Dispersing New Physics

Free Meson Seminar
TIFR

November 19th 2020

Matthew McCullough

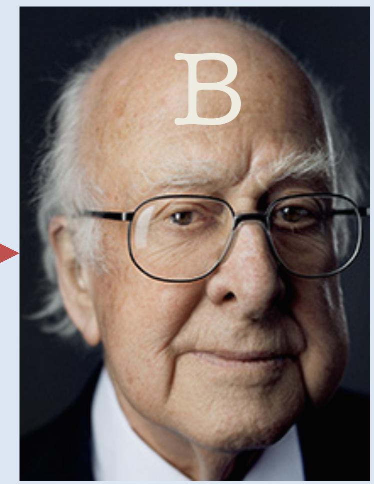
Part I: How does the Higgs move?

Based on 1903.07725 with Gian Giudice, Admir Greljo.

How Does the Higgs Move?

Surprisingly, we don't really know how the Higgs boson gets from:

to



This is especially true if it is only a very short distance. What is the propagator?

Källén-Lehmann - Recap

We can find the general form of a propagator for any scalar operator in QFT. Consider the two-point correlation function

$$\langle \Omega | \mathcal{O}(x) \mathcal{O}(y) | \Omega \rangle$$

And the complete set of momentum eigenstates of the Hamiltonian

$$\hat{P}^{\mu} | X \rangle = p_X^{\mu} | X \rangle$$

Which, due to completeness, gives us unity:

$$1 = \sum_X \int d\Pi_X | X \rangle \langle X |$$

Källén-Lehmann – Step 1

Unitarity!

$$\langle\Omega|\mathcal{O}(x)\mathcal{O}(y)|\Omega\rangle = \sum_X \int d\Pi_X \langle\Omega|\mathcal{O}(x)|X\rangle\langle X|\mathcal{O}(y)|\Omega\rangle$$

$$= \sum_X \int d\Pi_X \langle\Omega|e^{i\hat{P}x}\mathcal{O}(0)e^{-i\hat{P}x}|X\rangle\langle X|e^{i\hat{P}y}\mathcal{O}(0)e^{-i\hat{P}y}|\Omega\rangle$$

$$= \sum_X \int d\Pi_X e^{ip_X(y-x)}|\langle\Omega|\mathcal{O}(0)|X\rangle|^2 \quad .$$

We have basically just inserted 1 here.

Källén-Lehmann – Step 2

Positive Norm!

$$\langle\Omega|\mathcal{O}(x)\mathcal{O}(y)|\Omega\rangle = \int \frac{d^4p}{(2\pi)^4} e^{ip(y-x)} \left\{ \sum_X \int d\Pi_X \delta^4(p-p_X) |\langle\Omega|\mathcal{O}(0)|X\rangle|^2 \right\} \quad .$$

We have just inserted delta function. Rewriting

$$\sum_X \int d\Pi_X \delta^4(p-p_X) |\langle\Omega|\mathcal{O}(0)|X\rangle|^2 = 2\pi\theta(p^0)\rho_{\mathcal{O}}(p^2)$$

Gives: $\langle\Omega|\mathcal{O}(x)\mathcal{O}(y)|\Omega\rangle = \int \frac{d^4p}{(2\pi)^3} e^{ip(y-x)} \theta(p^0)\rho_{\mathcal{O}}(p^2)$

$$= \int_0^\infty dq^2 \rho_{\mathcal{O}}(q^2) D(x,y,q^2) \quad .$$

Källén-Lehmann – Step 3

Causality!

$$\langle\Omega|T\{\mathcal{O}(x)\mathcal{O}(y)\}|\Omega\rangle = \langle\Omega|\mathcal{O}(x)\mathcal{O}(y)|\Omega\rangle\theta(x^0 - y^0) + \langle\Omega|\mathcal{O}(y)\mathcal{O}(x)|\Omega\rangle\theta(y^0 - x^0)$$

And the Feynman propagator satisfies:

$$D(x,y,q^2)\theta(x^0 - y^0) + D(y,x,q^2)\theta(y^0 - x^0) = \int \frac{d^4p}{(2\pi)^4}\frac{ie^{ip(y-x)}}{p^2 - q^2 + i\epsilon}$$

Finally gives:

$$i\langle\Omega|T\{\mathcal{O}(x)\mathcal{O}(y)\}|\Omega\rangle = \int \frac{d^4p}{(2\pi)^4} e^{-ip(x-y)}\Pi_{\mathcal{O}}(p^2)$$

where

$$\Pi_{\mathcal{O}}(p^2) = -\int_0^\infty dq^2 \frac{\rho_{\mathcal{O}}(q^2)}{p^2 - q^2 + i\epsilon} \quad .$$

Källén-Lehmann – Step 3

Causality!

$$\langle\Omega|\ldots\rangle|\Omega\rangle = \langle\Omega|\mathcal{O}(x)\mathcal{O}(y)|\Omega\rangle\theta(x^0-y^0) + \langle\Omega|\mathcal{O}(y)\mathcal{O}(x)|\Omega\rangle\theta(y^0-x^0)$$

A ... ator satisfies:

These are the steps for deriving the KL representation. As you can see, they haven't assumed anything more than causality, unitarity, relativity.

Finally give...

$$i\langle\Omega|T\{\mathcal{O}(x)\mathcal{O}(y)\}|\Omega\rangle = \int \frac{\ldots}{(2\pi)^4}\ldots$$

where $$\Pi_{\mathcal{O}}(p^2) = -\int_0^\infty dq^2 \frac{\rho_{\mathcal{O}}(q^2)}{p^2-q^2+i\epsilon} \quad .$$

How Does the Higgs Move?

We know, thanks to Källén-Lehmann, that the propagator is, in full generality:

$$\langle 0|T\{h(z)h(0)\}|0\rangle = i\int d^4p\; e^{-ipz}\int_0^\infty dq^2\,\frac{\rho_h(q^2)}{p^2-q^2+i\epsilon}$$

This encodes information on correlations in the Higgs field between two space-time points. For a free field

$$\rho_h(q^2) = \delta(q^2 - m_h^2)$$

and we have measured the position of the pole!

How Does the Higgs Move?

In the Standard Model we have interactions, thus:

$$\rho_h(q^2) = \rho_{\mathrm{SM}}(q^2)$$

The right hand side is at least calculable...

But we don't know all fields in nature, thus all we can say in full generality is that

$$\rho_h(q^2) = \rho_{\mathrm{SM}}(q^2) + \rho_X(q^2)$$

and

$$\rho_h(q^2) \geq 0 \quad .$$

How Does the Higgs Move?

Returning to the momentum-space propagator:

$$\Delta_h(p^2) = \int_0^\infty dq^2 \frac{\rho_h(q^2)}{p^2 - q^2 + i\epsilon}$$

The density of states is associated with the poles and branch cuts at the mass scale of new (multi)particle Hamiltonian eigenstates. Thus, if the BSM states are heavy

$$\rho_X(q^2 < M^2) = 0$$

We may make some general statements.

How Does the Higgs Move?

Expanding the propagator in small momenta we have:

$$\Delta_h(p^2) = \Delta_{\text{SM}}(p^2) - \frac{1}{M^2}\sum_{n=1}^{\infty} c_n \left(\frac{p^2}{M^2}\right)^{n-1}$$

where

$$c_n = M^2 \int_0^1 dx\, \rho_X(M^2/x)\, x^{n-2} \,.$$

Some comments...

How Does the Higgs Move?

