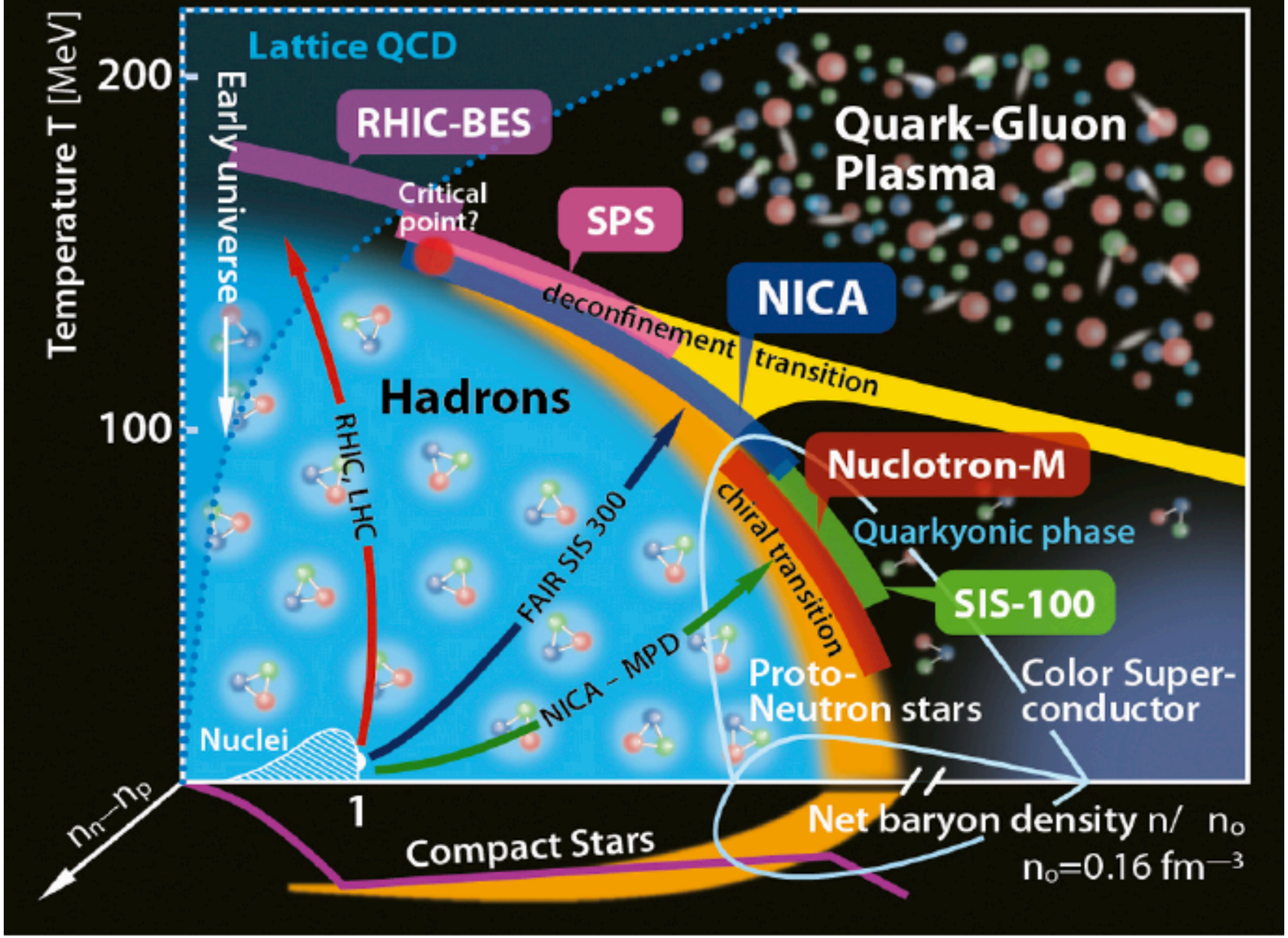


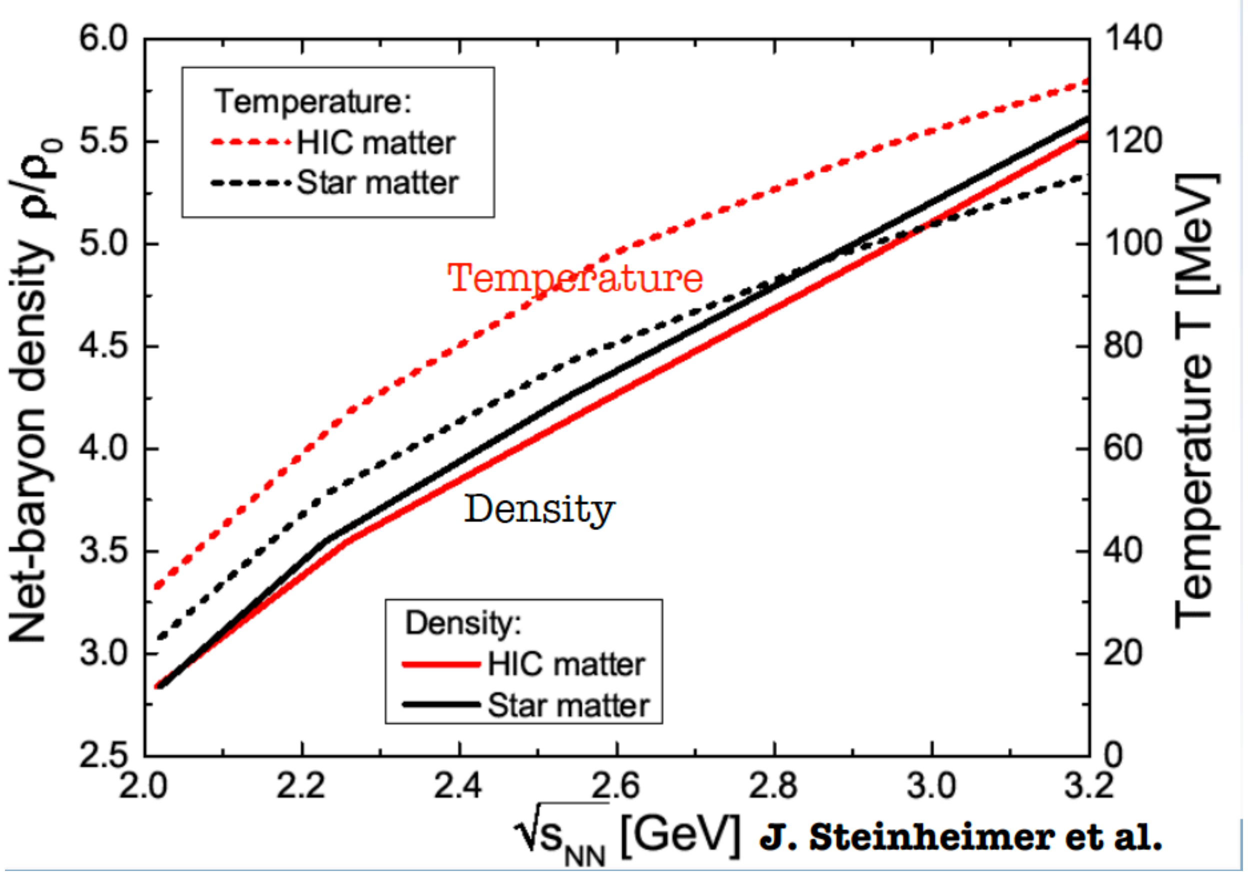

Work with Sanjay Reddy PRL 122 (2019) 122701;
with Srimoyee Sen and Kiesang Jeon, arXiv:1908.04799

