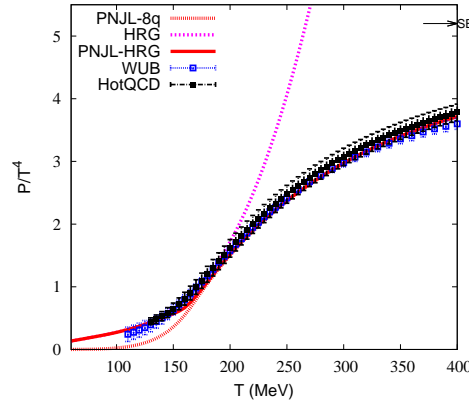
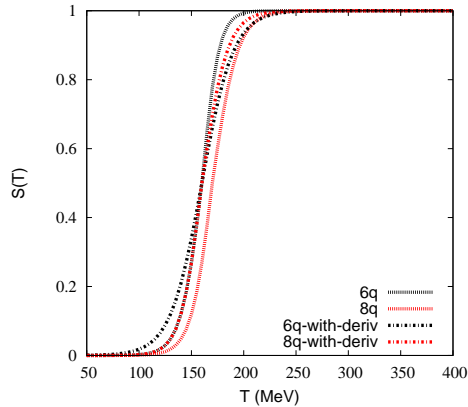


RR *et al.*, PRC 90 034909 '14.

- Included hard core repulsion between hadrons to obtain fluctuations
- Reasonable agreement with Lattice QCD data for intermediate T .



PNJL+HRG vs Continuum LQCD



● Total pressure $\rightarrow P(T) = S(T)P_{PNJL}(T) + (1 - S(T))P_{HRG}(T)$

● The switching function is given as,

$$S(T) = \left(1 + \exp \left[-\frac{T - T_S}{\Delta T_S(T)} \right] \right)^{-1}$$

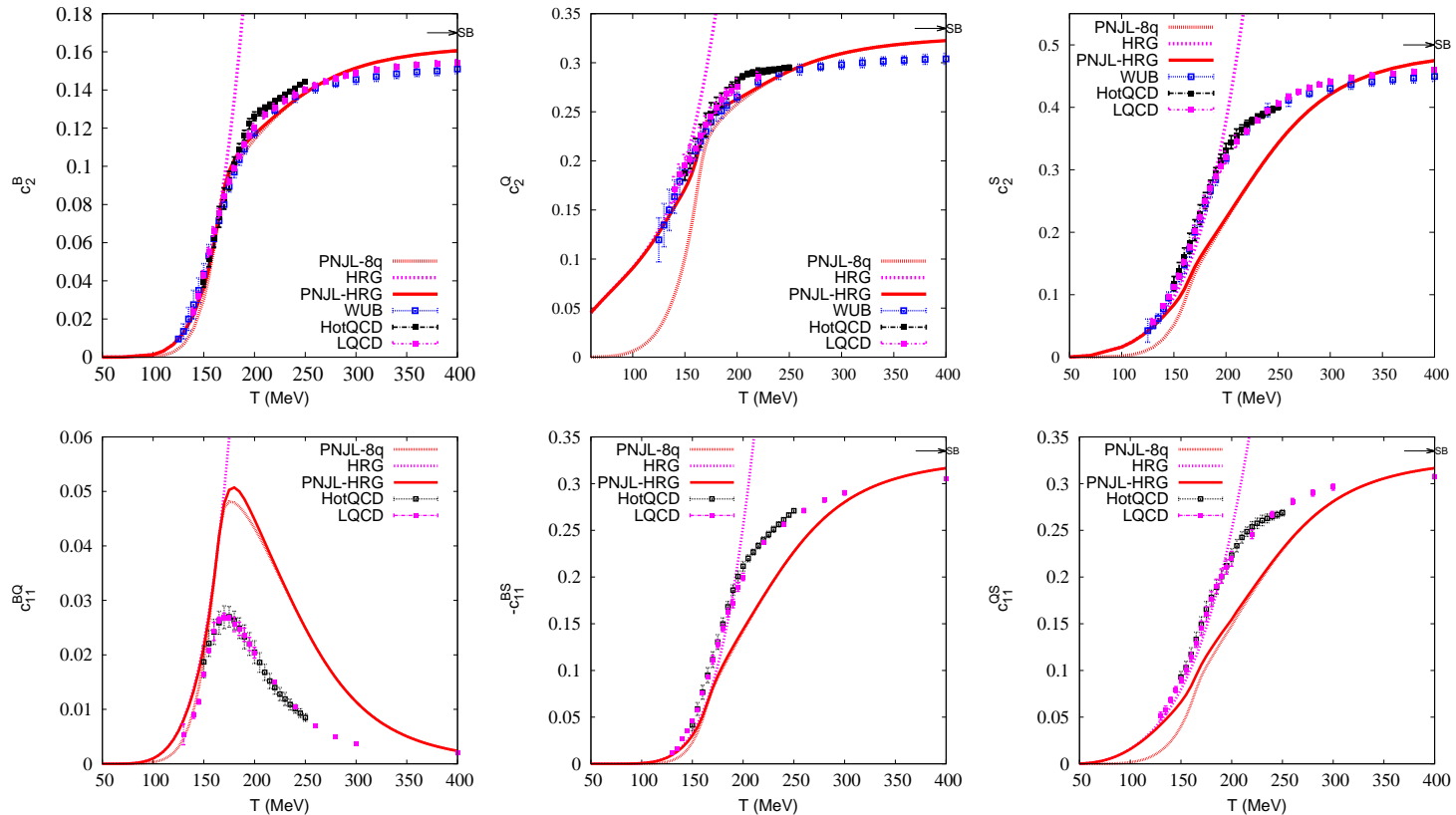
HotQCD: Phys. Rev. D 86, 034509 (2012).

