Kalman Filter and Tracking

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Track reconstruction

1. Identification of “hits”
2. Track finding
3. Track fitting
4. Track filtering

Track Finding

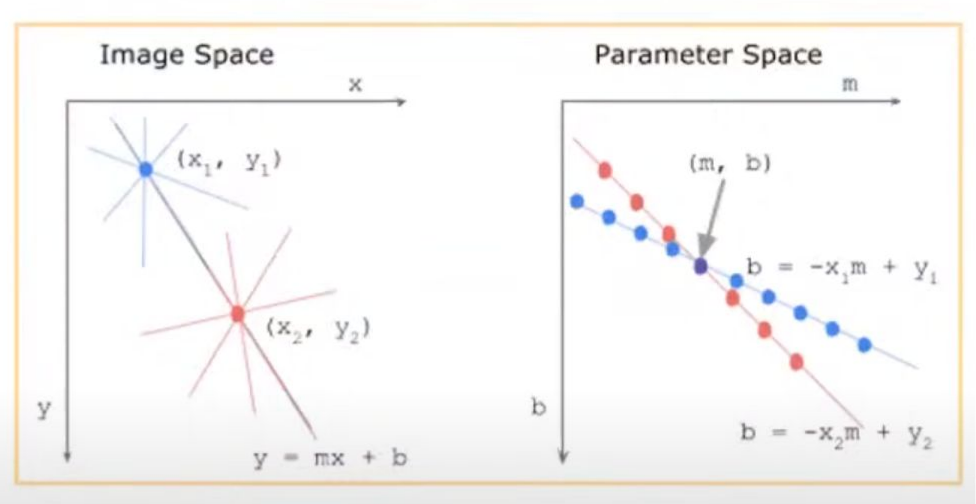
Hough transformation:

If points lie in y=mx+b, then

Transform into (m,b)

Usually transform in (r,Φ)

to use $\Phi = \Phi_0 - (0.3Bq/p_T).r$



Tracking fitting

Let's start with choosing track parameters

- The parameters should be continuous with respect to small changes of the trajectory.
- The choice of track parameters should have the local expansion of the track model into a linear function.
- The uncertainties of the estimated parameters should follow a Gaussian distribution as closely as possible.

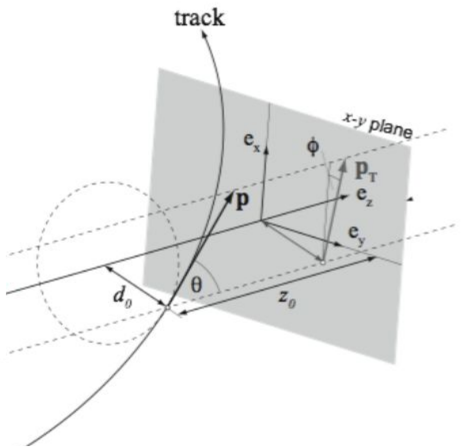
Track Parameters

Without magnetic field

$(x , y , \tan\theta_x , \tan\theta_y , Q/p)$

With magnetic field

$(d_0 , \phi_0 , z_0 , \tan\theta , Q/p_T)$



Naive Approach: Least Square Fitting

Let the measurements be m_i and parameters be p_i. Initials guess of parameters is p_A

Now,

$$m_i = f(p)$$

$$m_i = f(p_A) + (\partial f/\partial p_i)(p - p_A)$$

$$\chi^2 = \Sigma(m_i - f(p_A) + A(p - p_A))^2/\sigma_i^2$$

$$= (m_i - f(p_A) + A(p - p_A))^T V^{-1} (m_i - f(p_A) + A(p - p_A))$$

$$= (\Delta m_i + A(p - p_A))^T V^{-1} (\Delta m - A(p - p_A))$$

where $\Delta m_i = m_i - f(p_A)$ and V is covariance matrix of m_i

Least Square Fitting (continued)