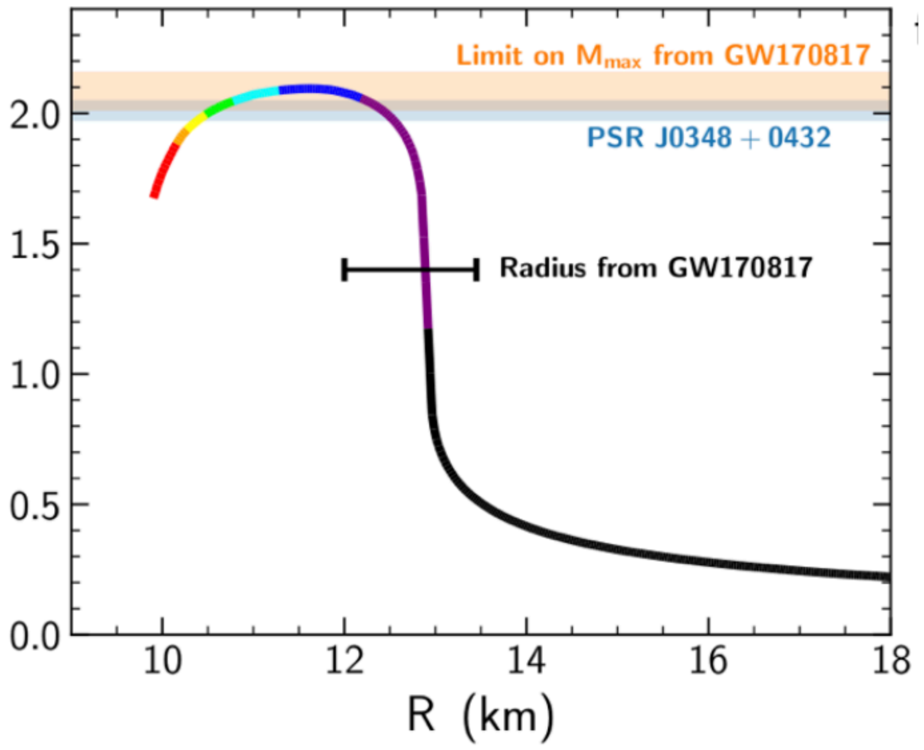
We only have compelling knowledge of the properties of QCD matter at high T and very small baryon density,.