WUB: J. High Energy Phys. 01 138 (2012).

PNJL + HRG model: [RR et. al](#), PRC 99 045207 '19.



PNJL+HRG vs Continuum LQCD



HotQCD: Phys. Rev. D 86, 034509 (2012).

WUB: J. High Energy Phys. 01 138 (2012).

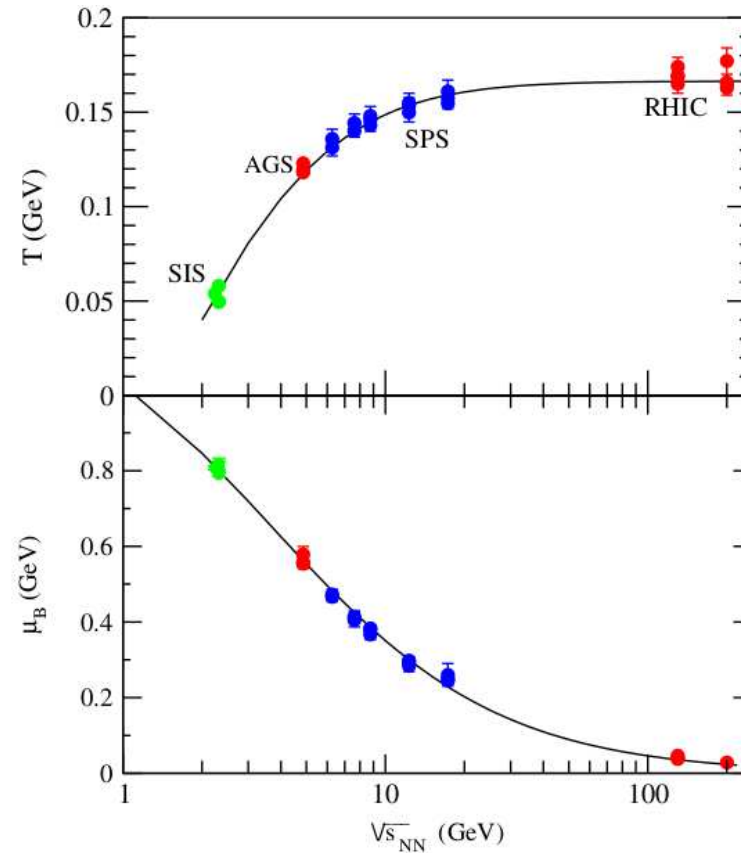
LQCD: Phys. Rev. D 92, 114505 (2015).

PNJL + HRG model: [RR et. al](#), PRC 99 045207 '19.



HRG at Freezeout I

From hadron multiplicity data:

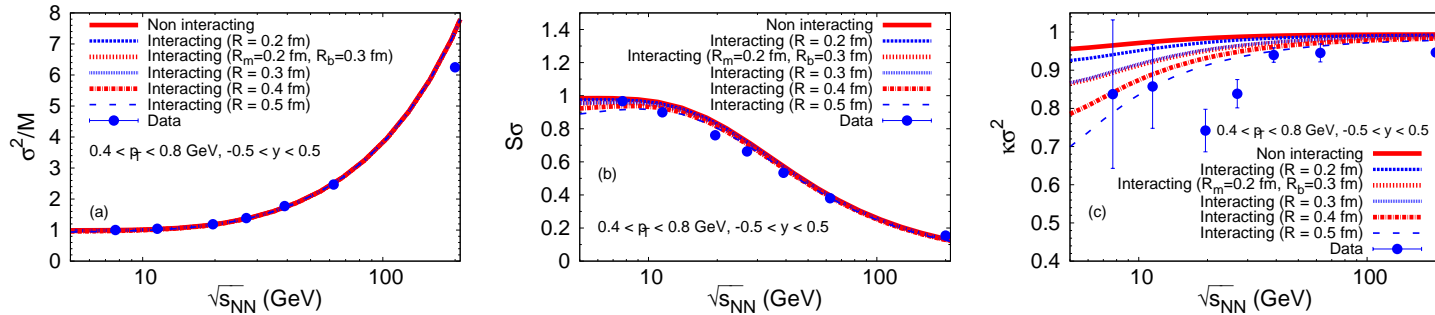


J. Cleymans *et al.*, PRC 73, 034905 (2006)

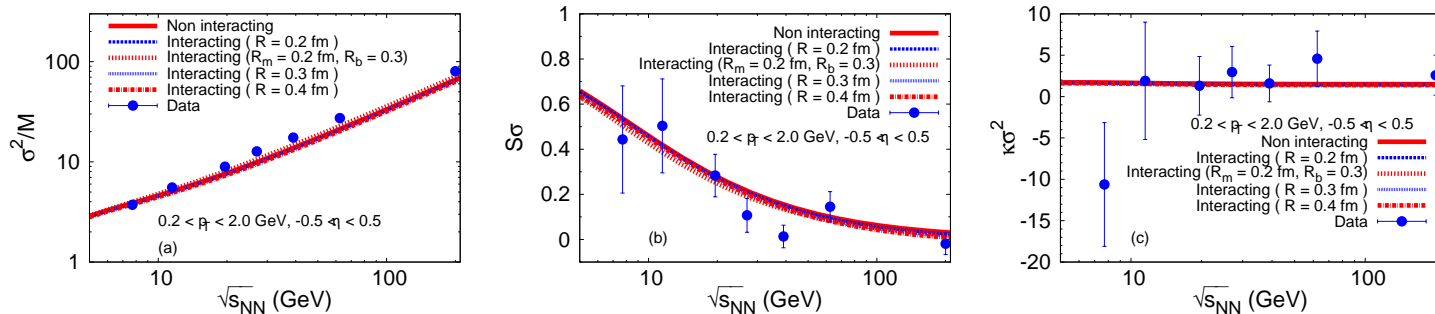
What happens for ratios of fluctuations with the freezeout data ??



HRG at Freezeout II



Net-proton data: X. Luo (for the STAR collaboration), Nucl. Phys. A 904-905,911c (2013)

