Z_N symmetry and confinement-deconfinement transition in SU(N) Higgs theory

Minati Biswal

Institute Of Physics, Bhubaneswar, India.

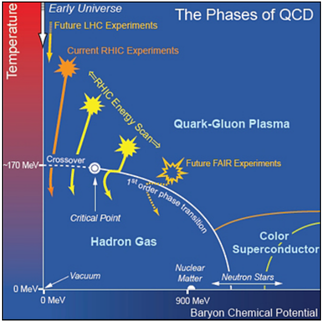
Collaborators: S. Digal, P.S. Saumia

Outline

- Introduction.
- Z_N symmetry (with matter fields).
- Simulation study of Z_N symmetry in presence of Higgs.
- N_τ (number of temporal lattice sites) dependence continuum limit.
- Summary.

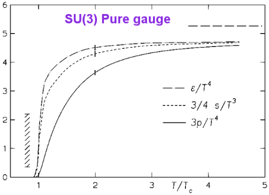
Introduction

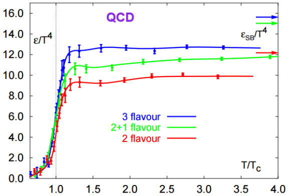
- It is expected that at high enough temperatures hadrons melt into quark-gluon plasma (QGP).



- These conditions existed in the early universe. Recently heavy ion collision experiments are able to reach such extreme condition.

Introduction





- Theoretical studies in Quantum Chromodynamics (QCD) show that the melting proceeds via a transition known as confinement-deconfinement (CD) transition. [1]
- The CD transition is present in all SU(N) [$N \geq 2$] gauge theories like QCD and Electroweak theory.

[1] *F. Karsch*, Lect.Notes Phys. 583 (2002) 209-249 (arXiv:hep-lat/0106019).

Introduction

- In pure SU(N) gauge theories the CD transition is described by order parameter, the average of Polyakov loop ($\langle L \rangle$) and the Z_N symmetry. Order of the transition depends on N.
- In QCD, at '$\mu = 0$' the presence of dynamical quarks (in the fundamental representation) break this symmetry explicitly. The CD transition is a cross-over for realistic quark masses.
- In Electroweak theory, the presence of Higgs and other matter fields break this symmetry explicitly.
- The dependence of this explicit symmetry breaking on the bare parameters of the theory is not completely understood.

Introduction

- In QCD the explicit symmetry breaking grows with decrease in the quark mass. However this dependence of explicit symmetry breaking on m_q does not explain physics in the chiral limit.
- In SU(N) Higgs theory there are very few non-perturbative studies on explicit breaking of Z_N symmetry. And also it is important to understand the similarities (differences) between bosonic and fermionic matter as to how they affect the Z_N symmetry.
- So we study Z_N symmetry in SU(N) Higgs theory by Monte Carlo simulations of the CD transition.
- Our main result is that the explicit symmetry breaking is vanishingly small in part of the Higgs symmetric phase. Preliminary results indicate that the explicit symmetry breaking vanishes in the entire Higgs symmetric phase.

Z_N symmetry

- Partition function of a pure SU(N) gauge theory at high temperature ($T = \frac{1}{\beta}$) is

$$\mathcal{Z} = \mathrm{Tr} e^{-\beta H} = \int dA \langle A | e^{-\beta H} | A \rangle = \int_{\mathbf{bc}} DA e^{-S(A)} \tag{1}$$

$$S(A) = \int_0^\beta d\tau \int_V d^3x \left\{ \frac{1}{2} \mathrm{Tr} \left(F^{\mu\nu} F_{\mu\nu} \right) \right\} \tag{2}$$

- Where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + ig[A_\mu, A_\nu]$

- The allowed A's in the path integral are periodic in β,

$$A_\mu(\vec{x}, 0) = A_\mu(\vec{x}, \beta) \tag{3}$$

Contd...