Solution is

$$p = p_o + (A^T V^{-1} A)^{-1} A^T V^{-1} (m - f(p_o))$$

Features:

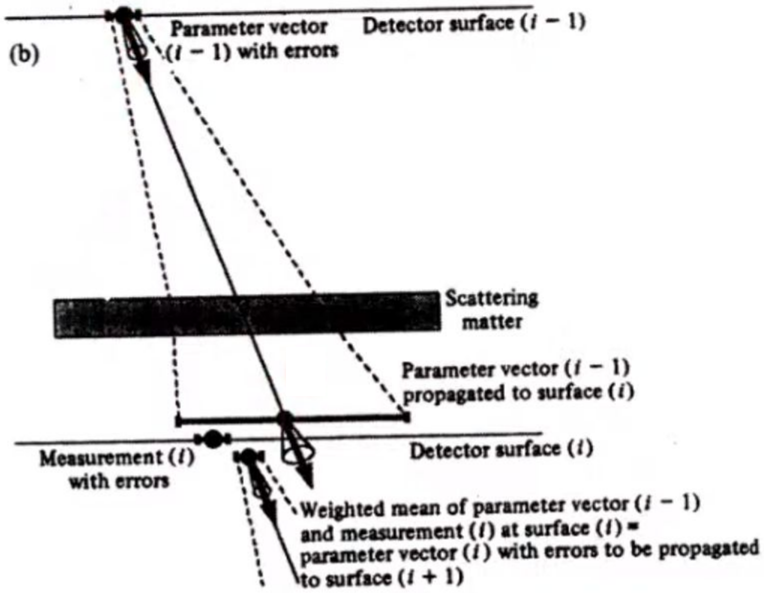
- Global fit
- Works well if function f is (sufficiently) linear and if the measurements m_i follow a normal distribution.
- $cov(p) = (A^T V^{-1} A)^{-1}$

Better Approach: Kalman filter

Game changer: Adopt “progressive” thinking

Three types of operations:

- Filtering is the estimation of the "present" state vector, based upon all "past" measurements.
- Prediction is the estimation of the state vector at a ' future" time.
- Smoothing is the estimation of the state vector at some time in the "past" based on all measurements taken up to the "present" time.



Algebra of Kalman filter

- **State vector at any step is the combination of extrapolation from previous measurement and measurement at that point,**

$$p_k^k = K_k^1\, p_k^{k-1} + K_k^2\, m_k, \qquad p_k^{k-1} = F_{k-1}\, p_{k-1}$$

where, K_k^1 and K_k^2 are two weight factors, p_k^{k-1} is the expected state vector from previous measurements

- **Weight factors is calculated (for true state vector, p) from the minimisation of**

$$\chi^2 = \left(m_k - f(p)\right)^T V^{-1} \left(m_k - f(p)\right) + \left(p - p_k^{k-1}\right)^T \left(C_k^{k-1}\right)^{-1} \left(p - p_k^{k-1}\right)$$

$$V = \left(V_k + A_k\, C_k^{k-1}\, A_k^T\right)$$

$$C_k^{k-1} = F_{k-1}\, C_{k-1} F_{k-1}^T + Q_{k-1}$$

$$K_k = C_k^{k-1}\, A_k^T \left(V_k + A_k\, C_k^{k-1}\, A_k^T\right)^{-1}$$

$$p_k = F_{k-1}\, p_{k-1} + K_k\left(m_k - A_k\, F_{k-1}\, p_{k-1}\right) = \left(I - K_k\, A_k\right) p_k^{k-1} + K_k\, m_k$$

$$C_k = \left(I - K_k\, A_k\right) C_k^{k-1}$$

where F is propagator matrix of state vector, C is covariance matrix of parameter,V is error matrix of measurements and Q is noise matrix due to MS and energy loss

Kalman filter

Advantages:

The linear approximation of the track model needs to be valid only over a short range

No large matrices have to be inverted.

Cons:

The track parameters are known with optimal precision only after the last step of the fit

Thank You!