We have strong evidence for the equation of state for very small T and high T from static properties of neutron stars and their collisions.

All the rest is at best educated conjecture and at worst idle speculation. Later in the talk I will engage in some speculation

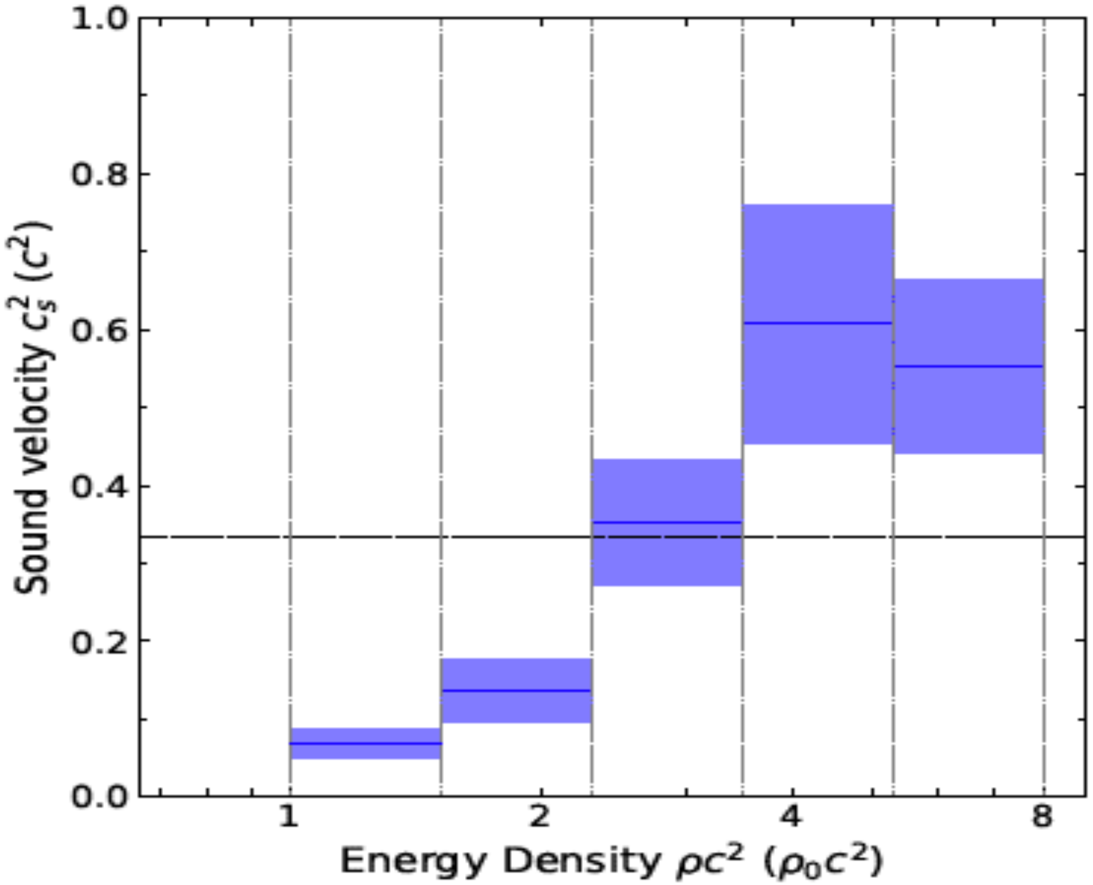
***It is not what you don't know that gets you in trouble, it is what you think you know but you don't. Mark Twain***




**Comparison of neutron star collisions and heavy ion collisions**

**There are constraints from both neutron star collisions and static properties of neutron stars**




Y. Fujimoto, K. Fukushima,
K. Murase

As a result of LIGO experiments, and more precise measurement of neutron star masses, the equation of state of nuclear matter at a few time nuclear matter density is tightly constrained

Typically sound velocity approaches and perhaps exceeds

$$v_s^2 = 1/3$$

at a few times nuclear matter density

Tews, Carlso, Gandolfi and Reddy; Kojo; Anala, Gorda, Kurkela and Vorinen

Sound velocity of order one has important consequences
For zero temperature Fermi gas:

$$\frac{n_B}{\mu_B dn_B/d\mu_B} = v_s^2$$

where the baryon chemical potential includes the effects of nucleon mass

$$\frac{\delta\mu_B}{\mu_B} \sim v_s^2 \; \frac{\delta n_B}{n_B}$$

So if the sound velocity is of order one, an order one change in the baryon density generates a change in the baryon number chemical potential of order the nucleon mass

For nuclear matter densities

$$\mu_B - M \sim \frac{\Lambda_{QCD}^2}{2M} \sim 100 \; MeV$$

Large sound velocities will require very large intrinsic energy scales, and a partial occupation of available nucleon phase space because density is not changing much while Fermi energy changes a lot

Kaiser, Meissner and Weise

It may be true that this is a consequence of complicated nuclear interactions, and one can construct model computations that do this, but strong questions about self-consistency.
We do need to understand nuclear matter better as we approach hard core density