- $S(A)$ and $\mathcal{Z}$ are invariant under the gauge transformation $V(\vec{x},\tau)$, A_μ transforms

$$A_\mu \longrightarrow V A_\mu V^{-1} - \frac{i}{g}\left(\partial_\mu V\right) V^{-1} \tag{4}$$

- $V(\vec{x},\tau)$ need not be periodic, as long as it satisfies the following eqn.

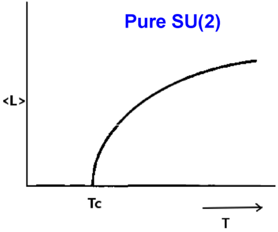
$$V(\vec{x},\tau=0) = z V(\vec{x},\tau=\beta) \tag{5}$$

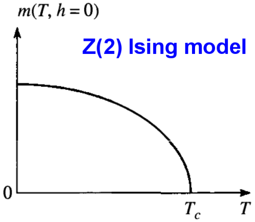
  Where $z \in Z_N$,with $z = \mathbb{1} exp(\frac{2\pi i n}{N})$, $n = 0, 1, 2...N-1$,
- Therefore, all the allowed gauge transformations at finite temperature are classified by Z_N group.
- Z_N is a symmetry of $\mathcal{Z}$.

- The Polyakov loop transforms nontrivially under Z_N.

$$L(\vec{x}) = \frac{1}{N}\mathrm{Tr}\left\{ P e^{\left(-ig\int_0^\beta A_0(\vec{x},\tau)d\tau\right)}\right\} \tag{6}$$

Under $Z_N, L \longrightarrow zL$.





- $\langle L \rangle$ is an order parameter for CD transition and it is analogous to the magnetization in a $Z(N)$ spin system.

Z_N symmetry (with matter fields)

- The action in presence of fundamental Higgs field is given by,

$$S[A,\Phi] = \int_0^{\beta} d\tau \int_V d^3x \left[\frac{1}{2}\mathrm{Tr}\left(F^{\mu\nu}F_{\mu\nu}\right) + \frac{1}{2}|D_\mu\Phi|^2 + \frac{m^2}{2}\Phi^\dagger\Phi + \frac{\lambda_0}{4!}(\Phi^\dagger\Phi)^2 \right. \tag{7}$$

- Being a bosonic field, $\Phi(\vec{x},0) = \Phi(\vec{x},\beta)$. Under above non-periodic gauge transformations, $\Phi'(0) \neq \Phi'(\beta)$ (when $z \neq \mathbb{1}$).
- It is not clear how this Z_N explicit breaking will affect the CD transition. Fluctuations of the gauge and Higgs fields need to be considered.

- For simulations we discretise the action on a 4-D Euclidean space on a $N_\tau \times N_s^3$ lattice,

$$S[A,\Phi] = \int_0^\beta d\tau \int_V d^3x \left[\frac{1}{2}\mathrm{Tr}\left(F^{\mu\nu}F_{\mu\nu}\right) + \frac{1}{2}|D_\mu\Phi|^2 + \frac{m^2}{2}\Phi^\dagger\Phi + \frac{\lambda_0}{4!}(\Phi^\dagger\Phi)^2\right. \tag{8}$$

$$x = na, \Phi(x) \to \frac{\sqrt{\kappa}\Phi_n}{a}, \lambda_0 \to \frac{\lambda}{\kappa^2}, m^2 \to \frac{(1-2\lambda-8\kappa)}{\kappa a^2} \tag{9}$$

$$S[U,\Phi] = \beta_g \sum_p \frac{1}{2}\mathrm{Tr}(2 - U_p - U_p^\dagger) - \kappa \sum_{\mu,n} Re\left[\mathrm{Tr}(\Phi^\dagger_{n+\mu}U_{n,\mu}\Phi_n)\right] + \sum_n \left[\frac{1}{2}\mathrm{Tr}\left(\Phi_n^\dagger\Phi_n\right) + \lambda\left(\frac{1}{2}\mathrm{Tr}\left(\Phi_n^\dagger\Phi_n\right) - 1\right)^2\right]. \tag{10}$$

