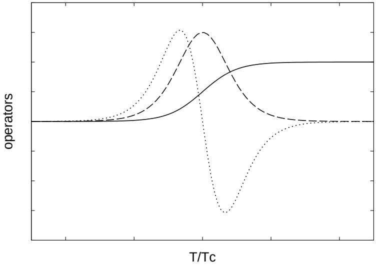
Effective Field Theory = universality+

Sourendu Gupta
with Rishi Sharma and Aminul Islam

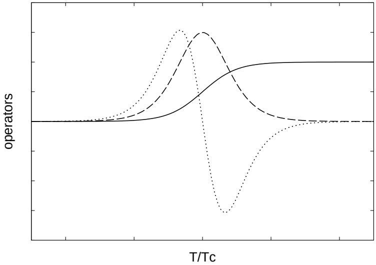
20 November, 2019

2005



First measurements and schematic picture of χ_2, χ_4, χ_6.
Discussed the form of an effective theory.

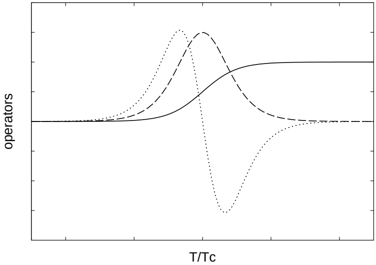
2005



First measurements and schematic picture of χ_2, χ_4, χ_6.
Discussed the form of an effective theory. The T dependence of χ_{n+2} looks like a single temperature derivative of χ_n.

$$\frac{\partial}{\partial T} \propto \frac{\partial^2}{\partial \mu^2}?$$

2005



First measurements and schematic picture of χ_2, χ_4, χ_6.
Discussed the form of an effective theory.

Gavai, Gupta PRD 72, 054006, 2005

Why use EFT when there is lattice?

EFTs can give a different language to understand the same physics. Deepens our understanding of physics: quark masses and chiral symmetry, pions as pseudo Goldstone bosons, currents and current (non-) conservation, are all descriptions of the same phenomena, but at different length/momentum scales.

Questions which are vague in one language can become precise in another. Does long distance physics near the QCD cross over see hadrons or quarks? Difficult to say using lattice methods. In EFT can ask how precise is bosonisation. **SG, Sharme, PRD 97, 2018**

Can suggest ways of approaching physics which lattice finds difficult to do. How to do analytic continuation of finite temperature correlation functions? **SG, Sharma, QM 2019**

The Wilsonian EFT approach

$$\text{QCD} \xrightarrow{\hspace{2em} integrate\ \ all\ \ virtual\ \ p \hspace{2em}} \text{Correlators (q)}$$

$$\xrightarrow{\hspace{2em} integrate\ \ p > \Lambda \hspace{2em}} \text{EFT} \longrightarrow \qquad \text{('')}$$

Doing part of the integrals (right arrows) does not change the global symmetries. So EFT has symmetries of QCD: universality.

The Wilsonian EFT approach

$$\text{QCD} \xrightarrow{\hspace{4em} integrate\ \ all\ \ virtual\ \ p \hspace{4em}} \text{Correlators (q)}$$

$$\xrightarrow{\hspace{3em} integrate\ \ p>\Lambda \hspace{3em}} \text{EFT} \longrightarrow \qquad ('')$$

Doing part of the integrals (right arrows) does not change the global symmetries. So EFT has symmetries of QCD: universality. In EFT Lagrangian, retain terms of dimension up to D. Then, correlators exactly reproduced up to $\mathcal{O}(q/\Lambda)^D$. More than just universality.

The Wilsonian EFT approach

$$\text{QCD} \xrightarrow{\hspace{10em} integrate\ \ all\ \ virtual\ \ p \hspace{10em}} \text{Correlators (q)}$$

$$\xrightarrow{\hspace{8em} integrate\ \ p>\Lambda \hspace{8em}} \text{EFT} \longrightarrow \qquad ('')$$

Doing part of the integrals (right arrows) does not change the global symmetries. So EFT has symmetries of QCD: universality. In EFT Lagrangian, retain terms of dimension up to D. Then, correlators exactly reproduced up to $\mathcal{O}(q/\Lambda)^D$. More than just universality.

Shortcut: since symmetries dictate the Lagrangian, write down all possible terms. Some unknown constants (LECs), matched to some correlators. Others are predictions.

Finite temperature EFT

Typical momenta in a relativistic gas is of order temperature, T.
For correlators at distances $\gg 1/T$, take UV cutoff $\Lambda = T_0 \simeq T$.
Consider Fermion fields, with dimension $\mathcal{N} = 4N_f N_c$.