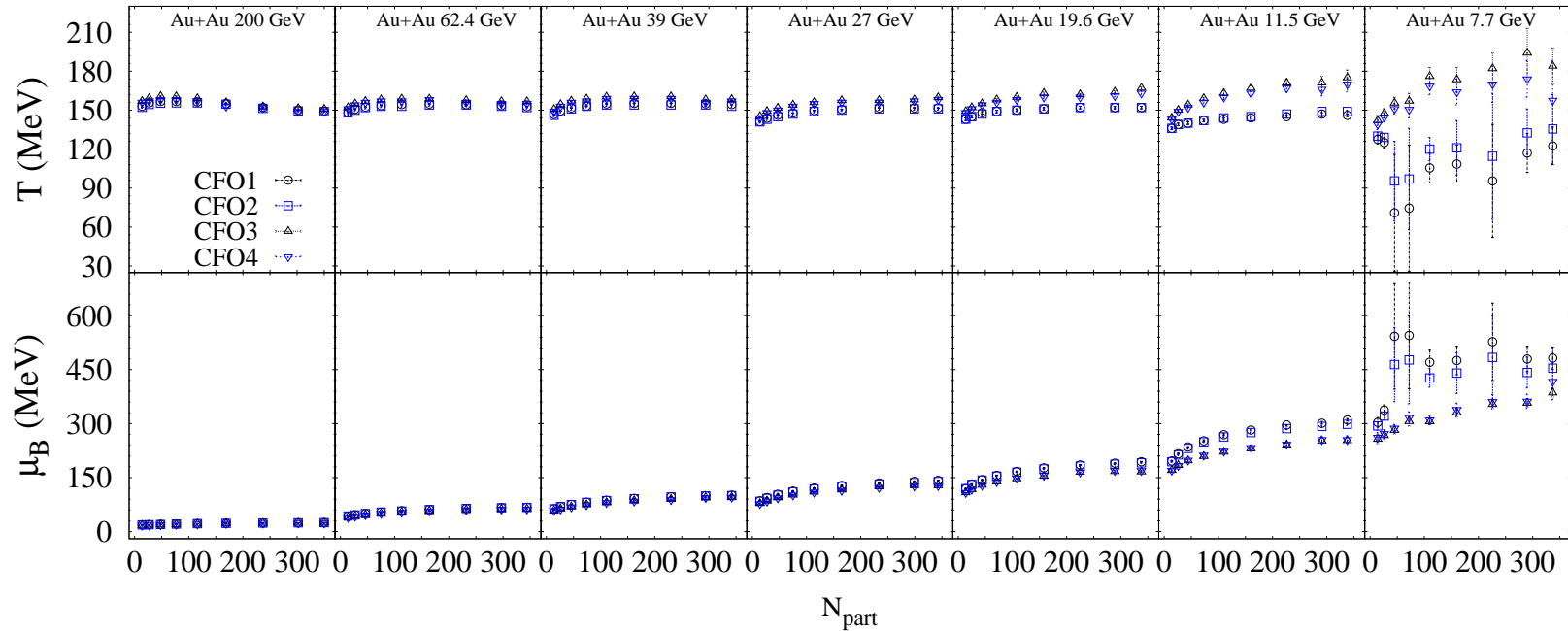


Net-charge data: L. Adamczyk et al. (STAR), Phys. Rev. Lett. 113, 092301 (2014)

HRG Results: RR *et al.*, PRC 90 034909 '14.



HRG at Freezeout III



Sets of parameters	Experimental data used	Model used
CFO1	$(\sigma^2/M)_{np}, (\sigma^2/M)_{nc}$	HRG
CFO2	$(\sigma^2/M)_{np}, (\sigma^2/M)_{nc}$	EVHRG
CFO3	$(\sigma^2/M)_{nc}, (S\sigma)_{np}, (S\sigma)_{nc}$	HRG
CFO4	$(\sigma^2/M)_{nc}, (S\sigma)_{np}, (S\sigma)_{nc}$	EVHRG

RR *et. al*, PRC 96 014902 '17.

Data from STAR: PRL 112, 032302 (2014); PRL 113, 092301 (2014)



Chemical Freezeout: Alternative approach

- Thermal density of i 'th Hadron is given as,

$$n_i = \frac{g_i}{(2\pi)^3} \int \frac{d^3 p}{\exp[(E_i - \mu_i)/T] \pm 1}.$$

- $\mu_i = B_i\mu_B + S_i\mu_S + Q_i\mu_Q$ is total chemical potential, g_i is the degeneracy factor.
- In **chemical equilibrium**, detected i 'th hadron's rapidity density,

$$\frac{dN_i}{dy} = \frac{dV}{dy} n_i(T, \mu_Q, \mu_B, \mu_S) \quad \Rightarrow \quad \frac{dN_i/dy}{dN_j/dy} = \frac{n_i}{n_j}$$

- Add external constraints,

$$\frac{\sum_i n_i(T, \mu_B, \mu_S, \mu_Q) Q_i}{\sum_i n_i(T, \mu_B, \mu_S, \mu_Q) B_i} = r$$

$$\sum_i n_i(T, \mu_B, \mu_S, \mu_Q) S_i = 0$$



Chemical Freezeout: Alternative approach

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- $\mu_i = B_i \mu_B + S_i \mu_S + Q_i \mu_Q$ is total chemical potential, g_i is the degeneracy factor.

- In **chemical equilibrium**,

$$\text{Minimize } \chi^2 \text{ w.r.t. } T \text{ and } \mu_B \text{ constructed from } \left(\frac{dN_i/dy}{dN_j/dy} - \frac{n_i}{n_j} \right)$$