Staring at this we may make some observations:

$$c_n = M^2 \int_0^1 dx\, \rho_X(M^2/x)\, x^{n-2}\ .$$

a) From positivity of density of states

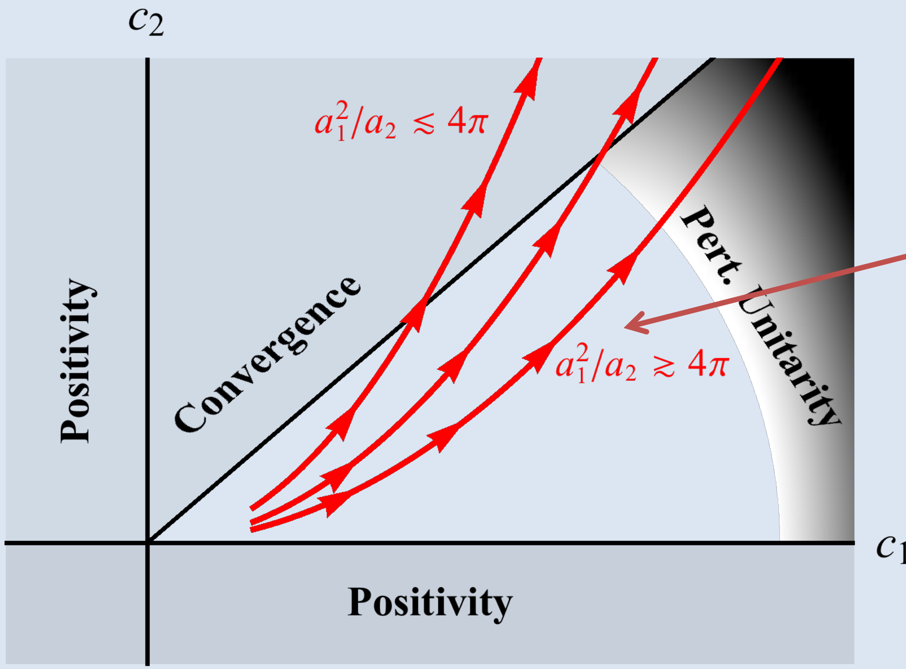
$$c_n \geqslant 0 \quad \forall n$$

b) From the integrand, we have a convergent series

$$c_n \geqslant c_{n+1} \quad \forall n$$

How Does the Higgs Move?

If one could measure leading and subleading Wilson coefficients in the momentum expansion

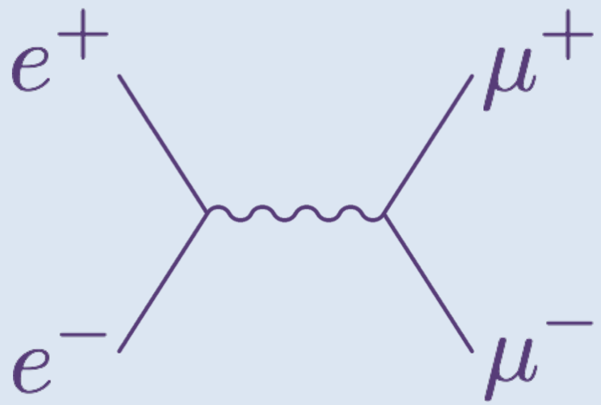


$$a_n = \frac{c_n}{M^{2n}}$$

it would be possible to extract constraints on the scale of UV-completion which are stronger than from Unitarity alone: $M^2 \leq a_2/a_1$.

Convergence Applied (a posteriori)

Consider the front-back asymmetry in low energy (E < 45 GeV), PEP, PETRA, TRISTAN:



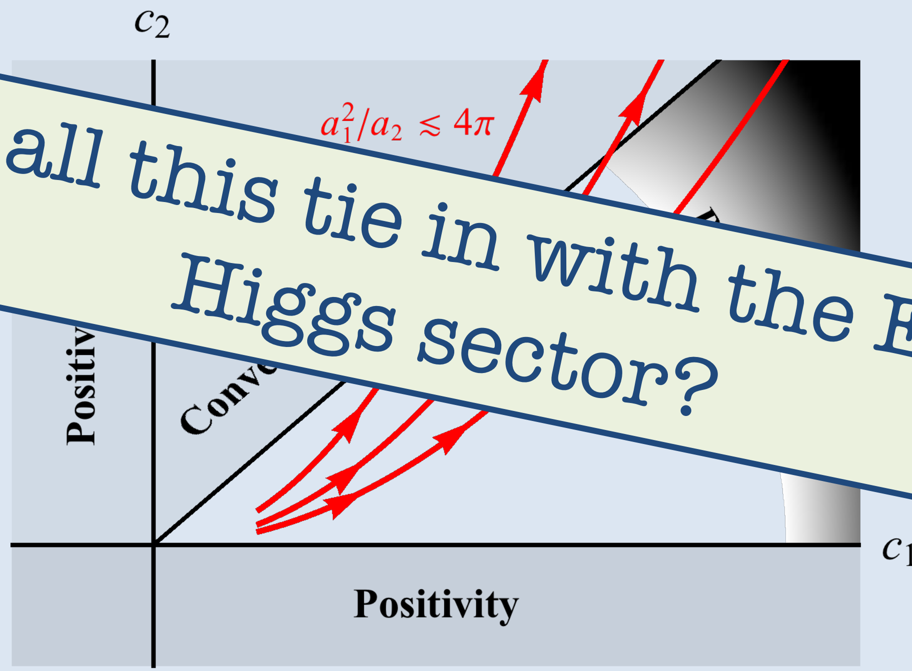
$$A_{FB}(s) = -\frac{3\,a_1\,s\,\left(1+\frac{a_2}{a_1}\,s\right)}{8\,\pi\,\alpha^2}$$

With this precision data alone, had we not already discovered the Z-boson, could have bounded, at 90%, the mass much better than from Unitary:

$$m_Z \lesssim 170\ \ \mathrm{GeV}$$

How Does the Higgs Move?

If one could measure leading and subleading Wilson coefficients in the momentum expansion



How doe all this tie in with the EFT of the Higgs sector?

it would be possible to extract constraints on the scale of UV-completion which are stronger than from Unitarity alone!

Stating a Well-Posed Question

To consider the full suite of heavy new physics possibilities, we need to go to full(ish) EFT...

$$\mathcal{O}_W = \frac{ig\, c_W}{2M^2}(H^\dagger \sigma^a \overleftrightarrow{D}^\mu H) D^\nu W^a_{\mu\nu} \qquad \mathcal{O}_{2B} = -\frac{c_{2B}}{4M^2}(\partial_\rho B_{\mu\nu})^2$$

$$\mathcal{O}_T = \frac{c_T}{2M^2}(H^\dagger \overleftrightarrow{D}^\mu H)^2$$

$$\mathcal{O}_{2G} = -\frac{c_{2G}}{4M^2}(D_\rho G^a_{\mu\nu})^2 \qquad \mathcal{O}_\Box = \frac{c_\Box}{M^2}|\Box H|^2 \qquad \mathcal{O}_{WW} = \frac{g^2\, c_{WW}}{M^2}|H|^2 W^{a\,\mu\nu} W^a_{\mu\nu}$$

$$\mathcal{O}_B = \frac{ig'\, c_B}{2M^2}(H^\dagger \overleftrightarrow{D}^\mu H)\partial^\nu B_{\mu\nu} \qquad \mathcal{O}_6 = \frac{c_6}{M^2}|H|^6 \qquad \mathcal{O}_{GG} = \frac{g_s^2\, c_{GG}}{M^2}|H|^2 G^{a,\mu\nu} G^a_{\mu\nu}$$

$$\mathcal{O}_H = \frac{c_H}{2M^2}\left(\partial^\mu |H|^2\right)^2 \qquad \mathcal{O}_R = \frac{c_R}{M^2}|H|^2 |D^\mu H|^2$$

$$\mathcal{O}_{BB} = \frac{g'^2\, c_{BB}}{M^2}|H|^2 B^{\mu\nu} B_{\mu\nu}$$

$$\mathcal{O}_{2W} = -\frac{c_{2W}}{4M^2}(D_\rho W^a_{\mu\nu})^2 \qquad \mathcal{O}_{WB} = \frac{gg'\, c_{WB}}{M^2} H^\dagger \sigma^a H B^{\mu\nu} W^a_{\mu\nu}$$

Operators like those above capture leading effects of heavy physics beyond the standard model. Probing them could reveal origins.

