QCD at and near finite isospin density

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Theoretical Physics Colloquium
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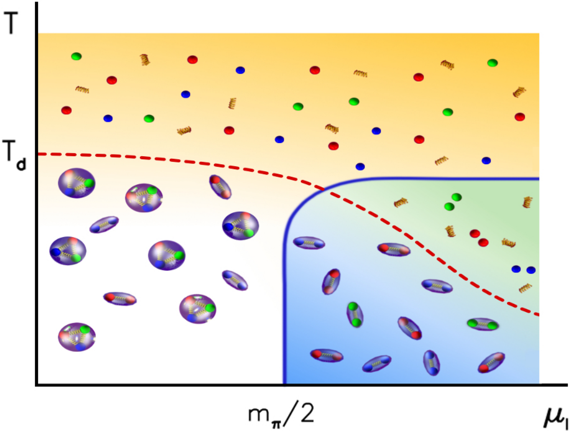
QCD phase diagram



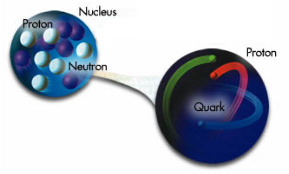
Outline

- introduction
  - QCD thermodynamics
  - finite isospin density
  - pion condensation
- lattice simulations
  - infrared problems and solutions
  - phase diagram
  - equation of state
- application: cosmological trajectories
- conclusions

Introduction

Strong interactions

- explain 99.9% of visible matter in the Universe



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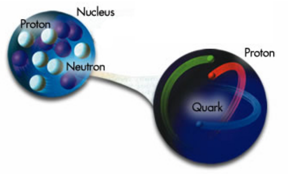

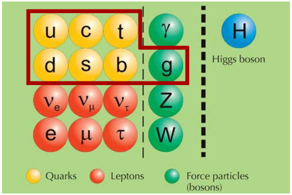

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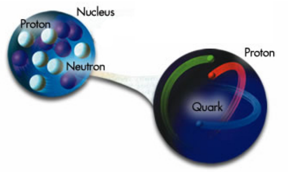

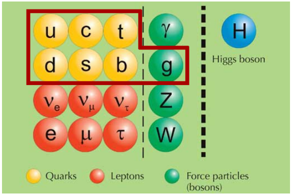

- elementary particles: quarks and gluons
- elementary fields: $\psi(x)$ and $A_\mu(x)$
- QCD Lagrangian

$$\mathcal{L}_{\rm QCD} = \frac{1}{4}\,\mathrm{Tr}\,F_{\mu\nu}(g_s, A)^2 + \bar{\psi}[\gamma_\mu(\partial_\mu + i g_s A_\mu) + m]\psi$$

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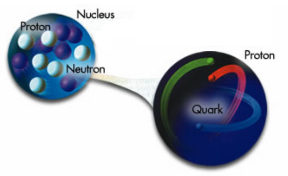

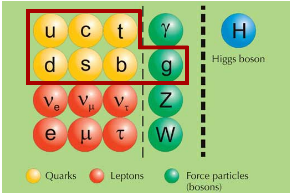

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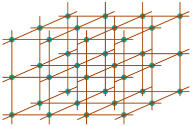
- $g_s = \mathcal{O}(1) \rightsquigarrow$ non-perturbative physics

Path integral and lattice field theory

- path integral Feynman '48

$$\mathcal{Z} = \int \mathcal{D}A_\mu \, \mathcal{D}\bar{\psi} \, \mathcal{D}\psi \, \exp\left(- \int \mathrm{d}^4x \, \mathcal{L}_{\mathrm{QCD}}(x) \right)$$

- discretize spacetime on a lattice with spacing a
  Wilson '74



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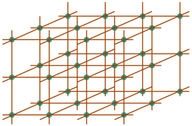
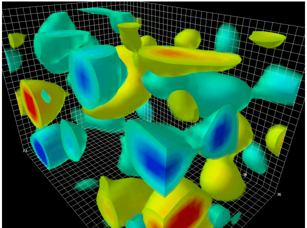


- Monte-Carlo algorithms to generate configurations
  animation courtesy D. Leinweber



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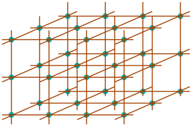

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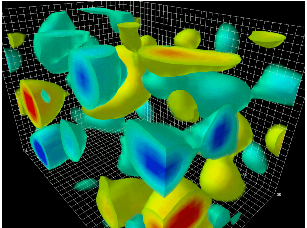
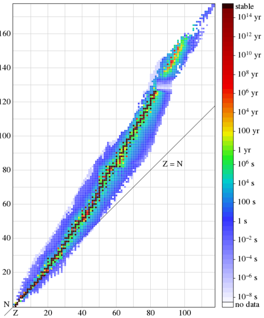
- 10^9-dimensional integrals $\rightsquigarrow$ high-performance computing

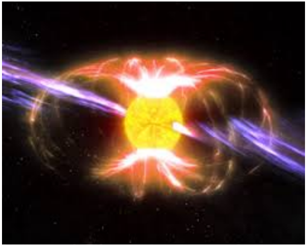


Isospin asymmetry

Isospin asymmetry: nuclei and neutron stars

- isospin asymmetry: $n_I \propto n_u - n_d$
  creates p^+-n asymmetry, excites π^+

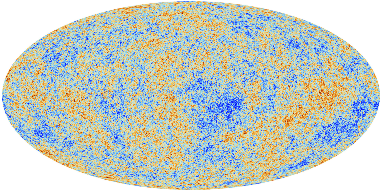




- proton to nucleon ratio in nuclei $\frac{Z}{A} \approx 0.4$
  but: 'neutron skin' near surface
- proton to nucleon ratio in interior of neutron stars $\frac{Z}{A} \approx 0.025$
- role of pion condensation ✎ Migdal et al '90