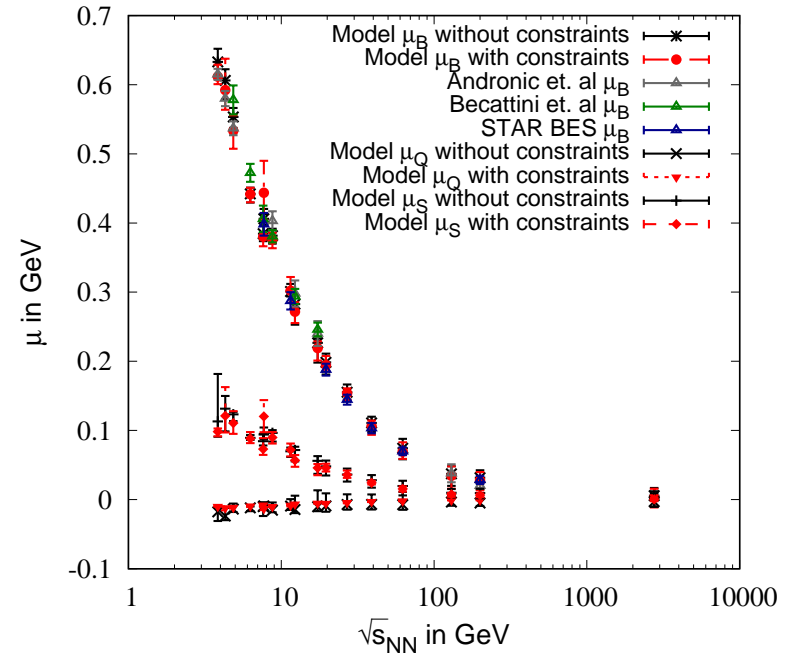
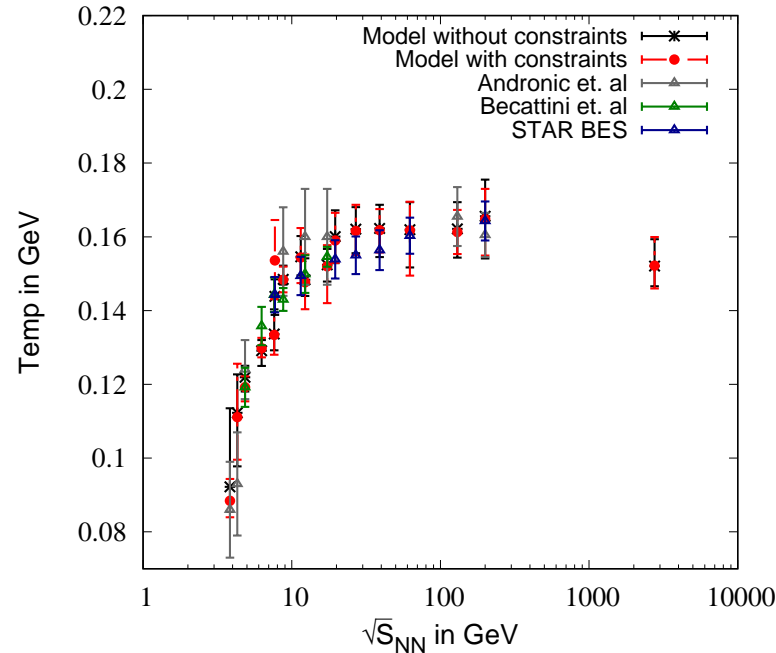
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Chemical Freezeout: Alternative approach