Another approach:

Almost free quasi-particle degrees of freedom

Simplest idea (which does not work):
A free gas of nucleons with nucleon mass going to zero

$$n_B = \frac{2}{3\pi^2} k_B^3 \qquad k_B = \sqrt{\mu_B^2 - M^2}$$

$\mu_B$ is the quark Fermi energy which increases as density increases. $\mu_B > M_0$

If the nucleon mass goes to zero, the density therefore

$$n_B > \frac{2}{3\pi^2} M_0^3 \sim 10 \ fm^{-3}$$

That is about 100 times that of nuclear matter

This can be understood in large N_c arguments:

$k_f \sim \Lambda_{QCD}$ requires $\mu_B - M_N \sim \Lambda_{QCD}/N_c$

Is it possible to get relativistic degrees of freedom with quarks?
Quarks should become important when

$$\mu_Q = \mu_B/N_c \sim \Lambda_{QCD}$$

The hypothesis of quarkyonic matter implies there is no confining transition, nor need there be any chiral transition that separates baryon matter from quark matter. Might they be smoothly connected? This would require a transition when the baryon Fermi energy is very close to the nucleon mass, so the transition may in principle occur quite close to nuclear matter density. To understand this, we do N_c counting

$$n_B^n = \frac{2}{3\pi^2} {k_n^F}^{\,3}$$

Density is of order one in power of N_c for baryon density computed both by both quark and nucleon degree of freedom

$$n_B^q = \frac{2}{3\pi^2} {k_f^q}^{\,3}$$

The problem is that if we take a constituent quark model

$$k_n^F = \sqrt{\mu_B^2 - M_N^2} = \sqrt{N_c^2 \mu_q^2 - N_c^2 M_q^2} = N_c k_q^F$$

If there is a continuous transition then the baryon density will have to remain fixed, so the chemical potential will change by of order N_c. The sound velocity is changing for a very non-relativistic system to a very relativistic one. The equation of state is therefore very stiff which is what is required from observation

$$\epsilon_B = M_N \Lambda_{QCD}^3 \sim N_c \Lambda_{QCD}^4$$

But quark density is of order N_c (each quark carries baryon number 1/N_c), so energy density can also be continuous

