X-ray Spectral Response

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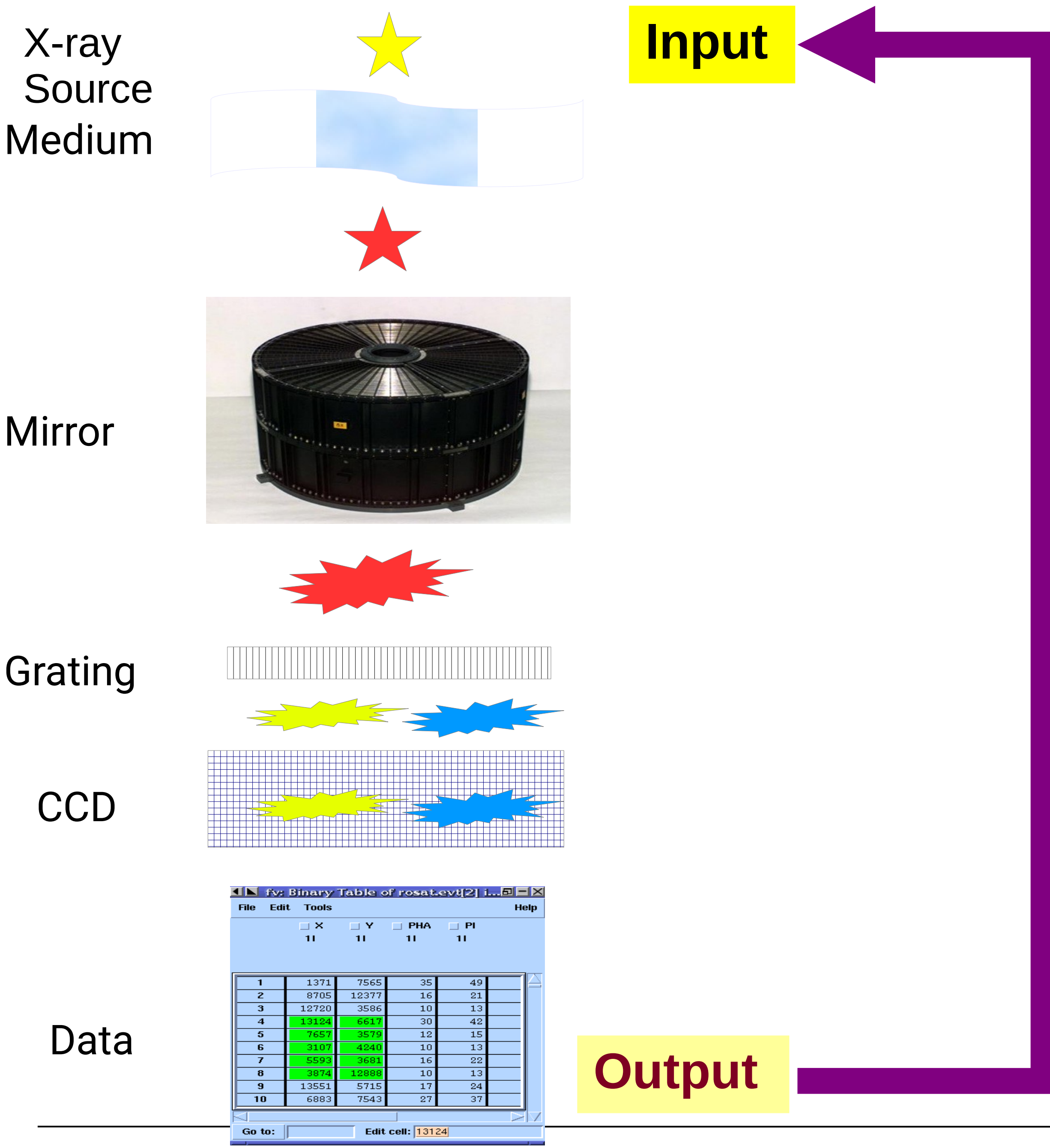
X-ray Astronomy