Isospin asymmetry: cosmology

- early Universe characterized by charge neutrality $n_Q = 0$,
  (almost perfect) baryon symmetry $n_B = 0$
  but lepton number n_L only weakly constrained by observations
  Oldengott, Schwarz '17



- weak equilibrium

$$d \leftrightarrow u\, e^- \, \bar{\nu}_e$$

large $n_L \leftrightarrow$ large $d - u$ asymmetry Abuki, Brauner, Warringa '09

Pion condensation

Isospin chemical potential

- quark chemical potentials (3-flavor)

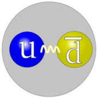
$$\mu_u = \frac{\mu_B}{3} + \frac{2\mu_Q}{3} \qquad \mu_d = \frac{\mu_B}{3} - \frac{\mu_Q}{3} \qquad \mu_s = \frac{\mu_B}{3} - \frac{\mu_Q}{3} - \mu_S$$

- pure isospin chemical potential

$$\mu_u = \mu_I \qquad \mu_d = -\mu_I \qquad \mu_s = 0$$

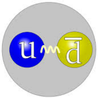
corresponds to $\mu_Q = 2\mu_I, \;\; \mu_B = -\mu_I, \;\; \mu_S = -\mu_I$

- pion chemical potential $\mu_\pi = \mu_u - \mu_d = 2\mu_I$



- isospin density $n_I = n_u - n_d$

Pion condensation

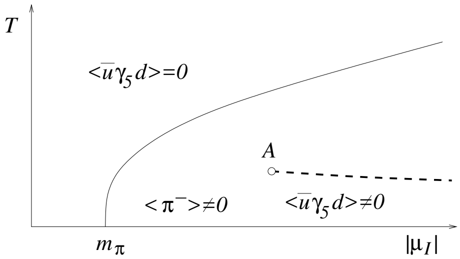


- QCD at low energies ≈ pions
  chiral perturbation theory
- chemical potential for charged pions: μ_π

at zero temperature $\mu_\pi < m_\pi$ vacuum state

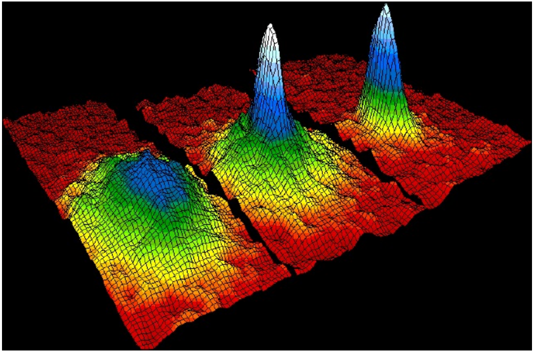
$\mu_\pi \geq m_\pi$ Bose-Einstein condensation

🔗 Son, Stephanov '00



Bose-Einstein condensate

- accumulation of bosonic particles in lowest energy state



🔗 Anderson et al '95 JILA-NIST/University of Colorado

- velocity distribution of Ru atoms at low temperature
  $\rightarrow$ peak at zero velocity (zero energy)

Symmetry breaking

- QCD with light quark matrix

$$M = \not{D} + m_{ud}\mathbb{1}$$

- chiral symmetry (flavor-nontrivial)

$$\mathrm{SU}(2)_V$$

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$$M = \not{D} + m_{ud}\mathbb{1} + \mu_I \gamma_0 \tau_3$$

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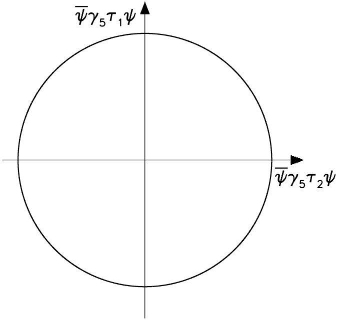

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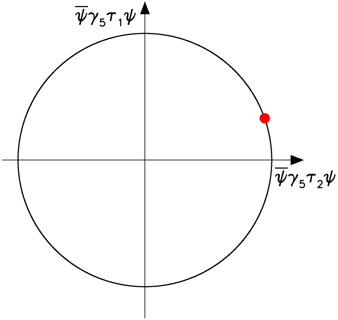

- spontaneously broken by a pion condensate

$$\left\langle \bar{\psi}\gamma_5\tau_{1,2}\psi \right\rangle = \left\langle \bar{u}\gamma_5 d \pm \bar{d}\gamma_5 u \right\rangle$$

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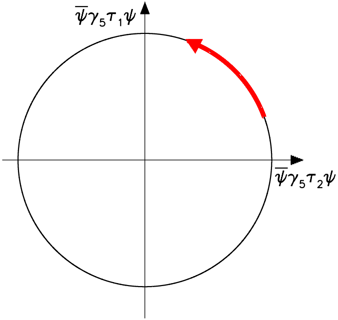

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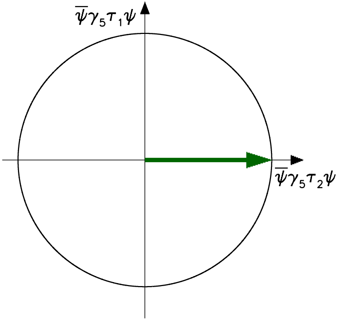

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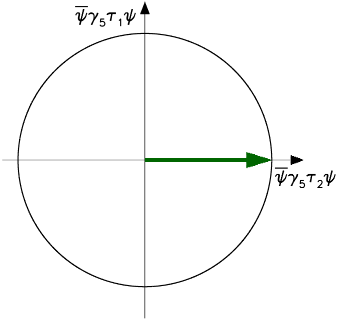

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- extrapolate results $\lambda \to 0$