$$\epsilon_Q \sim \Lambda_{QCD} n_q \sim N_c \Lambda_{QCD}^4$$

$$P \sim \frac{k_F}{M_B}\epsilon_N$$

The pressure on the other hand must jump by order N_c squared

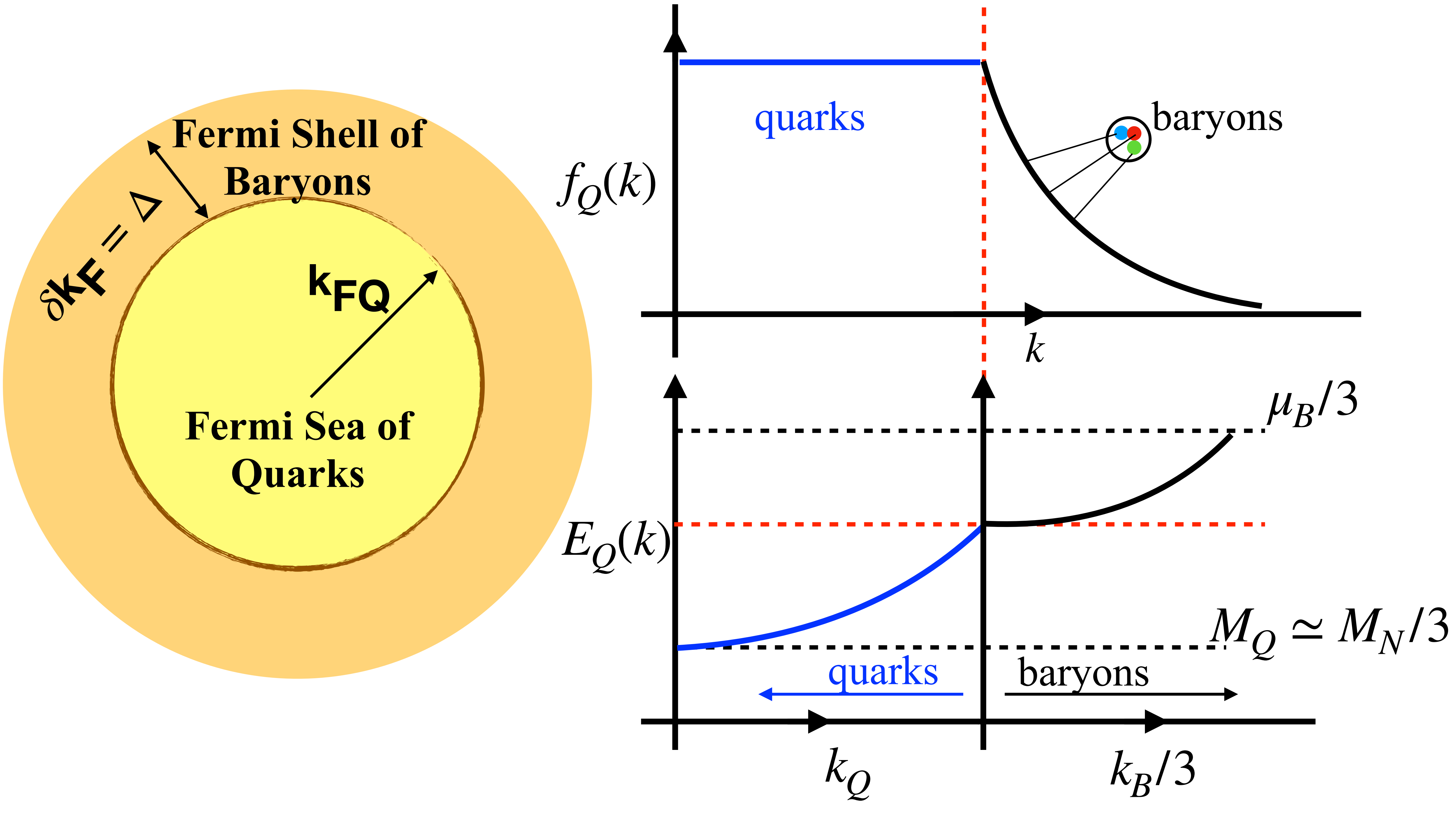
$$P \sim \epsilon_q$$

In ordinary first order phase transitions, the energy density and density jump but the pressure and chemical potential remain fixed

Here the energy density and density do not jump but the pressure and chemical potential jump.

The transition from nuclear matter to quarkyonic matter therefore involves a

**First Order Un-Phase Transition**



Fermi Shell of Baryons
$\delta k_F = \Delta$
$k_{FQ}$
Fermi Sea of Quarks
quarks
baryons
$f_Q(k)$
$k$
$\mu_B/3$
$E_Q(k)$
$M_Q \simeq M_N/3$
quarks
baryons
$k_Q$
$k_B/3$

# Simplest model for a First Order Un-Phase Transition with Quarkyonic Matter

McLerran and Reddy

Above some Fermi momentum, assume a Fermi surface shell develops. For momenta

$$k_F^B > k > k_F^B - \Delta$$

the degrees of freedom are nucleons. Near the Fermi surface there can be non-perturbative low momentum interactions.

For momentum

$$k < k_F^B - \Delta$$

The degrees of freedom are quarks.

When the Fermi shell first appear, Delta is of the order of the QCD scale. At high densities, it must narrow to be of order 1/N_C^2 because we require that we smoothly match to a QCD limit for a a finite shell thickness

$$k_F^{B\ 3} - (k_F^B - \Delta)^3 \sim k_F^{B\ 2}\Delta \sim N_C^2 \Lambda^2 \Delta \sim \Lambda^3$$