Organising Thoughts

Naïve dimensional analysis:

$$[H] = [A_\mu] = \frac{1}{LC} \quad , \quad [\psi] = \frac{1}{L^{3/2}C}$$

Fields carry not only dimension of inverse length, but also inverse coupling.

Fermi Scale

Interaction: $\mathcal{L} \sim \frac{\psi^4}{\Lambda^2}$

UV-completion

Dimension: $[\Lambda] = [G_F^{-1/2}] = \frac{[M_W]}{[g]}$

Coupling

Organising the UV

Higgs Only

$[g_*^0]$

$$\mathcal{O}_{\Box} = \frac{c_{\Box}}{M^2}|\Box H|^2$$

$[g_*^2]$

$$\mathcal{O}_H = \frac{c_H}{2M^2}\left(\partial^\mu |H|^2\right)^2$$

$$\mathcal{O}_T = \frac{c_T}{2M^2}(H^\dagger \overleftrightarrow{D}^\mu H)^2$$

$$\ldots \frac{c_R}{\ldots}|H|^2|D^\mu H|^2$$

$[g_*^4]$

$$\mathcal{O}_6 = \frac{c_6}{M^2}|H|^6$$

$$\mathcal{O}_{2G} = -\frac{c_{2G}}{4M^2}(D_\rho G^a_{\mu\nu})^2$$

Any new physics interacting primarily with Higgs and gauge sectors matches, at leading order, to these operators.

Mixed

$$\mathcal{O}_B = \frac{ig' c_B}{2M^2}(H^\dagger \overleftrightarrow{D}^\mu H)\partial^\nu B_{\mu\nu}$$

$$\mathcal{O}_W = \frac{ig\, c_W}{2M^2}(H^\dagger \sigma^a \overleftrightarrow{D}^\mu H)D^\nu W^a_{\mu\nu}$$

$$\mathcal{O}_{GG} = \frac{g_s^2 c_{GG}}{M^2}|H|^2 G^{a,\mu\nu}G^a_{\mu\nu}$$

$$\mathcal{O}_{WB} = \frac{gg' c_{WB}}{M^2}H^\dagger \sigma^a H B^{\mu\nu}W^a_{\mu\nu}$$

$$\mathcal{O}_{WW} = \frac{g^2 c_{WW}}{M^2}|H|^2 W^{a\,\mu\nu}W^a_{\mu\nu}$$

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Organising the UV

Higgs Only

$[g_*^0]$

$$\mathcal{O}_\Box = \frac{c_\Box}{M^2}|\Box H|^2$$

$[g_*^2]$

$$\mathcal{O}_H = \frac{c_H}{2M^2}\left(\partial^\mu |H|^2\right)^2$$

$$\mathcal{O}_T = \frac{c_T}{2M^2}(H^\dagger \overleftrightarrow{D}^\mu H)^2$$

$$\mathcal{O}_R = \frac{c_R}{M^2}|H|^2|D^\mu H|^2$$

$[g_*^4]$

$$\mathcal{O}_6 = \frac{c_6}{M^2}|H|^6$$

Gauge Only

$$\mathcal{O}_{2G} = -\frac{c_{2G}}{4M^2}(D_\rho G^a_{\mu\nu})^2$$

$$\mathcal{O}_{2W} = -\frac{c_{2W}}{4M^2}(D_\rho W^a_{\mu\nu})^2$$

$$\mathcal{O}_{2B} = -\frac{c_{2B}}{4M^2}(\partial_\rho B_{\mu\nu})^2$$

Mixed

$$\mathcal{O}_B = \frac{ig' c_B}{2M^2}(H^\dagger \overleftrightarrow{D}^\mu H)\partial^\nu B_{\mu\nu}$$

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$$\mathcal{O}_{BB} = \frac{g'^2\, c_{BB}}{M^2}|H|^2 B^{\mu\nu}B_{\mu\nu}$$

$$\mathcal{O}_{\Box} = \frac{c_{\Box}}{M^2} |\Box H|^2$$

The lowest coupling-dimension Higgs-only operator.

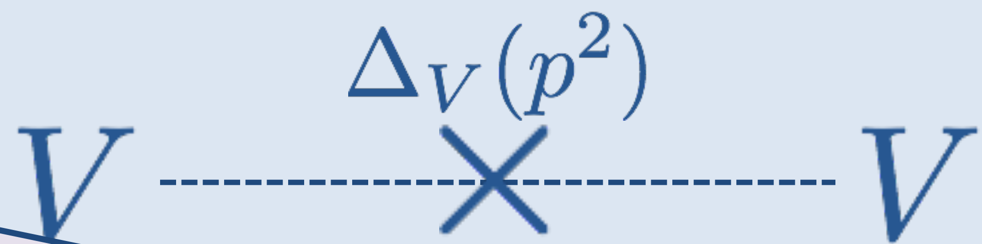
$$\mathcal{O}_{\Box} = \frac{c_{\Box}}{M^2} |\Box H|^2$$

Parameterises BSM deviations in how the Higgs moves.

Oblique Corrections

Oblique corrections have been a formidable toolkit in the effort to explore the electroweak sector.

- S-parameter
- T-parameter
- Y-parameter



We’ve been here before…

The latter two contribute to amplitudes in a
growing” manner:

$$\Delta_W(p^2) \approx \frac{1}{p^2 - M_W^2} - \frac{\hat{W}}{M_W^2}$$

Making these oblique parameters an excellent target for hadron colliders…

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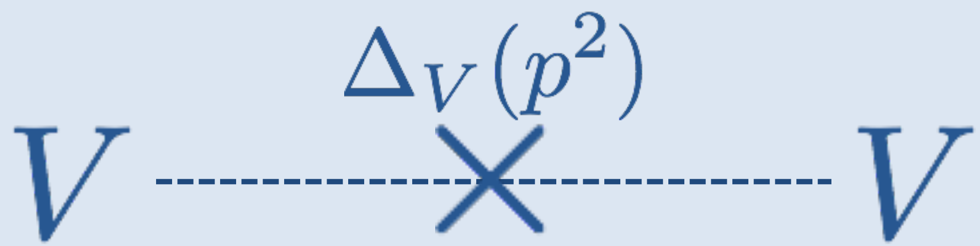

The latter two contribute to amplitudes in an "energy-growing" manner:

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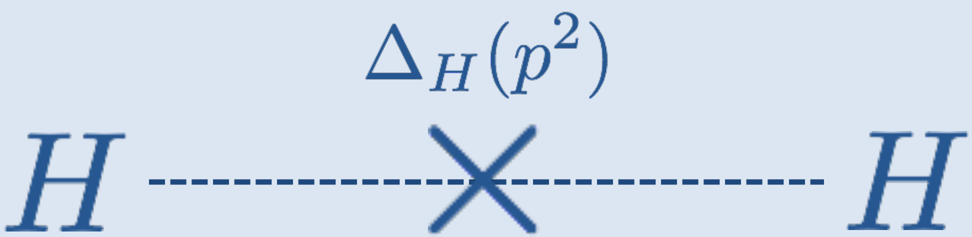
Oblique Corrections

Makes sense to extend to the Higgs sector. Especially since the Higgs can easily interact with new states…

- H-parameter:



1903.07725

This also contributes to amplitudes in an "energy-growing" manner:

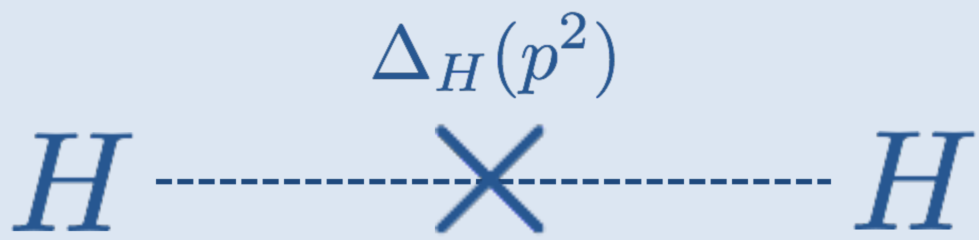
$$\Delta_H(p^2) \approx \frac{1}{p^2 - m_h^2} - \frac{\hat{H}}{m_h^2} + \ldots$$

However, one needs to take the Higgs off-shell, which isn't easy…

Oblique Corrections

Makes sense to extend to the Higgs sector. Especially since the Higgs can easily interact with new states...