Dictionary

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coset	$\mathrm{U}(1)$	
Goldstones	1	
spontaneous	$\left\langle \bar{\psi}\gamma_5\tau_2\psi \right\rangle$	
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- long story short: pion condensation 1/3 as challenging as the chiral limit of the QCD vacuum

Lattice simulations

On the lattice

- path integral

$$\mathcal{Z} = \int \mathcal{D}U \, e^{-S_g[U]} \; \det M_\ell \, \det M_s$$

- light quark matrix

$$M_\ell = \begin{pmatrix} \not{D}(\mu_I) + m & \lambda\gamma_5 \\ -\lambda\gamma_5 & \not{D}(-\mu_I) + m \end{pmatrix}$$

- hermiticity relation

$$\gamma_5 \tau_1 M_\ell \tau_1 \gamma_5 = M_\ell^\dagger \quad \rightarrow \quad \det M_\ell = \det(|\not{D}(\mu_I) + m|^2 + \lambda^2)$$

On the lattice

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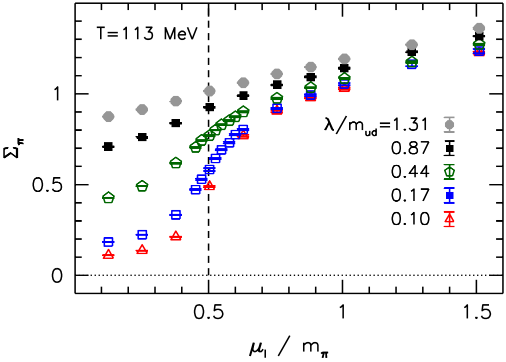
- staggered fermions: $\gamma_5 \to \eta_5$
- early studies Kogut, Sinclair '02 de Forcrand, Stephanov, Wenger '07 Endrődi '14 with unimproved actions

Pion condensate on the lattice

- measure full operator at nonzero λ (via noisy estimators)

  🔗 Brandt, Endrődi, Schmalzbauer '17

$$\Sigma_\pi \propto \left\langle \mathrm{Tr} M^{-1} \eta_5 \tau_2 \right\rangle$$



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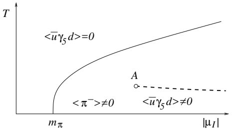

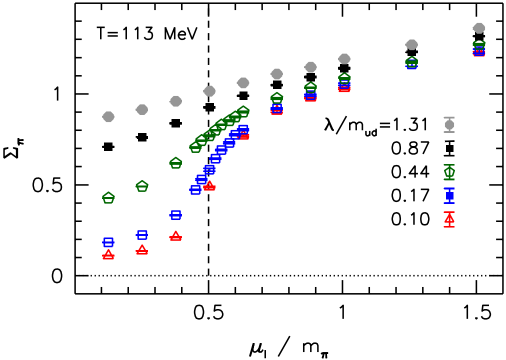

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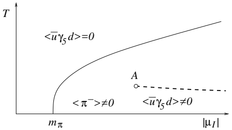

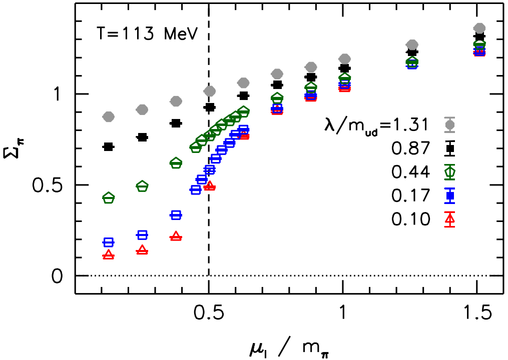

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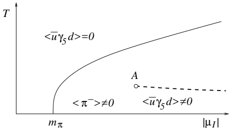

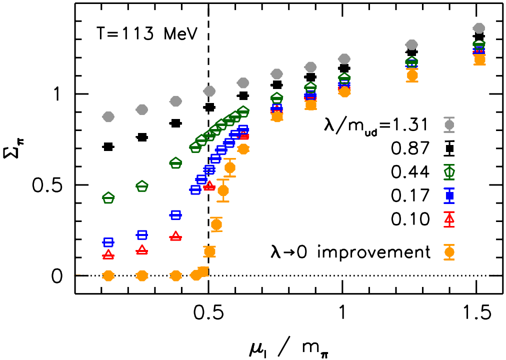

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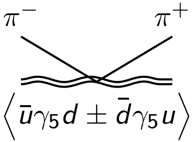
- extrapolation $\lambda \to 0$ very steep $\to$ improvement (see later)

Pion spectrum

- pion condensate carries electric charge (superconductor)
  $\rightsquigarrow$ electric charge eigenstates are not mass eigenstates



Pion spectrum

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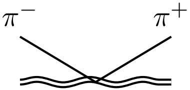
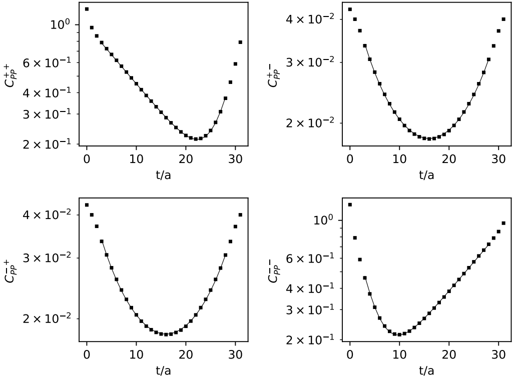
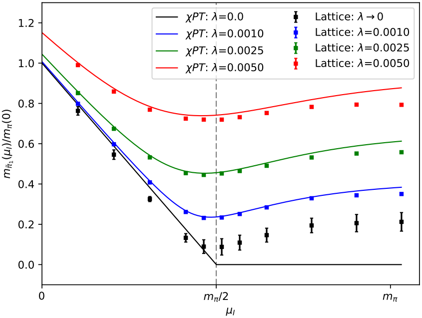