	X	Y	PHA	PI
	1I	1I	1I	1I
1	1371	7565	35	49
2	8705	12377	16	21
3	12720	3586	10	13
4	13124	6617	30	42
5	7657	3579	12	15
6	3107	4240	10	13
7	5593	3681	16	22
8	3874	12888	10	13
9	13551	5715	17	24
10	6883	7543	27	37

- **Image (spatial distribution)**
- **Spectrum**
- **Light curves (variability)**
- **Polarization**

How to go
from output to
input?

X-ray Detection Techniques

Two steps in X-ray detection technique

	Non-Imaging	Soft X-ray Imaging < 12 keV	Indirect Hard X-ray Imaging > 10 keV	Direct Hard X-ray imaging > 5 keV
1. Collection of X-rays from some portion of sky	Collimators	X-ray Optics	Coded Aperture Masks	Multi-layer optics
2. Detection of X-rays	Proportional counters, Scintillators, Semiconductor detectors	CCDs, MCPs, PSPCs	Position sensitive detectors (CZTI, PSPC)	Position sensitive detectors (CZTI)

X-ray data

- X-ray detectors measure individual photons. An X-ray photon recorded in the detector is called event.
- Basic data structure is a list of detected events, called the event file. Each event has a set of attributes
  - Arrival Time
  - Attributes related to energy (PHA, PI) (PI: PHA values corrected for gain)
  - Coordinates (if two dimensional detector)
  - Attributes related to discriminate good events from background or bad events (such as grade or pattern, status or flag in CCDs)

Sky Coordinates

- Initially event position on the detector which may consist of multiple CCDs
  (RAWX,RAWY) = event on CCD2 at position (132,500)
- Convert the raw detector position to a coordinate system fixed in the focal plane
- (RAWX,RAWY) on CCD2 => (DETX,DETY) e.g., (2345,3421) This conversion requires knowledge of pixel sizes and orientation of different CCDs
- Convert the focal plane detector coordinates to the sky coordinates
  (DETX,DETY) => (X,Y) e.g., (3560,4540) in pixels
- Sky coordinates are calculated for a tangent plane normal to the nominal pointing direction (RA_NOM,DEC_NOM). The sky (X,Y) coordinates can be converted to RA, DEC using WCS keyword.