- H-parameter:



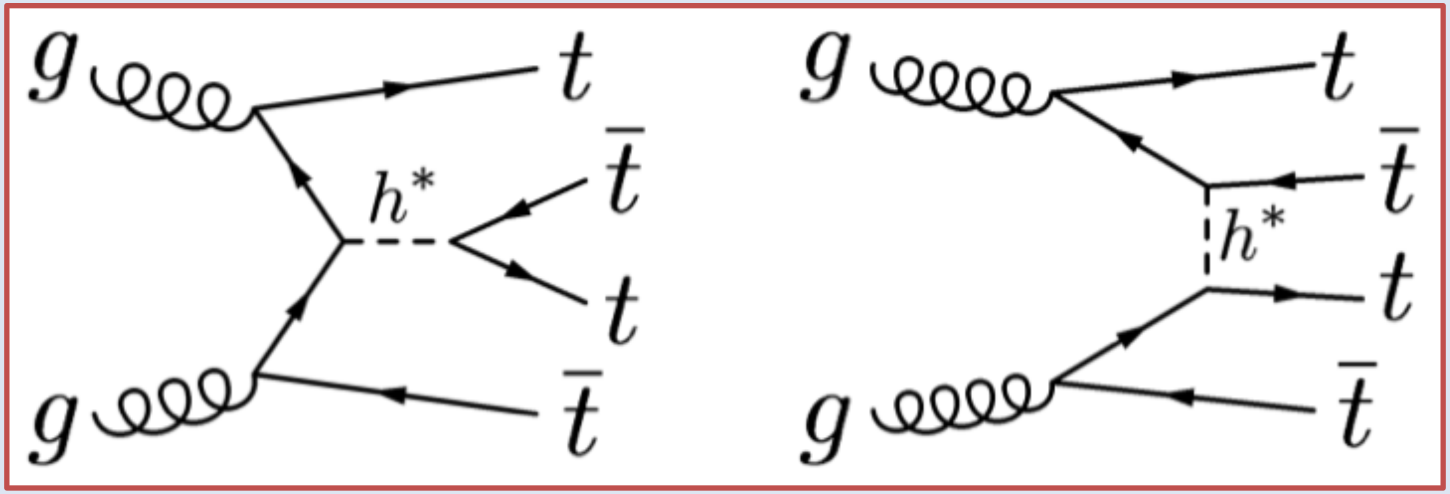
One can also translate basis to one in which this is a four-fermion operator and some more involving the Higgs

$$\mathcal{O} \propto \frac{\lambda^2 \hat{H}}{m_h^2}(\overline{\psi}\psi)^2$$

If new physics model interacts primarily with Higgs, then original basis may be better for interpretation purposes.

Oblique Corrections

Most promising avenue to take this Higgs off-shell is through four-top production:

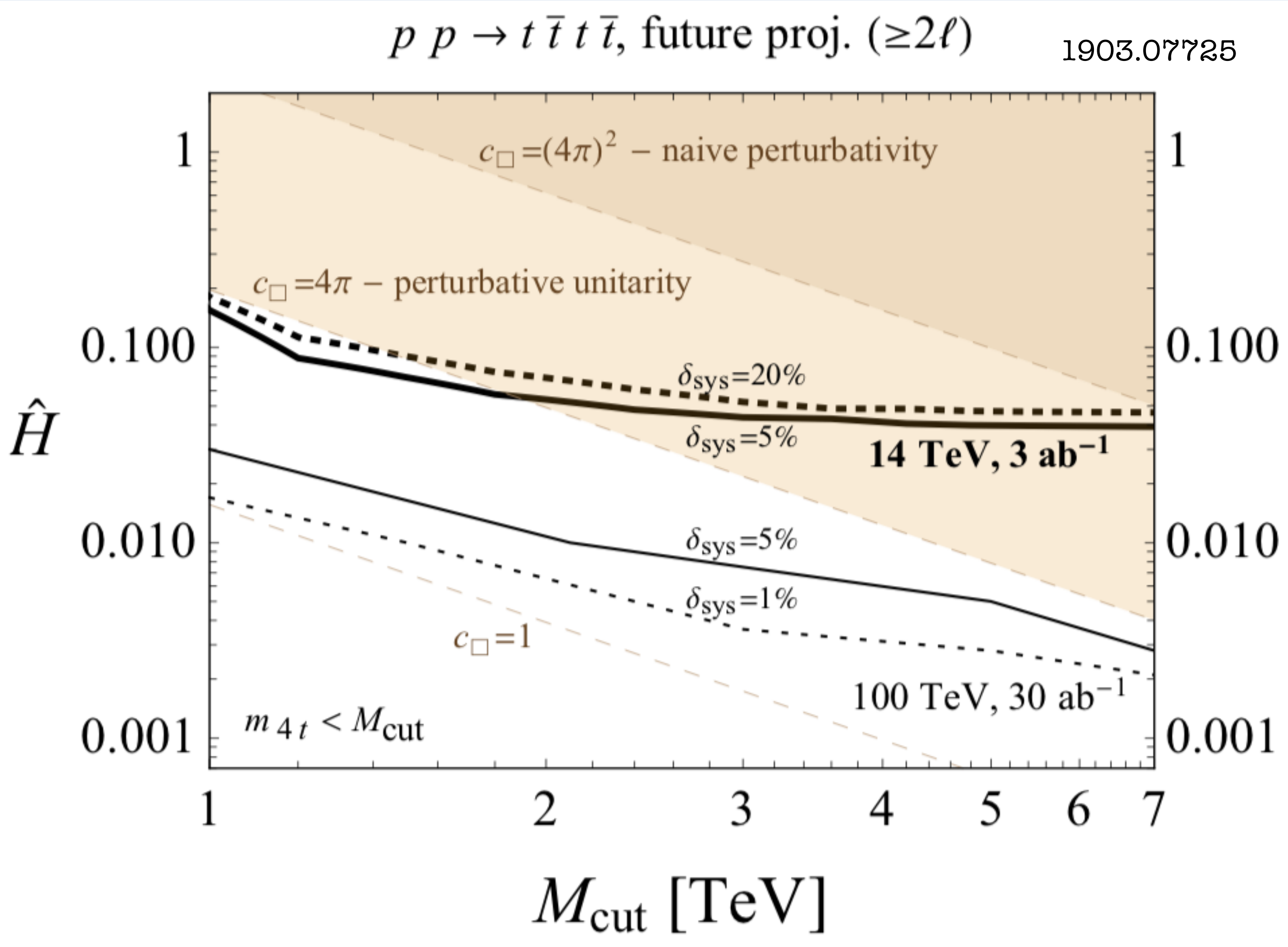


We may relate this Wilson coefficient to the scale of new physics as:

$$\frac{\hat{H}}{m_h^2} = \frac{c_\Box}{M^2}$$

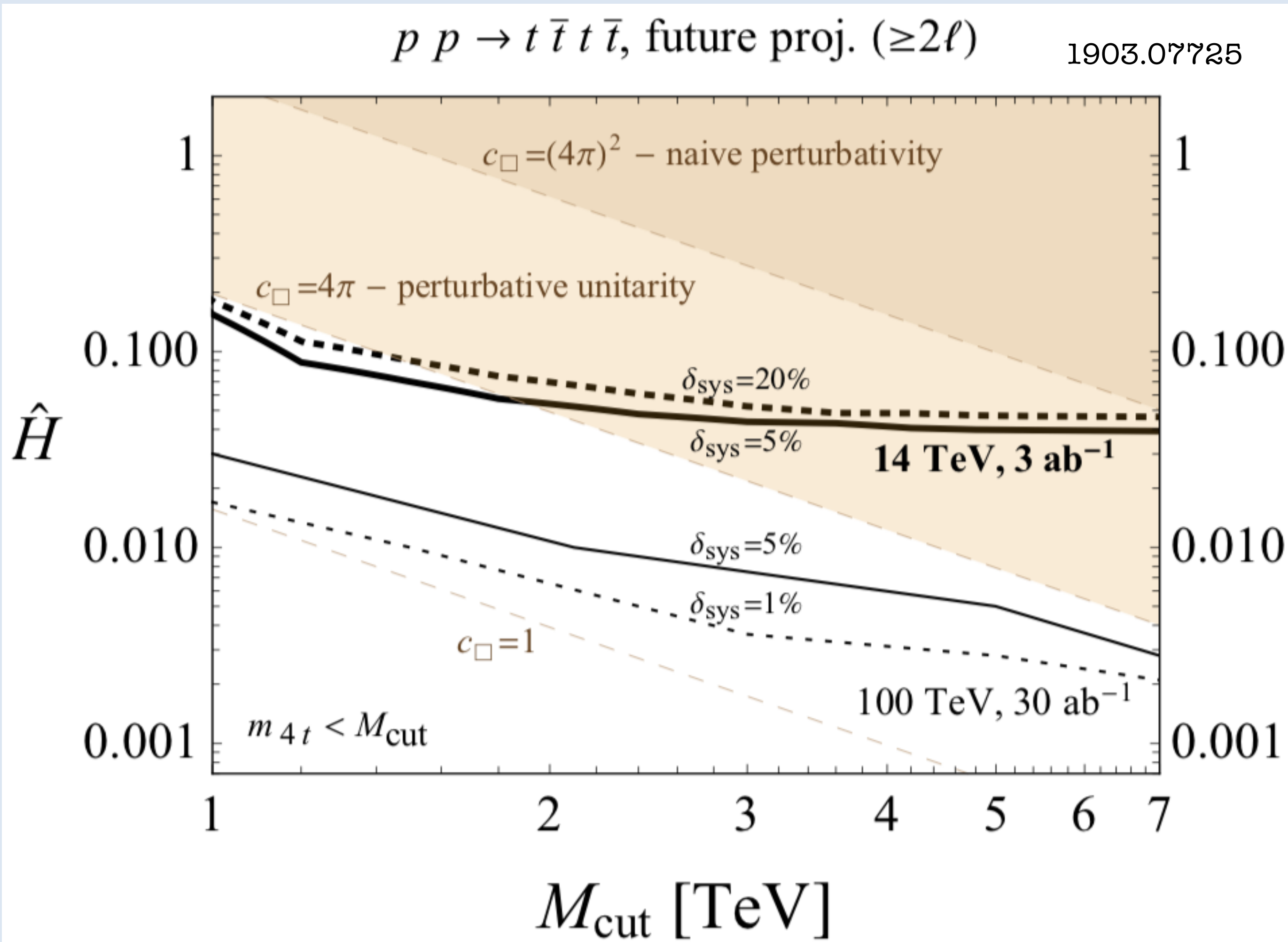
A Unique Operator

Our estimate suggests meaningful constraints are possible, but challenging at the HL-LHC:



A Unique Operator

Future proton colliders could do much much better:



A Unique Operator

CMS does better than our estimates:

Abstract

1908.06463

The standard model (SM) production of four top quarks ($t\bar{t}t\bar{t}$) in proton-proton collision is studied by the CMS Collaboration. The data sample, collected during the 2016–2018 data taking of the LHC, corresponds to an integrated luminosity of 137 fb^{-1} at a center-of-mass energy of 13 TeV. The events are required to contain two same-sign charged leptons (electrons or muons) or at least three leptons, and jets. The observed and expected significances for the $t\bar{t}t\bar{t}$ signal are respectively 2.6 and 2.7 standard deviations, and the $t\bar{t}t\bar{t}$ cross section is measured to be $12.6^{+5.8}_{-5.2}$ fb. The results are used to constrain the Yukawa coupling of the top quark to the Higgs boson, y_t, yielding a limit of $|y_t/y_t^{SM}| < 1.7$ at 95% confidence level, where y_t^{SM} is the SM value of y_t. They are also used to constrain the oblique parameter of the Higgs boson in an effective field theory framework, $\hat{H} < 0.12$. Limits are set on the production of a heavy scalar or pseudoscalar boson in Type-II two-Higgs-doublet and simplified dark matter models, with exclusion limits reaching 350–470 GeV and 350–550 GeV for scalar and pseudoscalar bosons, respectively. Upper bounds are also set on couplings of the top quark to new light particles.

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Still a long way off meaningfully probing how the Higgs propagates…

Convergence and forward scattering amplitudes.

Based on 1903.07725 with Gian Giudice, Admir Greljo.

Forward Scattering Amplitudes

Convergence goes beyond Källén-Lehmann.

Consider the forward scattering amplitude:

$$\mathcal{M}(s) = \int_0^\infty dq^2 \left(\frac{F(q^2)}{s+q^2+i\epsilon} - \frac{F(q^2)}{s-q^2-i\epsilon} \right) + \mathrm{Poly}(s)$$

The Taylor series coefficients are

$$b_{n>l/2} = \frac{1}{2n!} \frac{d^{2n}\mathcal{M}(s)}{ds^{2n}}\bigg|_{s=0}$$

and also satisfy convergence...

$$M^4 \frac{b_{n+1}}{b_n} \leqslant 1$$

tending to unity as n grows.

Forward Scattering Amplitudes

Convergence goes beyond Källén-Lehmann.

C[...] the forward scattering amplitude:

$$\ldots \frac{F(q^2)}{\ldots} \Big) + \mathrm{Poly}(s)$$

This applies to any scalar forward scattering amplitude…. The EFT expansion in energy is absolutely convergent. A high-power term cannot be larger than the ones that came before.

and also satisfy converge[...]

$$M^4 \frac{b_{n+1}}{b_n} \leqslant 1$$

tending to unity as n grows.

Forward Scattering Amplitudes

Convergence goes beyond Källén-Lehmann.

Consider the forward scattering amplitude:

$$\mathcal{M}(s) = \int^{\infty} dq^2 \left(\frac{F(q^2)}{s + q^2 + i\epsilon} - \frac{F(q^2)}{s - q^2 - i\epsilon} \right) + \mathrm{Poly}(s)$$

See also 1903.08664, 2011.00037.

The Taylor coefficients are

$$b_{n>l/2} = \frac{1}{2n!} \frac{d \cdots}{ds^{2n}} \Big|_{s=0}$$

and also satisfy convergence...

$$M^4 \frac{b_{n+1}}{b_n} \leqslant 1$$

tending to unity as n grows.

Amusing Application

The forward limit of the Veneziano amplitude is:

$$\mathcal{M}(s) \propto s \tan\left(\alpha' \frac{\pi s}{2}\right)$$

which gives, upon expanding,

$$R_n = -\frac{\pi^2}{(2n+2)(2n+1)} \frac{2^{2n+2}-1}{2^{2n}-1} \frac{\mathcal{B}_{2n+2}}{\mathcal{B}_{2n}}$$

and the dispersion relation gives

$$\lim_{n\to\infty} (2n+2)(2n+1) \frac{2^{2n}-1}{2^{2n+2}-1} \frac{\mathcal{B}_{2n}}{\mathcal{B}_{2n+2}} = -\pi^2 \quad .$$

Amusing Application

The forward limit of the Veneziano amplitude is:

$$\mathcal{A}(s) \propto s \tan\left(\alpha' \frac{\pi s}{2}\right)$$

This result was derived relatively recently in number theory, and gives a cute connection between convergence, strings, and number theory.

and the dispersion relatio...

$$\lim_{n\to\infty} (2n+2)(2n+1)\frac{2^{2n}-1}{2^{2n+2}-1}\frac{\mathcal{B}_{2n}}{\mathcal{B}_{2n+2}} = -\pi^2 .$$

Part II: Dispersing the Fifth Force

Based on 2009.12399 with Hannah Banks.