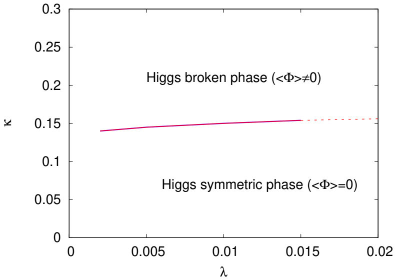
- The discretised action is given by,

$$S[U,\Phi] = \beta_g \sum_p \frac{1}{2}\mathrm{Tr}(2 - U_p - U_p^\dagger) - \kappa \sum_{\mu,n} Re\left[\mathrm{Tr}(\Phi^\dagger_{n+\mu} U_{n,\mu} \Phi_n)\right] + \sum_n \left[\frac{1}{2}\mathrm{Tr}\left(\Phi_n^\dagger \Phi_n\right) + \lambda\left(\frac{1}{2}\mathrm{Tr}\left(\Phi_n^\dagger\Phi_n\right) - 1\right)^2\right]. \tag{11}$$

- Here $\beta_g = \frac{2N}{g^2}$, link $U_{n,\mu} = e^{iagA_{n,\mu}}$. Φ_n is the field at site n. $U_p = U_{n,\mu}U_{n+\mu,\nu}U^\dagger_{n+\nu,\mu}U^\dagger_{n,\nu}$.
- To study the Z_N symmetry, we compute the distribution of the Polyakov loop and other properties using Monte Carlo simulations [2],

[2] *M. Biswal, S. Digal and P. S. Saumia,* **Nucl. Phys. B 910, 30 (2016).**

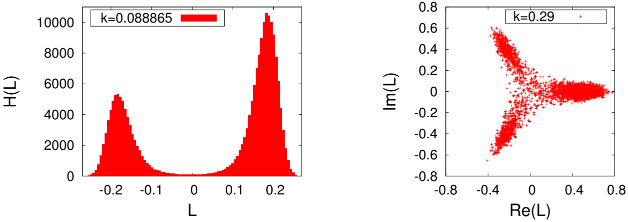
Results



- In this Higgs phase diagram, the Higgs symmetric ($\langle\Phi\rangle = 0$) and broken phase ($\langle\Phi\rangle \neq 0$) are separated by the Higgs transition line.
- We compute the Polyakov loop distribution at various points on this phase diagram to study the Z_N symmetry.
- Since the CD transition behaviour has been observed to be sensitive to N_τ, we consider larger N_τ for some values of the bare parameters.

Polyakov loop distribution

(close to Higgs transition line)



(Away from Higgs transition line)

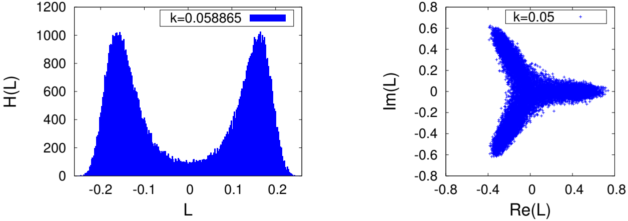
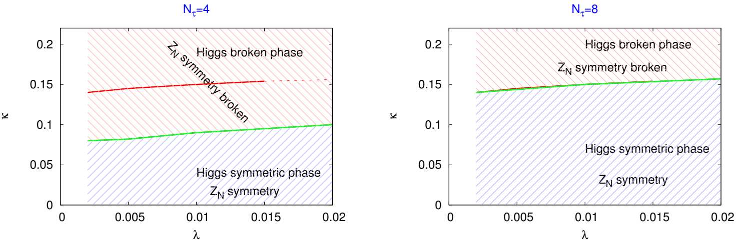