A reasonable parameterization is

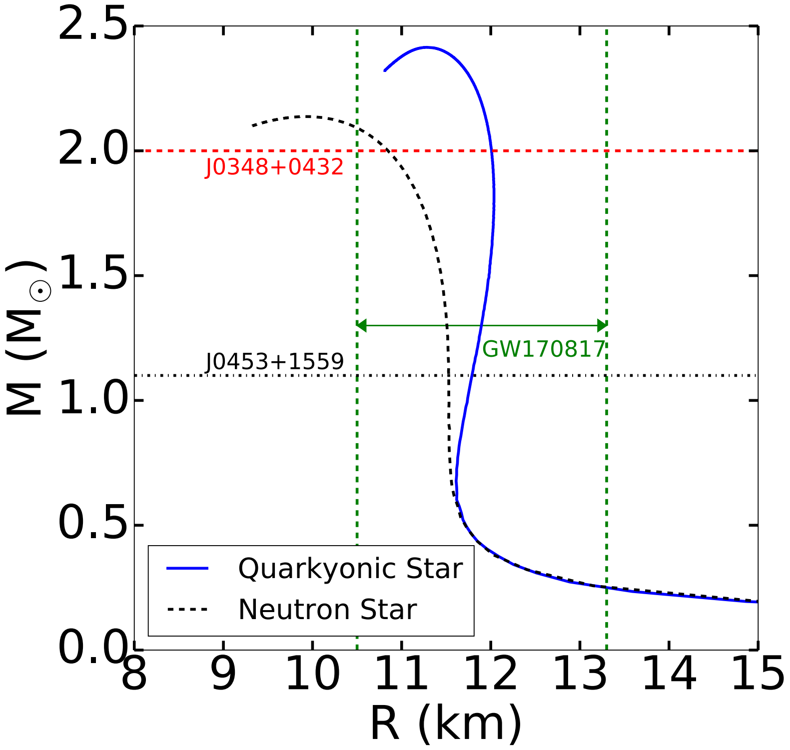
$$\Delta = \frac{\Lambda^3}{k_B^2}$$

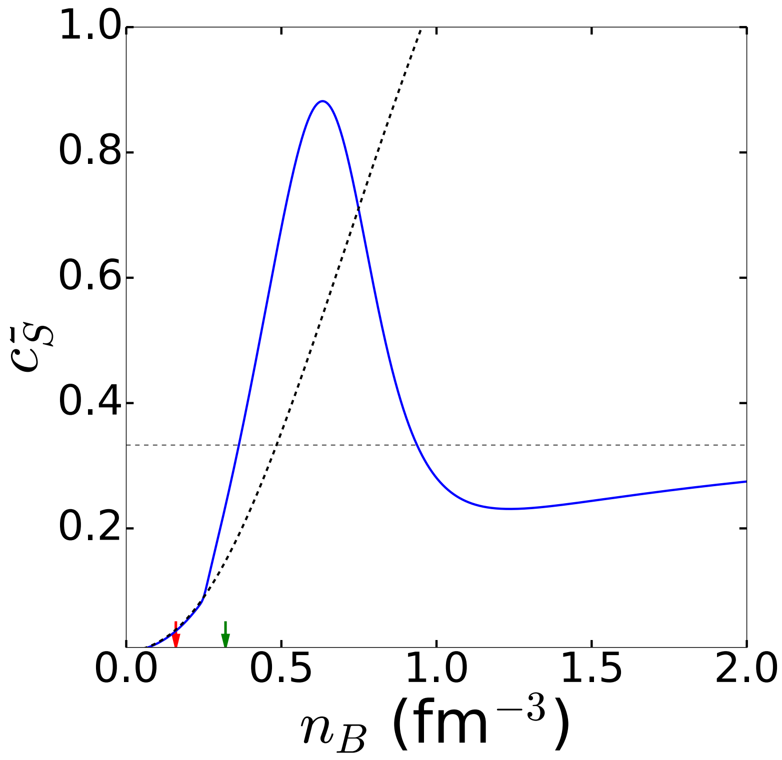
But if the density really becomes a constant then the sound velocity diverges.
to have a finite limit with v^2 < 1 need to include a correction

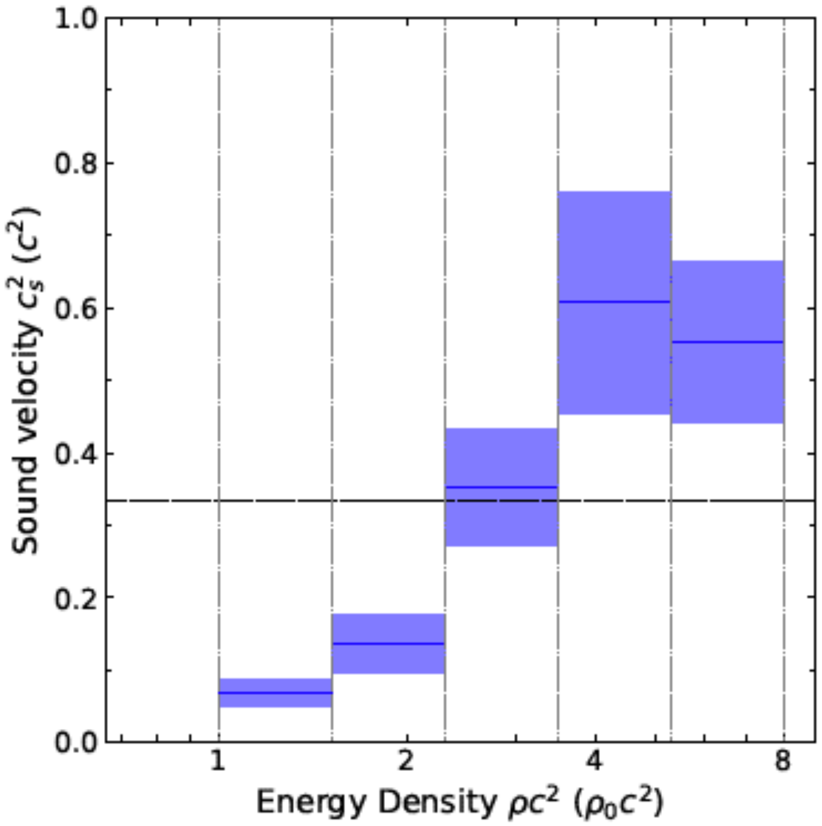
$$\Delta = \frac{\Lambda^3}{k_B^2} + \kappa \frac{\Lambda}{N_c^2}$$

This correction is important only at very high density where the quark contribution is of the order of the baryons

In explicit computation, we used a phenomenological equation of state for nuclear matter that is hard, and matched at a few times nuclear density




Y. Fujimoto, K. Fukushima, K. Murase

Work with Srimoyee Sen,
And Kiesang Jeon

## A simple model: Hard core nucleons plus quarks (No interactions between quarks and nucleons)

$$\epsilon = \epsilon_N(n^N_N) + \epsilon_Q(n^N_Q)$$

$$n^N = n^N_N + n^N_Q$$

Minimize energy density at fixed total baryon number density

$$\mu_N = d\epsilon_N/dn^N_N = d\epsilon^N_Q/dn^N_Q$$

Assume a hard core gas of nucleons:

$$\mu_N - M = \kappa \frac{M}{N_c^2} \left\{ (1 - n^N_N/n_0)^{-\gamma} - 1 \right\}$$

