Quark and gluon structure of hadrons computed from QCD

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BNL

Workshop on QCD in the nonperturbative regime-2019
TIFR, Mumbai

In collaboration with X. Gao, T. Izubuchi, L. Jin, C. Kallidonis, S. Mukherjee, P. Petreczky, C. Shugert, S. Syritsyn

Outline

- Introduction
- LaMET formalism: quasi- and pseudo-PDF
- How it’s made: From Euclidean 3-point function to PDF
- Progress, future and conclusions

First principle understanding of Parton structure of hadrons?

QCD at different resolutions

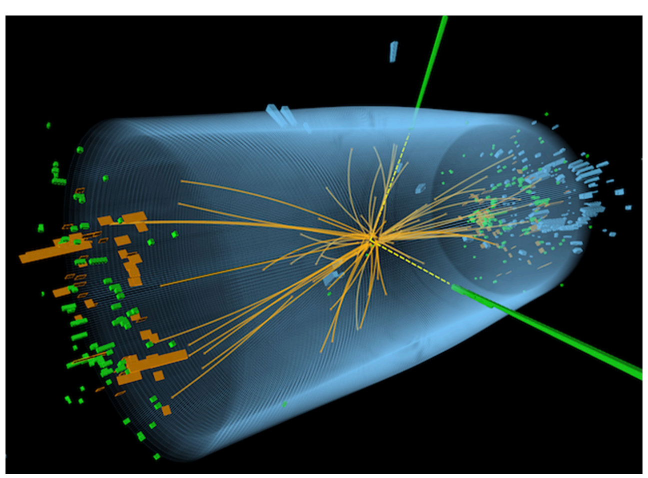
Asymptotic freedom

Confinement and mass-gap

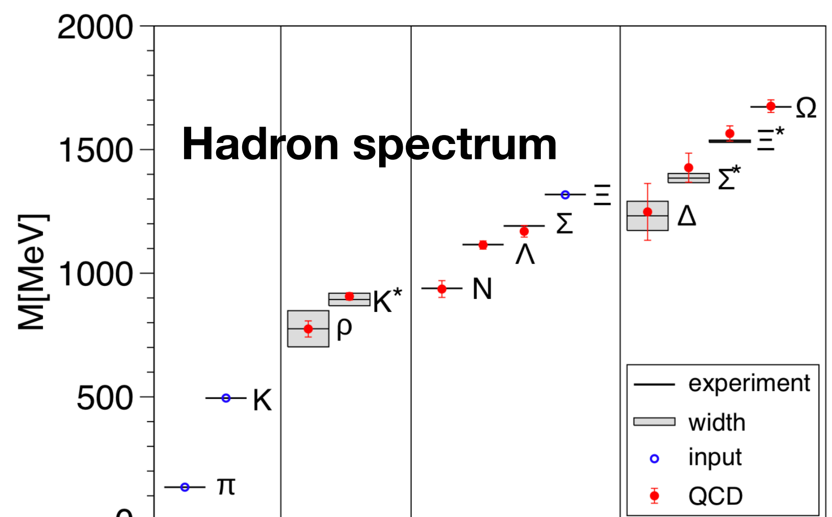
?

PDF +
Fragmentation +
Perturbation theory

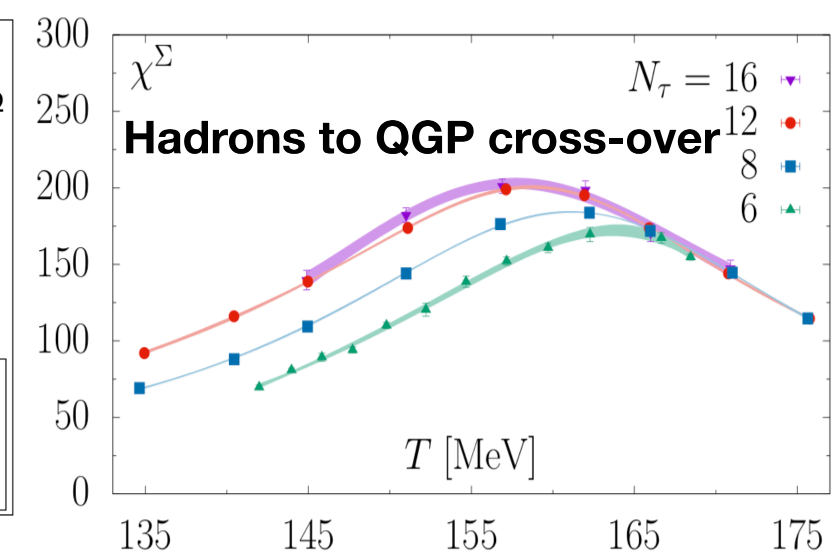
Lattice



CERN

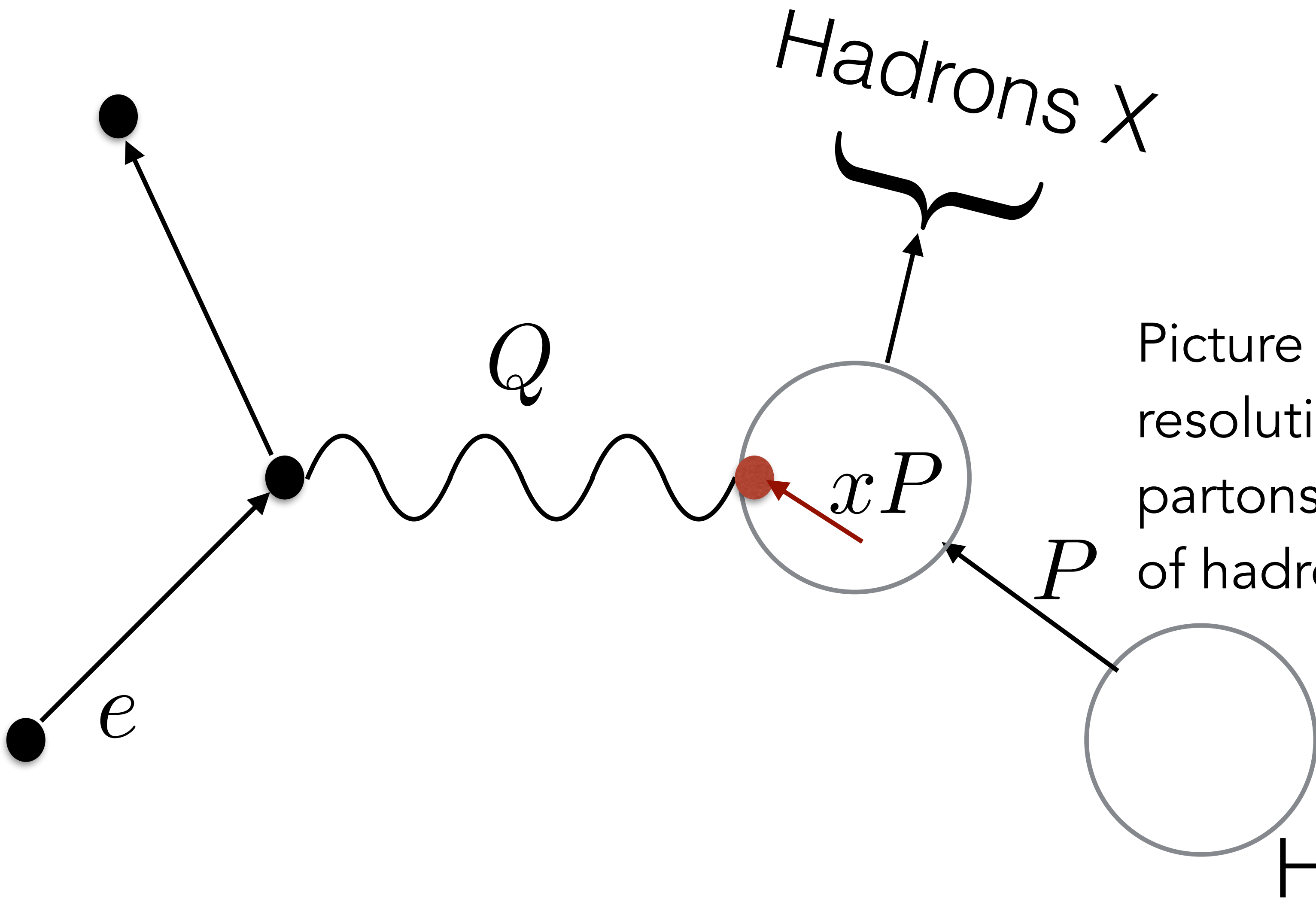


Fodor et al ‘08



HotQCD collaboration ‘19

Parton Distribution Functions



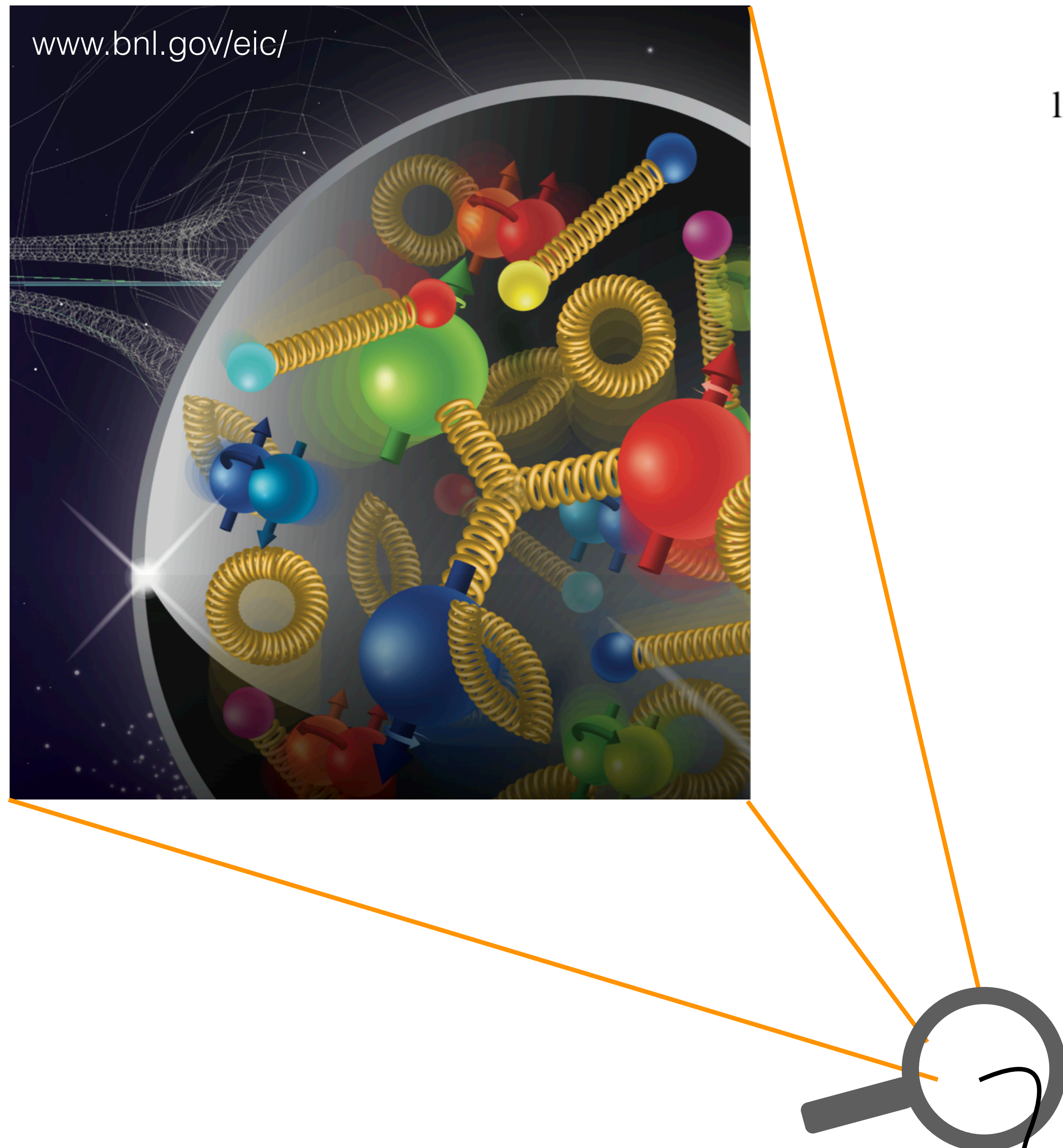
Parton Model (Feynman '69)

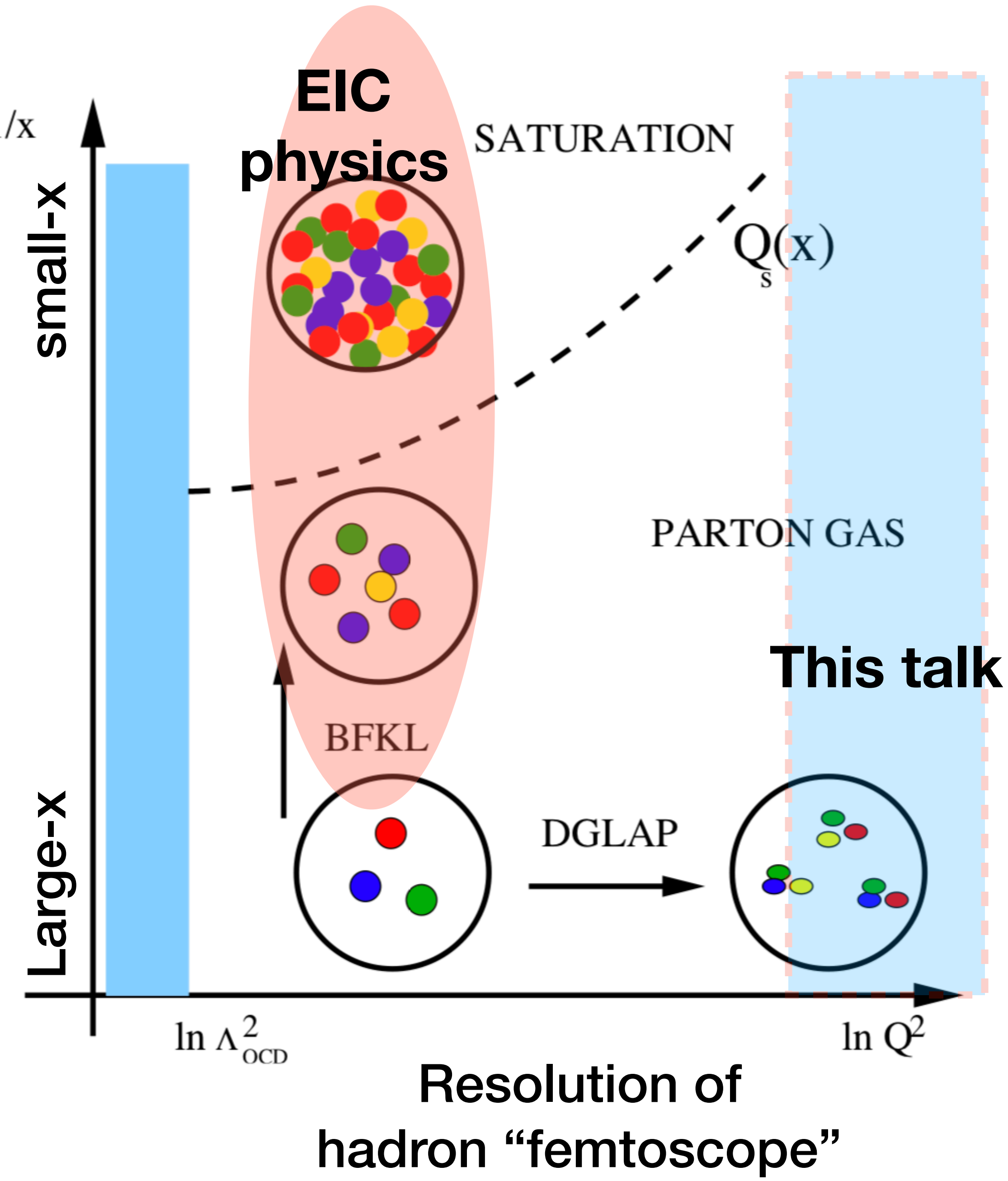
Picture of Hadron as imaged using a high-resolution probe = ensemble of massless partons each carrying fraction x of hadron momentum.

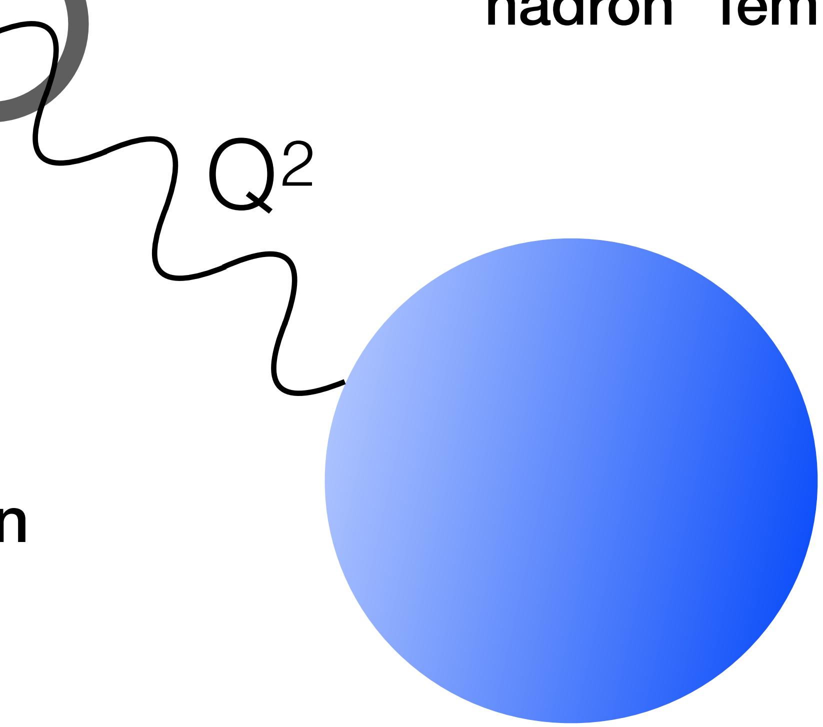
Factorization between UV and IR in the Bjorken limit:

$$\sigma = \sum_i f_i(x, Q^2) \circledast \sigma \{eq_i(xP) \to eq_i(xP+q)\}$$

"**Total inclusive cross-section** = **probability to find a parton X cross-section to scatter from a parton**"







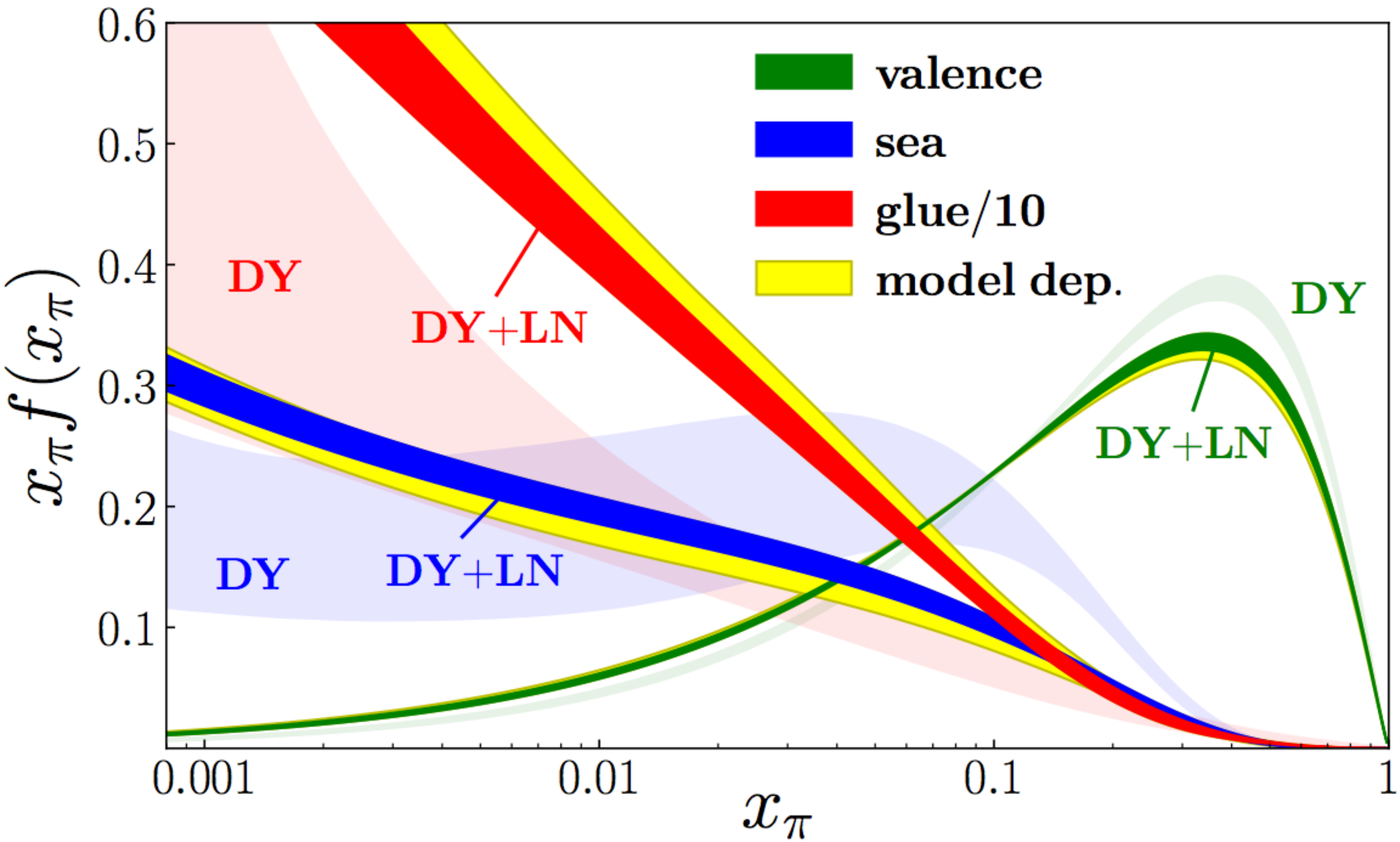
This talk: A simpler 1D structure of a very fast moving hadron along the direction of its motion

Valence PDF of $\pi^+(u\bar{d})$

$$f_v^\pi(x) = f_u(x) - f_{\bar{u}}(x) = f_u(x) - f_d(x), \quad 0 < x < 1$$

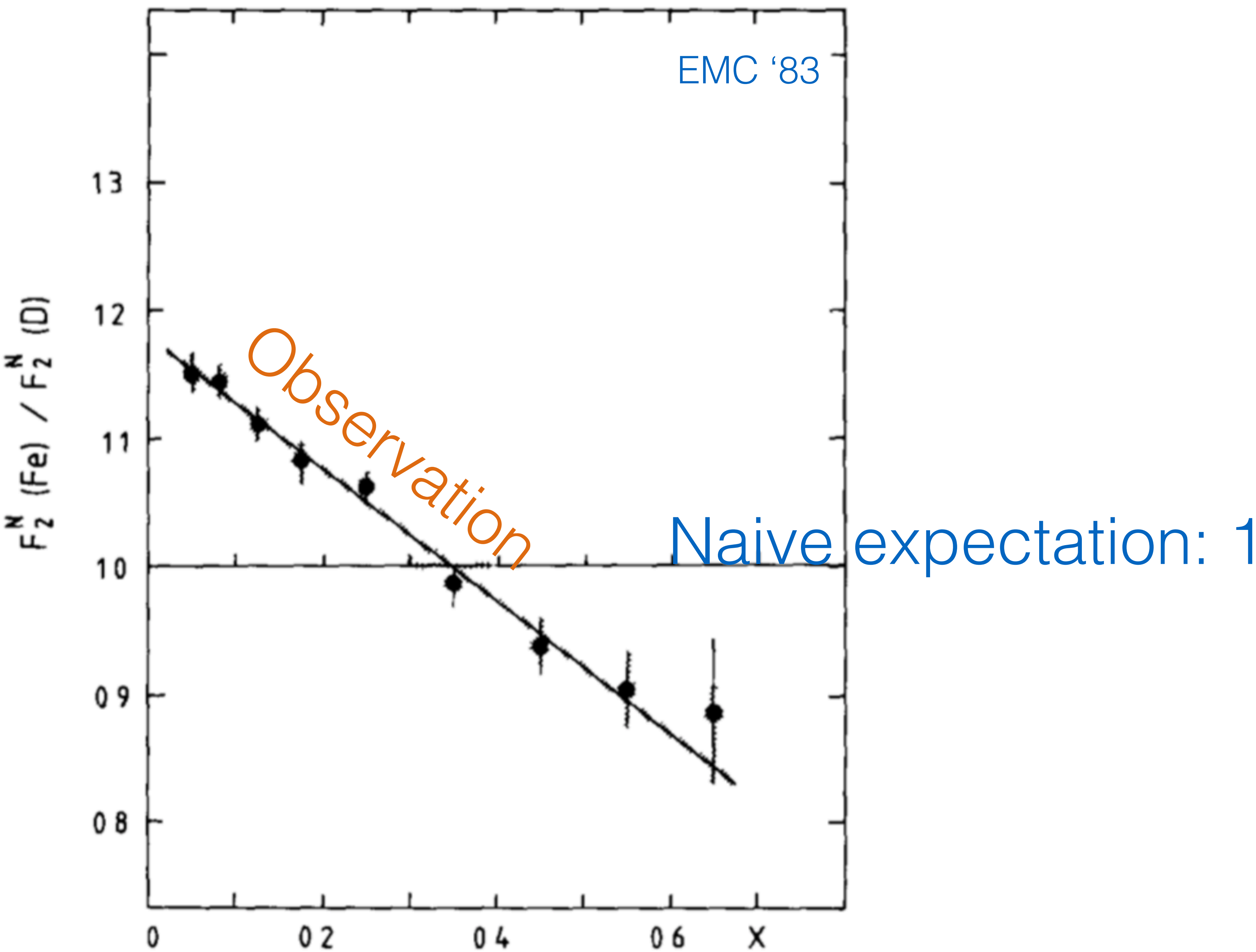
(sea+valence) **(sea)** **(Isospin symmetry)**

Experimental determination by P. C. Barry et al, 2018



Why do first principle computation at all?