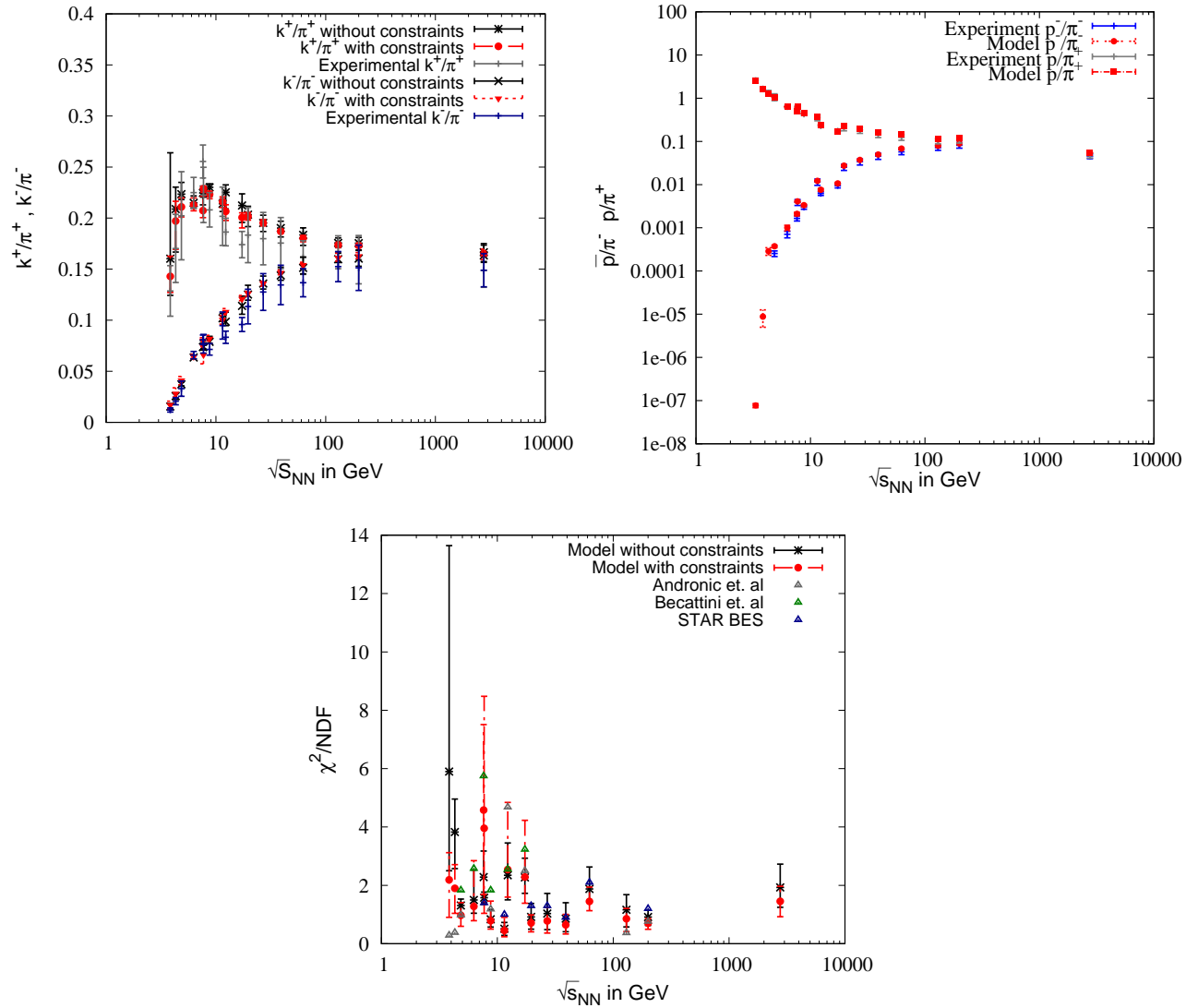


- AGS, SPS, RHIC and LHC (2.76 TeV) data have been used.
- Study has been performed for mid-rapidity data of most central collision of these \sqrt{s} .
- Yield of (π^\pm , k^\pm and p , \bar{p} , Λ , $\bar{\Lambda}$, Ξ^\pm) were used for fitting.
- We have not used Ω^\pm yield as it is not available for most of the \sqrt{s} .

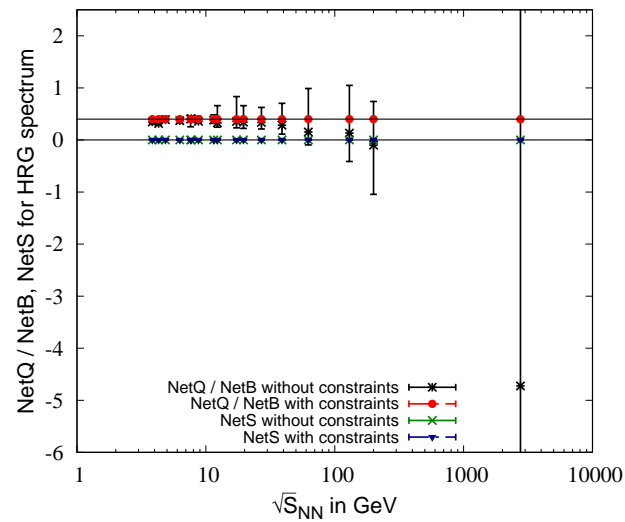
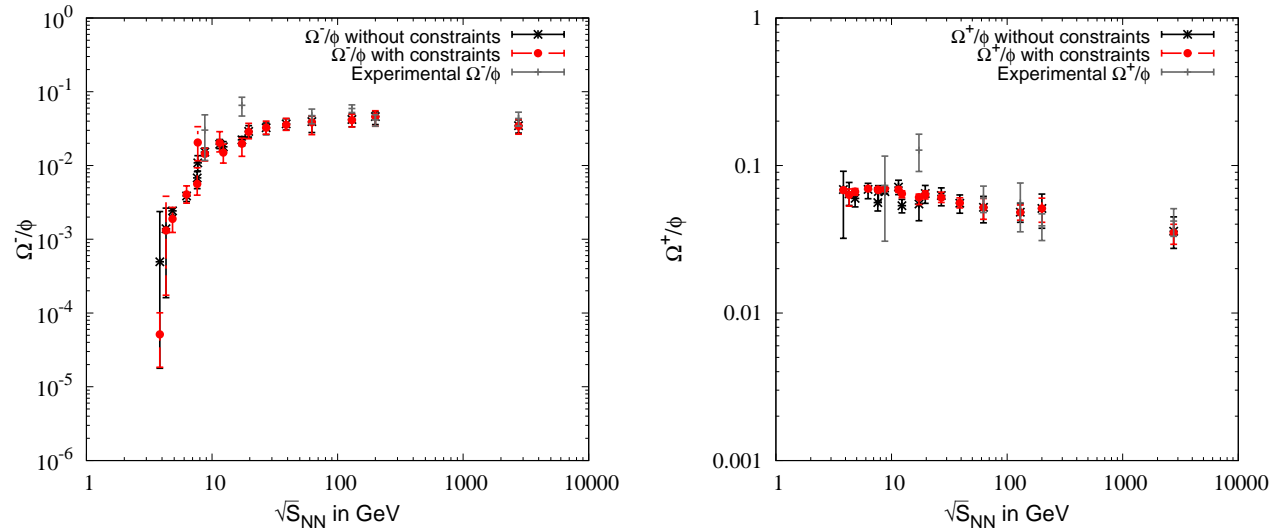
RR et. al. arXiv: 1911.04828