If we have a gas only of nucleons, then the sound velocity will diverge as the nucleon density approaches the hard core density

$$v_s^2 = \gamma \frac{n_N^N/n_0}{1 - n_N^N/n_0} \left\{ 1 + \frac{N_c^2}{\kappa} (1 - n_N^N/n_0)^\gamma \right\}^{-1}$$

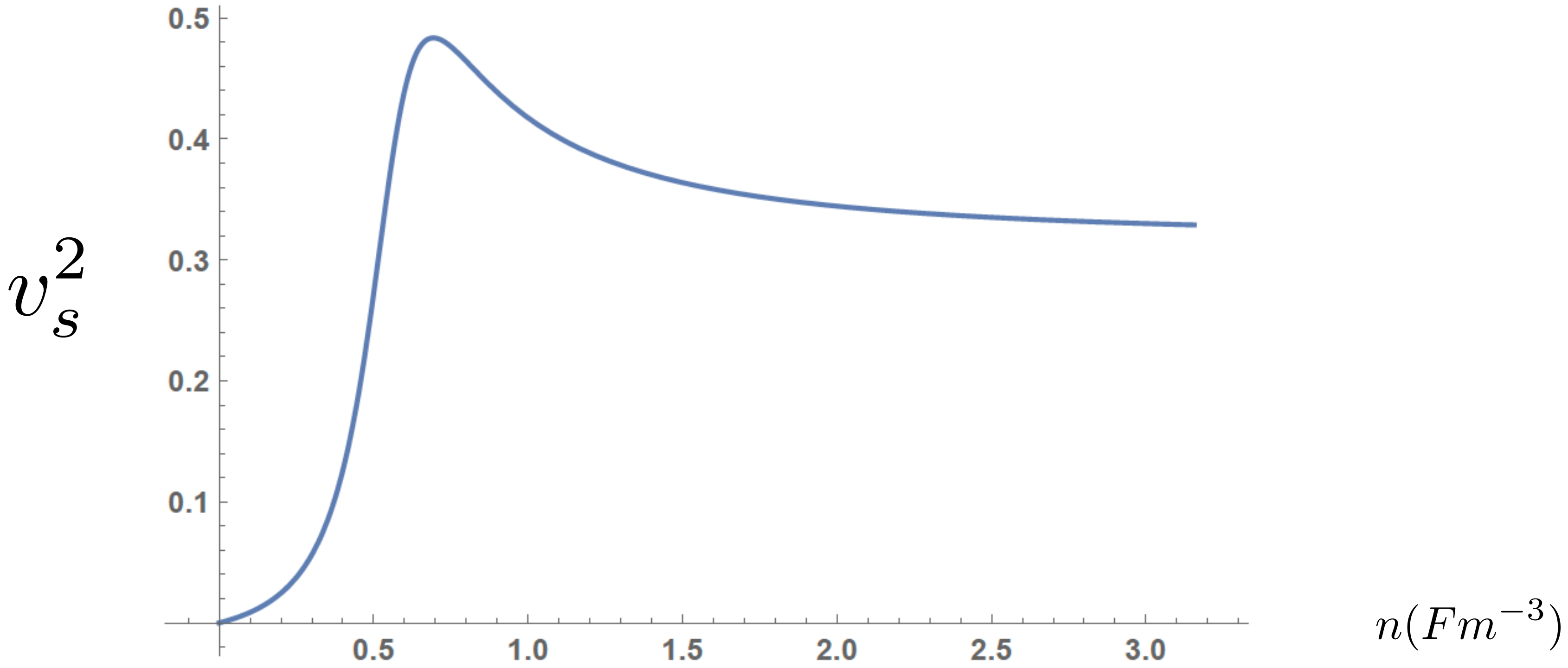
This is because in the evaluation of the sound velocity, the density stops growing near the hard core density, as a function of chemical potential,

$$v_s^2 = \frac{n_N^N}{\mu_N \; dn_N^N/d\mu_N}$$

With quarks:

$$v_s^2 = \frac{n_N^N + n_Q^N}{\mu_N (dn_N^N/d\mu_N + dn_Q^N/d\mu_N)}$$

And the quark density continues to grow as the nucleon density saturates




Mean field model for nucleons with a free gas of quarks for gamma = .7
and hard core density of .6