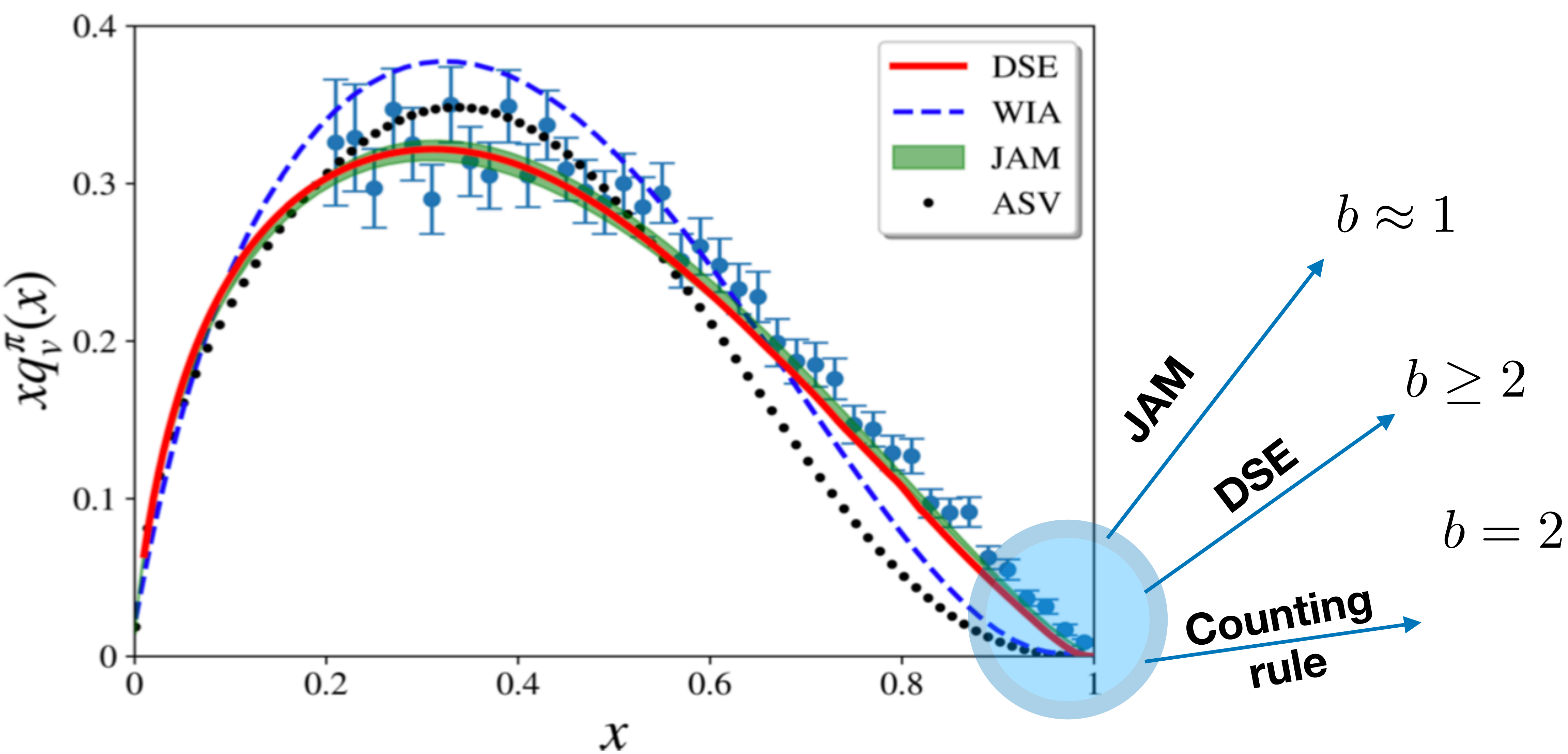
Long standing question-1: Do partons inside a proton within a nucleus know about other nucleons? **Yes (EMC effect)**



Why do first principle computation at all?

Long standing question-2: Valence PDF of $\pi^+(u\bar{d})$

Key physics issue is x=1 behavior: $\lim_{x \to 1} f_v^{\pi}(x) \sim (1-x)^b$

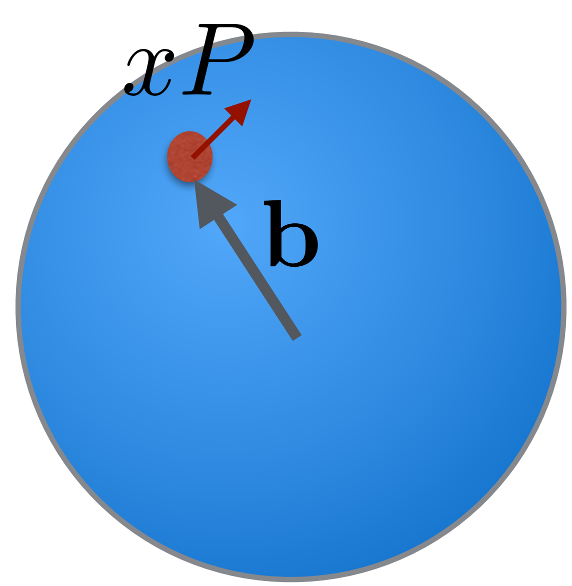


First principle calculation essential!

Why do first principle computation at all?

Very little experimental info available on
Generalized Parton Distribution Function (GPD):

$$H(x, Q^2, \Delta) \sim \tilde{H}(x, Q^2, \mathbf{b})$$



Why were PDFs not directly computed from QCD before?

and

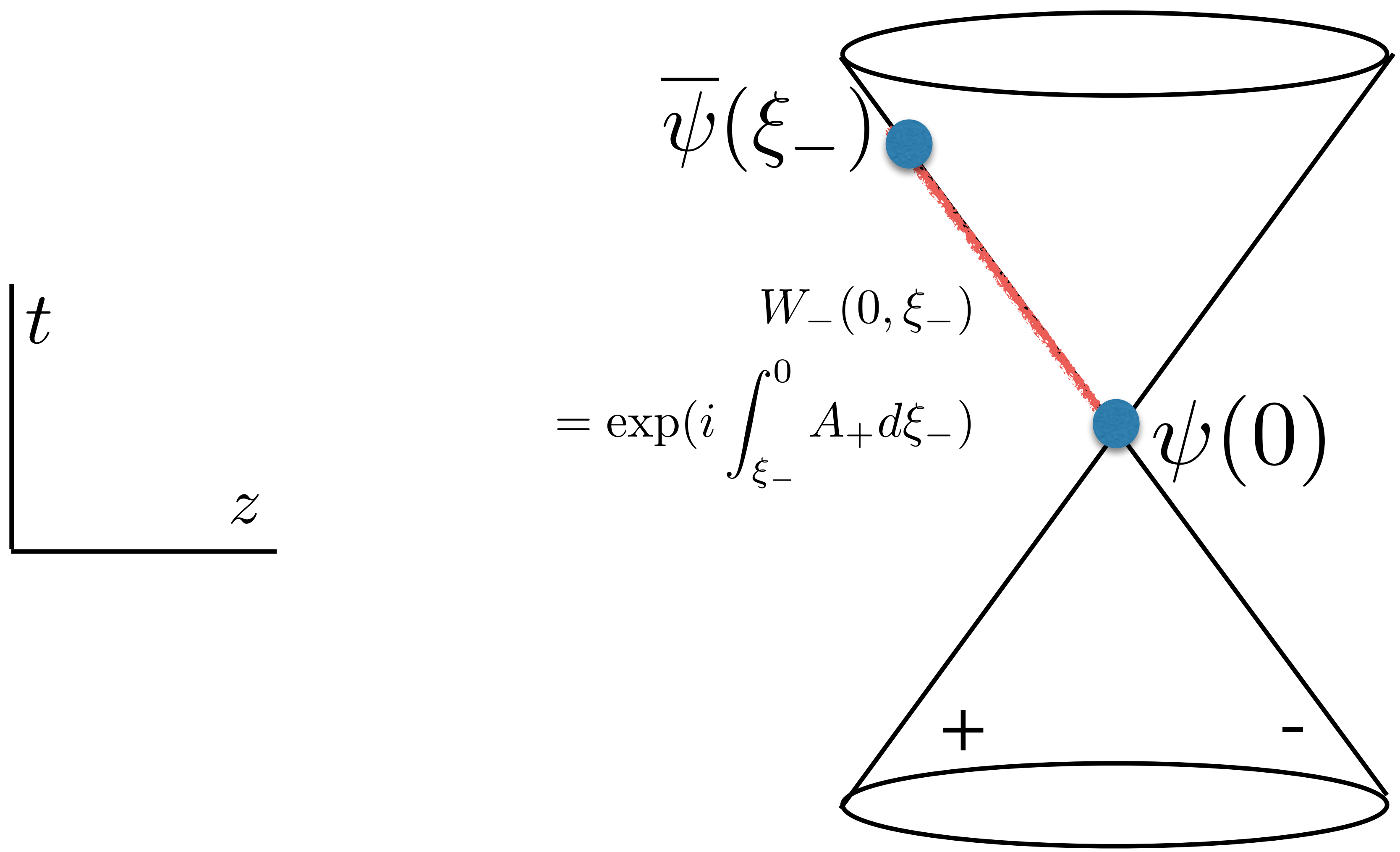
What has changed now?

PDF as light-like separated q-$\overline{\text{q}}$ correlation

Field theoretic Gauge-invariant and Lorentz invariant construction: (Soper '77)

$$f(x) = \int \frac{d\xi^-}{4\pi} e^{-ixP^+\xi^-} \langle H(P)|\overline{\psi}(\xi_-)\gamma_+ W_-(0,\xi_-)\tau\psi(0)|H(P)\rangle$$

= "Number of on-shell massless partons with energy x P^+"



…however, a problem for lattice

Projecting to hadron state is easy on lattice, but requires $t \to i\ t$

$$\lim_{\Delta t \to \infty} e^{-H_{\rm QCD}\Delta t} \hat{O}_h(t=0, \mathbf{P})|\Omega\rangle \propto |h(\mathbf{P}, E)\rangle$$

But presence of unequal time separation between $\psi(0)$ and $\psi(\xi_-)$ sandwiched between hadron states is a **sign problem** for Euclidean lattice.

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Definitely tractable on a quantum computer

Is it really out of reach of current lattice computations?

Resolutions:

- Compute moments of PDF which are related by OPE to twist-2 local operators.

Martinelli and Sachrajda '88, W. Detmold et al, d > '01

$$\left\langle H(P) \middle| \psi(x)\gamma_{\{\mu}D_{\nu}\dots D_{\rho\}}\psi(x) - \mathrm{trace} \middle| H(P) \right\rangle = P_{\mu}\dots P_{\rho}\langle x^n \rangle$$

! Mixing of operators on lattice due to rotational symmetry breaking not tractable

- Quasi-PDF approach (this talk), pseudo-PDF and factorization of lattice cross-sections.

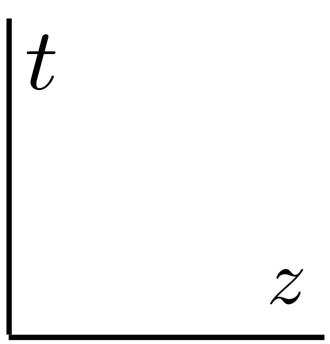
X. Ji '13, A. Radyushkin '17, Ma and Qiu '17

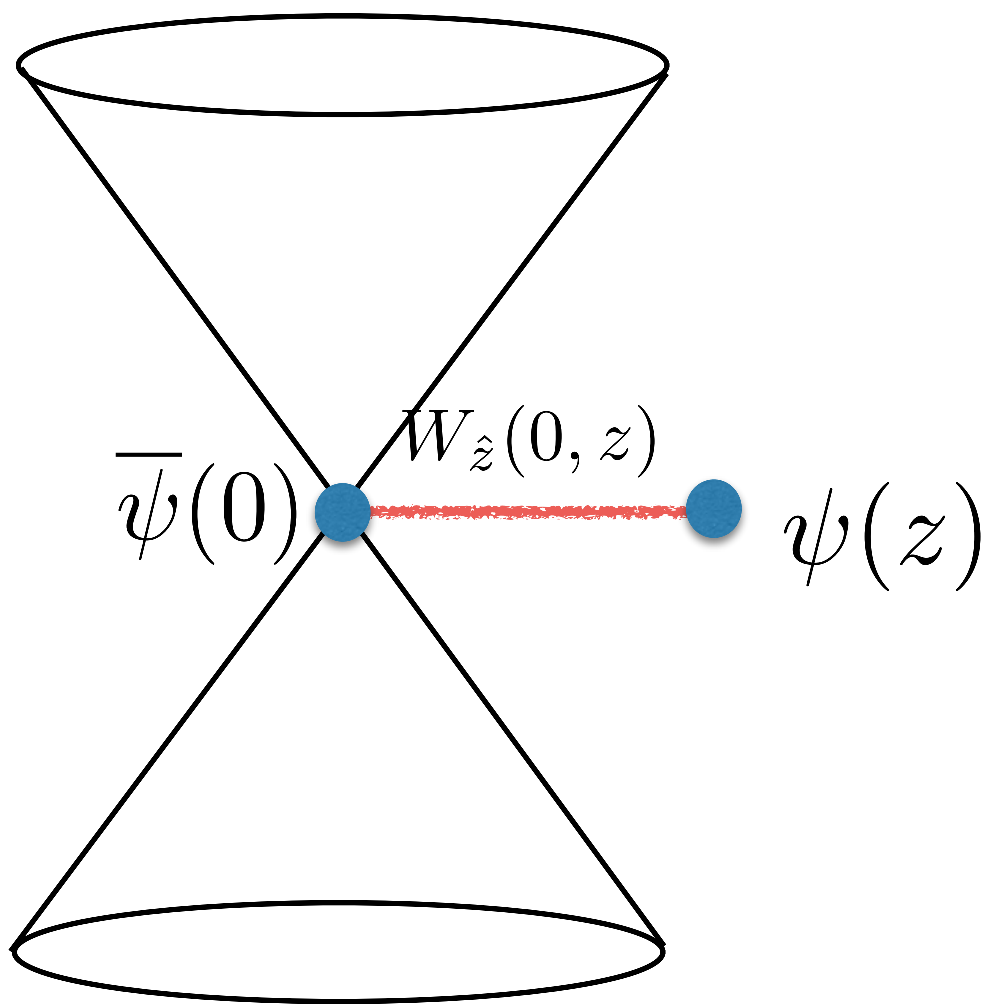
quasi-PDF approach to obtain PDF using Euclidean lattice

Equal time correlation function that can be determined on lattice:

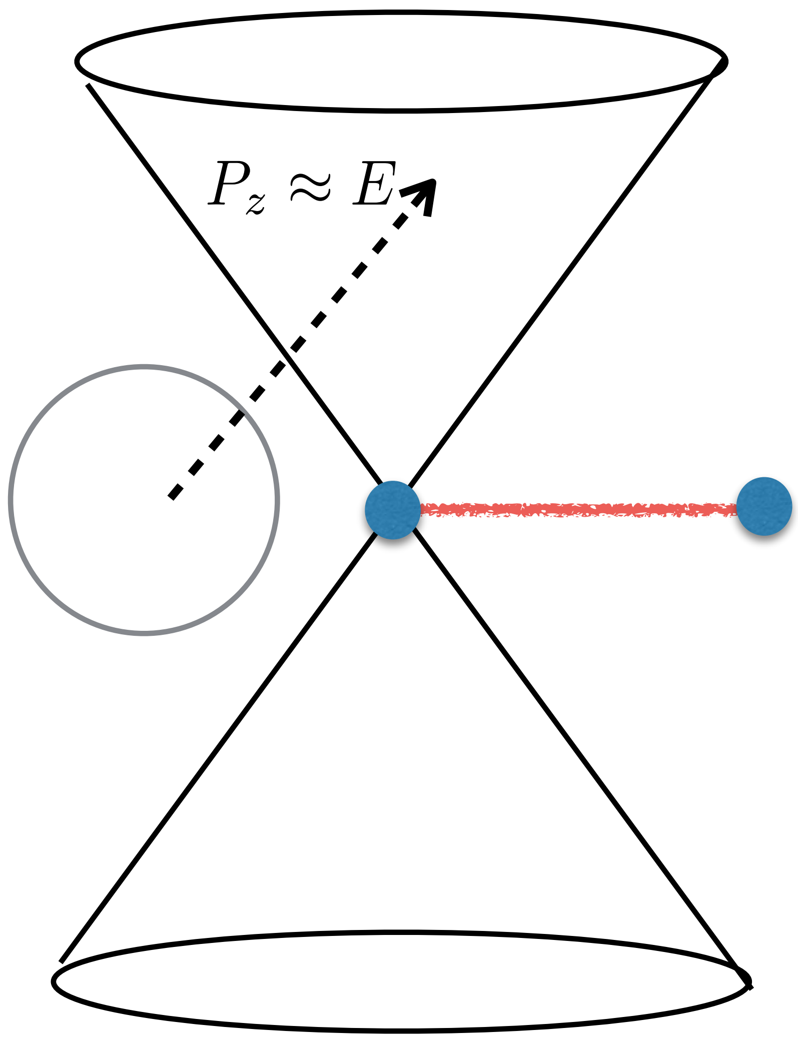
$$\tilde{q}(x) = \int \frac{dz}{4\pi} e^{-ixP_z z} \langle H(P_z, E)|\overline{\psi}(0)\gamma_\mu W_{\hat{z}}(0,z)\tau\psi(z)|H(P_z, E)\rangle$$

for $\mu = z \text{ or } t.$



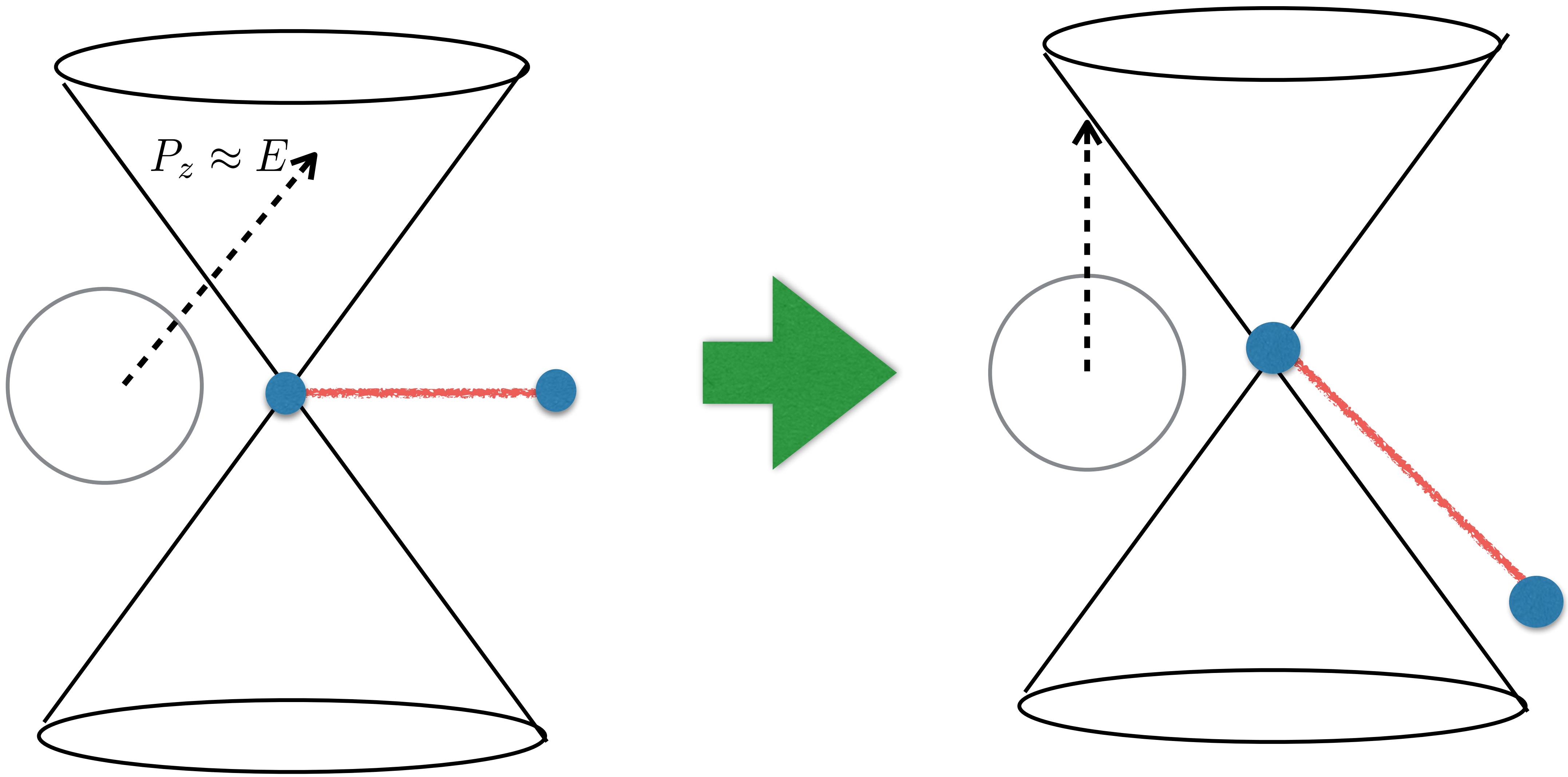


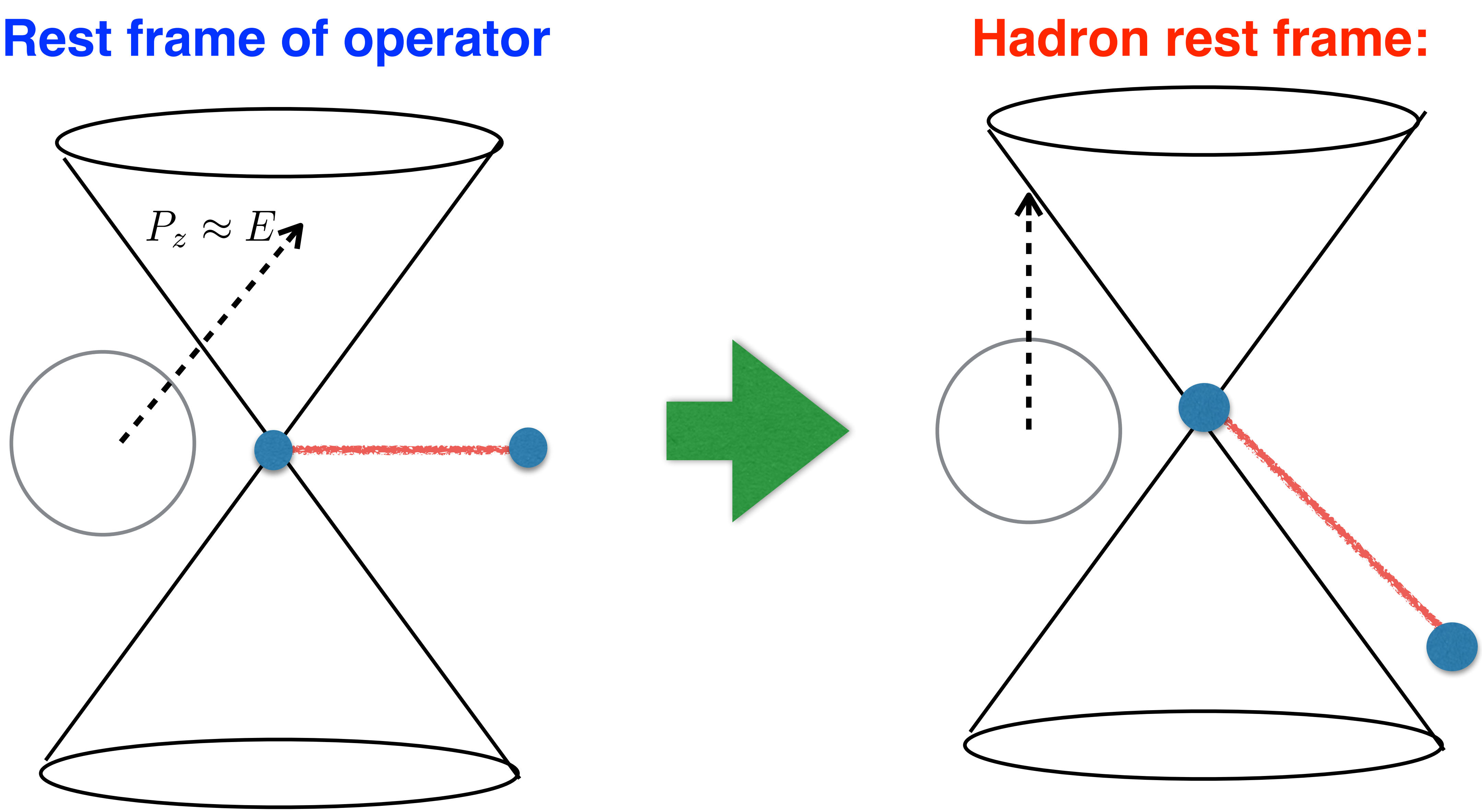
Rest frame of operator



Rest frame of operator

Hadron rest frame:





z^2 is Lorentz invariant. But the typical z contributing to Fourier transform at fixed x is $z_{typ} \sim 1/P_z$ and so $|z_{typ}|$ is power suppressed.