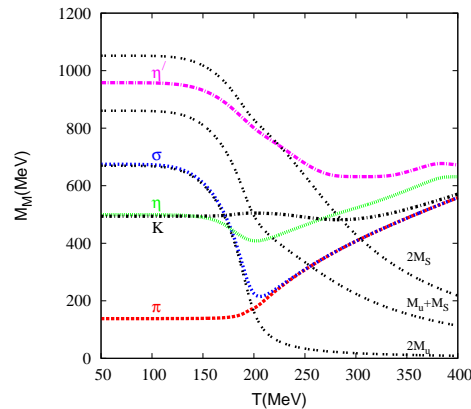
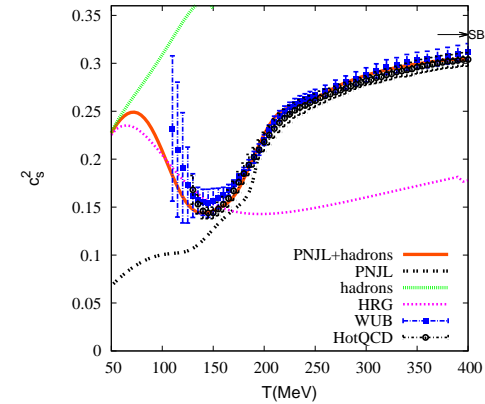
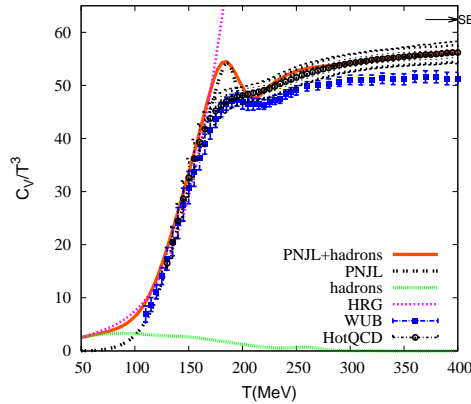
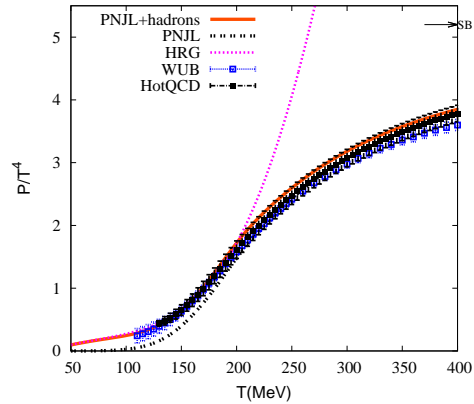
Chemical Freezeout: Alternative approach



Chemical Freezeout: Alternative approach



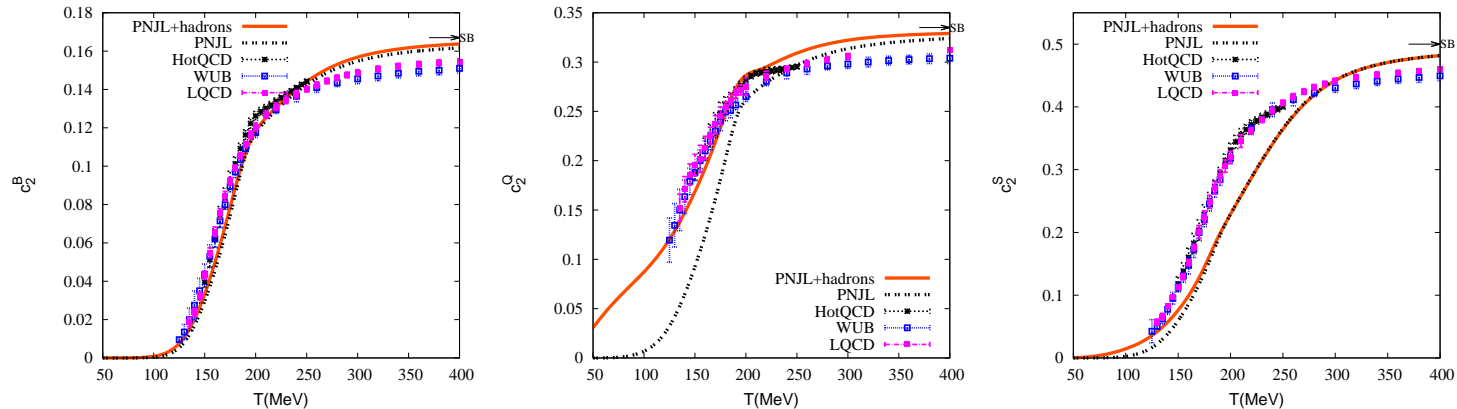
PNJL beyond Mean Field vs Continuum LQCD