Dark Sectors

Evidence for dark matter is now overwhelming

- Rotation curves
- CMB
- Large scale structure
- Velocity dispersions
- Gravitational lensing (Bullet Cluster)
- ....

Yet we have no clue what it is at the particle level!

?

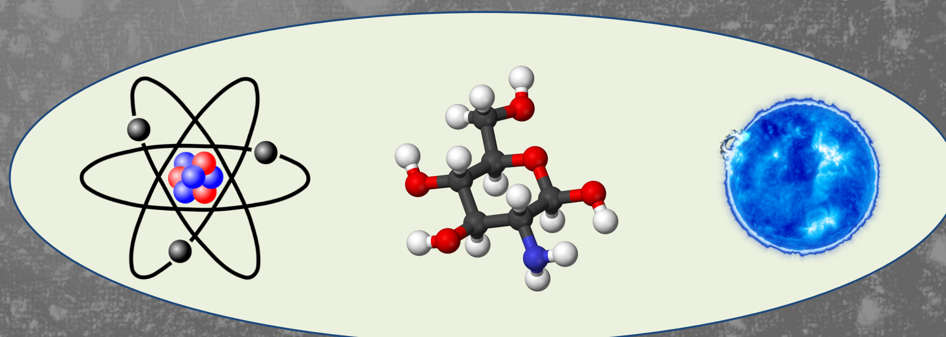
Only 18% of all matter in Universe is visible.

$e \quad u \quad d \quad Z \quad h$

$\mu \quad c \quad s \quad g$

$\tau \quad t \quad b \quad \gamma \quad W$

Within that 18% we observe extraordinary complexity.

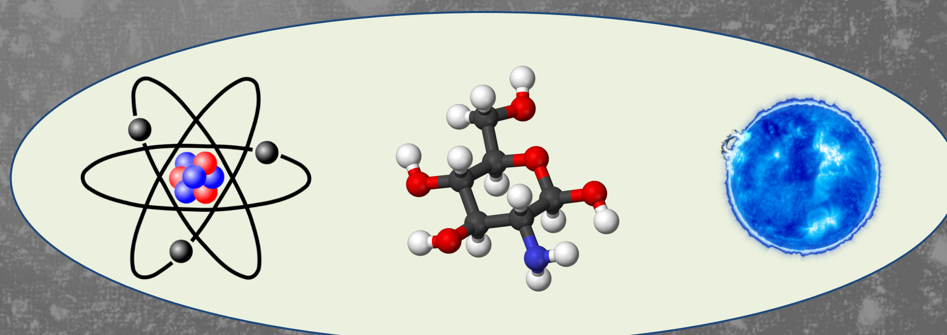


The photon, despite not being matter itself, gave us our first tool to explore the visible sector.

Only 18% of all matter in Universe is visible.

e u d Z h μ c s g τ t b γ W

Within that 18% we observe extraordinary complexity.



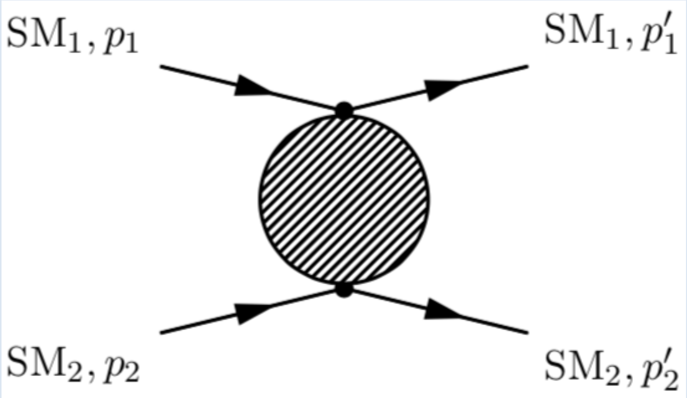
Similarly, it may be the light mediators, or other states, that open the window to the dark sector.

Dispersing Fifth Forces

Suppose some SM states are coupled to some hidden sector operator as

$$\mathcal{L} = \lambda \overline{\Psi}_{\rm SM} \Psi_{\rm SM} \mathcal{O}_{\rm HS}$$

then diagrams such as



will generate a fifth force, which is captured by

$$\langle \mathcal{O}_{\rm HS}(x) \mathcal{O}_{\rm HS}(y) \rangle$$

Dispersing Fifth Forces

A general two-point function like

$$\langle \mathcal{O}_{\rm HS}(x)\mathcal{O}_{\rm HS}(y)\rangle$$

is completely captured by the KL representation!

We can insert this into Born's approximation

$$V(\boldsymbol{r}) = -\frac{1}{4M^2}\int d^3\boldsymbol{q}\ \frac{\mathcal{M}^{\rm NR}}{(2\pi)^3}e^{i\boldsymbol{q}\cdot\boldsymbol{r}} \quad ,$$

and employ the KL representation for the propagator in the matrix element

$$\Delta(q) = 2\int_0^\infty \mu d\mu\ \frac{\rho(\mu^2)}{q^2-\mu^2+i\epsilon} \quad ,$$

Dispersing Fifth Forces

...to derive the general form of a scalar fifth force, regardless of the form of the hidden sector (perturbative, strongly coupled, minimal, complex, whatever):

$$V(r) = -\frac{\lambda^2}{2\pi r}\int_0^\infty \mu d\mu \;\rho(\mu^2) e^{-\mu r} \quad .$$

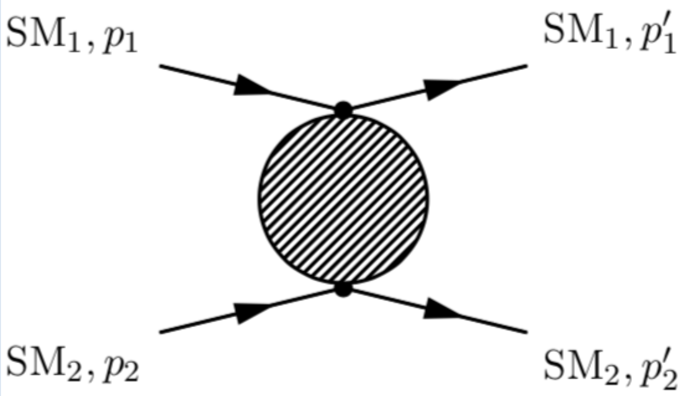
Note that positivity of density of states implies attractive force. Also, differentiating w.r.t. distance means no turning points, implying a monotonic dependence on distance.

Examples

For a given model we may readily extract the spectral density using the optical theorem

$$\rho(q^2) = -\frac{1}{\pi}\mathrm{Im}\{\Delta(q)\} \quad .$$

Thus all we need is the imaginary part of the loop in:

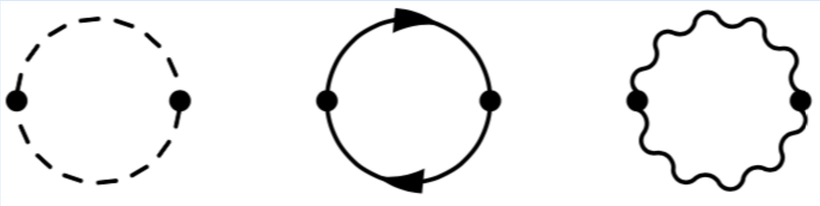


Examples

Simple one-loop examples include:

(A) $\frac{1}{\Lambda}\mathcal{O}_{SM}|\phi|^2$ (B) $\frac{1}{\Lambda^2}\mathcal{O}_{SM}\bar{\psi}\psi$ (C) $\frac{m^2}{\Lambda^3}\mathcal{O}_{SM}|V|^2$ (D) $\frac{1}{\Lambda^3}\mathcal{O}_{SM}\partial_\mu\phi^*\partial^\mu\phi$

Extracting the imaginary part of the loops



there is no loop integral required since

$$2\mathrm{Im}\{\mathcal{M}(A\to A)\} = \sum_X \int d\Pi_X (2\pi)^4\delta^4(p_A - p_X)|\mathcal{M}(A\to X)|^2 \quad .$$

which enables straightforward extraction.