Sky coordinates

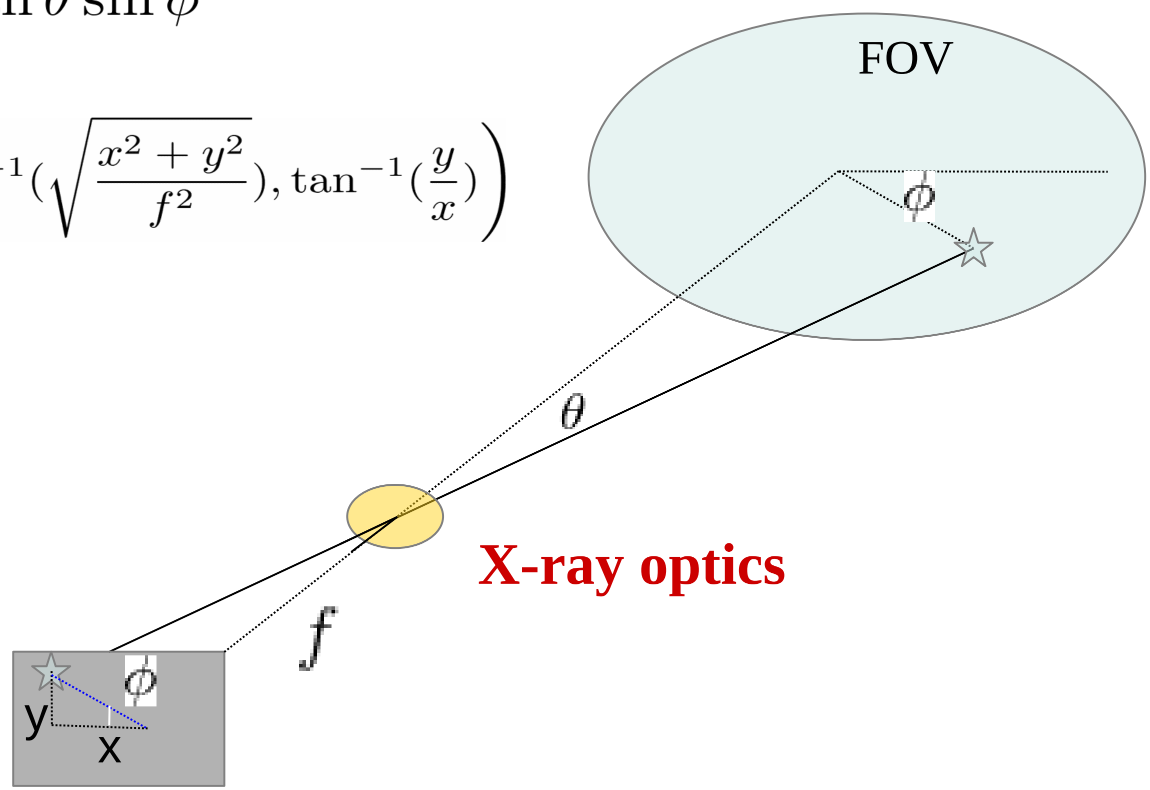
- Source function $S(\theta,\phi)$: Spatial distribution of the incident radiation

Focal plane coordinates

$$x = f\tan\theta\cos\phi$$

$$y = f\tan\theta\sin\phi$$

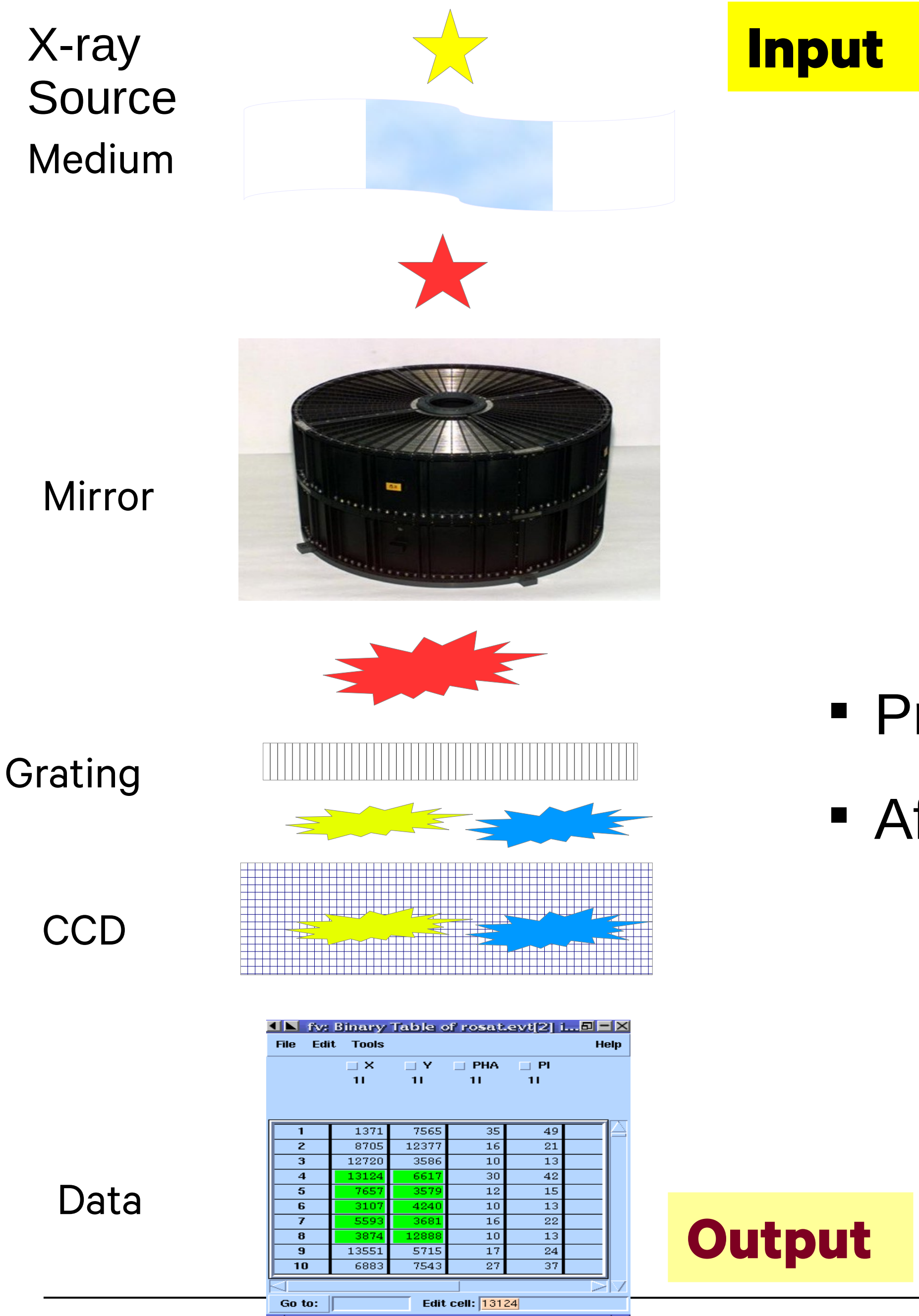
$$S(\theta,\phi) \rightarrow S(x,y) = S\left(\tan^{-1}(\sqrt{\frac{x^2+y^2}{f^2}}), \tan^{-1}(\frac{y}{x})\right)$$