Kinetic terms (dimension 4)

Lorentz invariance broken by special frame (heat bath) but theory is relativistic ($p_0 \gg T$). Retain P, CP, and CPT and spatial rotations. Kinetic terms are

$$\begin{aligned} \mathcal{L}_4 &= \overline{\psi}\gamma_4\partial_4\psi && \text{(fixes field normalization)} \\ &+ d^4\overline{\psi}\not{\nabla}\psi && \text{(new LEC)} \end{aligned}$$

Also satisfies the SU(N$_f$)$_L \times$SU(N$_f$)$_R$ chiral symmetry expected with massless quarks. Naturalness implies that $d^4 \simeq 1$. At $T = 0$ symmetry dictates $d^4 = 1$.
We will specialize to $N_f = 2$.

Interactions: dimension 6 terms

$$
\begin{aligned}
\mathcal{L}_6 \quad &= \frac{d^{61}}{T_0^2}\left[(\overline{\psi}\psi)^2 + (\overline{\psi}i\gamma_5\tau^a\psi)^2\right] \\
&+ \frac{d^{62}}{T_0^2}\left[(\overline{\psi}\tau^a\psi)^2 + (\overline{\psi}i\gamma_5\psi)^2\right] + \frac{d^{63}}{T_0^2}(\overline{\psi}\gamma_4\psi)^2 + \frac{d^{64}}{T_0^2}(\overline{\psi}i\gamma_i\psi)^2 \\
&+ \frac{d^{65}}{T_0^2}(\overline{\psi}\gamma_5\gamma_4\psi)^2 + \frac{d^{66}}{T_0^2}(\overline{\psi}i\gamma_5\gamma_i\psi)^2 \\
&+ \frac{d^{67}}{T_0^2}\left[(\overline{\psi}\gamma_4\tau^a\psi)^2 + (\overline{\psi}\gamma_5\gamma_4\tau^a\psi)^2\right] \\
&+ \frac{d^{68}}{T_0^2}\left[(\overline{\psi}i\gamma_i\tau^a\psi)^2 + (\overline{\psi}i\gamma_5\gamma_i\tau^a\psi)^2\right] \\
&+ \frac{d^{69}}{T_0^2}\left[(\overline{\psi}iS_{i4}\psi)^2 + (\overline{\psi}S_{ij}\tau^a\psi)^2\right] + \frac{d^{60}}{T_0^2}\left[(\overline{\psi}iS_{i4}\tau^a\psi)^2 + (\overline{\psi}S_{ij}\psi)^2\right]
\end{aligned}
$$

10 more LECs with full SU(2)$\times$SU(2) chiral symmetry!

Renormalization

LECs are defined by fitting observed correlation functions for a fixed T_0. Their values may change when you change T_0. This is part of the renormalization group flow from QCD.

Choosing a value of T_0 is a choice of the renormaliation point. We choose T_0 to be the critical temperature of QCD.

The cross over temperature measurable on the lattice, along with a few correlation functions, also measured on the lattice, at consistent values of the QCD parameters, then fix LECs.

Statistical errors in lattice measurements translate to induced errors on the LECs. Systematic errors? Lattice has UV cutoff $1/a$, EFT has UV cutoff T_0. If $aT_0 \gg 1$ then systematic errors (power corrections) can be kept under control.

SG and Sharma, PRD 97, 2018

Phase diagram in mean-field approximation

Mean field $\Sigma \propto \langle\overline{\psi}\psi\rangle$ breaks chiral symmetry. Simple: 10 couplings collapse to one. Second order transition at finite temperature; expected since quark masses are taken to be zero. $SU(2)_A$ fluctuations give pion properties at finite temperature: connection with lattice.

Introduce a chemical potential in the usual way. A baryon chemical potential does not break the chiral symmetry, so line of second order transitions—

$$\left(\frac{T_c(\mu)}{T_0}\right)^2 = 1 - \frac{1}{3\pi^2}\left(\frac{\mu}{T_0}\right)^2.$$

Absolute prediction, independent of LECs.

Curvature of the critical line not bad (but not terribly good either). So maybe we forgot something which changes this slightly? **SG and Sharma, PRD 97, 2018**

Symmetry breaking

Chiral symmetry broken in QCD by quark masses:

$$\mathcal{L}_3^A = d^3 T_0 \overline{\psi}\psi$$

Dimension 3 terms break symmetry in the UV, generate all possible symmetry breaking terms of dimension 3 and higher in the IR by RG flow.