Converse: Small x at fixed P_z → Larger $|z_{typ}|$ ⇨ Effect of Λ_{QCD}, M, z^2

Issue of limits

In 3+1d, PDF operator already is on the light-cone before regularization and renormalization.

On 4d lattice…

- one has finite lattice spacing ***a***
- At any finite ***a***, ***q(x)*** has to be renormalized at a scale P^R
- Take ***a*** → 0 first, then $P_z \to \infty$

Perturbative matching (or LaMET)

- Perturbative matching between ***q(x,*** P_z***,*** P^R***)*** in a regulator independent renormalization scheme at finite P_z to the infinite momentum MS-bar PDF ***f(x,*** μ^2***)***

$$q(x; P_z, P^R) = \int_{-1}^{1} \frac{dy}{|y|} C\left(\frac{x}{y}, \frac{\mu}{yP_z}, \frac{P_\perp^R}{P_z^R}, \frac{yP_z}{P_z^R}\right) f(x, \mu)$$

with the matching coefficient $C(\xi) = \delta(1-\xi) + \alpha_S(\mu) C^{(1)}(\xi)$

X. Ji '13, Stewart and Zhao '17

An alternate perspective: pseudo Ioffe-time distribution

Pseudo-Ioffetime distribution:

$$\left\langle H(P)\middle|\overline{\psi}(0)\gamma_t W_{\hat{z}}(0,z)\psi(z)\middle|H(P)\right\rangle \equiv \mathcal{M}(P.z, z^2)$$

Lorentz invariance

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Lorentz invariance

Approach light-cone by taking z^2 =0 at fixed P.z

$$\lim_{z^2\to 0}\mathcal{M}(\nu, z^2) \to M(\nu,\mu) = \int_{-1}^{1} f(x,\mu)e^{ix\nu}dx$$

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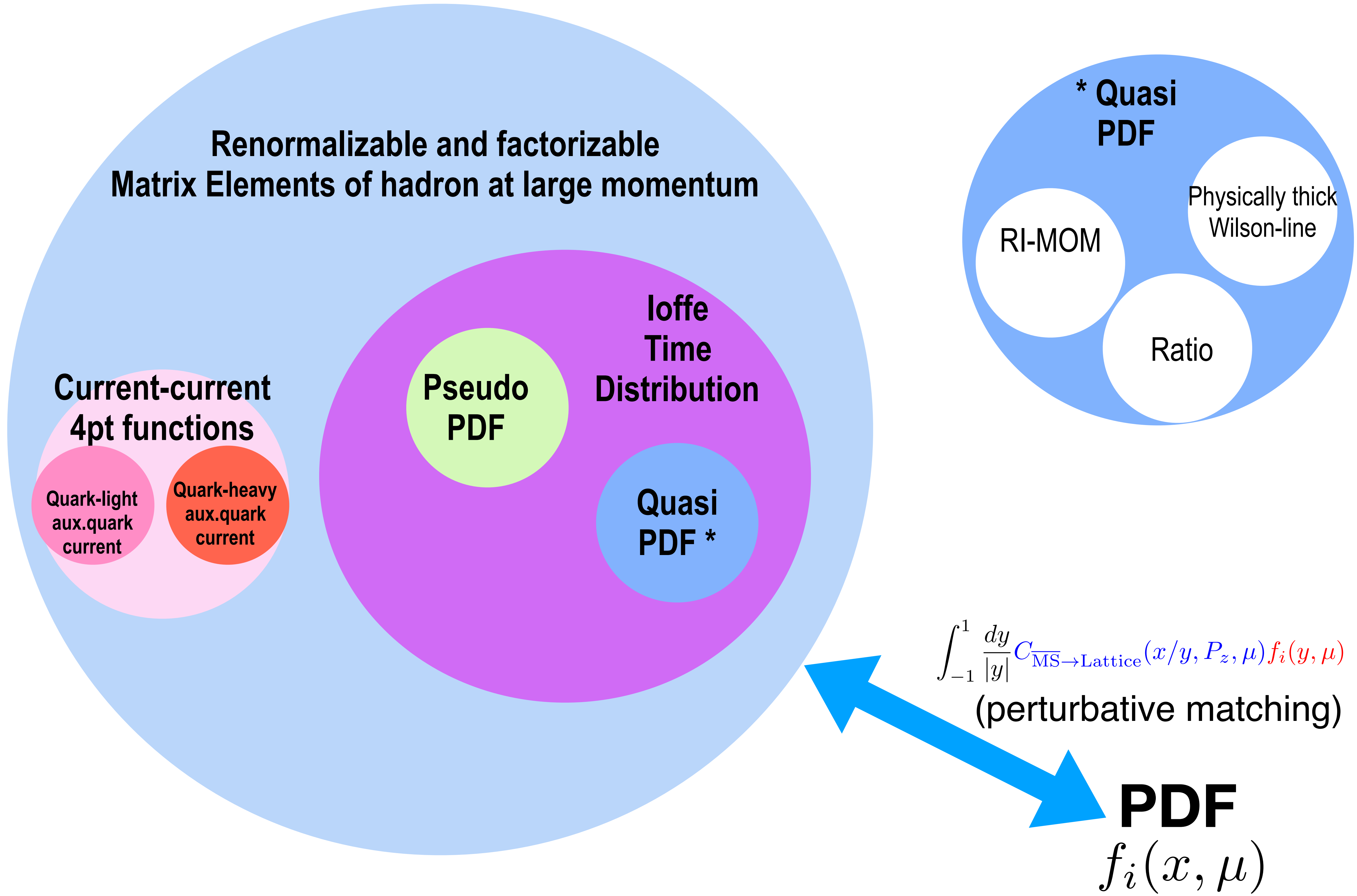
$$\lim_{z^2\to 0}\mathcal{M}(\nu, z^2) \to M(\nu,\mu) = \int_{-1}^{1} f(x,\mu)e^{ix\nu}dx$$

Concretely:

$$\mathcal{M}(\nu, z^2) = \sum_{\nu}\frac{\nu^n}{n!}C_n(\mu^2 z^2)\frac{\partial^n M(\nu',\mu)}{\partial\nu'^n}\Big|_{\nu'=0}$$

Pseudo-Ioffe Wilson coeffs Ioffe

A unified framework to reconstruct PDF from lattice



Y.Q.Ma and J.W.Qiu, PRL120 (2018),022003

A. Radyushkin, PRD98 (2017),034025

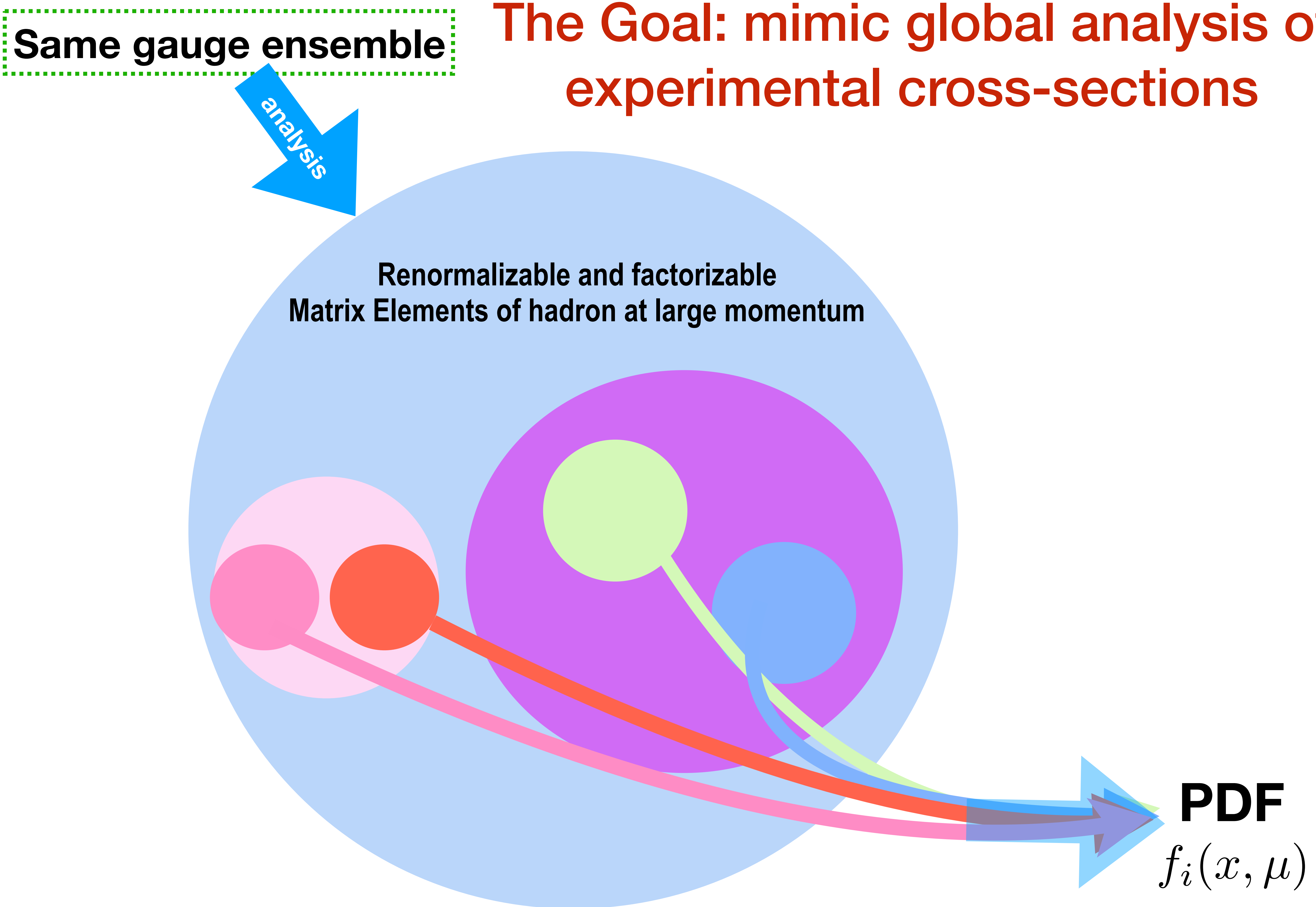
X. Ji, PRL110 (2013), 262002

W. Detmold and C.J. D. Lin, PRD73 (2006) 014501
V. M. Braun and D. Muller, EPJ C55, 349 (2008)

K.F. Liu and S.J.Dong, PRL 72(1994), 1790

R.S. Sufian et al, PRD (2019), 074507

The Goal: mimic global analysis of experimental cross-sections



LaMET*

1. Ensure $P_z \gg M$ and $P_z \gg \Lambda_{\mathrm{QCD}}$
2. Reliable extraction of M.E. of a fast moving hadron
3. First take continuum limit, then take $P_z \to \infty$
   In practice: Keep $(aP_z) < 1$ and renormalize
4. Matching currently to 1-loop order for quasi-PDF. Enough?
5. Extract PDF without using large quark-antiquark separations, and without modeling bias.

***Conditions apply**

Computing the pion valence PDF

Lattice Set-up

1. **Lattice ensemble A :** a=0.06 fm, 48×64^3 HISQ sea-quarks (HotQCD ensemble)

   1-HYP smeared Wilson-Clover valence quarks

   Valence pion mass M_π = 0.3 GeV

   Hadron momenta: P_z = 0.43, 0.86, 1.29, 1.72 GeV $< a^{-1}$ = 3.2 GeV and $> M_{pi}$

   Quark-antiquark separations, z, that enter analysis < 0.72 fm

   Results from this ensemble: Phys.Rev. D100 (2019), 034516

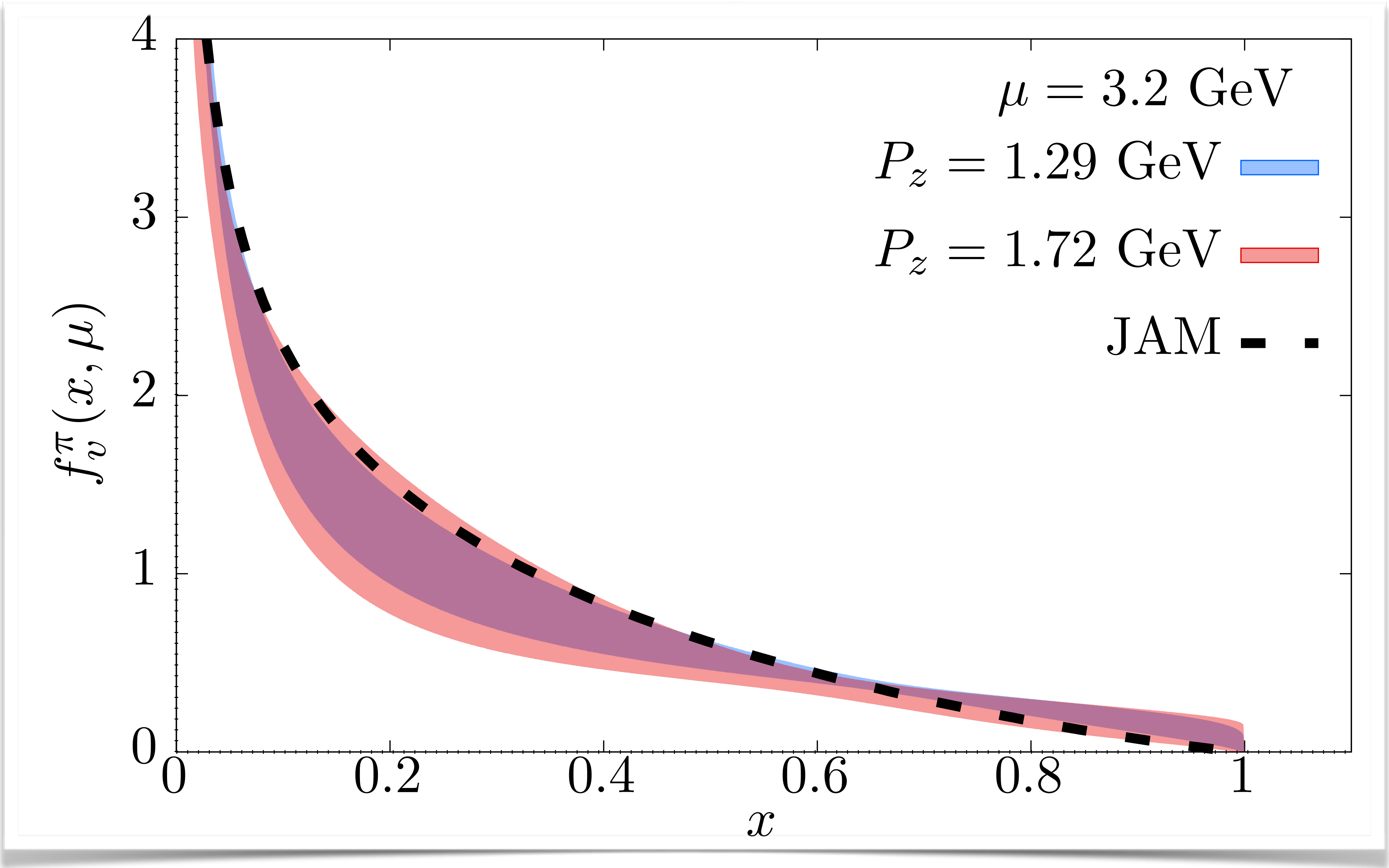
2. **Lattice ensemble B :** a=0.04 fm, 64×64^3

   Hadron momenta: P_z = 0.48, 0.97, 1.45, 1.94 GeV $< a^{-1}$ = 4.8 GeV

   This ensemble: Ongoing work

Glossary:
sea quark: (noun) det(D) used in Monte Carlo.
valence quark: (noun) D^{-1} used in propagators.
HYP : (Abbr.) A procedure to suppress UV lattice-like gluons.

Valence PDF of pion starting from quasi-PDF



BNL-SBU-UConn Collaboration, PRD100 (2019), 034516

$$\frac{\left\langle H(t_{\text{sink}})\ \overline{\psi}(0)\ W(0,z)\ \psi(z)\ H^\dagger(0)\right\rangle}{\left\langle H(t_{\text{sink}})H^\dagger(0)\right\rangle}$$

$t_{\text{sink}} \to \infty$

$$h(z, P_z) = \left\langle P_z \middle| \overline{\psi}(0)\ W(0,z)\ \psi(z) \middle| P_z \right\rangle$$

Renormalize

$$h^R(z, P_z, P^R)$$

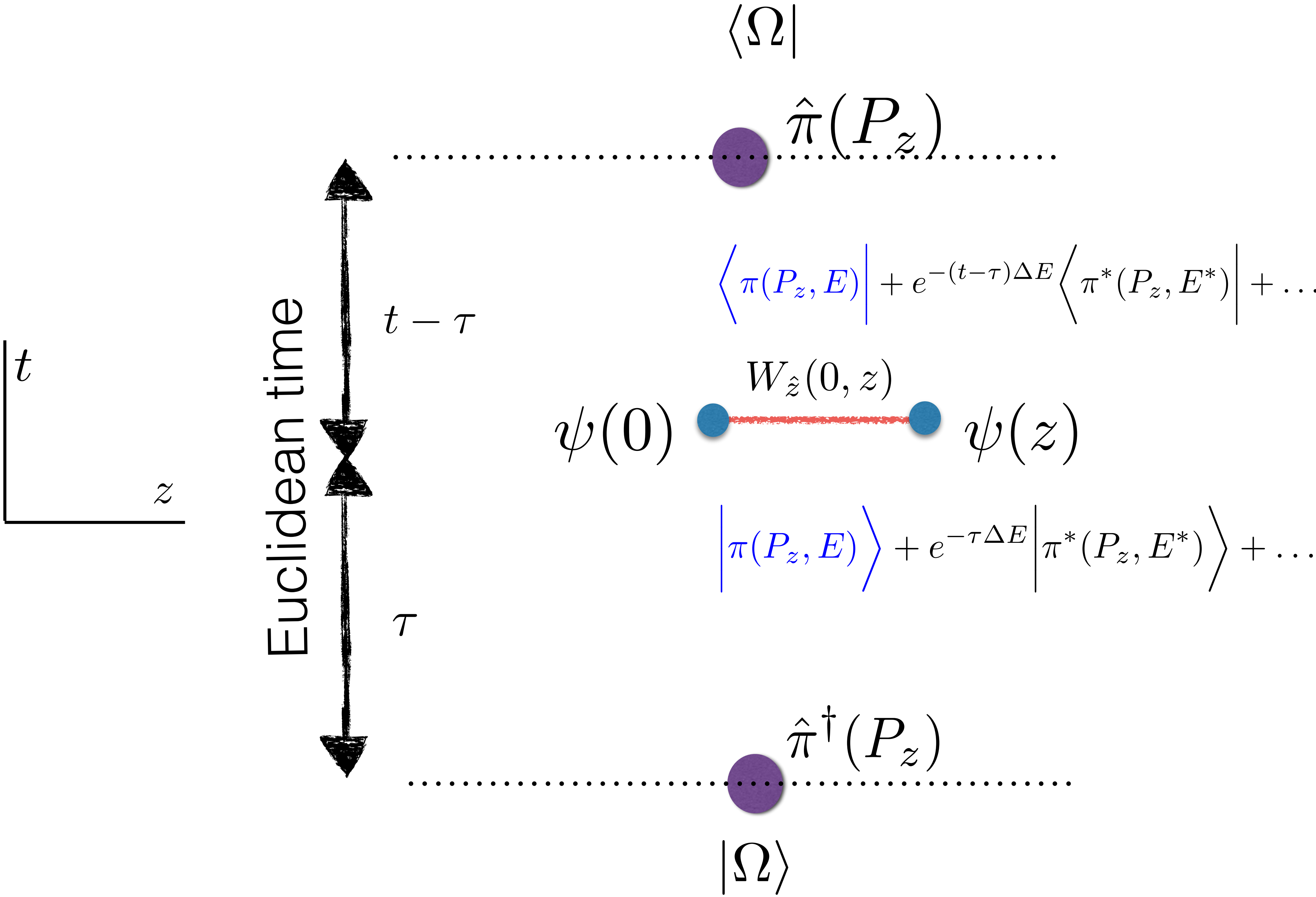
Fourier z to conjugate x
at fixed P_z

$$\tilde{q}(x, P_z, P^R)$$

LaMET

$$f(x, \mu^2)$$

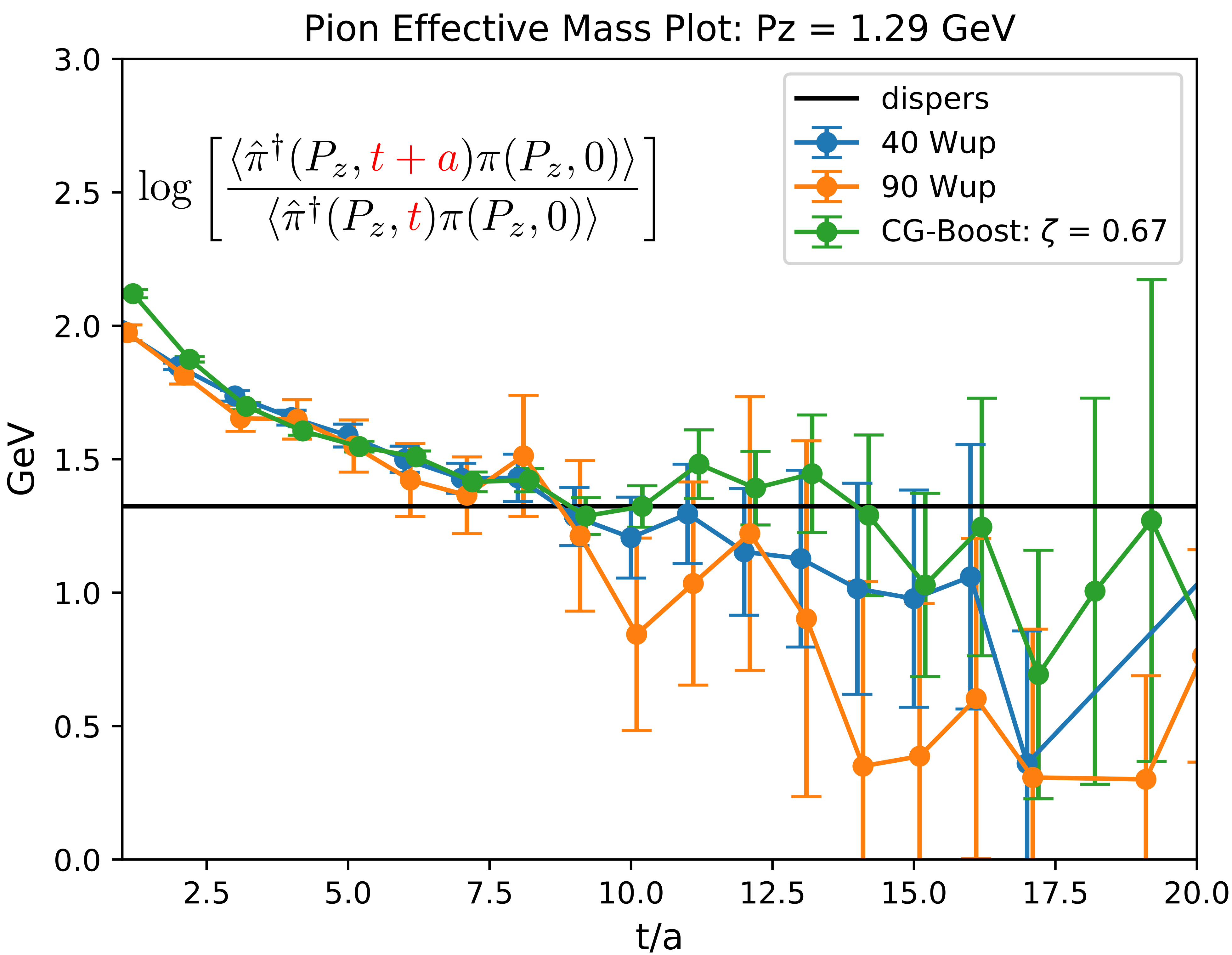
Set up of the ‘measurement’