Incident energy: PHA and PI

- Energy-sensitive detectors usually record the "pulse height" of an event. The pulse height is proportional to the photo-electron charge induced by event, which is proportional to the energy of the photon. A detector assigns a digitized pulse height to each event (pha).
- The detector gain is the relation between detector pulse height and the incident photon energy.
- Generally the gain is non-uniform across an imaging detector and also varies with time. In order to compare events from different regions of a detector, one must correct for the gain variation (gain map).
- To compare events due to the same source but at different times, one must correct for time variation of the gain.
- PHA corrected for gain becomes PI.

X-ray Astronomy



- Products NOT in physical units
- Affected with instrument characteristics

Instrumental Response

- Effect of instruments is included in the analysis steps as response files.
- Observed counts *C(X,Y,PI)* in a given pixel and PI bin result from a source flux $S(X_S, Y_S, E, t)$ $\mathrm{photons\ cm^{-2}\ s^{-1}\ keV^{-1}\ arcmin^{-2}}$ for a given sky position at energy *E* and time *t*. *S* and *C* are related by
-
- $$C(X,Y,PI) = \int\int\int\int R(X,Y,PI,X_S,Y_S,E,t).S(X_S,Y_S,E,t)dX_SdY_SdEdt$$
- R is called instrumental response and has unit cm^2.
- R is a measure of the chance of a photon from the sky position (X_S, Y_S) with energy E at time t being detected as a count in pixel (X,Y) and channel PI.