CP symmetry broken by putting a baryon chemical potential

$$\mathcal{L}_3^B = \mu\overline{\psi}\gamma_4\psi$$

Dimension 3 terms break symmetry in the UV, again generate all possible CP symmetry breaking terms of dimension 3 and higher in the IR through the RG flow.

Higher dimensional symmetry breaking terms

$$
\begin{aligned}
\mathcal{L}_6^A &= +\frac{d_A^{61}}{T_0^2}\left[(\overline{\psi}\psi)^2 - (\overline{\psi}i\gamma_5\tau^a\psi)^2\right] + \frac{d_A^{62}}{T_0^2}\left[(\overline{\psi}\tau^a\psi)^2 - (\overline{\psi}i\gamma_5\psi)^2\right] \\
&\quad +\frac{d_A^{67}}{T_0^2}\left[(\overline{\psi}\gamma_4\tau^a\psi)^2 - (\overline{\psi}\gamma_5\gamma_4\tau^a\psi)^2\right] \\
&\quad +\frac{d_A^{68}}{T_0^2}\left[(\overline{\psi}i\gamma_i\tau^a\psi)^2 - (\overline{\psi}i\gamma_5\gamma_i\tau^a\psi)^2\right] \\
&\quad +\frac{d_A^{69}}{T_0^2}\left[(\overline{\psi}iS_{i4}\psi)^2 - (\overline{\psi}S_{ij}\tau^a\psi)^2\right] \\
&\quad +\frac{d_A^{60}}{T_0^2}\left[(\overline{\psi}iS_{i4}\tau^a\psi)^2 - (\overline{\psi}S_{ij}\psi)^2\right], \\
\mathcal{L}_6^B &= \frac{d_B^{61}}{T_0^2}(\overline{\psi}\psi)(\overline{\psi}\gamma_4\psi) + \frac{d_B^{62}}{T_0^2}(\overline{\psi}\tau^a\psi)(\overline{\psi}\gamma_4\tau^a\psi)
\end{aligned}
$$

8 more LECs!

Two mean fields

Introduce mean fields for symmetry breaking: $\Sigma \propto \langle \overline{\psi}\psi \rangle$ for chiral, and $\Gamma \propto \langle \overline{\psi}\gamma_4\psi \rangle$ for CP symmetry breaking. Three independent combinations of LECs (α, α', and β).
Mean field Lagrangian

$$\begin{aligned} L &= \left(d^3 T_0 + \Sigma - \frac{\beta}{\alpha'} \Gamma \right) \overline{\psi}\psi \\ &\quad + \left(\mu + \Gamma - \frac{\beta}{\alpha} \Sigma \right) \overline{\psi}\gamma^4\psi \\ &\quad + \overline{\psi}\gamma^4 \partial_4 \psi + d^4 \overline{\psi} \not{\nabla} \psi + V(\Sigma, \Gamma) \end{aligned}$$

$\beta \propto d^3\mu / T_0$, therefore not relevant in the chiral limit.

The shape of the critical line

$$\frac{T_c(\mu)}{T_0} = 1 - \frac{1}{2}\,\kappa\,\left(\frac{\mu}{T_0}\right)^2 - \frac{1}{2}\,\kappa_4\,\left(\frac{\mu}{T_0}\right)^4 + \cdots$$

For a circle $\kappa_4 = 3\kappa^2$, so it is useful to compute

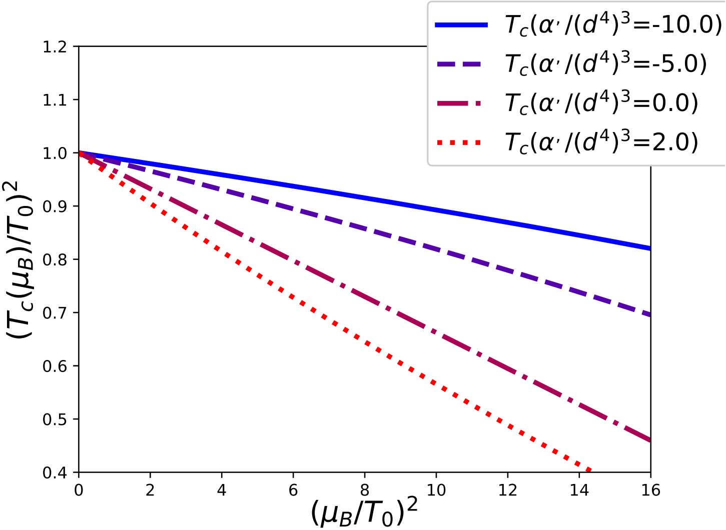
$$\widetilde{\kappa}_4 = \kappa_4 - 3\kappa^2.$$