- Total pressure $\rightarrow P(T, \mu) = P_{PNJL\text{Meanfield}}(T, \mu) + P_{PNJL\text{Hadrons}}(T, \mu)$
- The increasing hadron masses provide automatic switch



PNJL beyond Mean Field vs Continuum LQCD



- Total pressure $\rightarrow P(T, \mu) = P_{PNJL\text{Meanfield}}(T, \mu) + P_{PNJL\text{Hadrons}}(T, \mu)$
- The increasing hadron masses provide automatic switch
- Though hadronic region is satisfactory but significant difference in partonic sector

PNJL beyond mean field: [RR et. al. Under progress](#)

HotQCD: Phys. Rev. D 86, 034509 (2012).

WUB: J. High Energy Phys. 01 138 (2012).



Gluons + Glueballs ??

- Consider adjoint Polyakov Loop

$$L_A = \text{diag}(1, 1, e^{i(\phi_1 - \phi_2)}, e^{-i(\phi_1 - \phi_2)} e^{i(2\phi_1 + \phi_2)}, e^{-i(2\phi_1 + \phi_2)} e^{i(\phi_1 + 2\phi_2)} e^{-i(\phi_1 + 2\phi_2)})$$

- The partition function

$$\mathcal{Z} = \int \mathcal{D}\Phi \mathcal{D}\bar{\Phi} H \exp \left(-2V \int \frac{d^3p}{(2\pi)^3} \ln \left[1 + \sum_{n=1}^8 C_n e^{(-nE_g/T)} \right] \right)$$

where, H is the Haar measure is given by,

$$H = \frac{8}{9\pi^2} [1 - 6\bar{\Phi}\Phi + 4(\bar{\Phi}^3 + \Phi^3) - 3(\bar{\Phi}\Phi)^2]$$

and the coefficient C_n is given by,

$$C_8 = 1; C_1 = C_7 = 1 - 9\bar{\Phi}\Phi$$

$$C_2 = C_6 = 1 - 27\bar{\Phi}\Phi$$

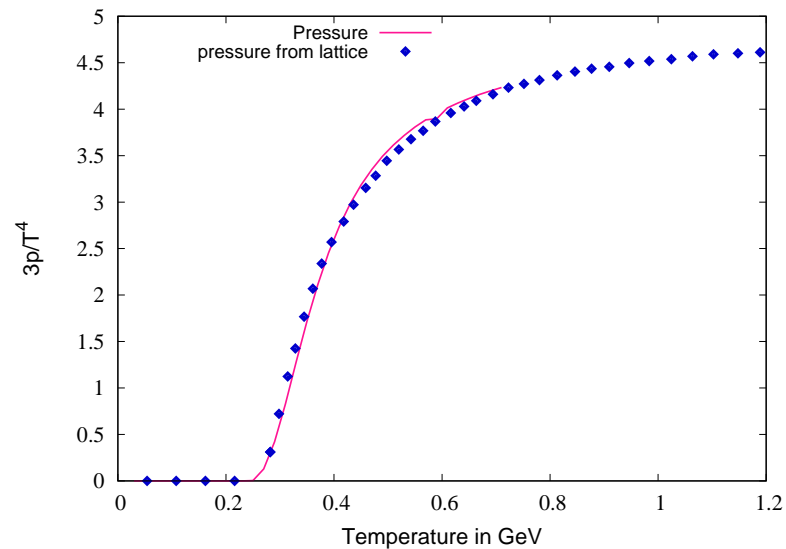
$$C_3 = C_5 = -2 + 27\bar{\Phi}\Phi - 81(\bar{\Phi}\Phi)^2$$

$$C_4 = 2[-1 + 9\bar{\Phi}\Phi - 27(\bar{\Phi}^3 + \Phi^3) + 81(\bar{\Phi}\Phi)^2]$$



Gluons + Glueballs ??

- A saddle point analysis yields negative pressure for $T < T_c$
RR et.al. J. Phys. G, 41 (2014) 025001
- Integrate over all possible constant field configurations



RR et. al. Under progress



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