Response for Imaging

- Image is created by summing over PI channels

$$C(X,Y)=\sum_{PI}C(X,Y,PI)=\int\int\int\int\left(\sum_{PI}R(X,Y,PI,X_S,Y_S,E,t)\right).S(X_S,Y_S,E,t)dX_SdY_SdEdt$$

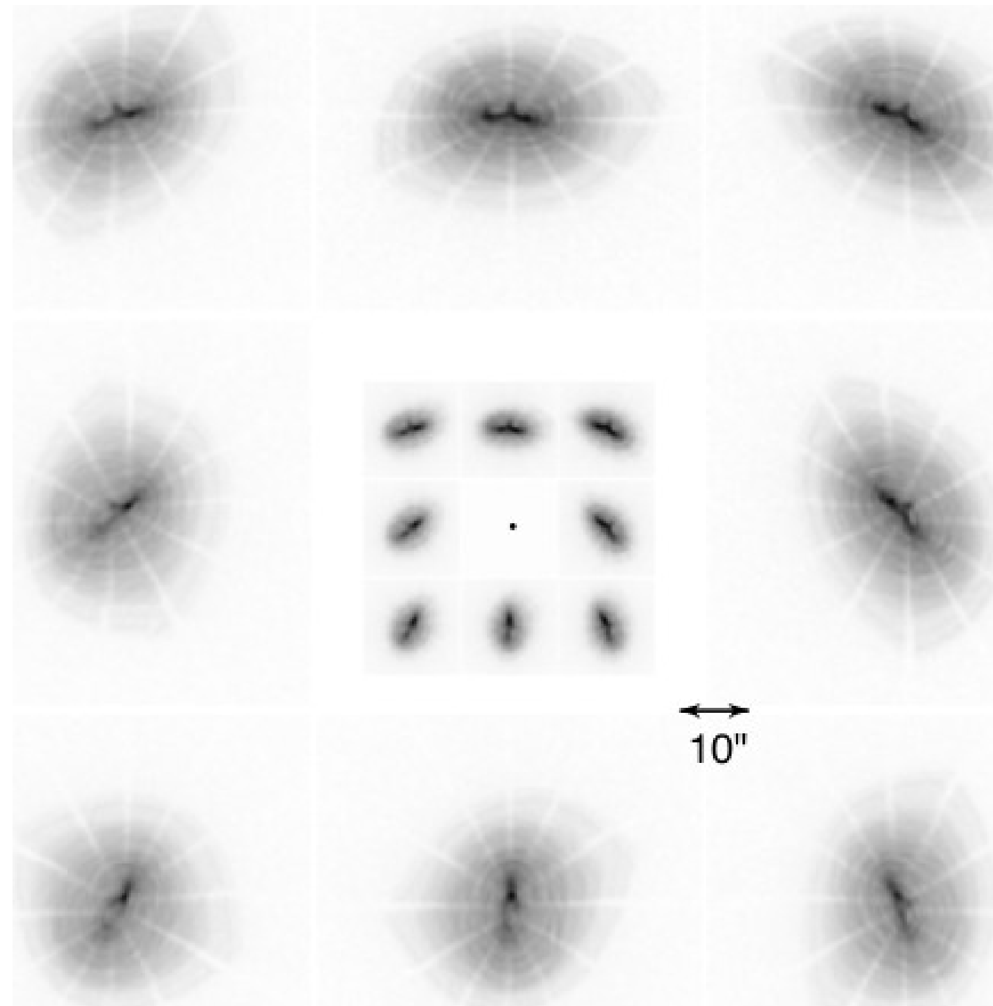
- $$=\int\int\int\int R_{image}(X,Y,X_S,Y_S,E,t).S(X_S,Y_S,E,t)dX_SdY_SdEdt$$
- The instrumental response for images can be split in two parts
-
- $$R_{image}(X,Y,X_S,Y_S,E,t)=PSF(r,\theta,X_S,Y_S,E).EA(X_S,Y_S,E,t)$$

$$\text{with}\quad \mathrm{r}^2=(\mathrm{X}-\mathrm{X_S})^2+(\mathrm{Y}-\mathrm{Y_S})^2\quad\text{and}\quad\theta=\arctan\left(\frac{\mathrm{Y}-\mathrm{Y_S}}{\mathrm{X}-\mathrm{X_S}}\right)$$