$$\left\langle \bar{u}\gamma_5 d \pm \bar{d}\gamma_5 u \right\rangle$$



Pion spectrum

- pion condensate carries electric charge (superconductor)
  $\rightsquigarrow$ electric charge eigenstates are not mass eigenstates
- pion correlator becomes a 2×2 matrix
- after diagonalization, lighter eigenstate is the Goldstone boson
  🔗 Endrődi, Theilig unpublished



comparison to χPT 🔗 Kogut et al '00

$\lambda \to 0$ improvement

Singular value representation

- pion condensate operator

$$\Sigma_\pi = \frac{\partial}{\partial\lambda} \log\det(|\not{D}(\mu_I) + m|^2 + \lambda^2) = \mathrm{Tr}\,\frac{2\lambda}{|\not{D}(\mu_I) + m|^2 + \lambda^2}$$

- singular values

$$|\not{D}(\mu_I) + m|^2\, \psi_i = \xi_i^2\, \psi_i$$

- spectral representation Brandt, Endrődi, Schmalzbauer '17

$$\Sigma_\pi = \frac{T}{V} \sum_i \frac{2\lambda}{\xi_i^2 + \lambda^2} \xrightarrow{V\to\infty} \int \mathrm{d}\xi\, \rho(\xi)\, \frac{2\lambda}{\xi^2 + \lambda^2} \xrightarrow{\lambda\to 0} \pi\rho(0)$$

first derived for $m = 0$ in Kanazawa, Wettig, Yamamoto '11

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first derived for $m = 0$ in Kanazawa, Wettig, Yamamoto '11

- compare to Banks-Casher-relation at $\mu_I = 0$

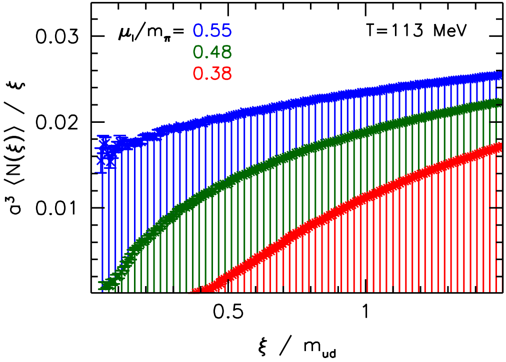
Dictionary

	pion condensation	vacuum chiral symmetry breaking
pattern	$\mathrm{U}(1)_{\tau_3} \to \varnothing$	$\mathrm{SU}(2)_L \otimes \mathrm{SU}(2)_R \to \mathrm{SU}(2)_V$
coset	$\mathrm{U}(1)$	$\mathrm{SU}(2)_A$
Goldstones	1	3
spontaneous	$\left\langle \bar{\psi}\gamma_5\tau_2\psi \right\rangle$	$\left\langle \bar{\psi}\psi \right\rangle$
explicit	$= \partial \log \mathcal{Z} / \partial \lambda$	$= \partial \log \mathcal{Z} / \partial m$
limit	$\lambda \to 0$	$m \to 0$
Banks-Casher	$\rho^{\lvert \not{D}(\mu_I) + m \rvert^2}(0)$	$\rho^{\not{D}}(0)$

Singular value density

- integrated spectral density

$$N(\xi) = \int_0^{\xi} d\xi' \rho(\xi'), \qquad \rho(0) = \lim_{\xi \to 0} N(\xi)/\xi$$



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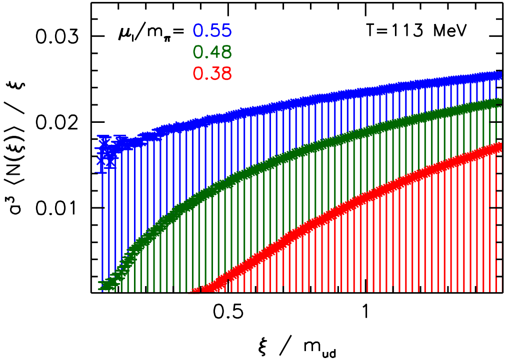

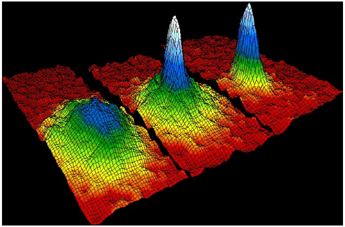

- compare $\rho(0)$ to velocity distribution around zero

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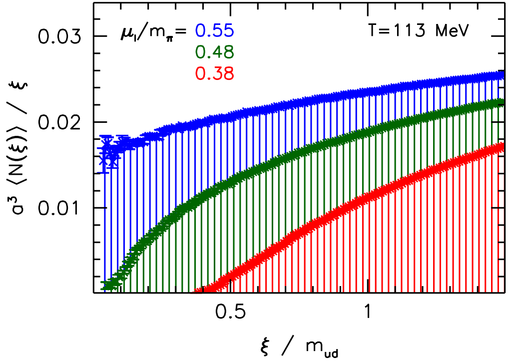
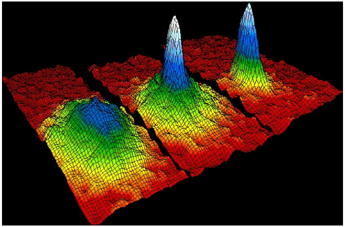


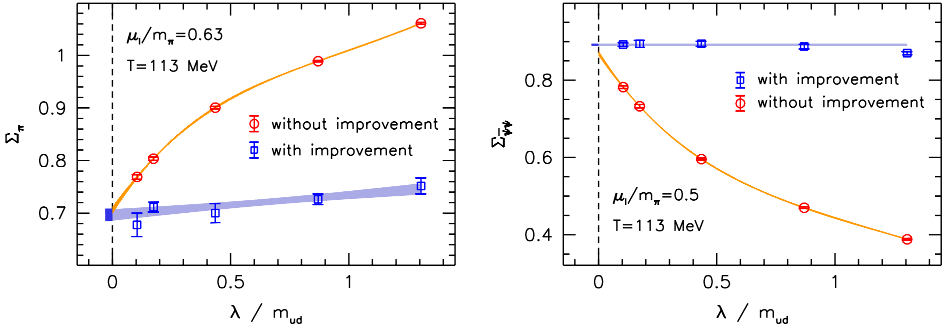


- compare $\rho(0)$ to velocity distribution around zero
- Bose-Einstein condensation!