***But the mean field model does not build in the Fermi exclusion principle.***

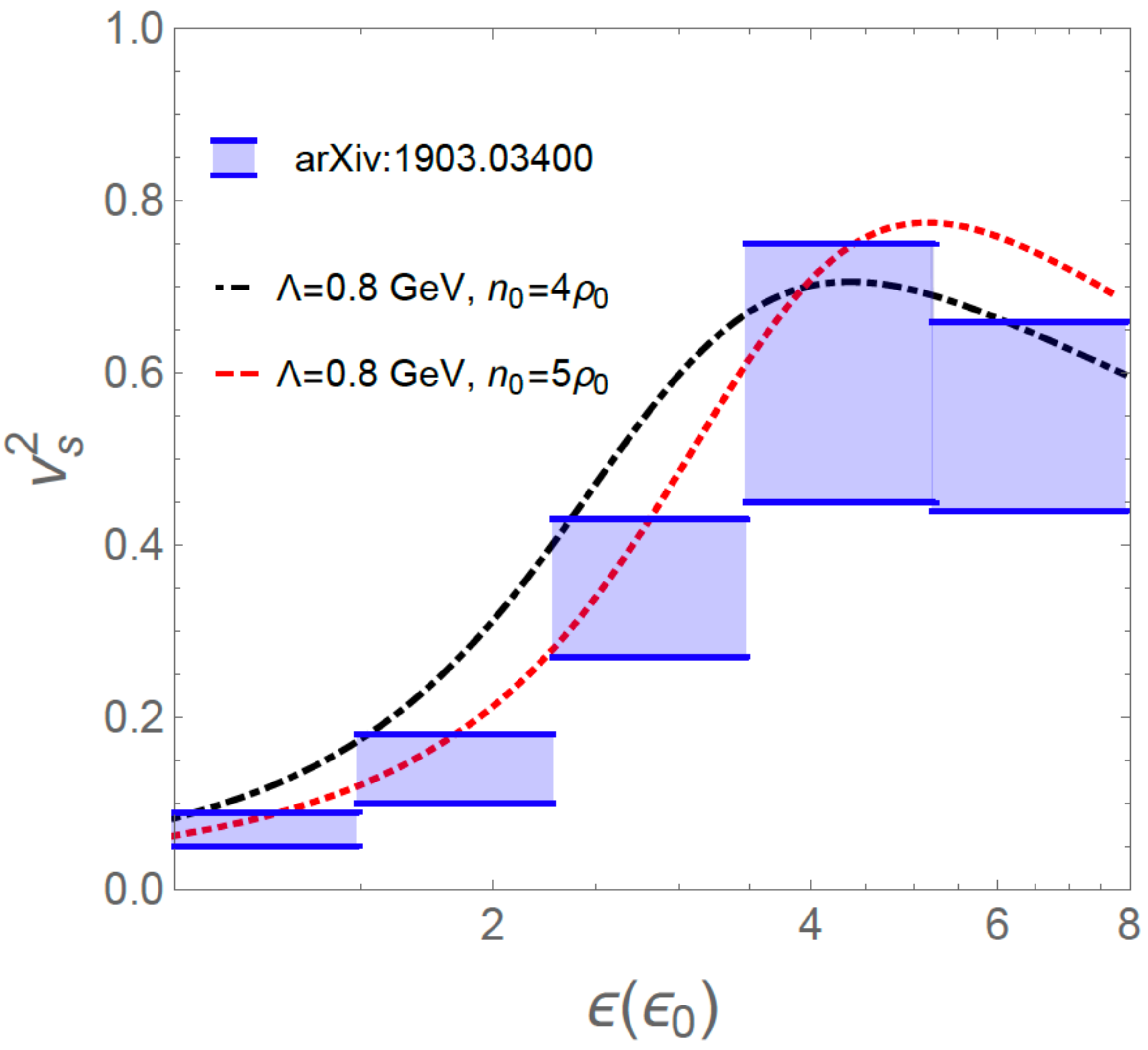
With Srimoyee Sen and Kiesang Jeon, we considered a gas of nucleons in a finite thickness Fermi shell surrounding a sea of quarks. The hard core nucleon interactions were modeled by taking the nucleon to be free nucleons in the volume excluded by the nucleonic cores

$$1/v_0 = n_0$$

$$V = V_{exc}/(1 - n_N^N/n_0)$$

(For technical reasons, at low quark density, we needed to soften the dependence of quark density upon Fermi momenta, for quark Fermi momenta less than the QCD scale)

For a pure nucleon, the baryon chemical potential will be singular as density approaches the hard core density. With both quarks and nucleons are present, it is energetically favorable to form a shell of nucleons. At a density a little less than the density of hard cores. Here the quarks become important and temper the rise in the sound velocity.




For reasonable choices of hard core density and the scale at which the quark density dependence softens, we get reasonable sound velocity profiles

**Main conclusion:**

**If a maximum in the sound velocity is conclusively determined from neutrons star studies, it very probably is due to a hard core nucleon density at about the density of the maximum, and the physics at and beyond the maximum involves quarks.**

Things that need further study: beta equilibrium and charge neutrality
(J. Magueron)
Strangeness

How to experimentally see this singular behavior of sound velocity and Fermi momentum distribution in lab experiments?

How to measure Fermi momentum distribution at high density?

Theoretically, getting the sound velocity less than 1 is clumsy, and in a really good treatment it should be automatic.
Meh?
Shrug?

**I think not!**