Choice of the creation operator $\hat{\pi}(P_z)$ is important

$\pi(x_0) = u(x_0)\gamma_5 \bar{d}(x_0)$...choose quark sources $\psi(x_0,t) \sim \int d^3k e^{ikx_0} e^{-\sigma^2 \frac{(k-\zeta P)^2}{2}} \tilde{\psi}(0)$

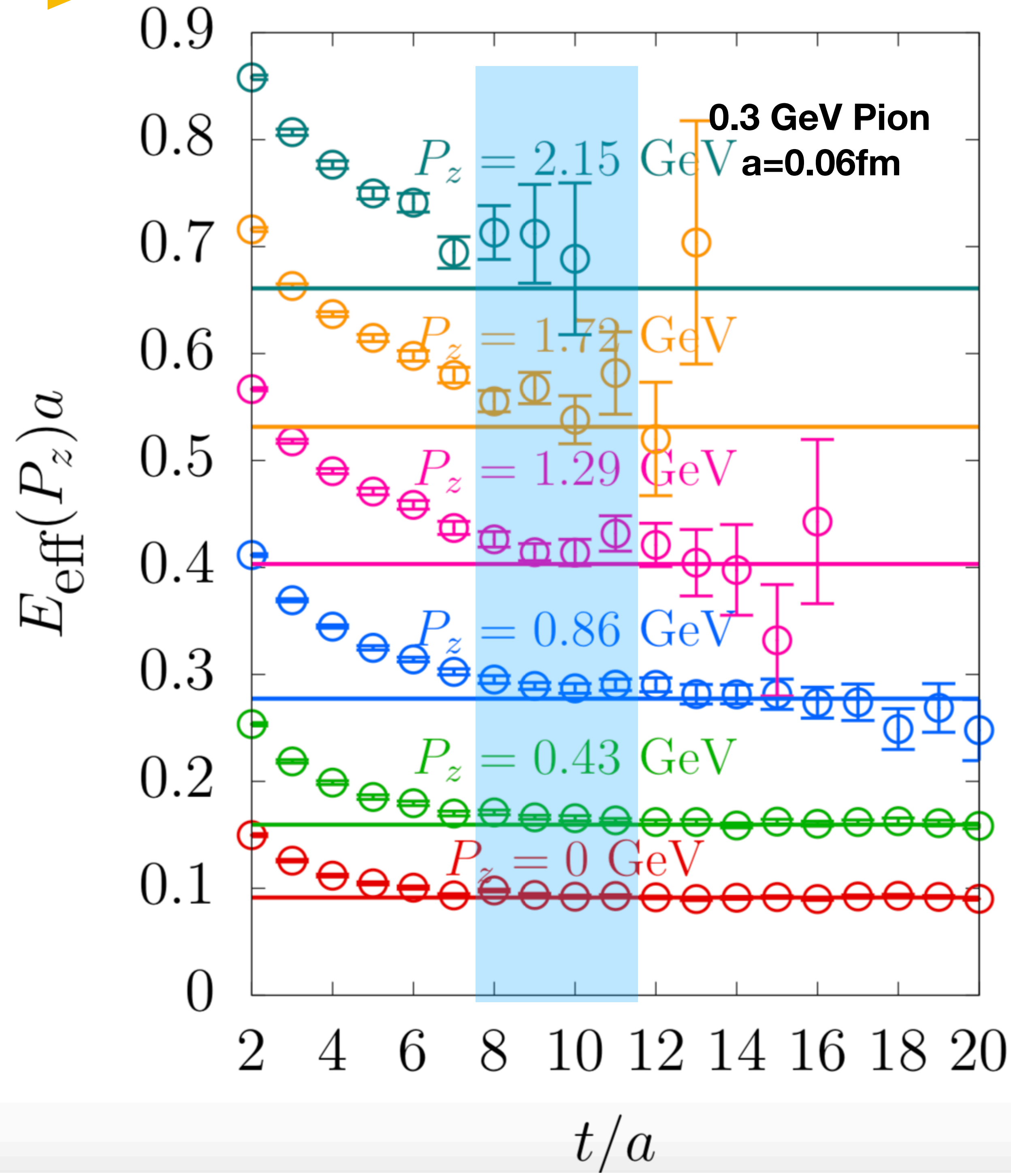
Bali et al '16



Ensuring $P_z \gg M_{\text{hadron}}, \Lambda_{\text{QCD}}$

Even with optimal smearing: T_{sink} ~ 8a-12a for pion on a=0.06 fm lattice

a=0.06 fm → Can push P_z ~ 3 GeV. But, only P_z =1.29 and 1.72 GeV usable

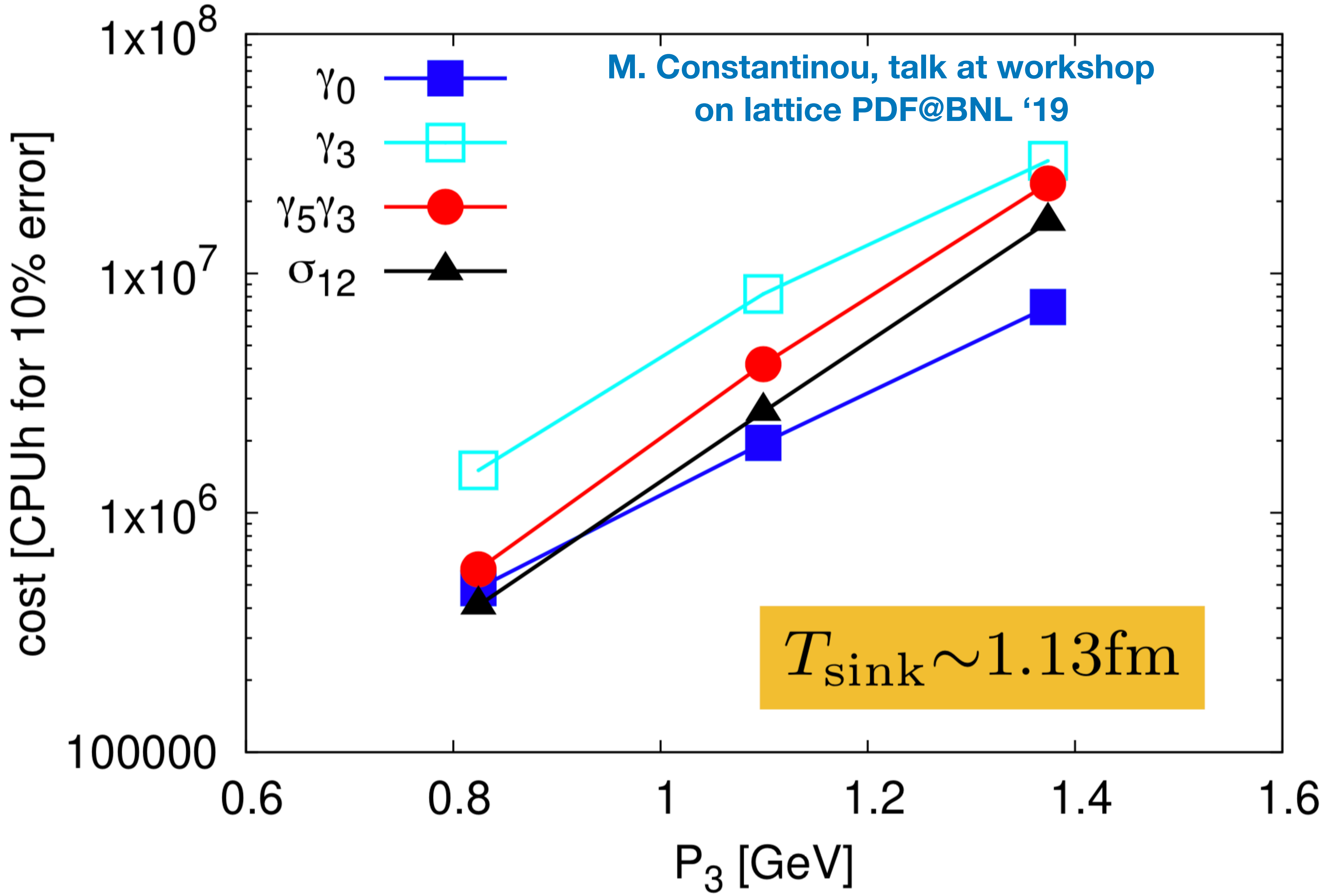


arXiv:1905.06349

Ensuring $P_z \gg M_{\mathrm{hadron}}, \Lambda_{\mathrm{QCD}}$

Can we beat the noise with statistics?

The required statistics at fixed T_{sink} exponentially increases with momentum



$$\frac{\left\langle H(t_{\text{sink}})\,\overline{\psi}(0)\,W(0,z)\,\psi(z)\,H^{\dagger}(0)\right\rangle}{\left\langle H(t_{\text{sink}})H^{\dagger}(0)\right\rangle}$$

$t_{\text{sink}} \to \infty$

$$h(z, P_z) = \left\langle P_z \middle| \overline{\psi}(0)\,W(0,z)\,\psi(z) \middle| P_z \right\rangle$$

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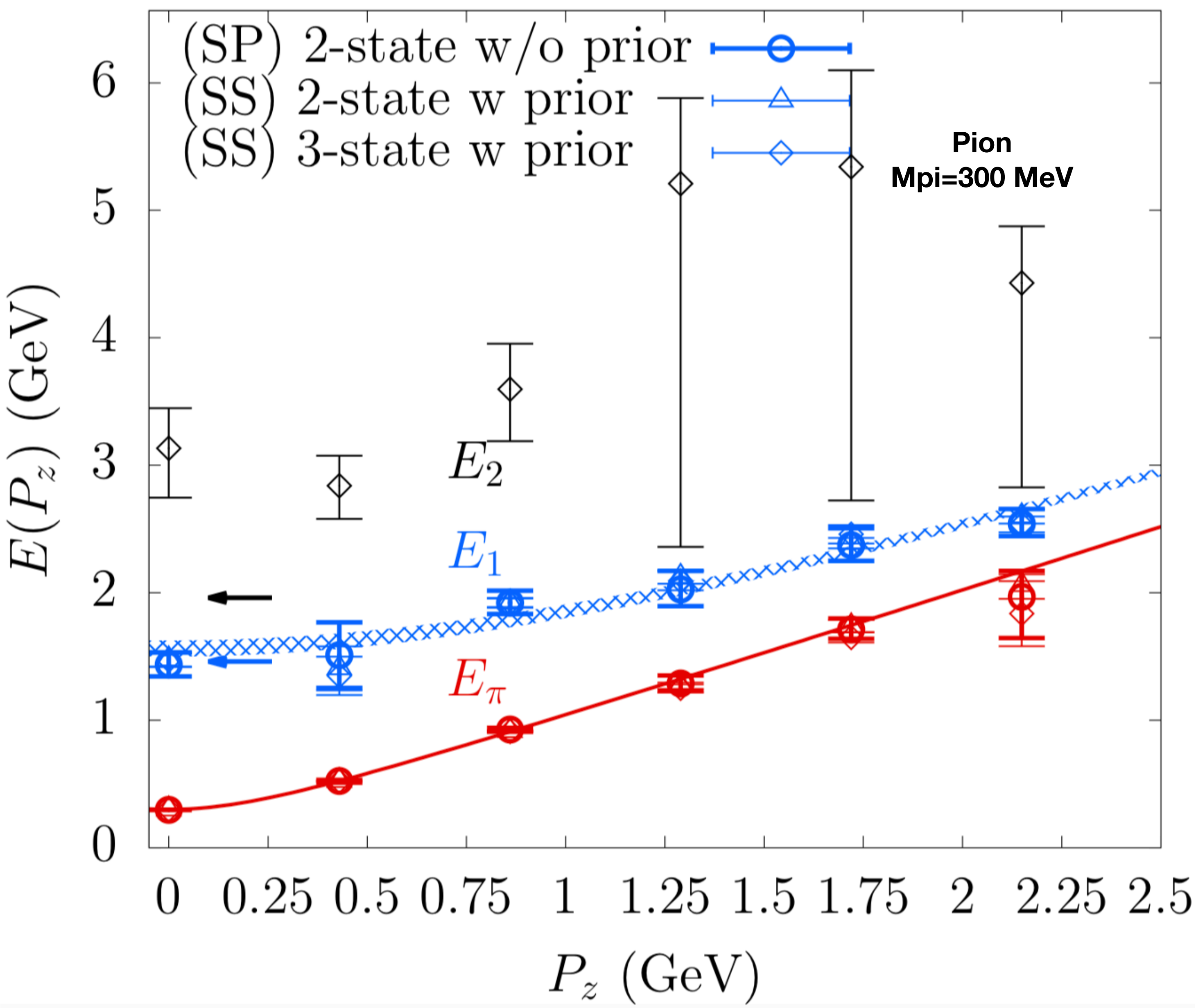
Reliable extraction of matrix element

- **Source of difficulty: increasing density of excited states for boosted hadrons.**

- Choice: amplitudes $\langle E_n|O_\Gamma(z)|E_0\rangle$ dependent on O_Γ. $\Gamma = \gamma_t$ better than γ_z

Boosted pion source: π , π_1 and "tower of states"
Boosted nucleon: nucleon and "tower of N-pi states" (ETMC, arXiv:1902.00587)



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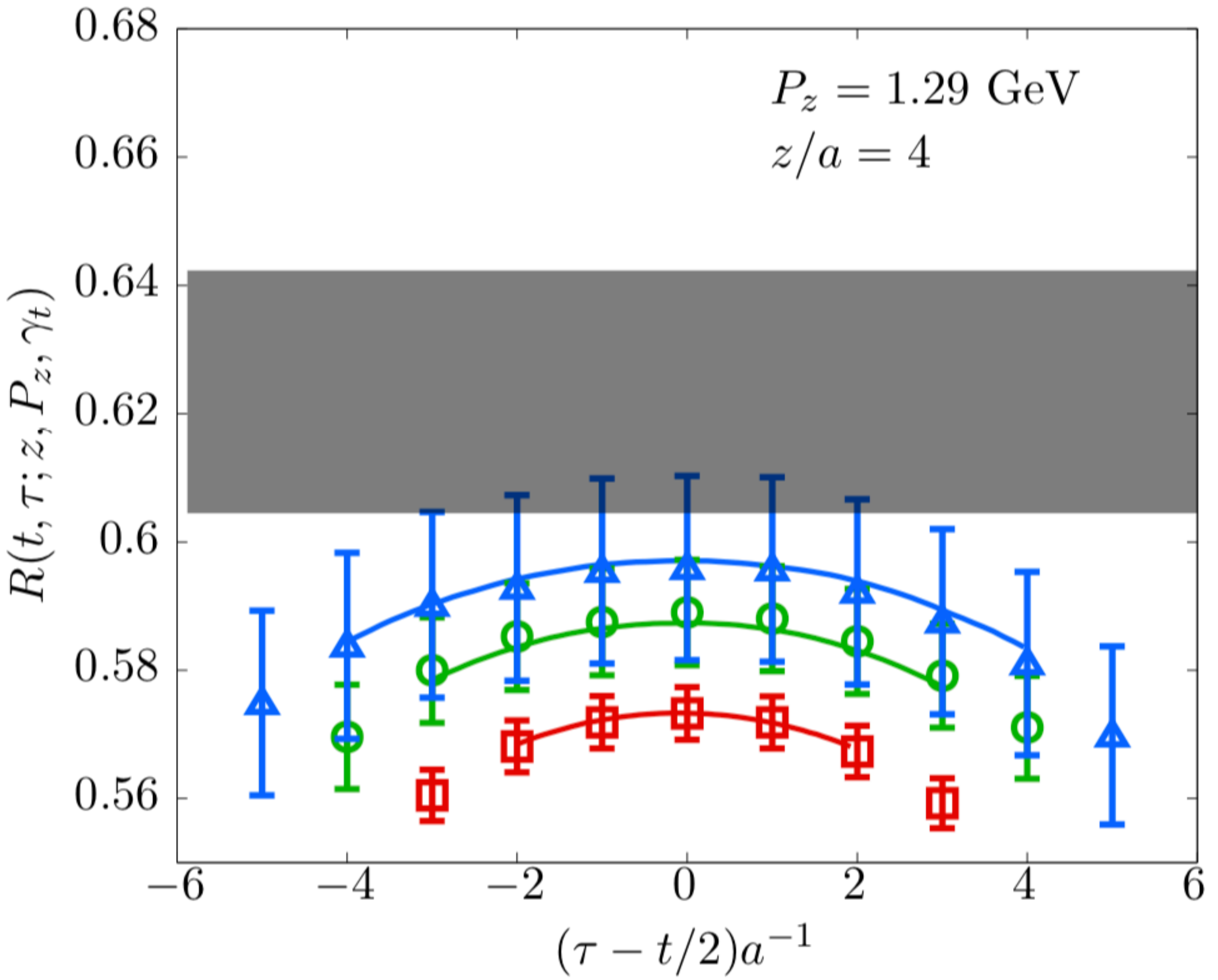
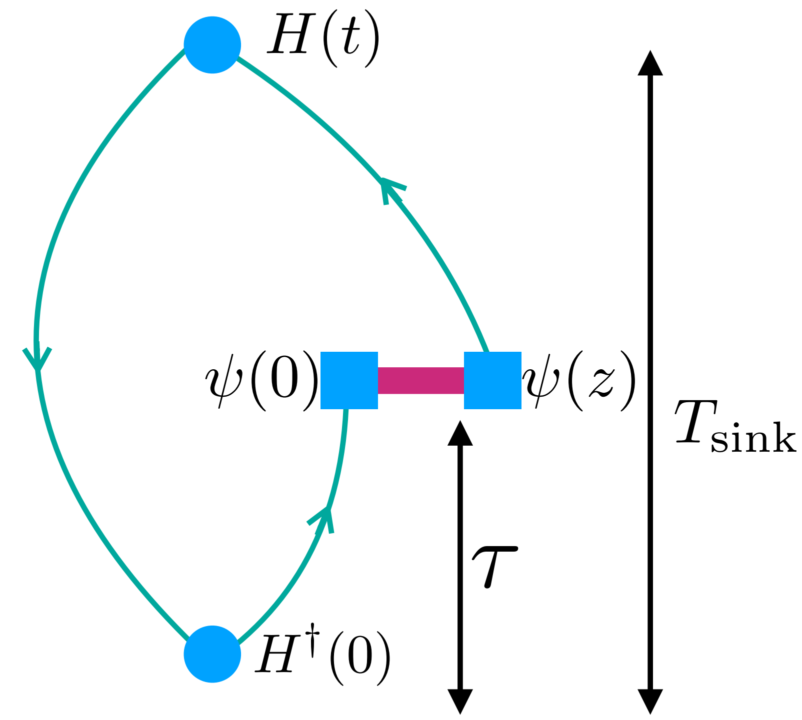
Reliable extraction of matrix element

- Strategies for quasi-PDF: plateau, 2- and 3-state fits to $T_{\rm sink}$ and τ dependence, summation methods.