Comparison between old and new methods

- improvement is crucial for reliable $\lambda \to 0$ extrapolation

  Brandt, Endrődi, Schmalzbauer '17

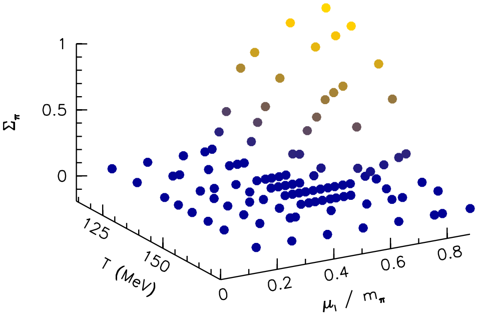


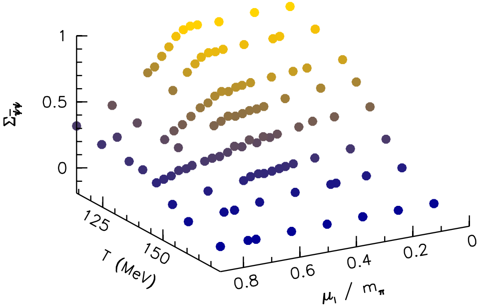
$$\Sigma_\pi = \frac{T}{V}\frac{\partial \log \mathcal{Z}}{\partial \lambda} \qquad \Sigma_{\bar\psi\psi} = \frac{T}{V}\frac{\partial \log \mathcal{Z}}{\partial m_{ud}}$$

Results: phase diagram

Condensates

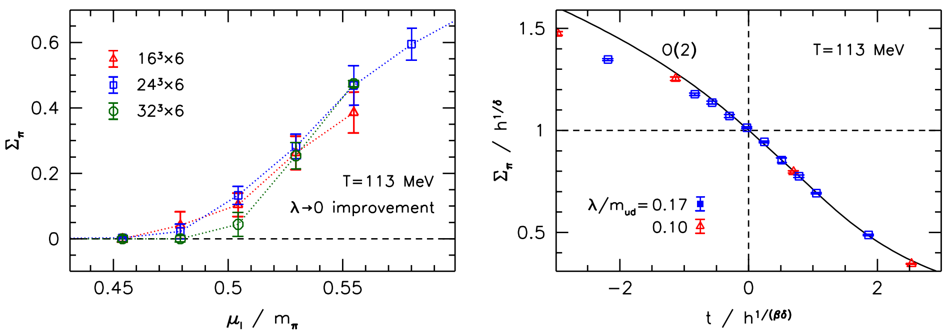
- pion and chiral condensate after $\lambda \to 0$ extrapolation





- read off chiral crossover $T_{pc}(\mu_I)$ and pion condensation boundary $\mu_{I,c}(T)$

Order of the transition



- volume scaling of order parameter shows typical sharpening
- collapse according to $\mathrm{O}(2)$ critical exponents 🔗 Ejiri et al '09

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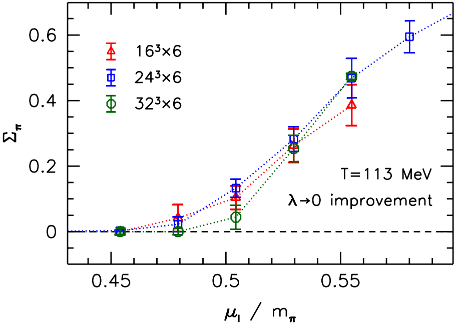
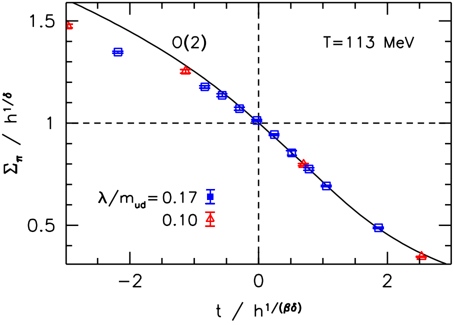
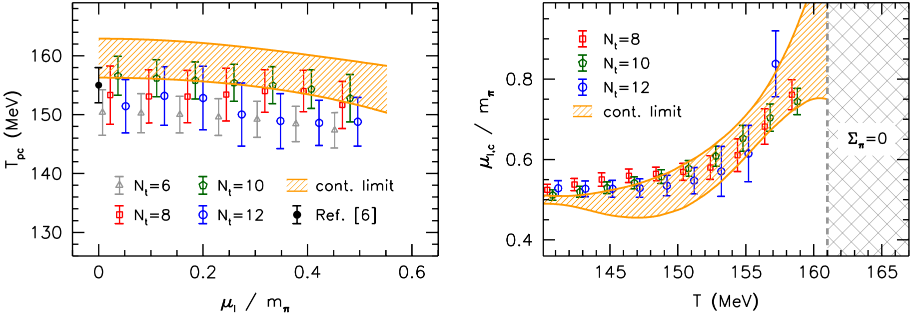