Examples

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The resulting density of states and potentials are

Operator	$\rho(\mu^2)$	$V(r)$
(A)	$\frac{\eta}{8\pi^2}\left(1-\frac{4m^2}{\mu^2}\right)^{\frac{1}{2}}\ \Theta(\mu^2-4m^2)$	$-\frac{\eta m}{8\Lambda^2\pi^3 r^2}K_1(2mr)$
(B)	$\frac{\mu^2\eta}{4\pi^2}\left(1-\frac{4m^2}{\mu^2}\right)^{\frac{3}{2}}\ \Theta(\mu^2-4m^2)$	$-\frac{3\eta m^2}{2\Lambda^4\pi^3 r^3}K_2(2mr)$
(C)	$\frac{\mu^4\eta}{32m^4\pi^2}\left(1+\frac{12m^4}{\mu^4}-\frac{4m^2}{\mu^2}\right)\left(1-\frac{4m^2}{\mu^2}\right)^{\frac{1}{2}}\ \Theta(\mu^2-4m^2)$	$-\frac{3m^3\eta(5+m^2r^2)}{8\Lambda^6\pi^3 r^4}K_3(2mr)$
(D)	$\frac{\mu^4\eta}{32\pi^2}\left(1-\frac{4m^2}{\mu^2}+\frac{4m^4}{\mu^4}\right)\left(1-\frac{4m^2}{\mu^2}\right)^{\frac{1}{2}}\ \Theta(\mu^2-4m^2)$	$-\frac{\eta}{8\Lambda^6\pi^3}\left(\frac{15m^3}{r^4}+\frac{m^5}{r^2}\right)K_1(2mr)$ - $\frac{\eta}{4\Lambda^6\pi^3}\left(\frac{15m^2}{r^5}+\frac{3m^4}{r^3}\right)K_2(2mr)$

Examples

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[illegible] states and potentials are

Mostly matching the results in 1710.00850, derived with more conventional methods.

Operator	[illegible]	$V(r)$
(A)	$\frac{\eta}{8\pi^2}\left(1-\frac{4m^2}{\mu^2}\right)$ [illegible]	[illegible]
(B)	$\frac{\mu^2\eta}{4\pi^2}\left(1-\frac{4m^2}{\mu^2}\right)^{\frac{3}{2}}\ \Theta(\mu^2-4m^2)$	[illegible]
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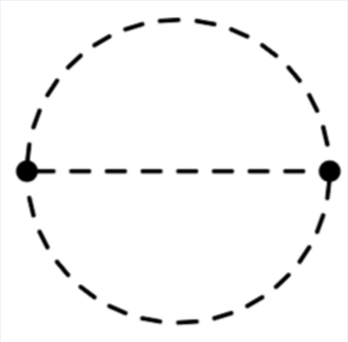
Examples

This makes it easy to go to higher loops. For instance:



gives, without any loop calculation,

$$\rho_3(\mu^2) = \frac{3\sqrt{(\mu - m)(\mu + 3m)}}{128\mu^2\pi^4}\left(\frac{(\mu - m)(\mu^2 + 3m^2)}{2}E(\tilde{k}) - 4m^2\mu K(\tilde{k})\right)\Theta(\mu^2 - 9m^2)$$

However, the real punchline is that the KL representation captures anything consistent with QFT fundamentals.

The Experimental Landscape

The standard suite of fifth force experimental observables can also be straightforwardly recast in a completely general form!

Molecular Spectroscopy

$$\Delta E_\psi = -\frac{\lambda^2}{2\pi} \int d^3\boldsymbol{r}\ \psi^*(r) \frac{1}{r} \left(\int_0^\infty d\mu\ \mu\ \rho(\mu^2) e^{-\mu r} \right) \psi(r)$$

Bouncing Neutrons

$$\delta V(z) = \frac{-\lambda^2 \rho_{\text{glass}}}{m_n} \int_0^\infty d\mu\ \frac{\rho(\mu^2) e^{-\mu z}}{\mu} \ .$$

The Experimental Landscape

Effective Planar Geometry Exps

$$F(s) = -\frac{2\pi R\lambda^2}{m_n^2}\int_0^\infty d\mu \, \frac{\rho(\mu^2)e^{-\mu s}}{\mu^2}\left(\rho_{\rm Au} + (\rho_{\rm sap} - \rho_{\rm Au})\, e^{-\mu\Delta}\right)\left(\rho_{\rm Au} + (\rho_{\rm pol} - \rho_{\rm Au})\, e^{-\mu\Delta}\right)$$

Cold Neutron Scattering

$$l_{\rm BSM}(\mathbf{q}) = 2m_N V(\mathbf{q}) = -2m_N\lambda^2\int_0^\infty d\mu^2\, \frac{\rho(\mu^2)}{|\mathbf{q}|^2+\mu^2}$$

Lunar Perihelion Precession

$$\delta\theta = \frac{\lambda^2}{Gm_n^2(1-\epsilon^2)}\int_0^\infty d\mu \; \mu \; \rho(\mu^2)e^{-\mu a}\left[1+\mu a+\frac{(\mu a)^2}{2}\right]$$

Limits on Perturbative Models

Simple one-loop examples include:

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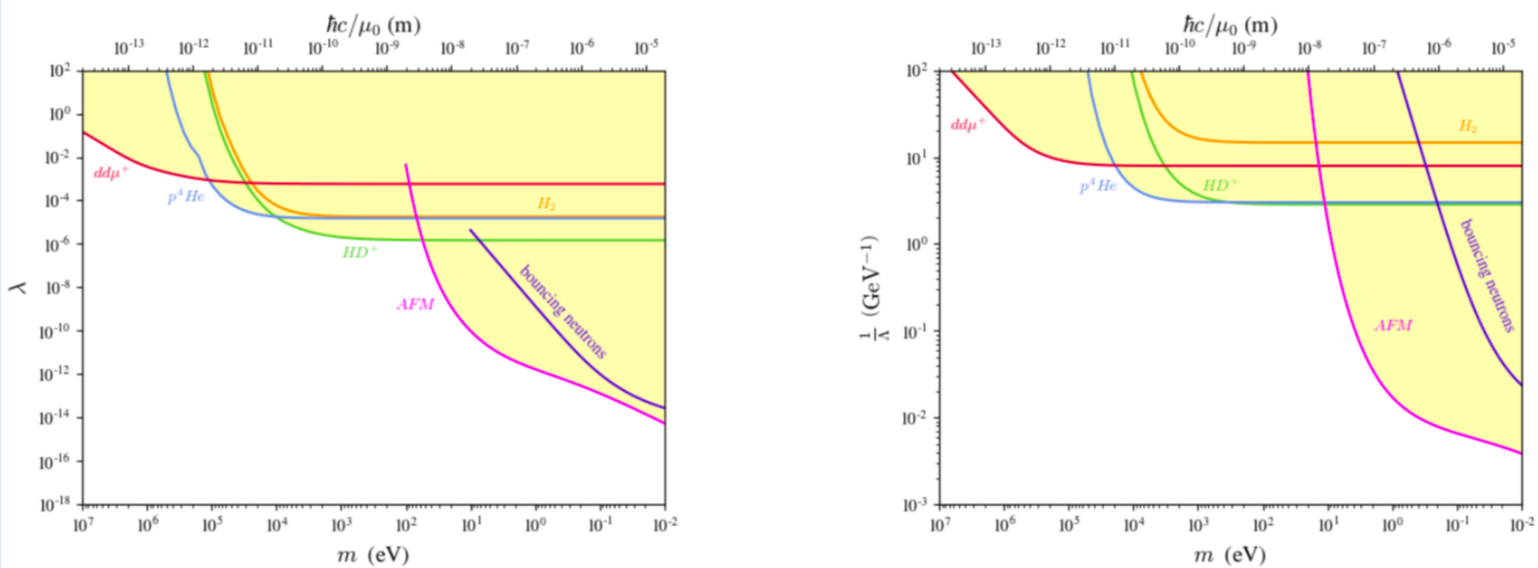
Which give constraints such as