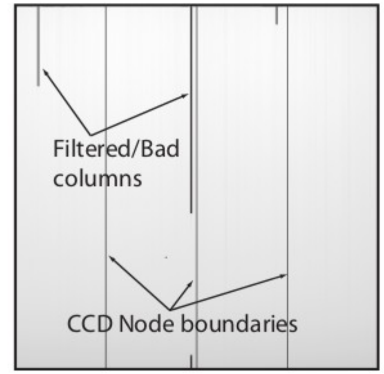
- PSF: Spatial resolution of the telescope as a probability distribution of event positions on the detector from a point source.
- Effective area or Exposure map EA: telescope area at (X_S,Y_S) at energy E and time t.

PSF

- Chandra PSF as a function of off-axis angle
  - Central dot on-axis PSF (0.5 arcsec in diameter)
  - Inner eight are 5 arcmin offaxis
  - Outer eight are 10 arcmin offaxis



Exposure Map



- Count image contains instrumental artefacts complicating surface brightness measurements
- Artefacts due to imperfections in the mirror and detector
- To create an image in photons/cm2/s/arcmin^2, assume delta function for PSF and independent of time and E

$$C(X,Y) = \int\int\int\int PSF(X,Y.X_S,Y_S,E,t)EA(X_S,Y_S,E,t).S(X_S,Y_S,E,t)dX_SdY_SdEdt$$

$$PSF(X,Y,X_S,Y_S,E,t) = \delta(X_S - X)\delta(Y_S - Y)$$

$$C(X,Y) = \int\int EA(X,Y,E,t).S(X,Y,E,t)\Delta A dEdt$$

ΔA = area of an image pixel in arcmin^2

- Assuming S is not variable and image is made at an energy E_0

$$S(X,Y,E_0) = \left(\frac{C(X,Y)}{\Delta A}\right) \Big/ \int EA(X,Y,E_0,t) \cdot dt$$

No PSF correction but takes care of vignetting and bad pixel effects

Spectroscopy

Spectra are created by summing over a region and time t assuming that S does not vary over the region or time.

$$C(PI) = \int \left(\int \int \int \int \int R(X, Y, PI, X_S, Y_S, E, t) \cdot dX_S \cdot dY_S \cdot dX \cdot dY \cdot dt \right) \cdot S_{spec}(E) \cdot dE$$

The response is split between a vector (the ancillary response file, ARF) with units of cm^2 and a matrix (the RMF), which is unitless.

$$C(PI) = T \int \mathrm{RMF}(PI, E) \cdot \mathrm{ARF}(E) \cdot S_{spec}(E) \cdot dE$$

T = Total good observing time

$$\mathrm{RMF}(PI, E) = \int \int \int R_{RMF}(X, Y, PI, E, t) \cdot dX \cdot dY \cdot dt$$

and

$$\mathrm{ARF}(E) = \int \int \int \int \int R_{ARF}(X, Y, X_S, Y_S, E, t) \cdot dX_S \cdot dY_S \cdot dX \cdot dY \cdot dt$$

X-ray spectroscopy

Suppose a detector measures *D(I)* counts in PI channel *I* from some **source** **(+background)**. Then