- volume scaling of order parameter shows typical sharpening
- collapse according to $\mathrm{O}(2)$ critical exponents Ejiri et al '09
- indications for a second order phase transition at $\mu_I = m_\pi/2$, in the $\mathrm{O}(2)$ universality class

Continuum extrapolations

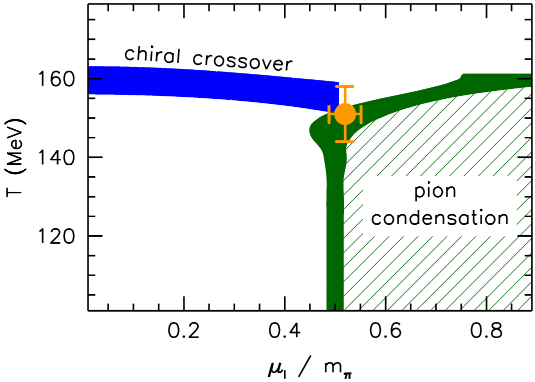
- compare (pseudo)critical temperatures for different lattice spacings $a = 1/(N_t T)$
- take continuum limit $a \to 0$ ($N_t \to \infty$)



Phase diagram

- meeting point of chiral crossover and pion condensation boundary: *pseudo-triple* point

  at $T_{pt} = 151(7)$ MeV, $\mu_{I,pt} = 70(5)$ MeV



Brandt, Endrődi, Schmalzbauer '17

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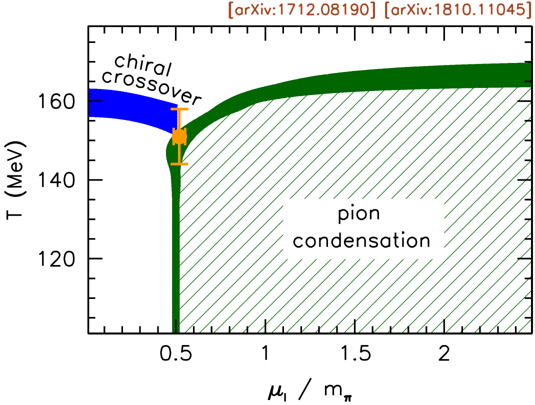

Brandt, Endrődi, Schmalzbauer '17 Brandt, Endrődi '18

Results: BCS phase

BCS superconductor

- perturbation theory at $\mu_I \to \infty$ indicates attractive $\bar{u} - d$ interaction in pseudoscalar channel Son, Stephanov '00
- $\langle \bar{u}\gamma_5 d \rangle \neq 0$ but deconfined: effective degrees of freedom are Cooper pairs and not pions
- BEC-BCS transition expected to be an analytic crossover

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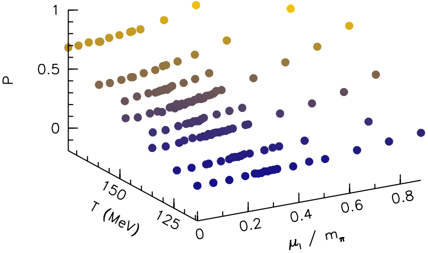
- similar to $\mu_B \to \infty$ expectations – can we detect this phase here?
- so far only indirect approaches involving:
  scaling of isospin density (two-color QCD) 🔗 Cotter et al. '12
  behavior of constituent quark mass 🔗 Adhikari et al. '18
  conformality of EoS 🔗 Carignano et al. '16
  ...

Deconfinement within BEC

- Polyakov loop shows steady rise as μ_I grows



Deconfinement within BEC

- Polyakov loop shows steady rise as μ_I grows
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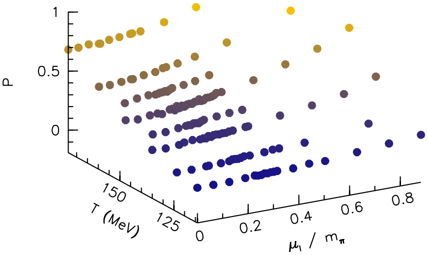

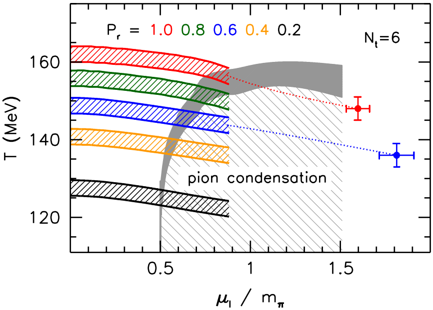

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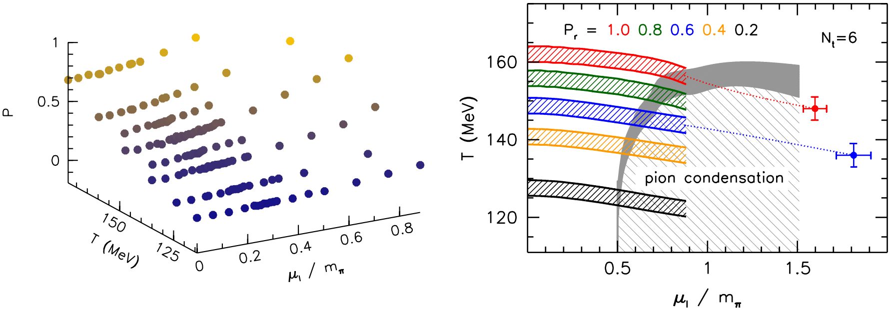