(i) Yukawa interaction

(ii) Interaction (A)

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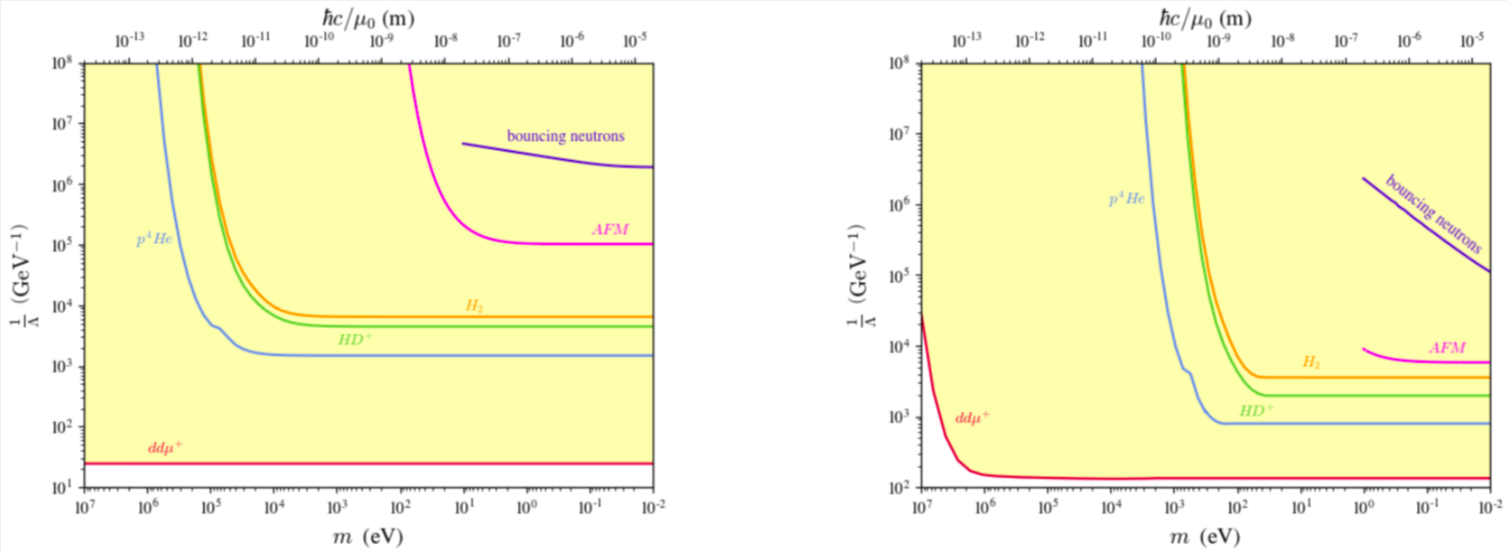
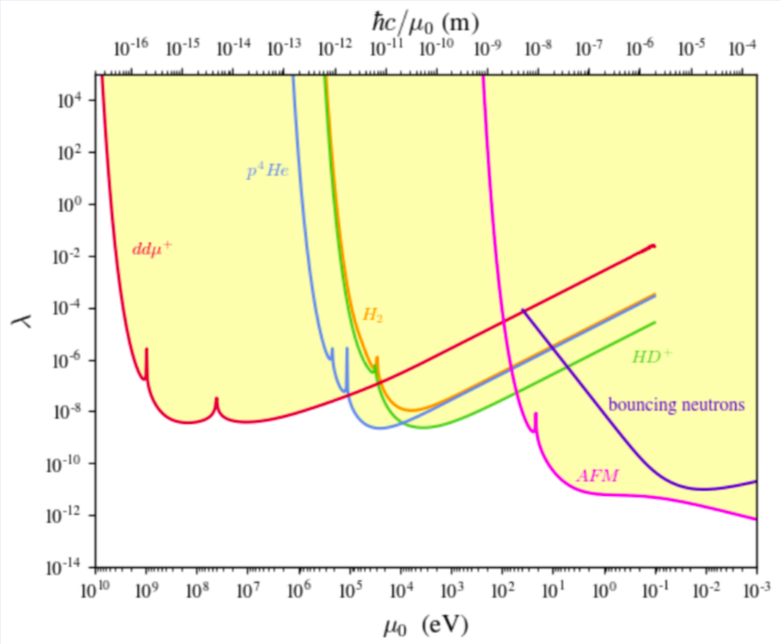


(v) Interaction (D)

(vi) $\frac{1}{\Lambda^2}\mathcal{O}_{SM}\phi^3 \equiv$ Interaction (E)

What QFT Violation Looks Like…

For a scalar force with a turning point, experimental limits change form significantly…



Don't have an actual concrete model of causality violation, so this is just for a simple toy model which has negative density of states regions.

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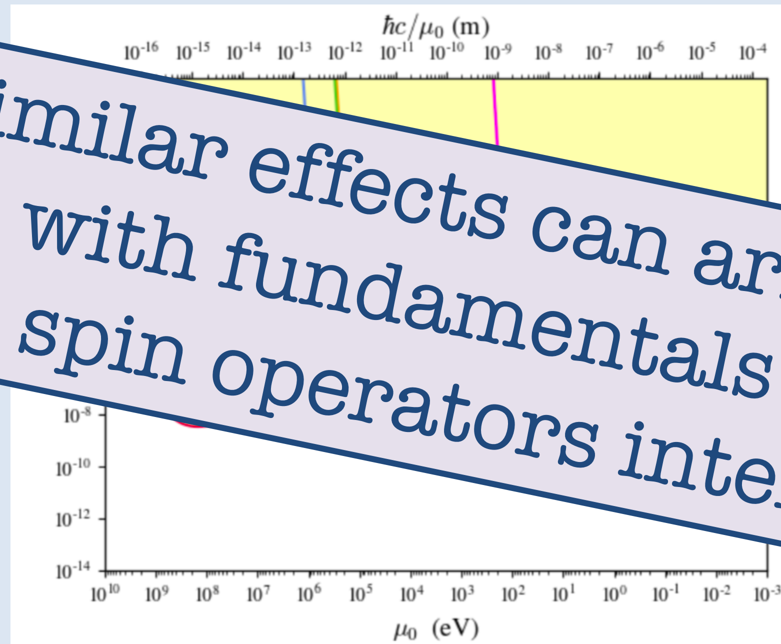

Note that similar effects can arise which are consistent with fundamentals if you have different spin operators interfering...

Don't have an actual concrete model of causality violation, so this is just for a simple toy model which has negative density of states regions.

Part III: Summary

In the early days of QFT some pioneers found beautiful, powerful, general results that are fully non-perturbative. Usually expressed in the form of dispersion relations like the KL rep.

Gunnar Källén
1926-1968

Harry Lehmann
1924-1998

These dispersion relations are powerful tools for understanding the possible structure of effective field theories, when new states are heavy and decoupled, leaving only irrelevant imprints on the IR theory.

Not only positivity

$$c_n \geqslant 0 \quad \forall n$$

but also convergence

$$c_n \geqslant c_{n+1} \quad \forall n$$

follows for two-point functions and forward scattering amplitudes.

When new states are not decoupled and enter physical observables on-shell we have few tools at hand. Often resort to toy models. However, dispersion relations still apply and allow us to make non-perturbative statements even for very light hidden sectors weakly coupled to us:

$$V(r) = -\frac{\lambda^2}{2\pi r}\int_0^\infty \mu d\mu \; \rho(\mu^2) e^{-\mu r} \quad .$$

As we look increasingly towards the hidden sector we should not be beguiled by toy models. We need a comprehensive search program and thus a comprehensive theoretical apparatus.

Maxwell's rotating vortices were the model that guided the development of the equations which arguably laid the foundations for the dawn of modern field theory in physics.

As we seek to look beyond the Standard Model, whether at the highest energies or the tiniest couplings, we should not let our models hide deeper truths which may be baked into quantum field theory, whether we know it or not.