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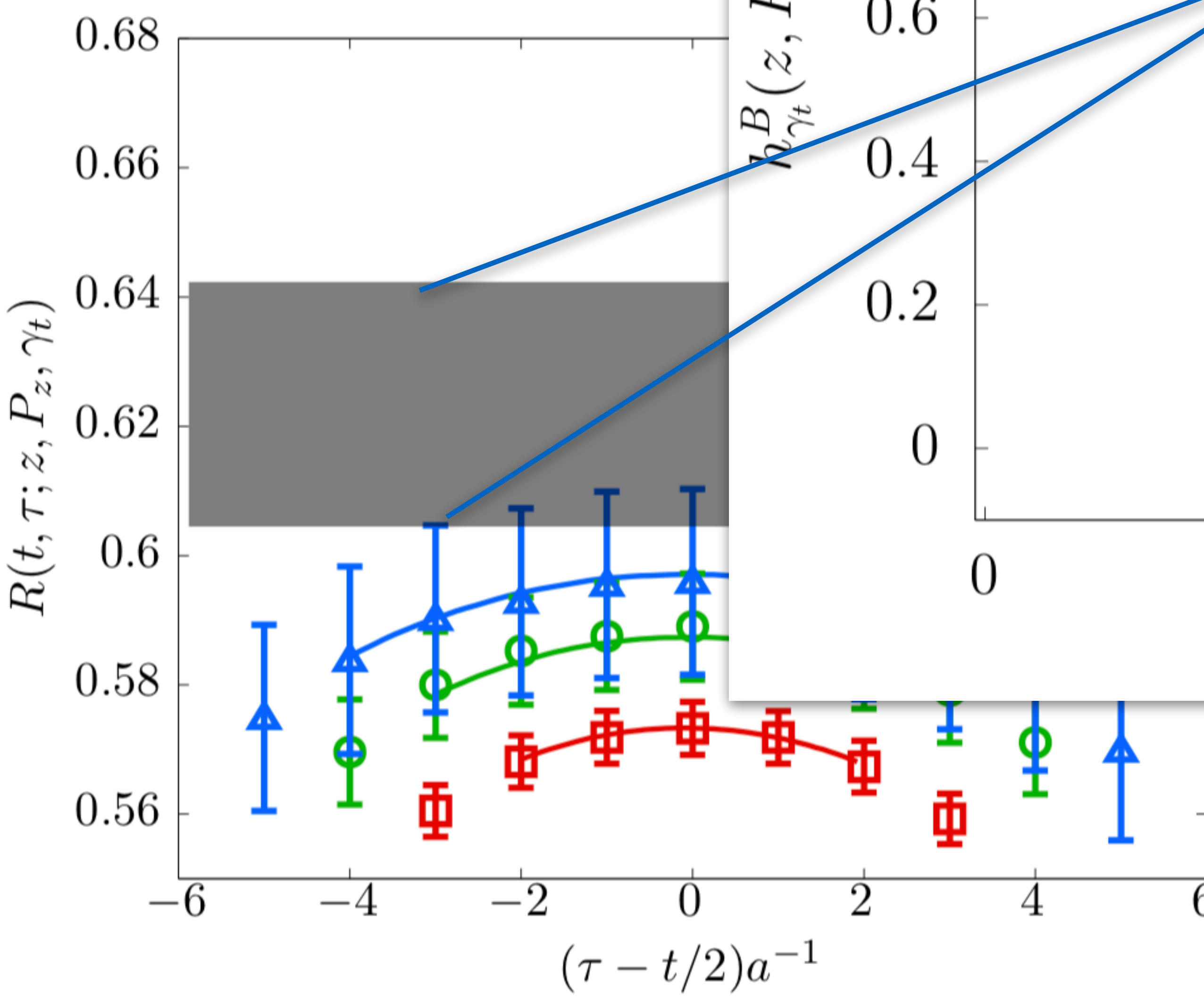

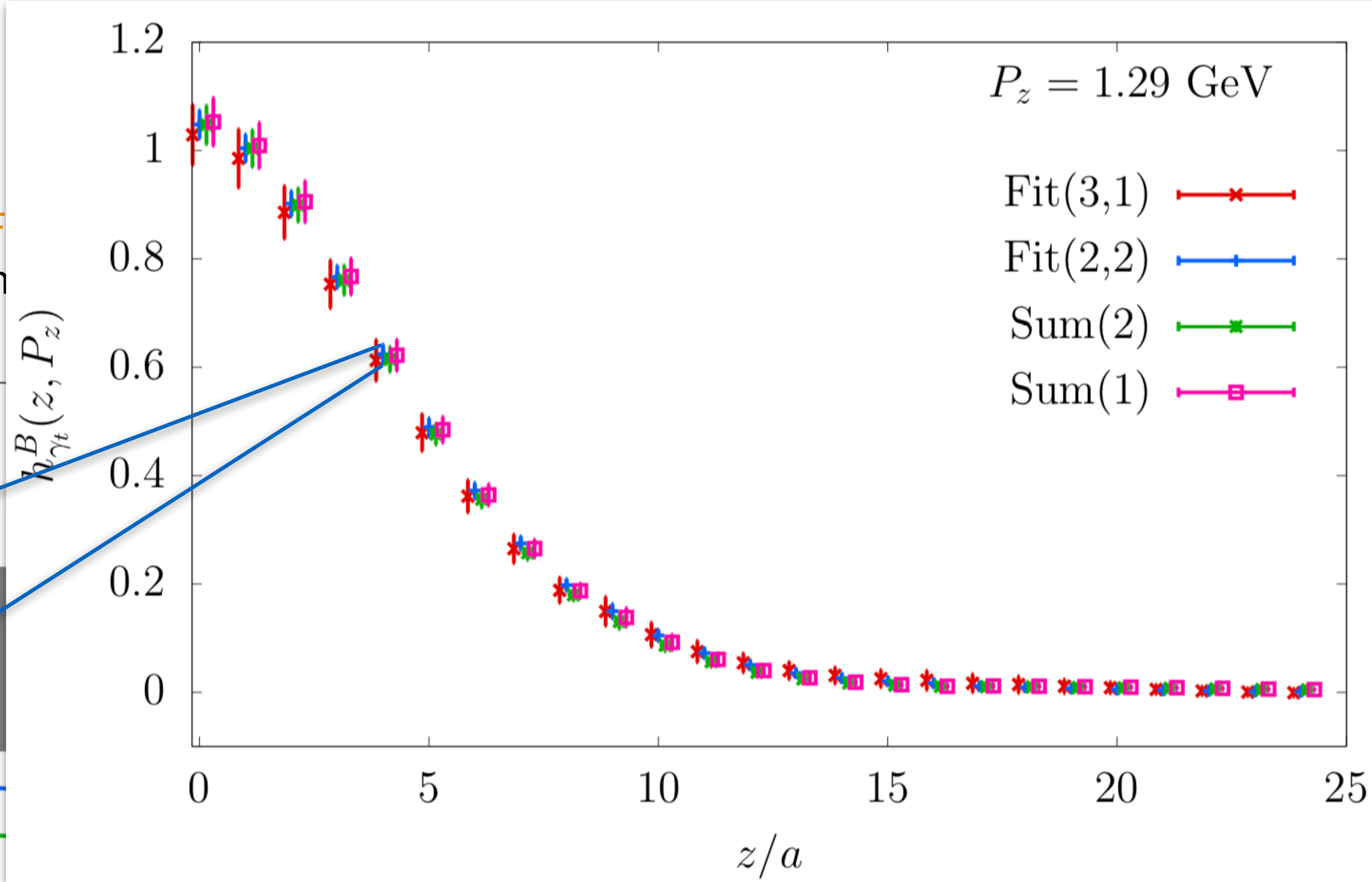

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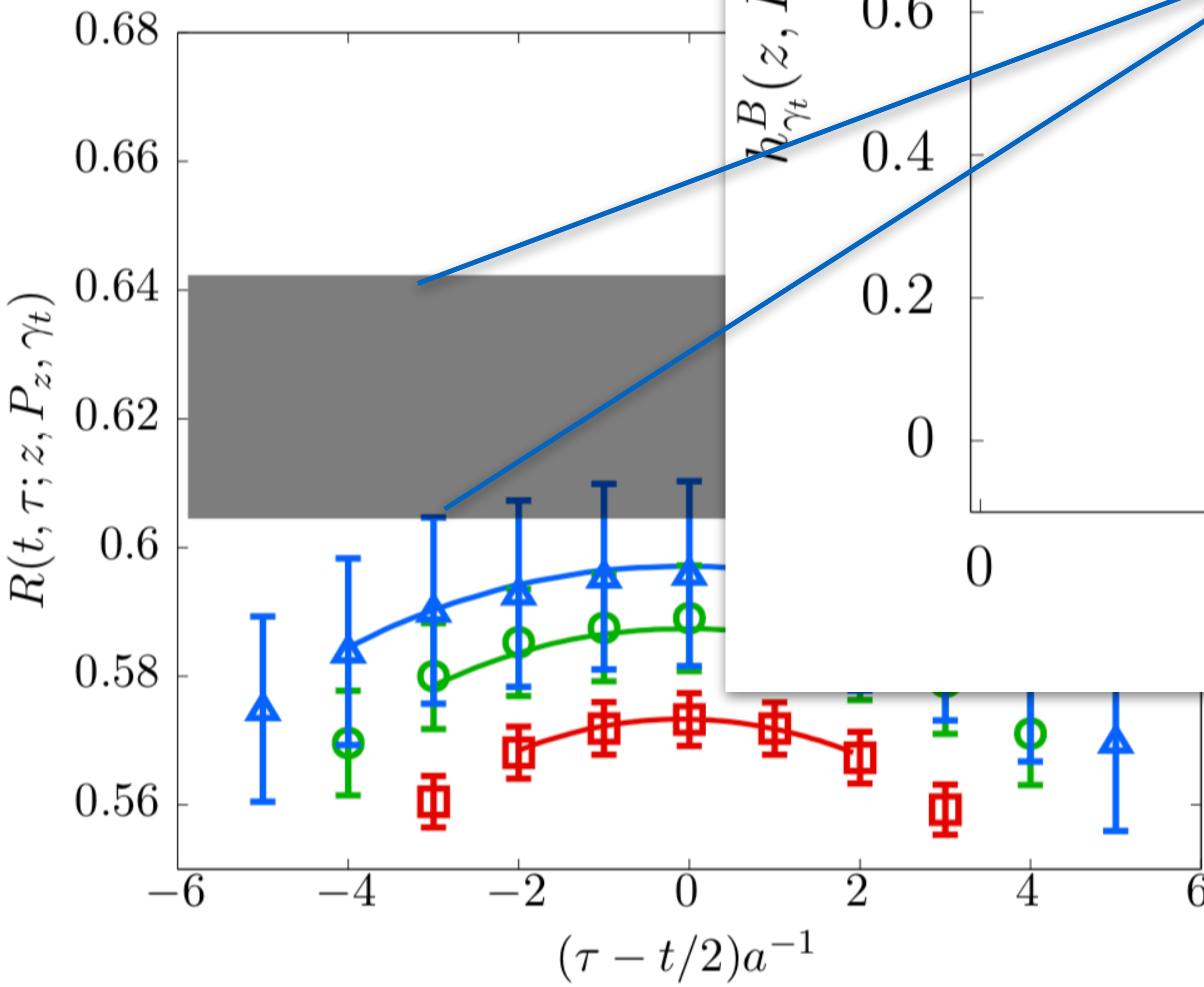

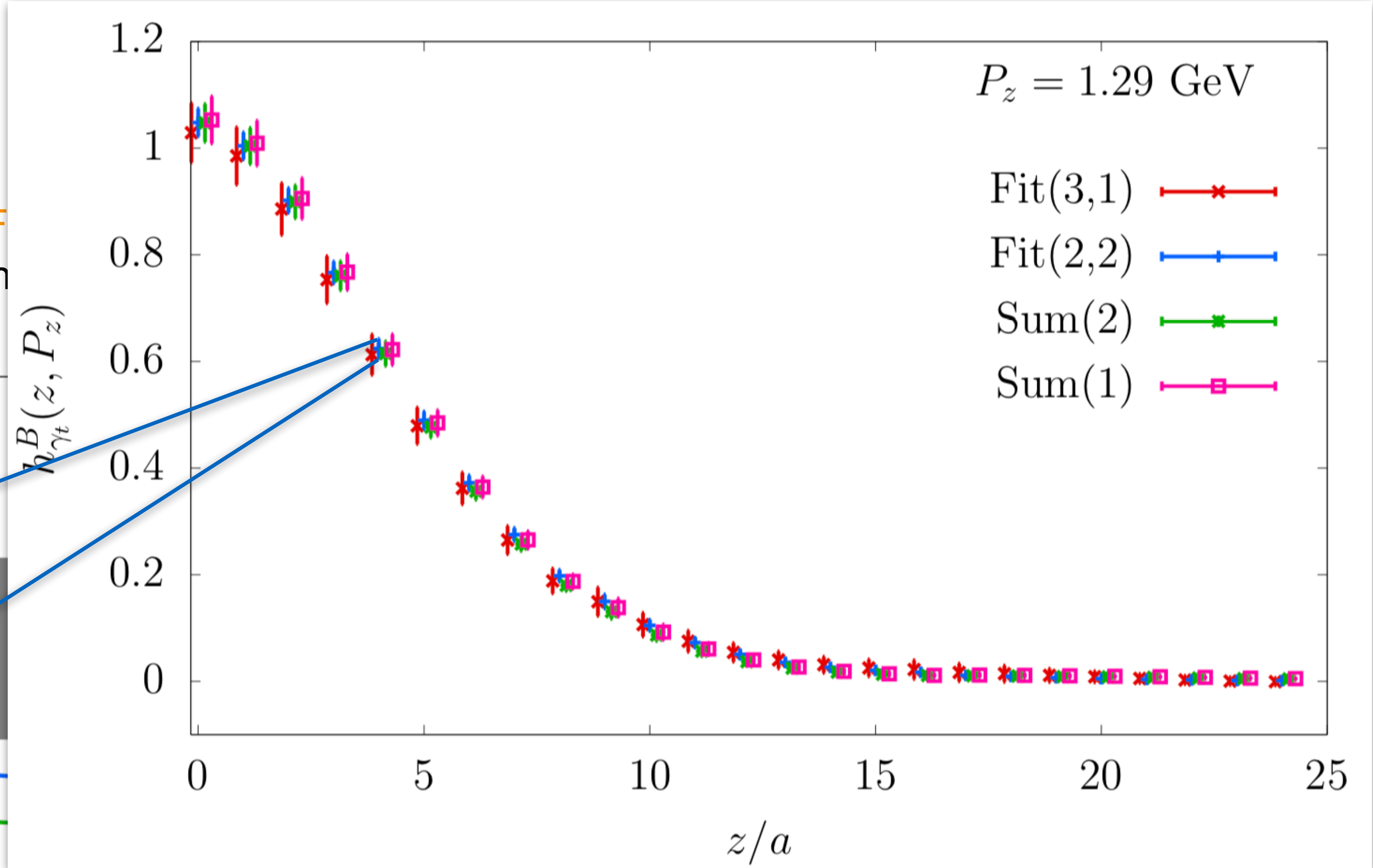

- Require finer lattices (a<0.05 fm) and larger P_z (>2GeV) ?
  Fits indispensable. Check for fit ansatz independence.

$$\frac{\left\langle H(t_{\text{sink}})\, \overline{\psi}(0)\, W(0,z)\, \psi(z)\, H^{\dagger}(0) \right\rangle}{\left\langle H(t_{\text{sink}}) H^{\dagger}(0) \right\rangle}$$

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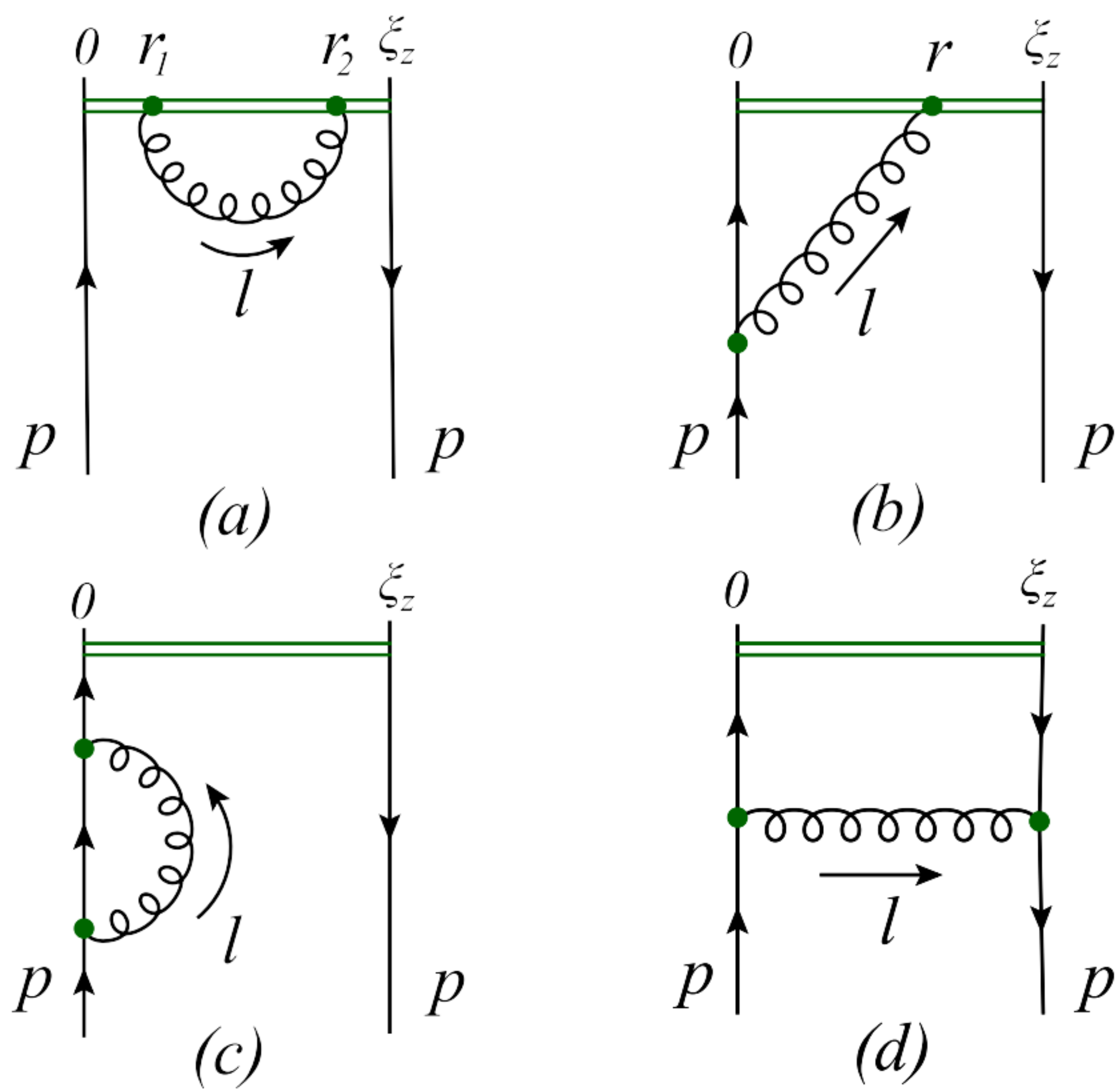
Renormalizability of bi-local quark bilinear

- Real-space quasi-PDF operator can be multiplicatively renormalized with a factor **Z(z)** (Ishikawa et al '17)



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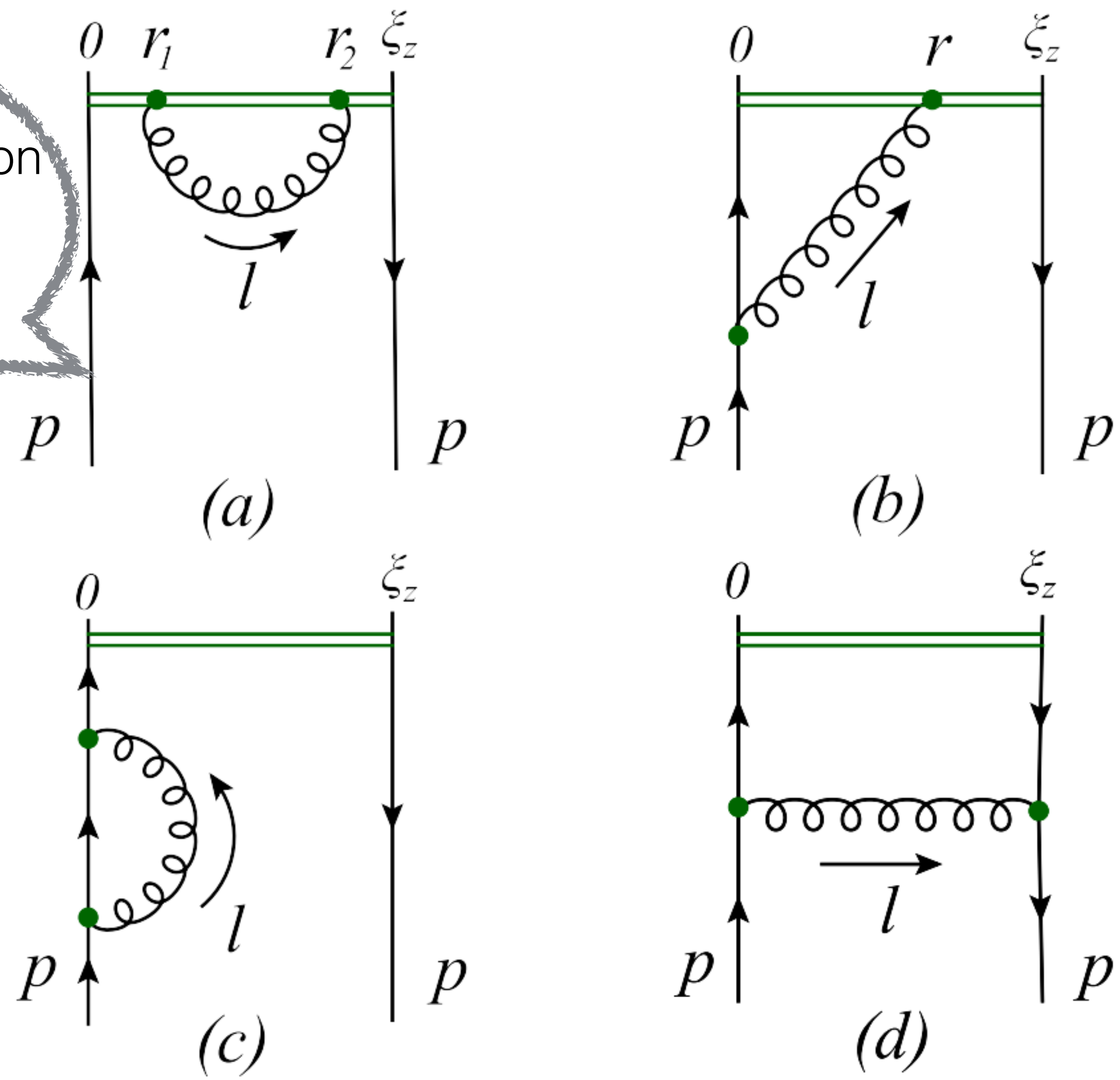
Divergent self-interaction part of Wilson loop:
$\sim e^{-c|z|}$



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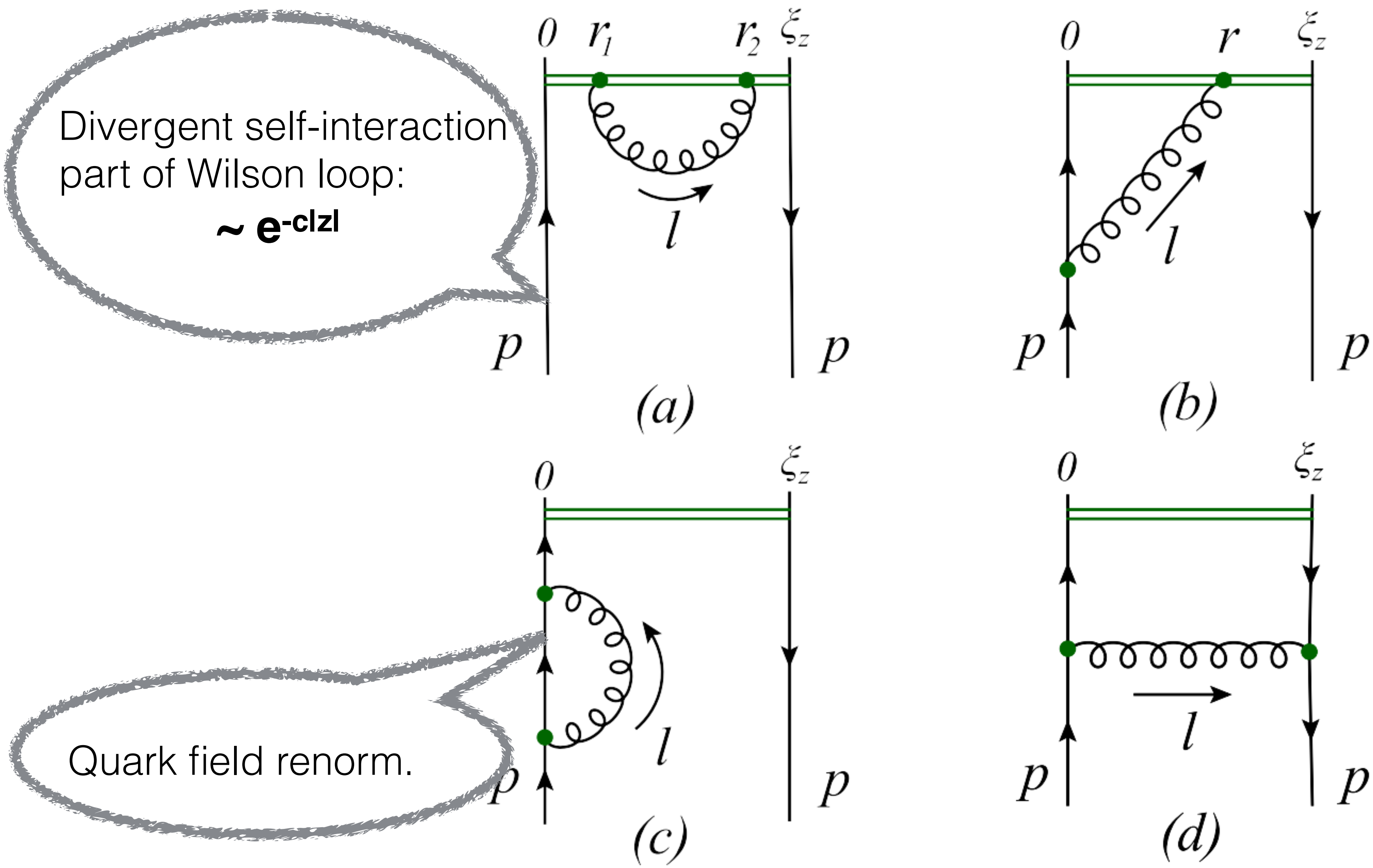