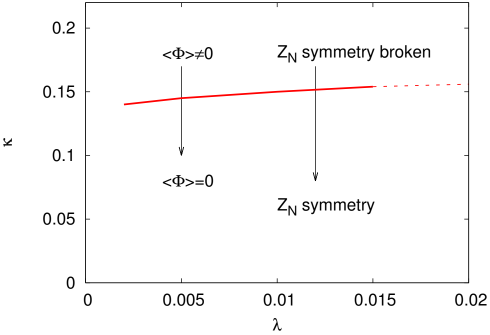
Figure: SU(2) and SU(3)

SU(N) Higgs theory N_τ dependence



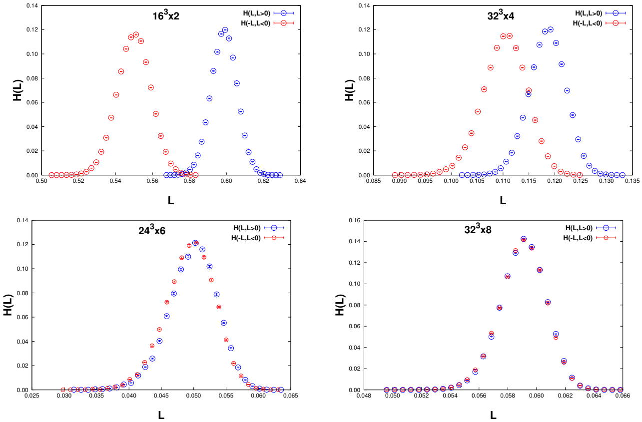
- In Higgs symmetric phase, $\langle \Phi \rangle = 0$ and in Higgs broken phase $\langle \Phi \rangle \neq 0$. Red line separates these two phases.
- Below green line Z_N symmetry exists and above the line Z_N symmetry explicitly broken. The Z_N symmetry breaking line will approach Higgs transition line for Higher N_τ.

Contd...



- Z_N symmetry explicit breaking decrease with decrease in κ.
- On the other hand, Higgs condensate decreases with decrease in κ.
- Therefore, we believe that the Higgs condensate plays the role of symmetry breaking field like external field in the Ising model.

- To study whether explicit symmetry breaking depends on N_τ, we consider SU(2) Higgs with $\lambda = 0$ and $ma = 0$.



- $H(L, L > 0)$ and $H(-L, L < 0)$ are converging with increase in N_τ. For $N_\tau = 8$ they are same.

Realization of Z_2 symmetry restoration.

- Under Z_2, $A \to A'$. But $\Phi \to \Phi' = V\Phi$ is not considered as Φ' is not periodic, so $S(A, \Phi) \neq S(A', \Phi)$.

$$S(A, \Phi) = S(A) + S(\Phi) + S_{\mathrm{I}}(A, \Phi) \tag{12}$$

- One can ask a question, if any Φ' exists for a given Φ which can compensate for the increase in the action.
- It seems with increase in N_τ a Φ' becomes available so that $S(A, \Phi) = S(A', \Phi')$ giving rise to restoration of Z_2 symmetry.
- We just don't know yet how Φ' and Φ are related for a given Z_2 (Z_N) action on A.

Summary.

- Our results suggest that the Higgs condensate plays a role of symmetry breaking field like external field in the Ising model.
- We believe increase in phase space of the Higgs field is responsible for Z_N restoration.
- It is expected that the explicit symmetry breaking is large for perturbative calculation incase of massless bosons, But surprisingly our simulation results show that the symmetry is restored.

References

- *F. Karsch*, **Lect.Notes Phys. 583 (2002) 209-249 (arXiv:hep-lat/0106019)**.
- *M. Biswal, S. Digal and P. S. Saumia*, **Nucl. Phys. B 910, 30 (2016)**.
- *B. Bunk.* **Nucl. Phys. B 42, 556 (1995)**.
- *A.D. Kennedy et.al* **PLB 156, 393 (1985)** .
- *M. Creutz.* **Phys. Rev. D 29, 306 (1984)**.
- *M. Biswal, M. Deka, S. Digal and P. S. Saumia*, **Phys. Rev. D 96, 014503 (2017)**.
- *N. Weiss*, **Phys. Rev. D 2667, 25 (1982)**.

Thank You