$$D(I) = T \int RMF(I,E)\, ARF(E)\, f(E)\, dE + B(I)$$

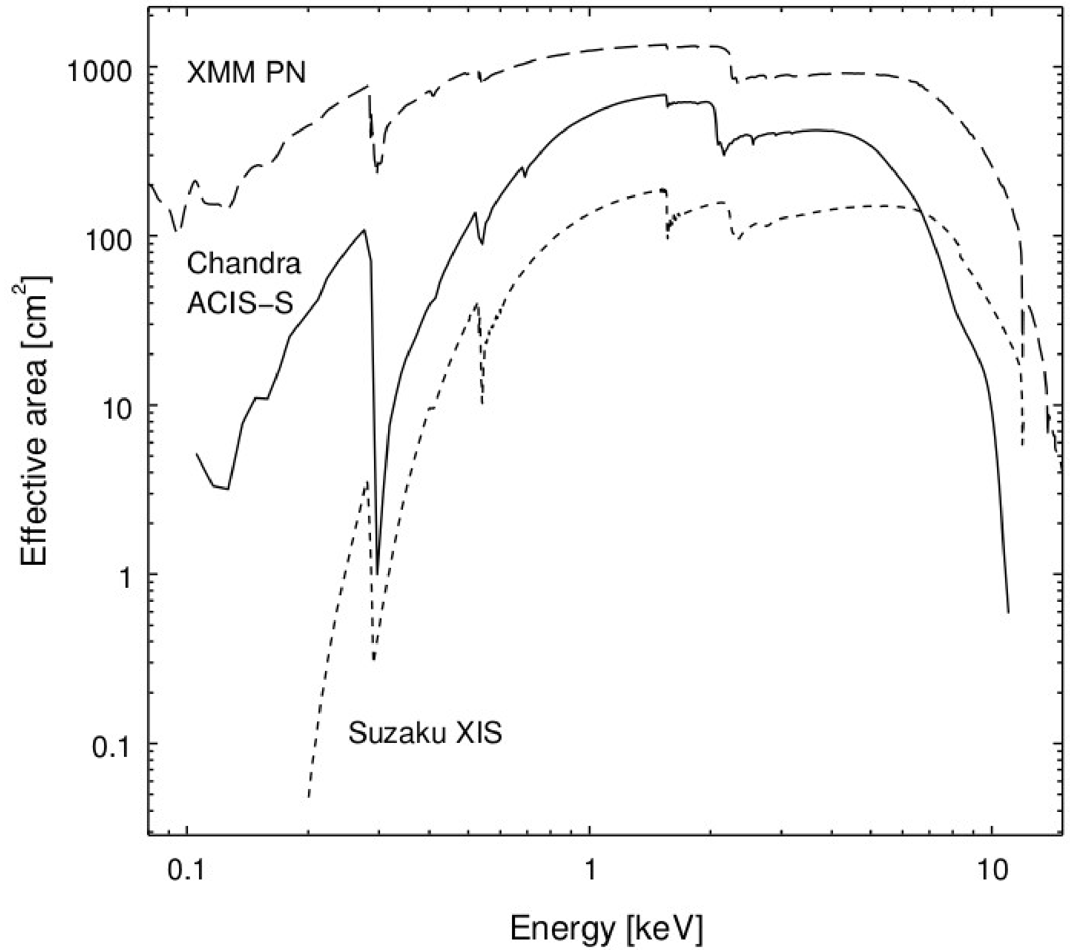
- T is the observation length (in seconds)
- *RMF(I,E)* is the ***redistribution matrix*** that gives the probability of an incoming photon of energy E being registered in channel I (dimensionless)
- *ARF(E)* is the energy-dependent **effective area of the telescope and detector** system (in cm^2).
- *f(E)* is the source flux at the front of the telescope (in photons/cm^2/s/keV)