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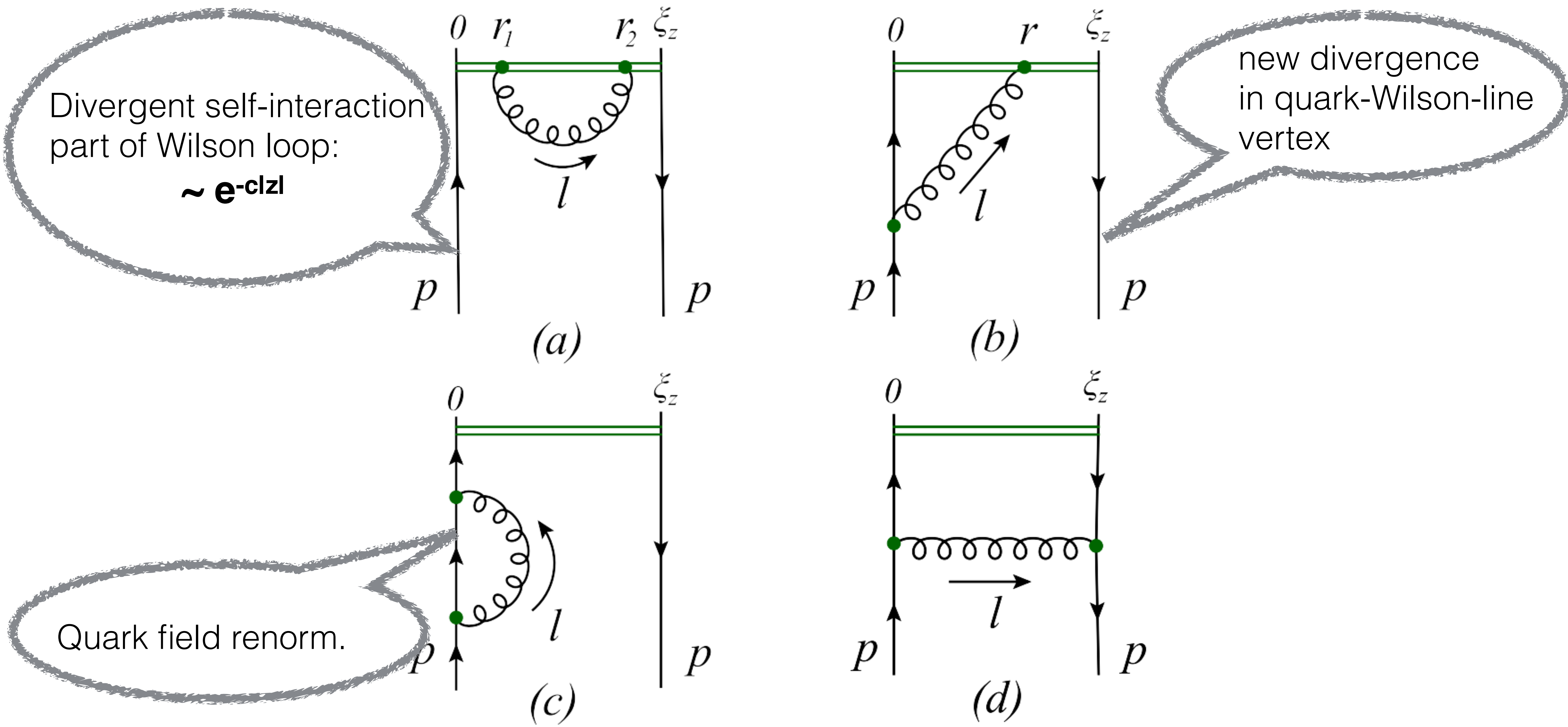

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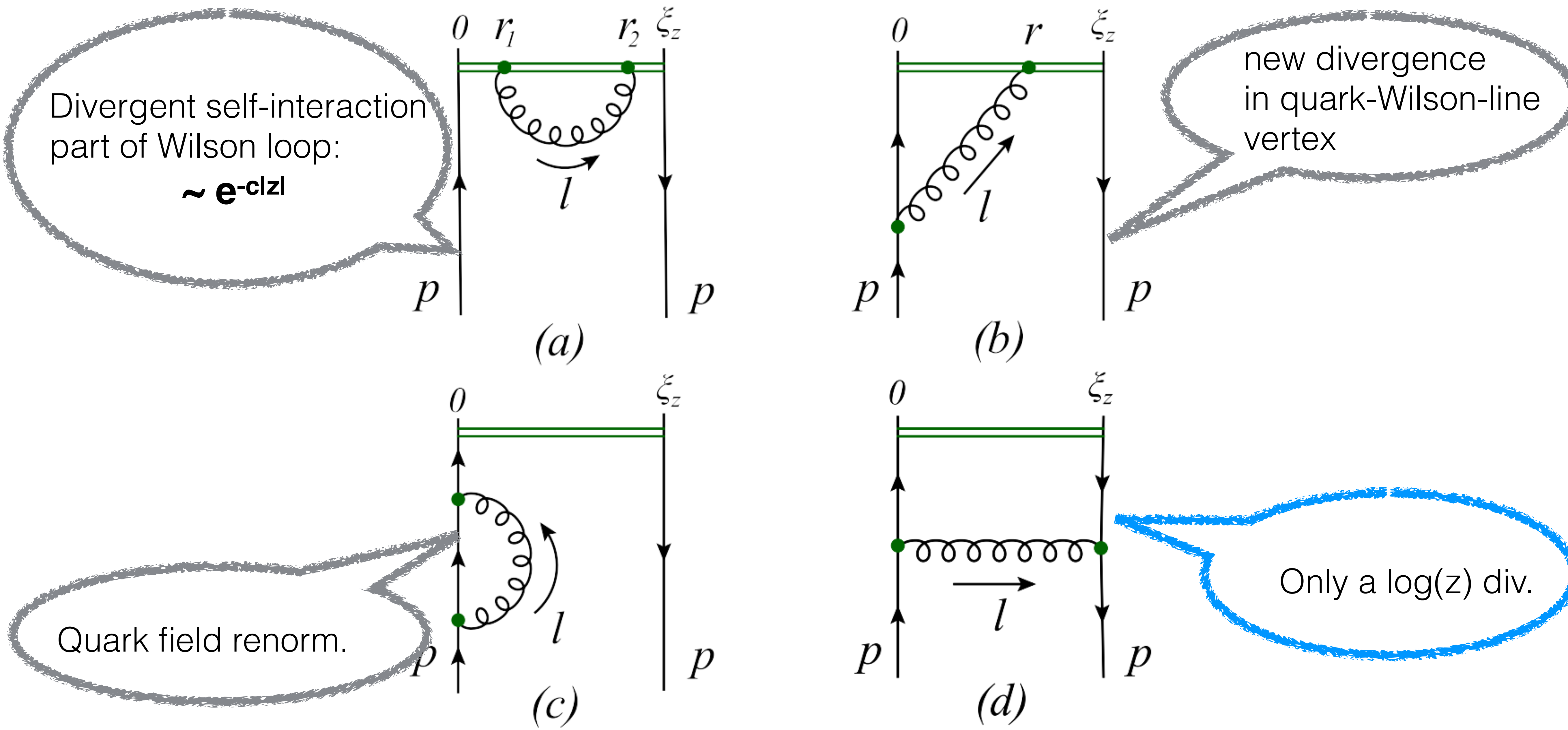

Renormalizability means:

$$h^{R}_{\gamma_t}(z;P_z,P^R) = Z_{\gamma_t\gamma_t}(z;P^R)\cdot h^{b}_{\gamma_t}(z;P_z,a)$$

renormalized hadron qPDF

bare hadron qPDF

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renormalized hadron qPDF

bare hadron qPDF

Non-perturbative Z-factor

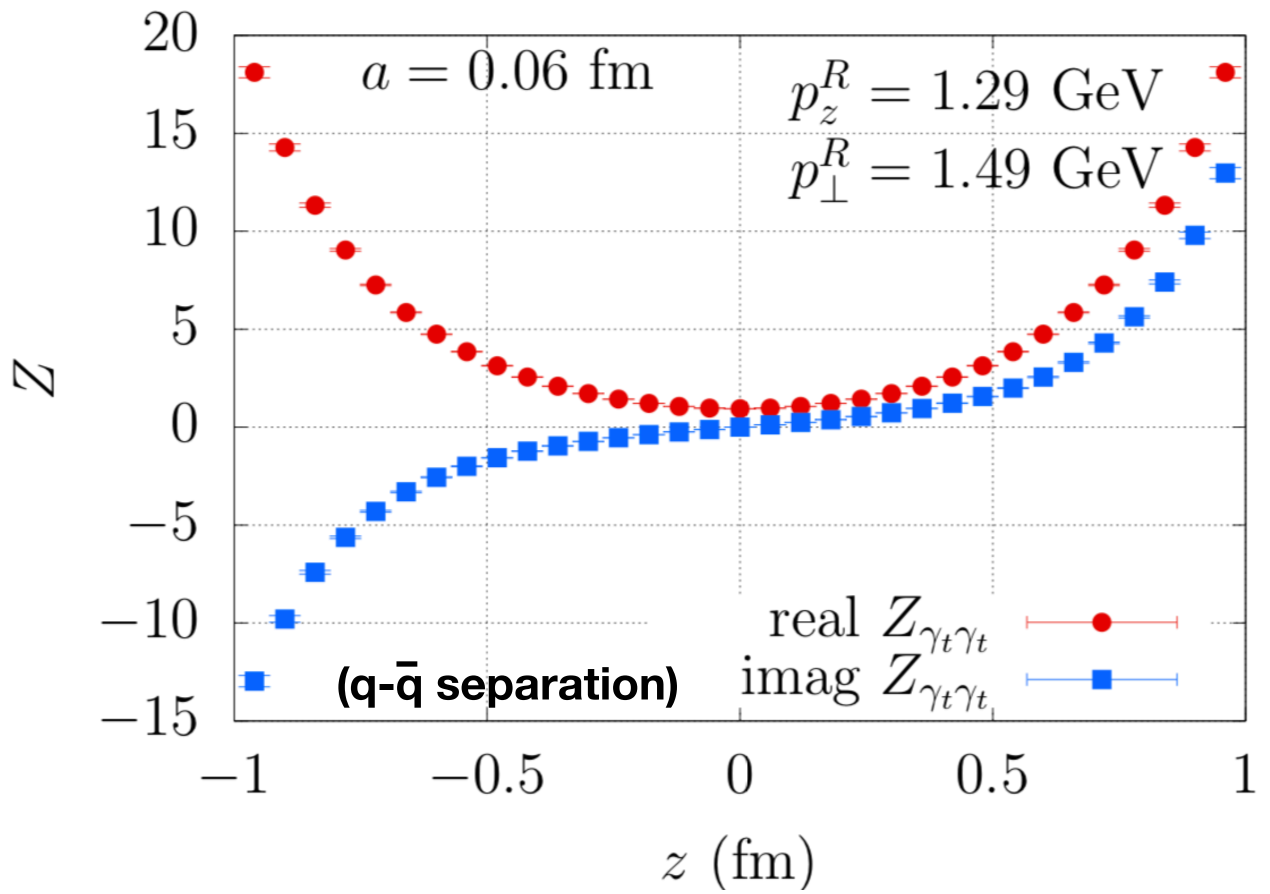
LaMET perturbative framework

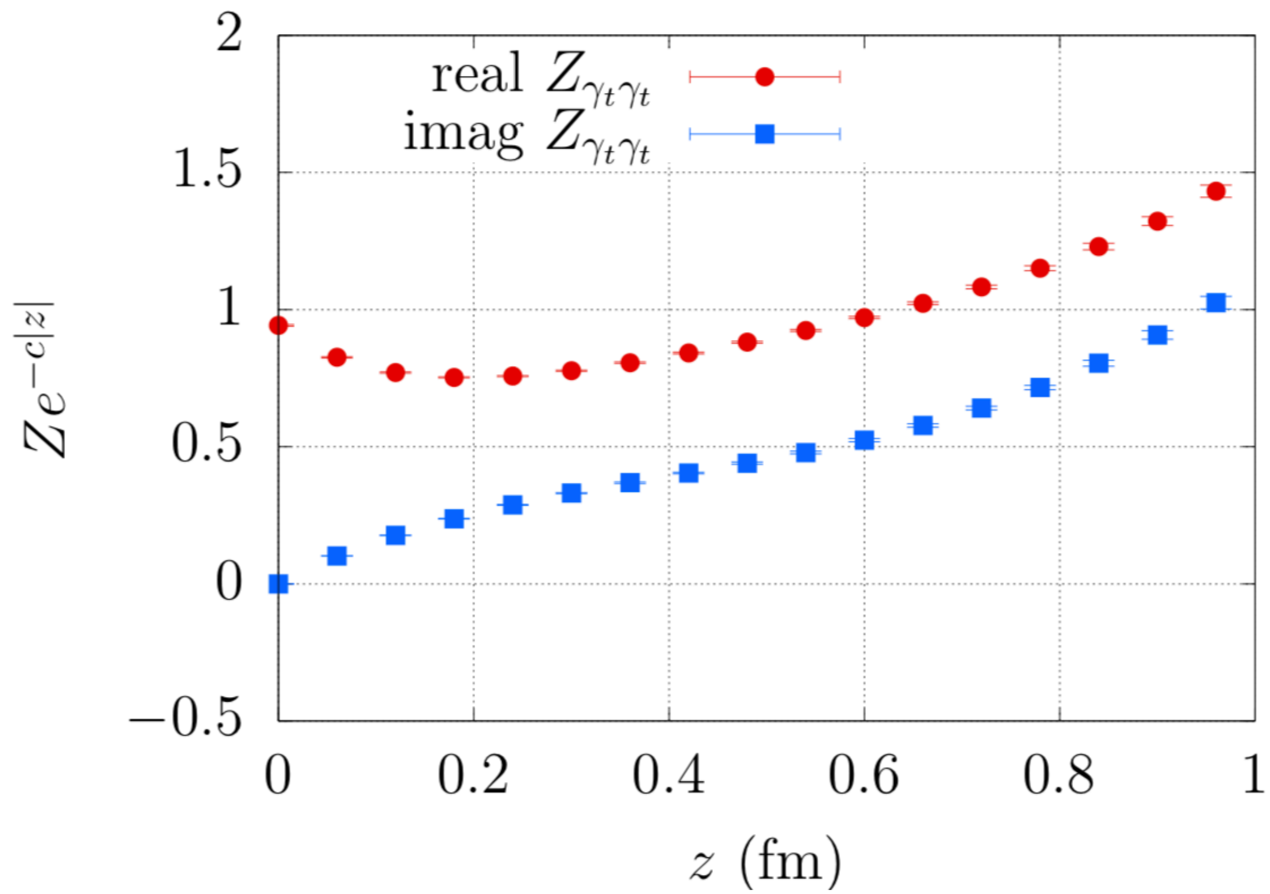
Perturbative renormalization and NPR compatible?

Renormalization in real-space → WL self-energy divergence $e^{-c|z|}$

Issue for 1-loop perturbation theory?

RI-MOM Z-factor for q-PDF → **RI-MOM Z-factor modulo WL divergence**





The residual piece modulo WL divergence is O(1)

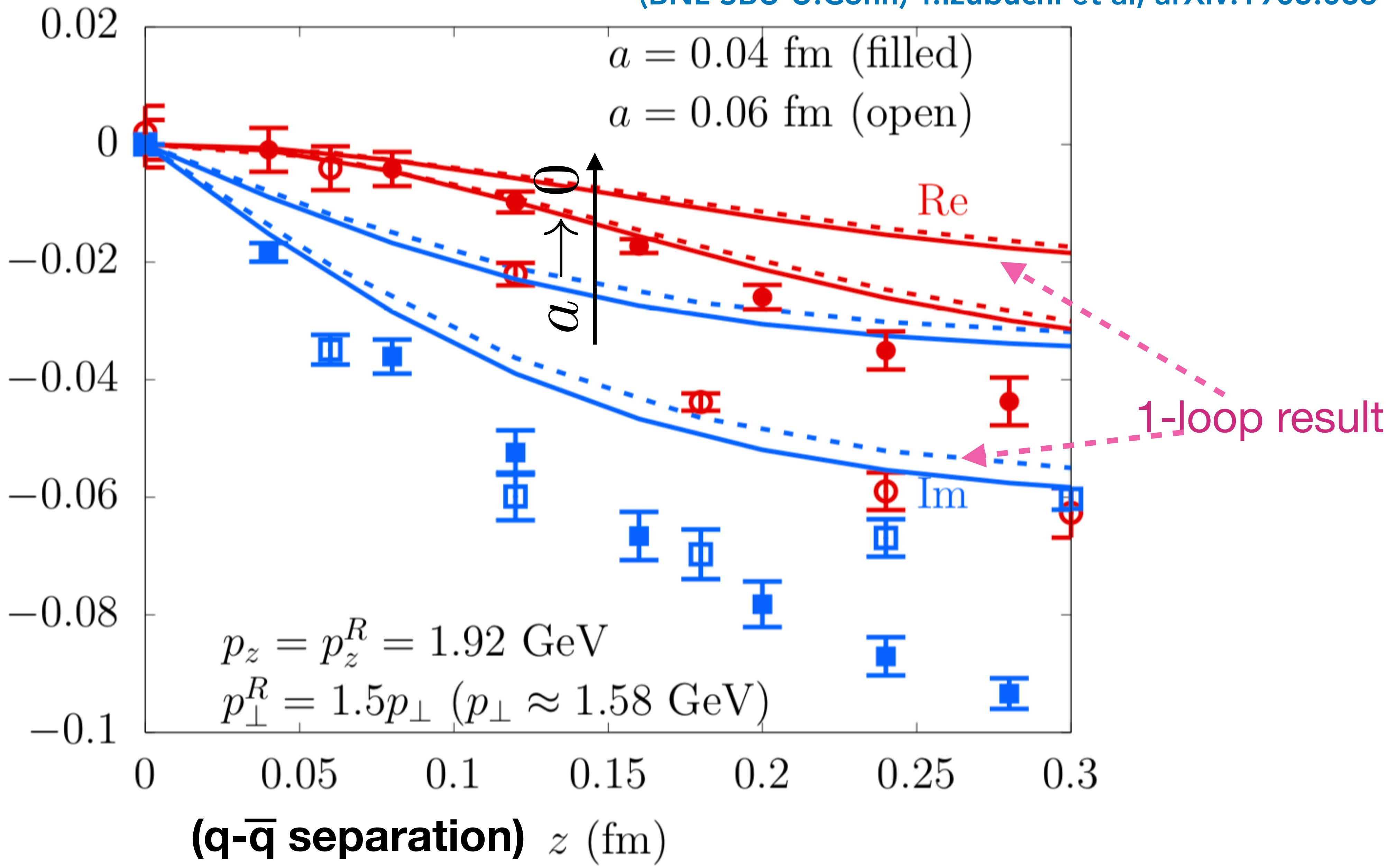
(BNL-SBU-U.Conn) T.Izubuchi et al, arXiv:1905.06349

Can this residual dependence on z be described by perturbation theory?

(BNL-SBU-U.Conn) T.Izubuchi et al, arXiv:1905.06349

“Z-factor modulo WL divergence”

$$\frac{Z(z,p^R) - Z(z,p)}{Z(z,p)}$$



➡ Only qualitative agreement between actual lattice data and 1-loop

➡ Both continuum limit as well as inclusion of 2-loop contributions could be important.

$$\frac{\left\langle H(t_{\text{sink}})\ \bar{\psi}(0)\ \overset{W(0,z)}{\longleftrightarrow}\ \psi(z)\ H^{\dagger}(0)\right\rangle}{\left\langle H(t_{\text{sink}})H^{\dagger}(0)\right\rangle}$$

$t_{\text{sink}} \to \infty$

$$h(z, P_z) = \left\langle P_z \middle|\ \bar{\psi}(0)\ \overset{W(0,z)}{\longleftrightarrow}\ \psi(z)\ \middle| P_z \right\rangle$$

Renormalize

$$h^R(z, P_z, P^R)$$

Fourier z to conjugate x at fixed P_z

$$\tilde{q}(x, P_z, P^R)$$

LaMET

$$f(x, \mu^2)$$

Reconstructing PDF from matrix elements

quasi-PDF as an example

Advantage: Model independent (?)
Disadvantage: No control over range of z

Renormalized qPDF Matrix element
$h^R(z, P_z, P^R)$

Fourier → Interpolate z and extrapolate to larger z

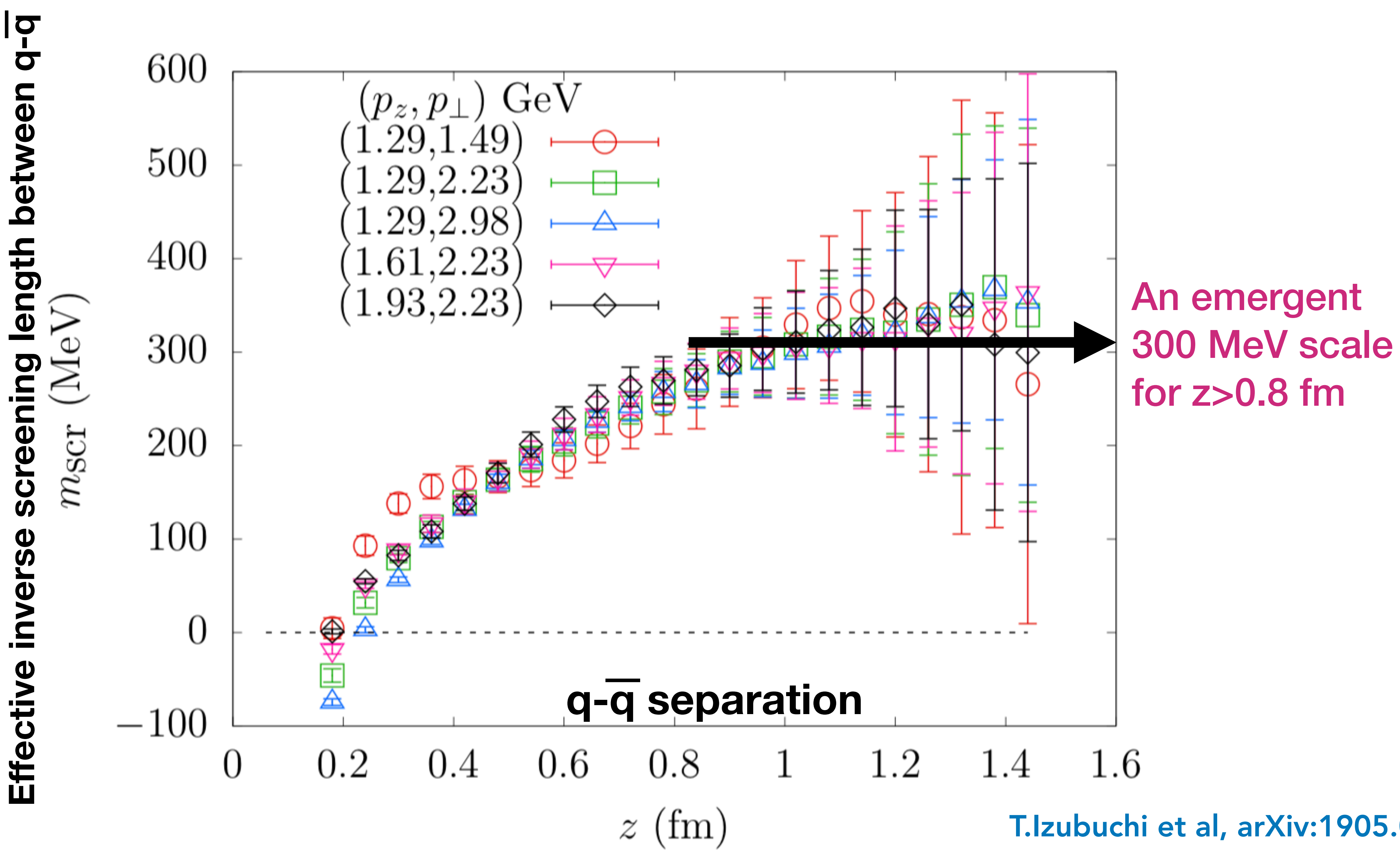
Quasi PDF
$\tilde{q}(x, P_z, P^R)$

Inverse 1-loop matching →

PDF
$f(x, \mu)$

Is it ok to use quasi-PDF at all values of z in matching?