- BEC + deconfinement ⇒ BCS at high μ_I and intermediate T

BCS gap Δ

- high-μ_I effective theory predicts 🔗 Kanazawa, Wettig, Yamamoto '12

$$\Delta^2 = \frac{2\pi^3}{9}\,\rho(0)$$

$\rho(\nu)$ is spectral density of complex eigenvalues $\not{D}\psi = \nu\psi$

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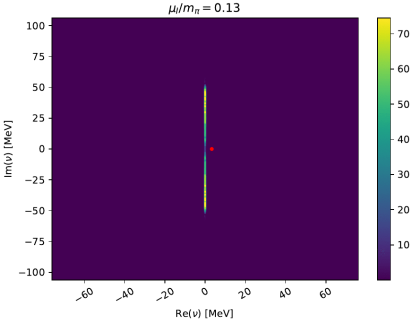
🔗 Brandt, Cuteri, Endrődi, Schmalzbauer '19



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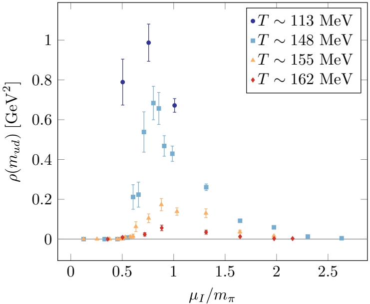
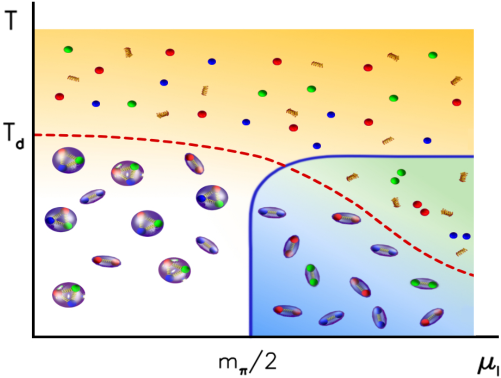
- preliminary results for $\rho(m_{ud} + i0)$ at the physical mass



severe lattice artefacts at high μ_I: finer lattices needed

Preferred phase diagram



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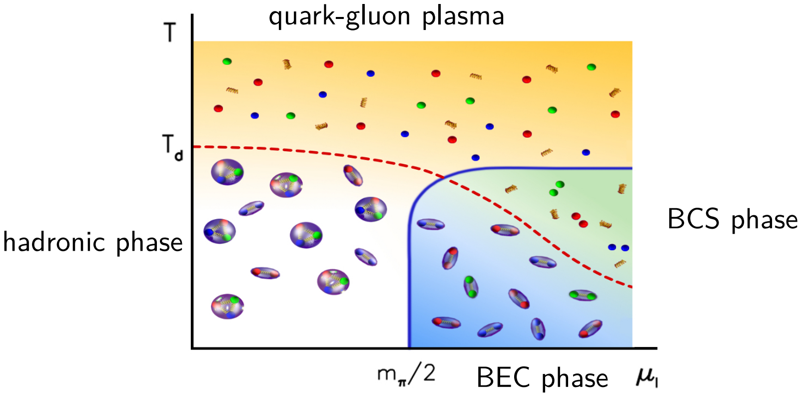

Results: equation of state

Equation of state

- equilibrium description of matter

$$\epsilon(p)$$

  relevant for:

  - neutron star physics (TOV equations)
  - cosmology, evolution of early Universe (Friedmann equation)
  - heavy-ion collision phenomenology (charge fluctuations)

- thermodynamic identities

$$p = \frac{T}{V}\log\mathcal{Z}, \qquad s = \frac{\partial p}{\partial T}, \qquad n_I = \frac{\partial p}{\partial \mu_I}, \qquad \epsilon = -p + Ts + \mu_I n_I$$

Equation of state on the lattice

- integral method to calculate differences

$$n_I = \frac{T}{V}\frac{\partial \log \mathcal{Z}}{\partial \mu_I}, \qquad p(T,\mu_I) - p(T,0) = \int_0^{\mu_I} \mathrm{d}\mu_I'\, n_I(\mu_I')$$

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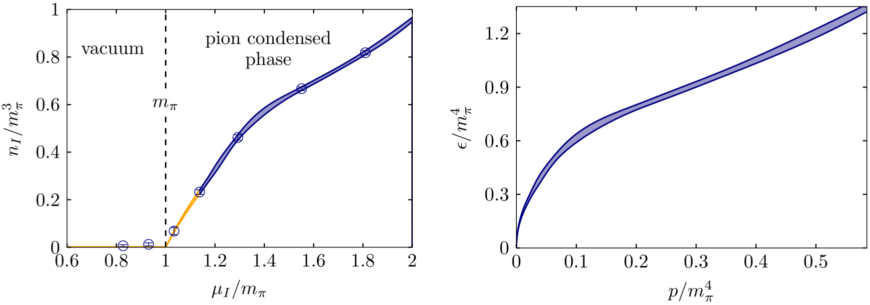
- results at $T \approx 0$ on one lattice spacing

Brandt, Endrődi, Fraga, Hippert, Schaffner-Bielich, Schmalzbauer '18



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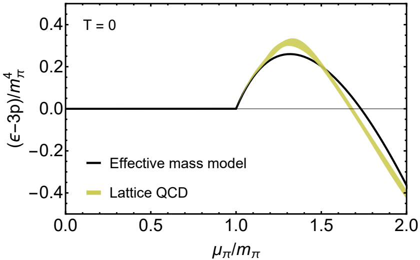
  Brandt, Endrődi, Fraga, Hippert, Schaffner-Bielich, Schmalzbauer '18

- interaction measure $I = \epsilon - 3p$, compared to a hadron resonance gas model including pion-pion interactions

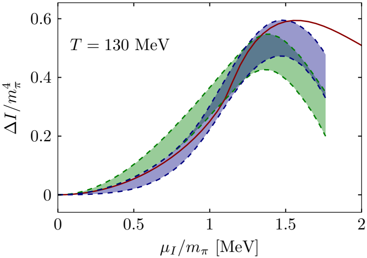
  Vovchenko, Brandt, Cuteri, Endrődi, Hajkarim, Schaffner-Bielich '20