A small value of this dimensionless shape parameter, $\widetilde{\kappa}_4$ says that the critical line is almost a critical circle:

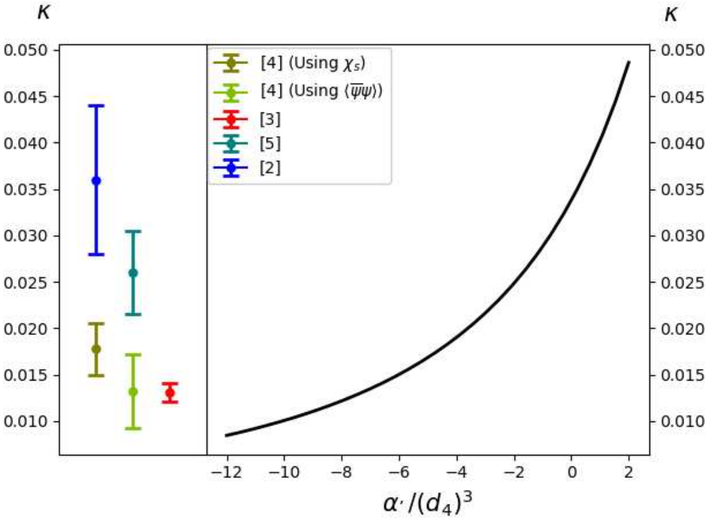
$$\left(\frac{T_c(\mu)}{T_0}\right)^2 = 1 - \kappa\,\left(\frac{\mu}{T_0}\right)^2$$

In the very special case that $\kappa_4 = \cdots = 0$, one has $\partial_T \propto \partial_\mu^2$. Generically, this is spoilt at larger μ even if $\kappa_4, \cdots$ are small.

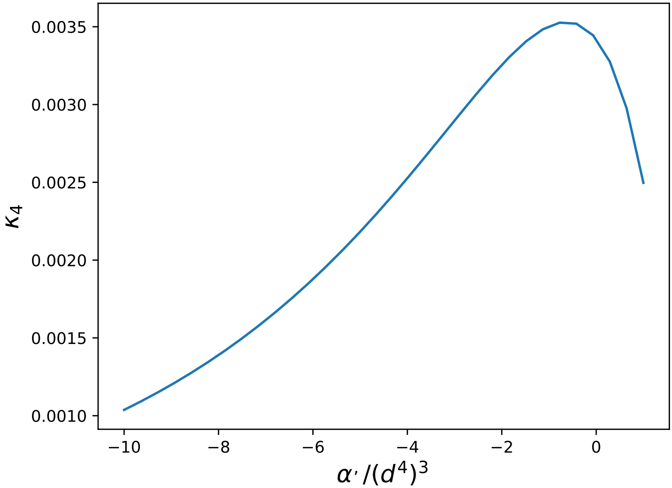
The critical line is almost a critical circle



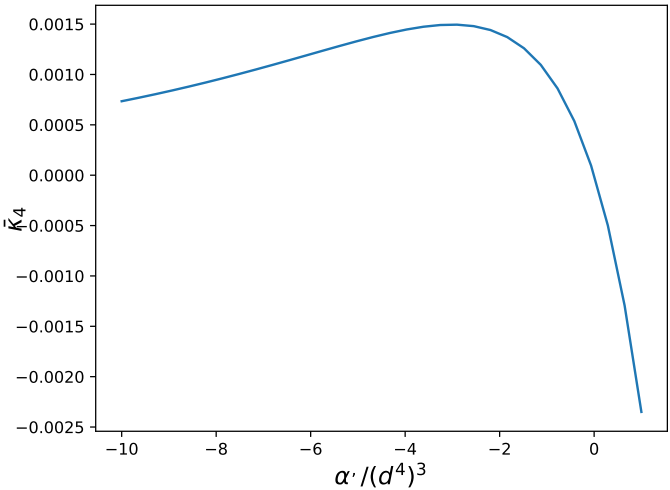
Curvature of the critical line



The fourth-order curvature



The fourth-order curvature



(Not the) conclusion

1. Symmetry breaking terms in the UV (QCD Lagrangian) through RG flow gives rise to a cascade of symmetry breaking terms of higher dimension. Symmetry breaking by dimension 3 terms could change the EFT significantly.
2. A small number of new parameters allow tuning to the observed lattice results on the curvature of the critical line.
3. The critical line is close to a critical circle. Simple predictions for higher order curvature, can be tested on the lattice. (Could that be the reason for the deviation of the imaginary chemical potential computation?)
4. Extension to finite quark mass simple.
5. Lots of new things to do

Crossover temperatures