Model NP effects as screening: $\langle q(p)| \bullet \overset{z}{—} \bullet |q(p)\rangle_{ren} \sim e^{-i\omega z} e^{-m_{\rm scr} z}$



T.Izubuchi et al, arXiv:1905.06349

Reconstructing PDF from matrix elements

An alternate approach

Disadvantage: Model dependent **(neural network?)**

Advantage: Control over range of z

Renormalized qPDF Matrix element

$h^R(z, P_z, P^R)$

← **Fit over range of z**

Model qPDF Matrix element

$h(z, P_z, P^R; a, b)$

← **Inv Fourier**

Quasi PDF

$\tilde{q}(x, P_z, P^R; a, b)$

← **1-loop matching**

Pheno inspired PDF

$$f(x, \mu; a, b) = Ax^a(1-x)^b$$

$$f(x, \mu; a, b) = Ax^a(1-x)^b$$

1-loop matching

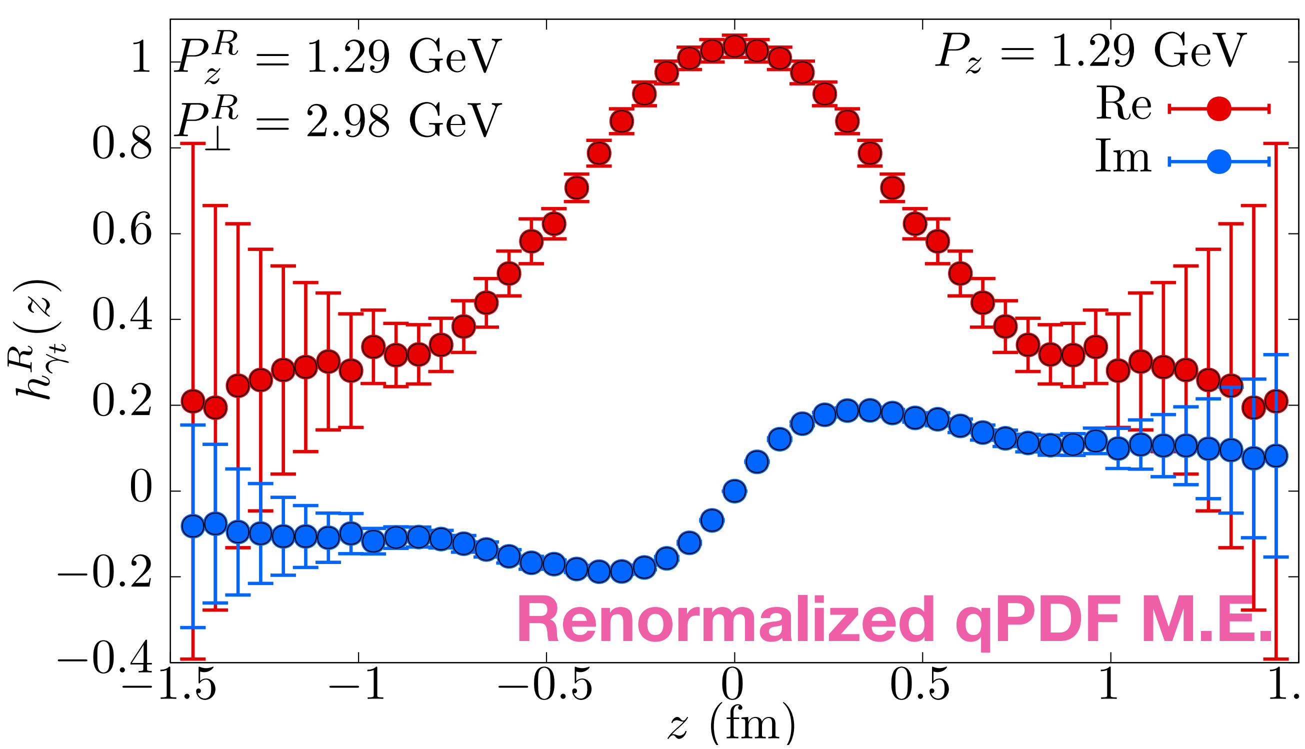
$$\tilde{q}(x, P_z, P^R; a, b)$$

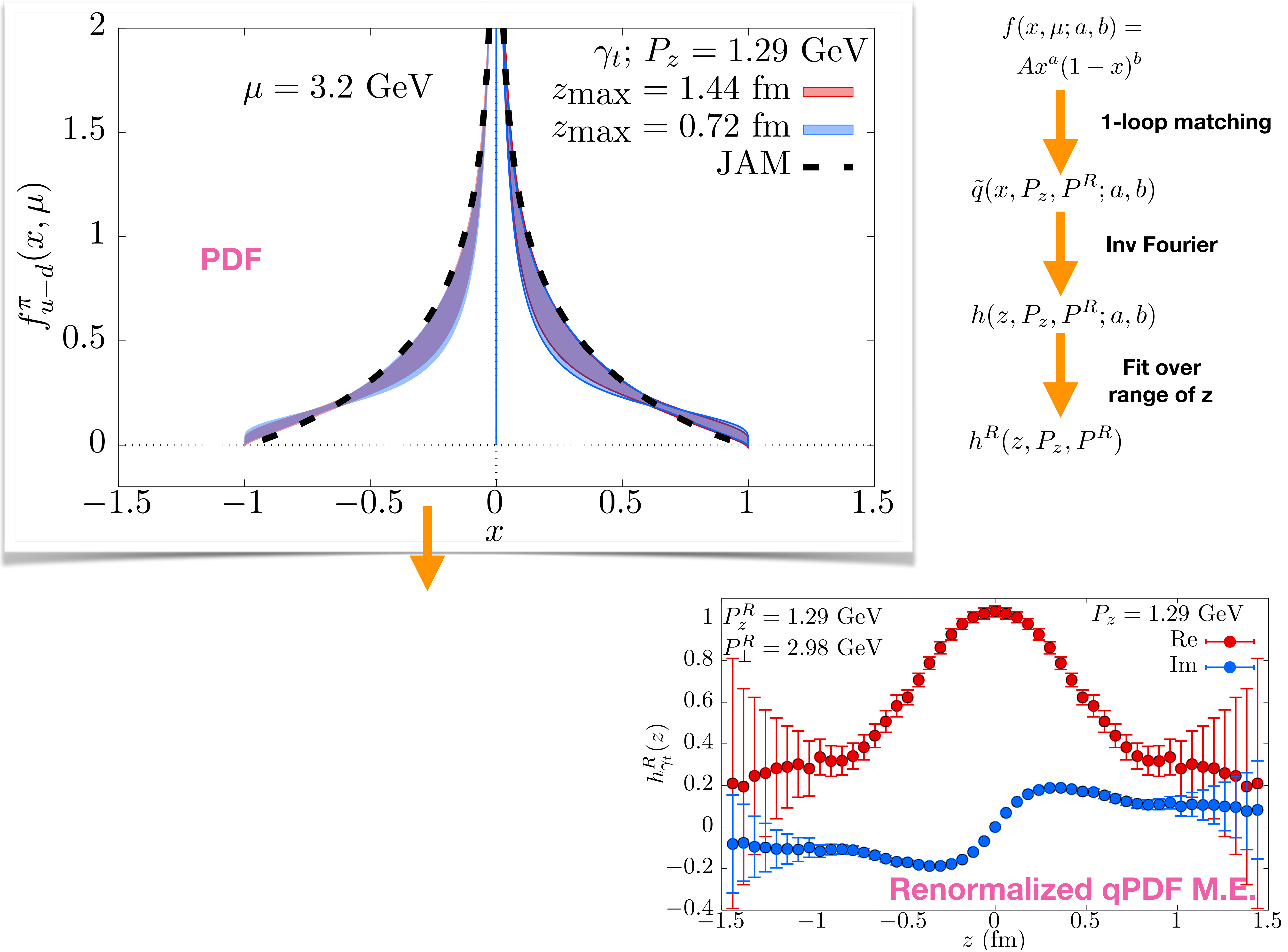
Inv Fourier

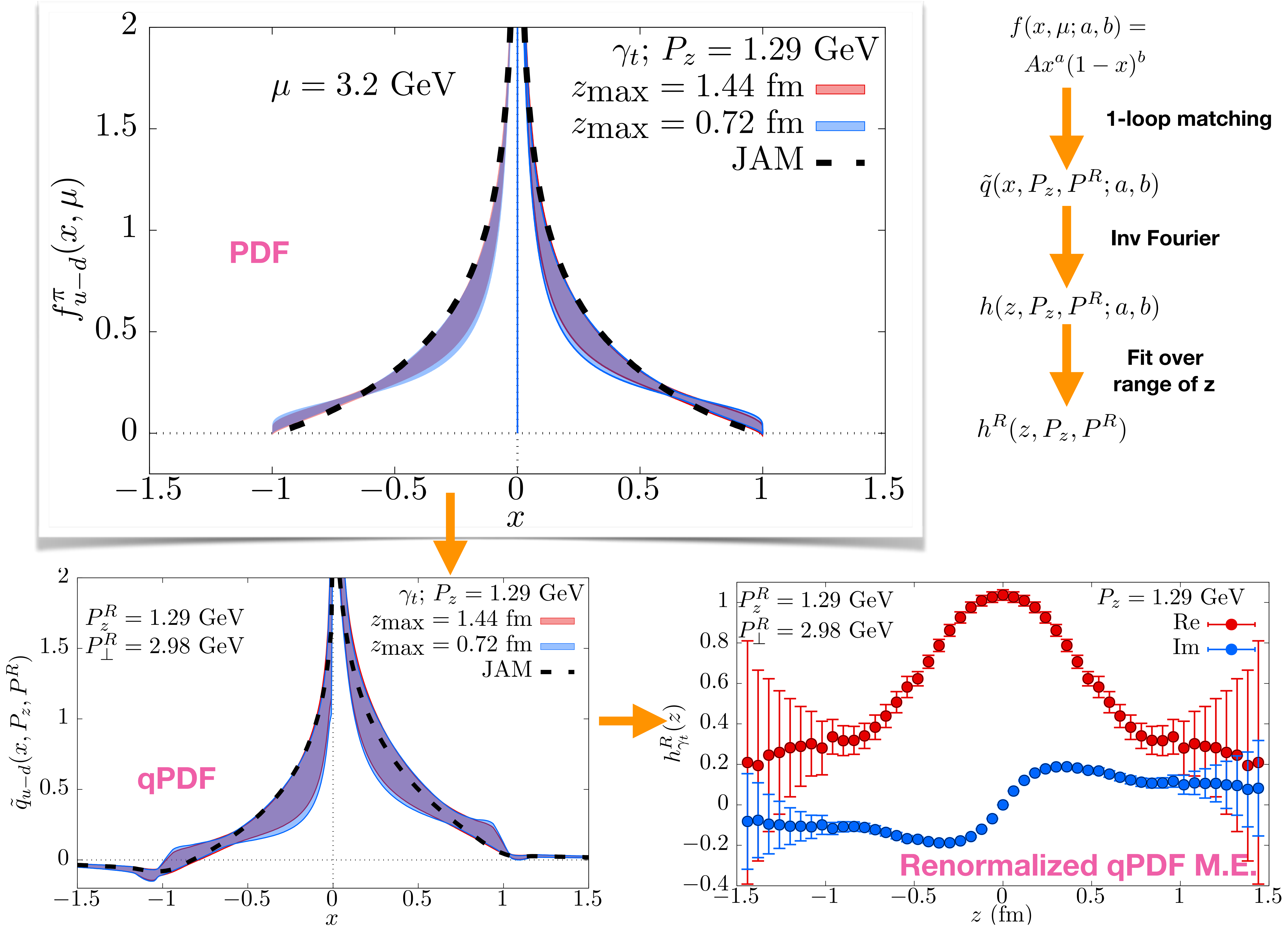
$$h(z, P_z, P^R; a, b)$$

Fit over range of z

$$h^R(z, P_z, P^R)$$




$\gamma_t;\ P_z = 1.29$ GeV
$\mu = 3.2$ GeV
$z_{\max} = 1.44$ fm
$z_{\max} = 0.72$ fm
JAM
PDF
$f^{\pi}_{u-d}(x,\mu)$
x
$f(x,\mu;a,b) =$
$Ax^a(1-x)^b$
1-loop matching
$\tilde{q}(x, P_z, P^R; a, b)$
Inv Fourier
$h(z, P_z, P^R; a, b)$
Fit over range of z
$h^R(z, P_z, P^R)$
$P_z^R = 1.29$ GeV
$P_\perp^R = 2.98$ GeV
$P_z = 1.29$ GeV
Re
Im
$h^R_{\gamma_t}(z)$
z (fm)
Renormalized qPDF M.E.



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$z_{\max} = 1.44$ fm
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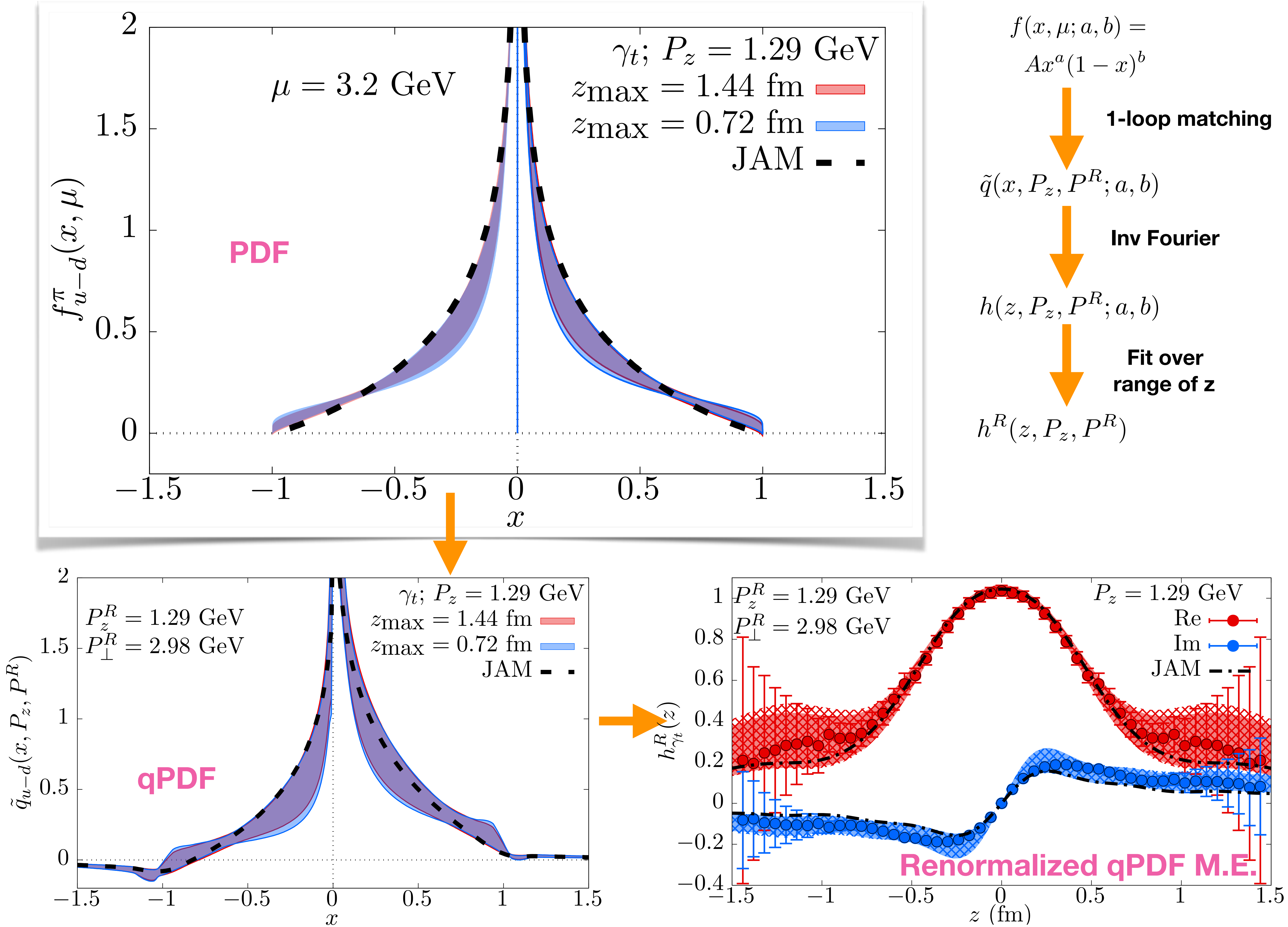
$f(x,\mu;a,b) = Ax^a(1-x)^b$
1-loop matching
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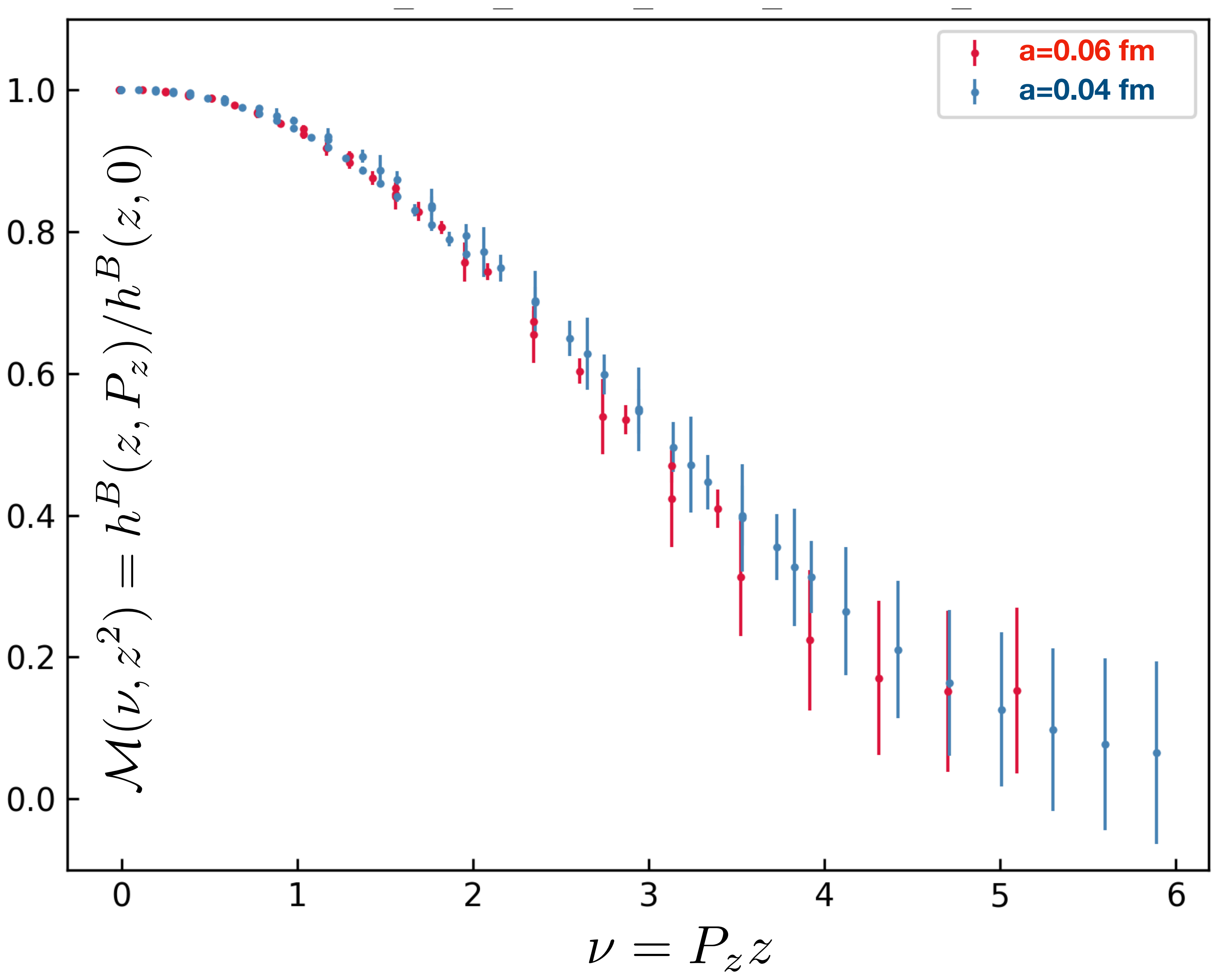
Renormalized qPDF M.E.

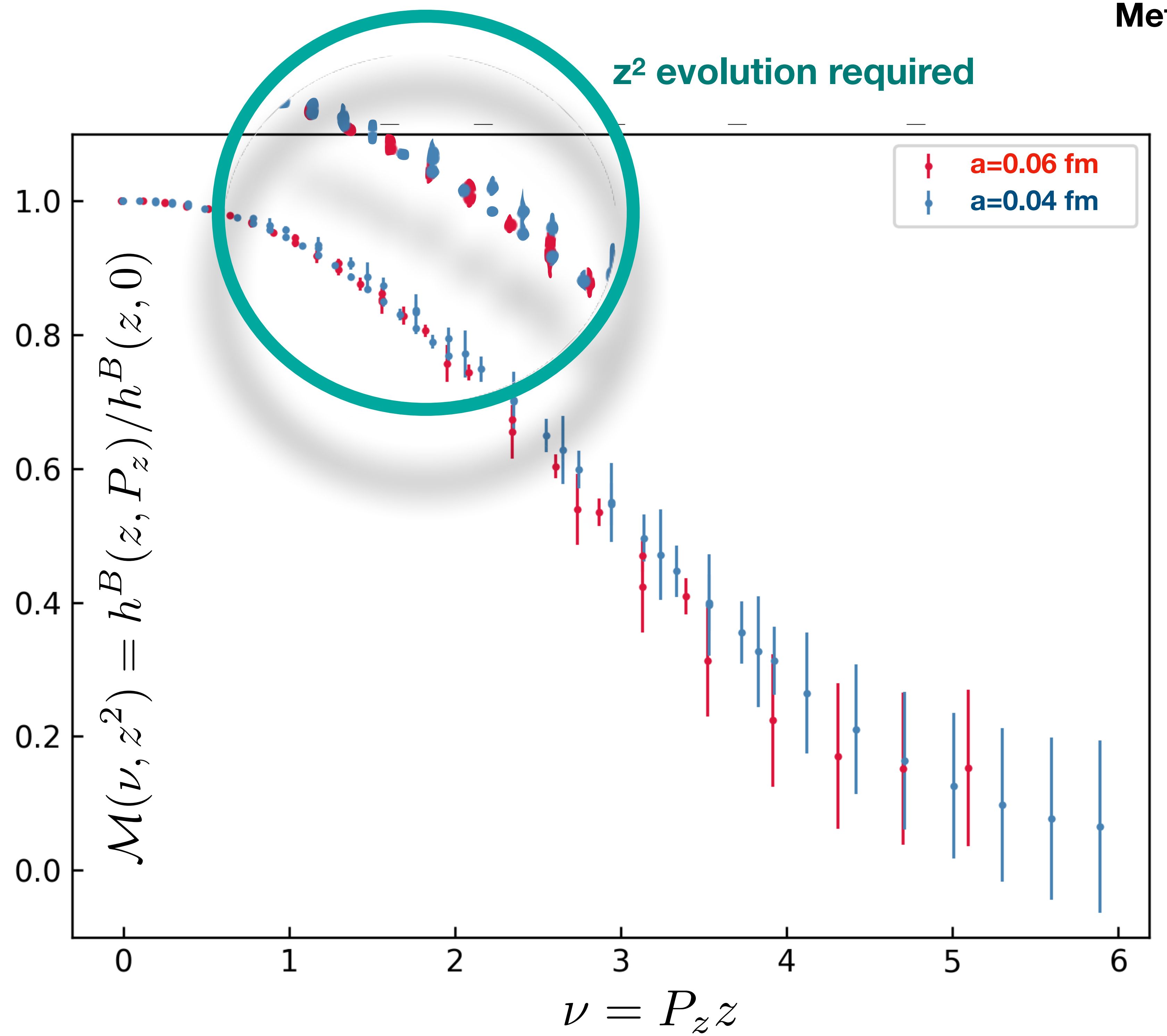
Analyzing reduced Ioffe-Time distribution