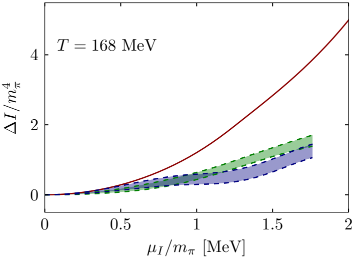


Equation of state on the lattice

- $\Delta I = I(T,\mu_I) - I(T,0)$ on two lattice spacings

  Vovchenko, Brandt, Cuteri, Endrődi, Hajkarim, Schaffner-Bielich '20





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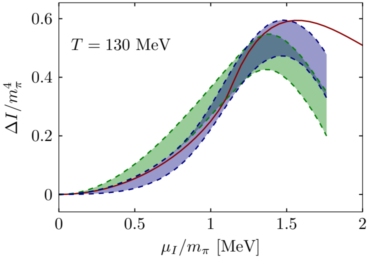
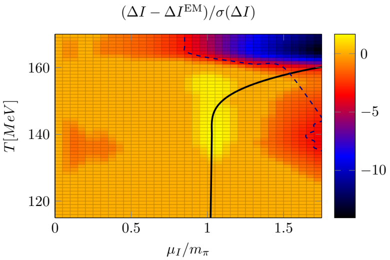


- comparison of ΔI with the effective HRG model
- validity range of the model is determined



Cosmological implications

Cosmic trajectories

- conservation equations for isentropic expansion

$$\frac{n_B}{s} = b, \quad \frac{n_Q}{s} = 0, \quad \frac{n_{L_\alpha}}{s} = l_\alpha \quad (\alpha \in \{e, \mu, \tau\})$$

- parameters: temperature plus chemical potentials

$$T, \quad \mu_B, \quad \mu_Q, \quad \mu_{L_\alpha}$$

- experimental constraints Planck coll. '15 Oldengott, Schwarz '17

$$b = (8.60 \pm 0.06) \cdot 10^{-11}, \qquad |l_e + l_\mu + l_\tau| < 0.012$$

(the individual l_α may have opposite signs)

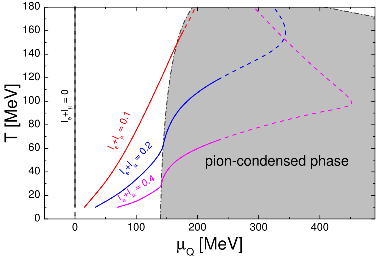
- complete EoS (neglecting QED interactions)

$$p = p_{\mathrm{QCD}} + p_{\mathrm{leptons}} + p_{\mathrm{photons}}$$

Cosmic trajectories

- cosmic trajectory enters BEC phase for lepton asymmetries allowed by observations

  🔗 Vovchenko, Brandt, Cuteri, Endrődi, Hajkarim, Schaffner-Bielich '20



Cosmic trajectories

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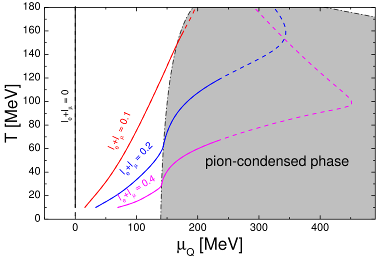
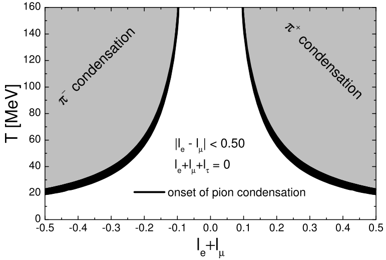
  Vovchenko, Brandt, Cuteri, Endrődi, Hajkarim, Schaffner-Bielich '20

- keeping $l_e + l_\mu + l_\tau = 0$, $l_e - l_\mu$ is not so important; the relevant condition is

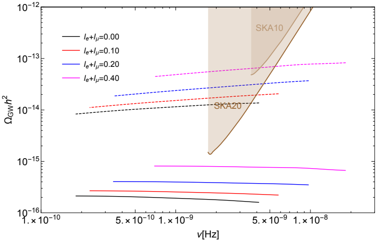
$$\boxed{|l_e + l_\mu| \gtrsim 0.1}$$

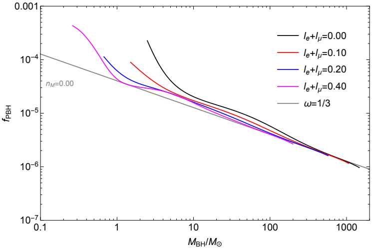




Signatures of the condensed phase

- relic density of primordial gravitational waves is enhanced with respect to amplitude at $l_e + l_\mu = 0$
- fraction of primordial black holes with mass below one solar mass is enhanced





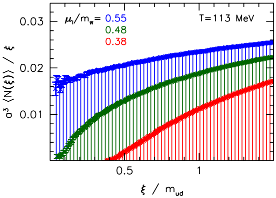
Vovchenko, Brandt, Cuteri, Endrődi, Hajkarim, Schaffner-Bielich ’20

Summary

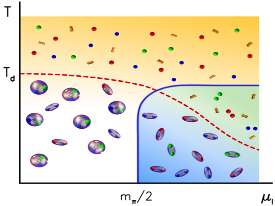
Summary

- BEC $\quad\leftrightarrow\quad \rho^{|\not{D}(\mu_I)+m|^2}(0) > 0$

  singular values useful for $\lambda \to 0$
  improvement of observables



- phase diagram and EoS for nonzero isospin asymmetry



- pions may condense in early Universe if lepton asymmetries are sizeable