Method: J. Karpie et al, JHEP 1811 (2018) 178



Analyzing reduced Ioffe-Time distribution

Method: J. Karpie et al, JHEP 1811 (2018) 178



Analyzing reduced Ioffe-Time distribution

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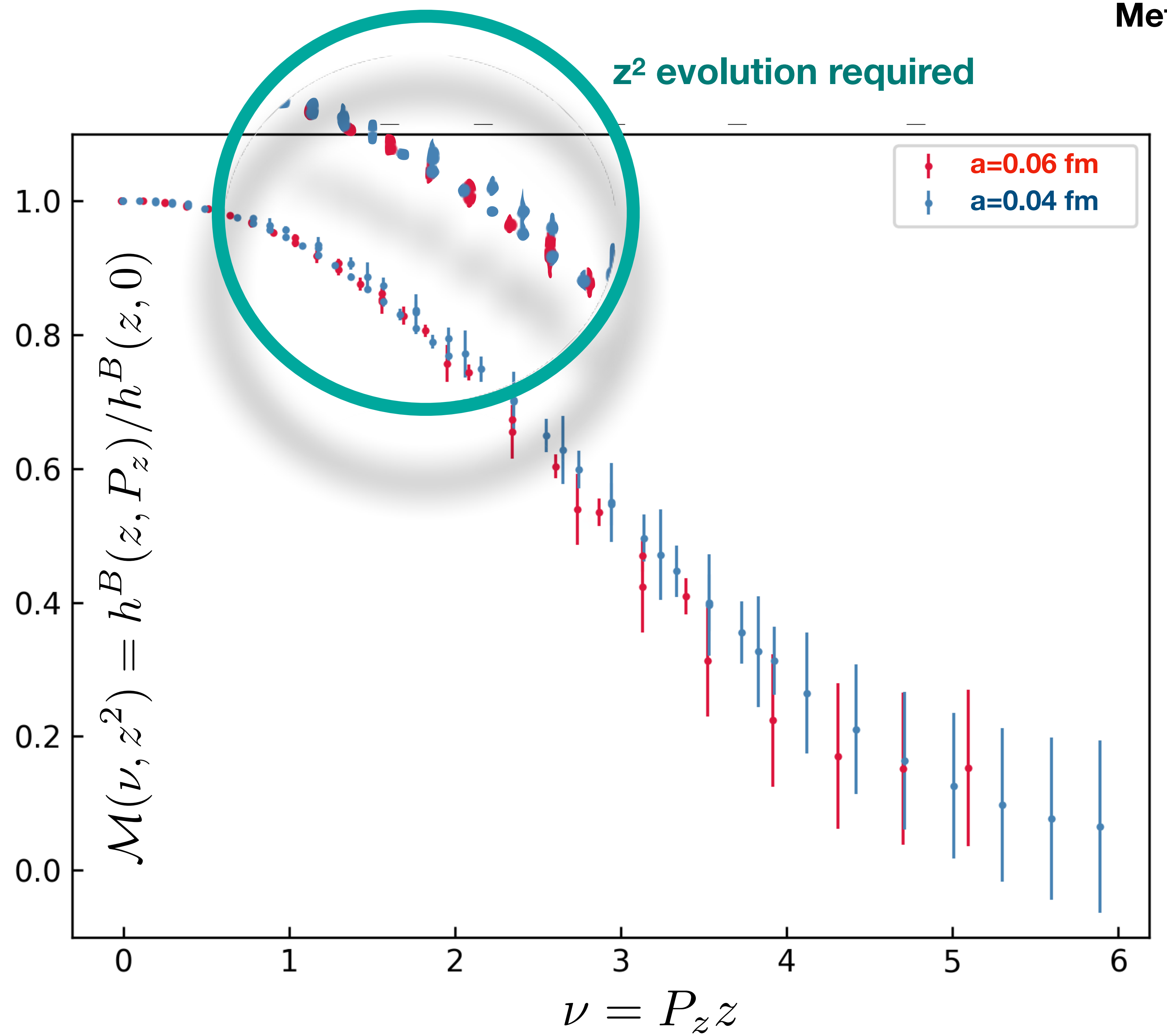


✷ **Fit**

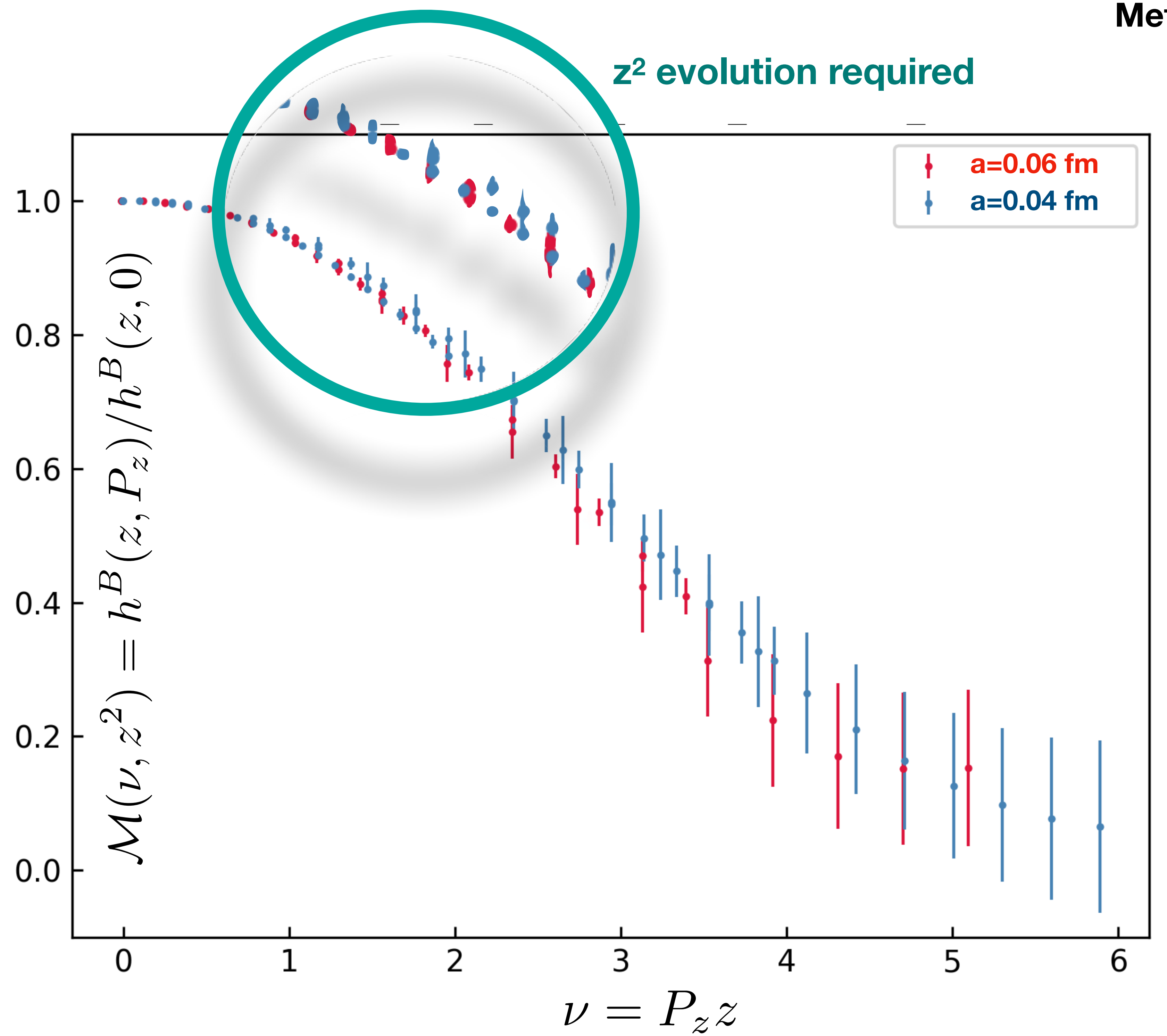
$$\mathcal{M}(\nu, z^2) = 1 + \nu^2 \mathcal{M}^{(2)}(z^2) + \nu^4 \mathcal{M}^{(4)}(z^2) + \dots$$

✷ **Without model PDF, determine**

$$\langle x^{2n} \rangle = \frac{\mathcal{M}^{(2n)}(z^2)}{c_{2n}(z^2, \mu)}$$

Analyzing reduced Ioffe-Time distribution

Method: J. Karpie et al, JHEP 1811 (2018) 178



Fit

$$\mathcal{M}(\nu, z^2) = 1 + \nu^2 \mathcal{M}^{(2)}(z^2) + \nu^4 \mathcal{M}^{(4)}(z^2) + \ldots$$

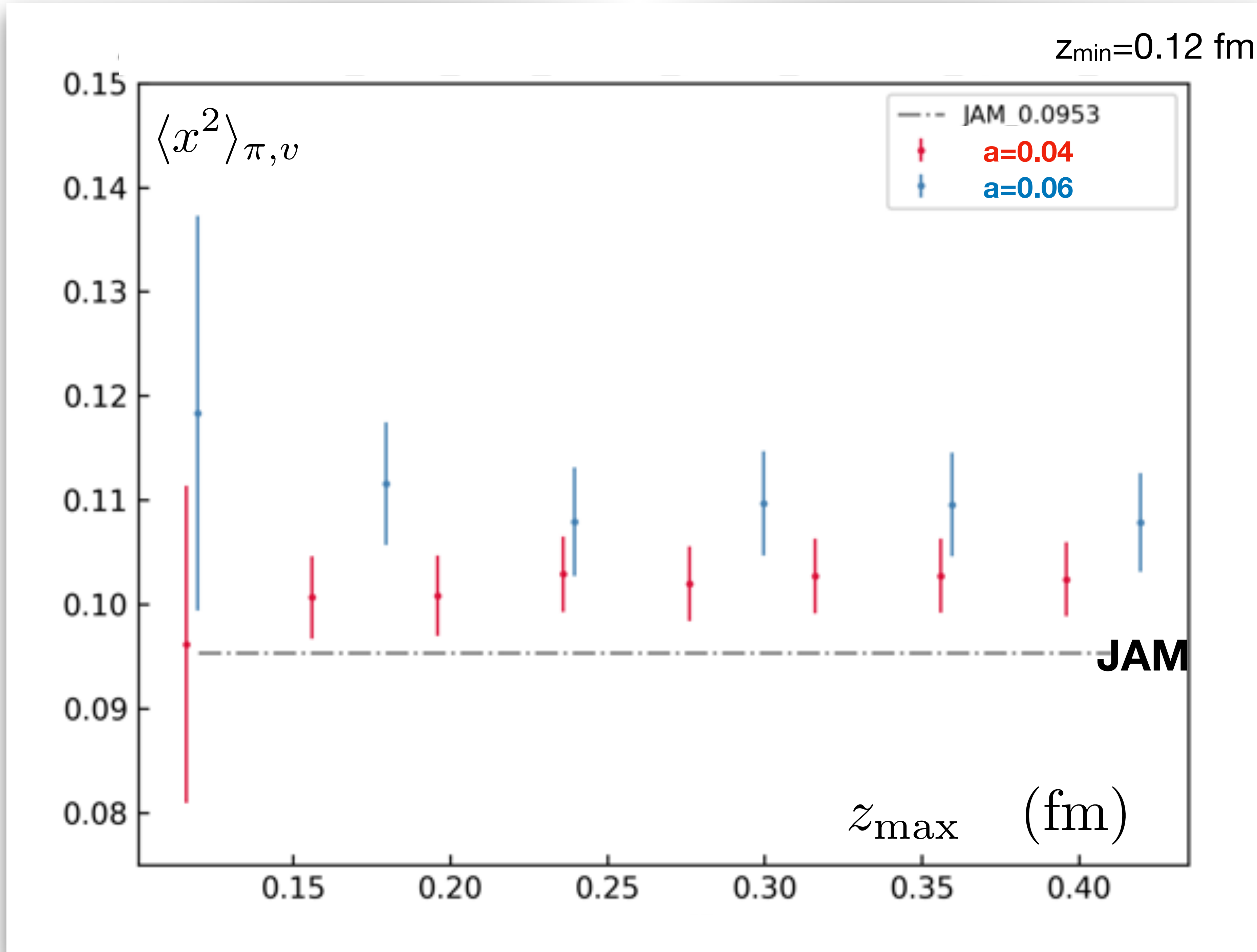
Without model PDF, determine

$$\langle x^{2n} \rangle = \frac{\mathcal{M}^{(2n)}(z^2)}{c_{2n}(z^2, \mu)}$$

A combined fit to data from $z \in [z_{\min}, z_{\max}]$

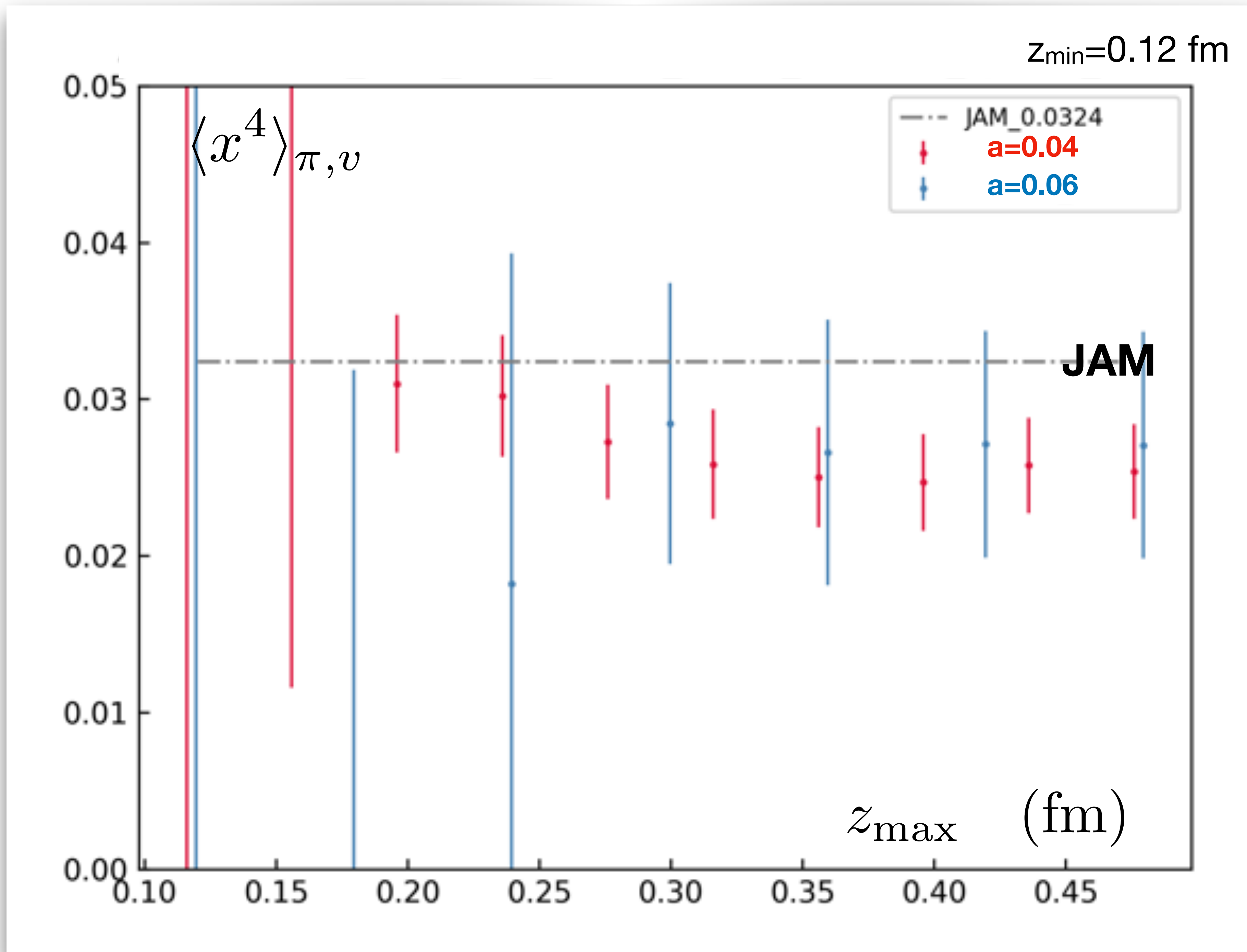
$$\mathcal{M}(\nu, z^2) = 1 + \nu^2 c_2(z^2, \mu) \langle x^2 \rangle + \nu^4 c_4(z^2, \mu) \langle x^4 \rangle + \ldots$$

$$\langle x^2 \rangle_{\pi,v}$$



Harder $(1-x)^b$ approach over a large-range of x~1? Then b↑ ⇒ $\langle x^n \rangle$↓

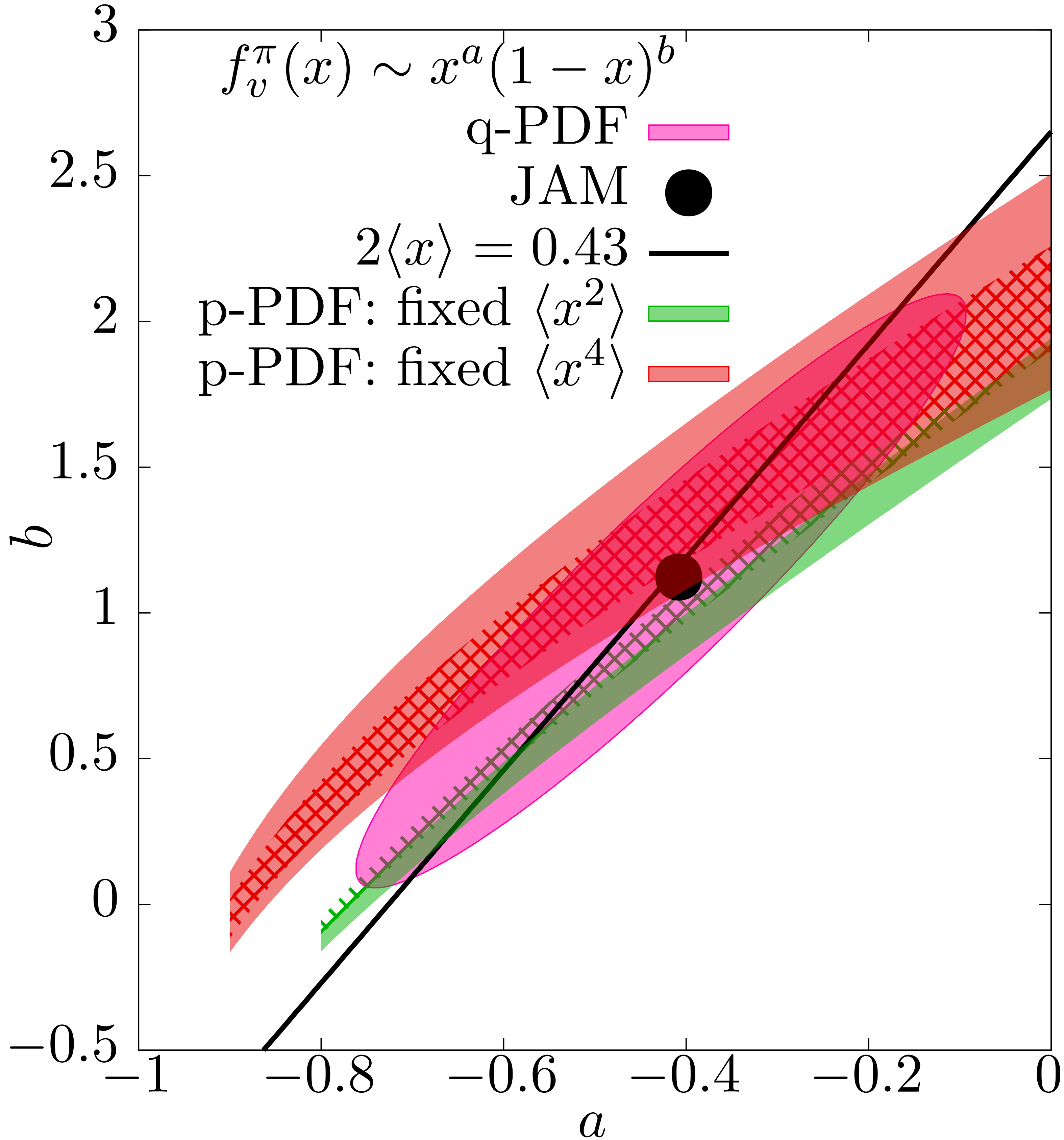
$$\langle x^4 \rangle_{\pi,v}$$



Not enough "curvature" in data to get fourth-derivative for z<0.25 fm

The region with signal gets pushed to larger-z for larger-n for <x^n>

Assuming <u>simple</u> PDF model: Constraints on small-x and large-x asymptotics



Question: Given a model

$$f_v(x) = \mathcal{N} x^a (1-x)^b$$

what values of (a,b) allowed by lattice data?

- **Data from 0.04,0.06 fm ensembles**
- **From quasi- and pseudo-PDF — matching at 1-loop and therefore different checks**
- **If a~-0.5 $\Rightarrow$ b~1**
- **Another take: the model for pion is *not* the simple ansatz, but *with* b>2 asymptotic behavior**

Advancements in computing gluon PDF

Choices of renormalizable gluon qPDFs h_0, h_1, and h_2 S. Zhao et al

$$h_1(z, P_z) = \langle P_z | F_t^{\mu}(z) \, W_{\mathrm{adj}}(0, z) \, F_{\mu t}(z) | P_z \rangle$$

Challenges

- Mixing with quark flavor singlets
- Statistics heavy. Improvement methods?

Not much has been done yet!

Summary and outlook

- **Theoretical framework of LaMET is now established.**
  Issues that remain are about practical implementation on lattice

Renormalization	Yes
$P_z \gg \Lambda_{\rm QCD}$	Yes
Target mass corrections	Yes
$P_z \ll 1/a$	~Ok
Excited state analysis:	$T_{\rm sink}$ <1 fm. Fits required
Continuum limit:	Not studied
Finite volume effects:	Not studied
Analysis methods:	No consensus
Matching:	1-loop. Convergence?

Things to look out for:

- Combined analysis of quasi-, pseudo- and other lattice cross-sections for PDF
- More work on GPDs and gluon PDFs — Still in very early stages
- Effects on global analysis of PDF — Sensitivity to determination of qPDF at even one value of x. **(T.J.Hobbs et al, 1904.00022)**
- Complementing experimental determination of PDF is ok. But, we also need to use lattice to *actually understand* why PDFs are the way they are!

Back-up slides

Renormalization conditions

Renormalization conditions

The renormalizability means:

$$h^R_{\gamma_t}(z; P_z, P^R) = Z_{\gamma_t\gamma_t}(z; P^R) \cdot h^b_{\gamma_t}(z; P_z, a)$$

renormalized hadron qPDF

bare hadron qPDF

Renormalization conditions

The renormalizability means:

$$h^R_{\gamma_t}(z;P_z,P^R)=Z_{\gamma_t\gamma_t}(z;P^R)\cdot h^b_{\gamma_t}(z;P_z,a)$$

renormalized hadron qPDF

bare hadron qPDF

Renormalization scheme independent conditions:

$$Z_{\gamma_t\gamma_t}(z;P^R)\cdot h^b_{quark}(z;P=P^R,a)=h_{free}(z;P^R)$$

barequark qPDF in full QCD

Tree level value

Renormalization conditions

The renormalizability means:

$$h^R_{\gamma_t}(z; P_z, P^R) = Z_{\gamma_t\gamma_t}(z; P^R) \cdot h^b_{\gamma_t}(z; P_z, a)$$

renormalized hadron qPDF

bare hadron qPDF

Renormalization scheme independent conditions:

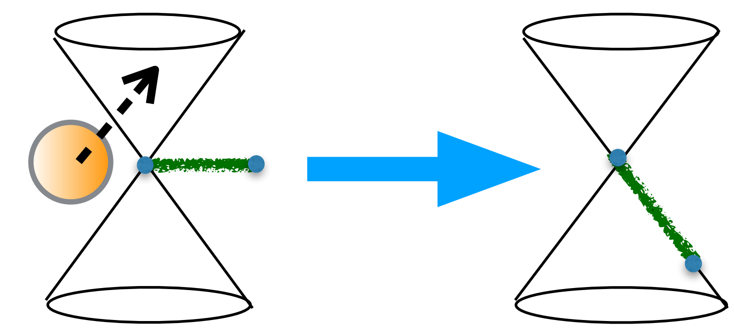
$$Z_{\gamma_t\gamma_t}(z; P^R) \cdot h^b_{quark}(z; P = P^R, a) = h_{free}(z; P^R)$$

barequark qPDF in full QCD

Tree level value

Implementable in lattice as well as pert. theory with off shell quark with $P^2 > 0$

From lattice correlator to PDF



$$\frac{\langle H(t_{\rm sink})\, \overline{\psi}(0)\; W(0,z)\; \psi(z)\, H^\dagger(0)\rangle}{\langle H(t_{\rm sink}) H^\dagger(0)\rangle}$$

$t_{\rm sink} \to \infty$

$$h(z, P_z) = \langle P_z | \overline{\psi}(0)\; W(0,z)\; \psi(z) | P_z \rangle$$

Renormalize

$$h^R(z, P_z, P^R)$$

$$\mathcal{M}(zP_z, z^2) = \frac{h(z,P_z)}{h(z,0)}$$

Reduced Ioffe time distribution

Fourier z to conjugate x at fixed P_z

Fourier zP_z to conjugate x at fixed z^2

Quasi-PDF $\tilde{q}(x, P_z, P^R)$

$\tilde{\mathcal{M}}(x, z^2)$ **pseudo-PDF**

LaMET

Light-ray OPE

$$f(x, \mu^2)$$

Lattice cross-section e.g.,

$$\langle P_z | j_V(0)\; j_A(z) | P_z \rangle$$