Instrumental Response

$$R(I, E) = \mathbf{RMF(I,E)} * \mathbf{ARF(E)}$$

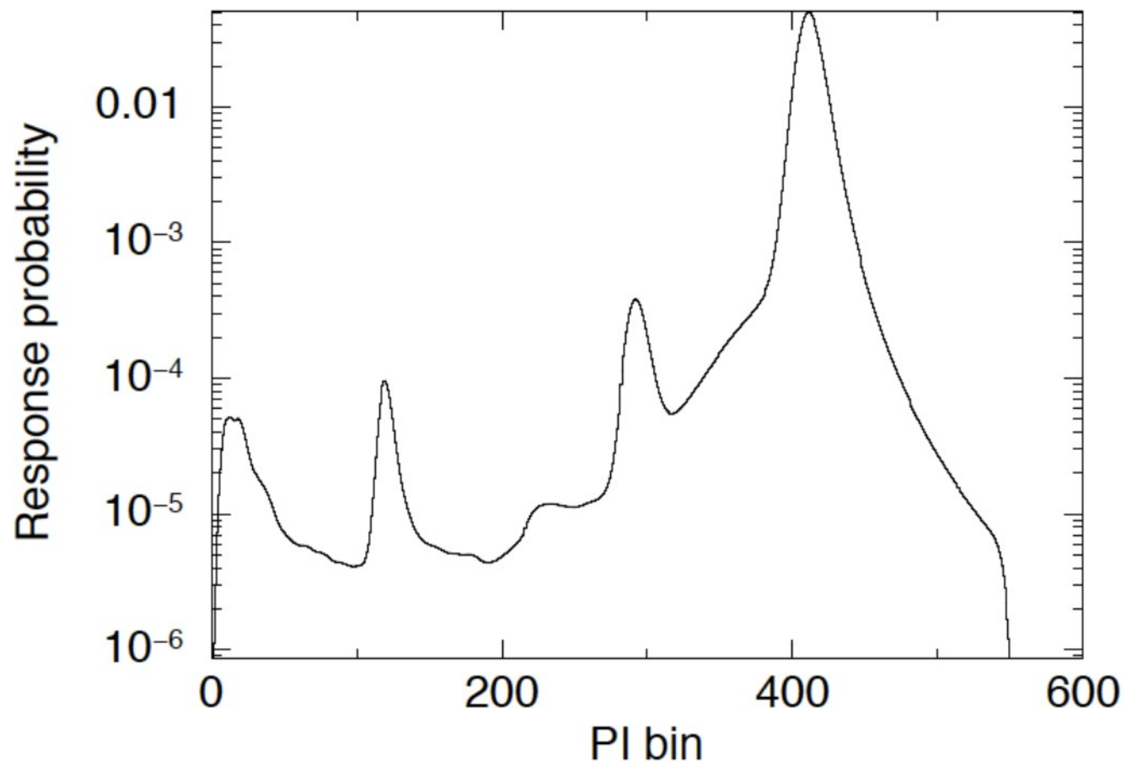
Response matrix | **Redistribution matrix** | ***Effective area***

- R(I,E) is the instrumental response and it includes effective area and redistribution matrix.
- Effective area is the amount of collecting area of the mirror+detector system. This usually depends on the position of the source relative to the "optical axis" of the imaging system and the energy of the incident photon.



Instrument Response: Photon redistribution

- R(I,E) also includes the photon redistribution.
- Redistribution Matrix RMF(I,E): This quantity gives the probability that a photon of energy *E* will be detected in a detector channel *I*. This is usually given as a matrix of photon probabilities.



X-ray Response

CCDs:

Absorption of photons in CCDs creates electron-hole pair.

- **Optical:** 1 pair / photon => collected charge ∝ intensity
- **X-rays:** $N_{pair} = N_e = E_X / w$ $w = 3.7\ eV\ \ for\ Si$

  Many pairs / photon => collected charge ∝ photon energy

  X-ray Spectroscopy

- Spectral resolution

$$\frac{\Delta E}{E} = \frac{\Delta N_{pair}}{N_{pair}} \sim \frac{\sqrt{N_{pair}}}{N_{pair}} = \frac{1}{\sqrt{N_{pair}}} \propto \frac{1}{\sqrt{E}}$$

$$\frac{\Delta E}{E} = 2.355\sqrt{\frac{3.65 \times F}{E}}$$

F~0.1 (fano factor)

$$\sigma_e^2 = F.N_e$$

~ 2% at 5.9 keV

X-ray CCD response

X-ray source : primary photopeaks Mn Kα, Kβ and unresolved complex of L lines

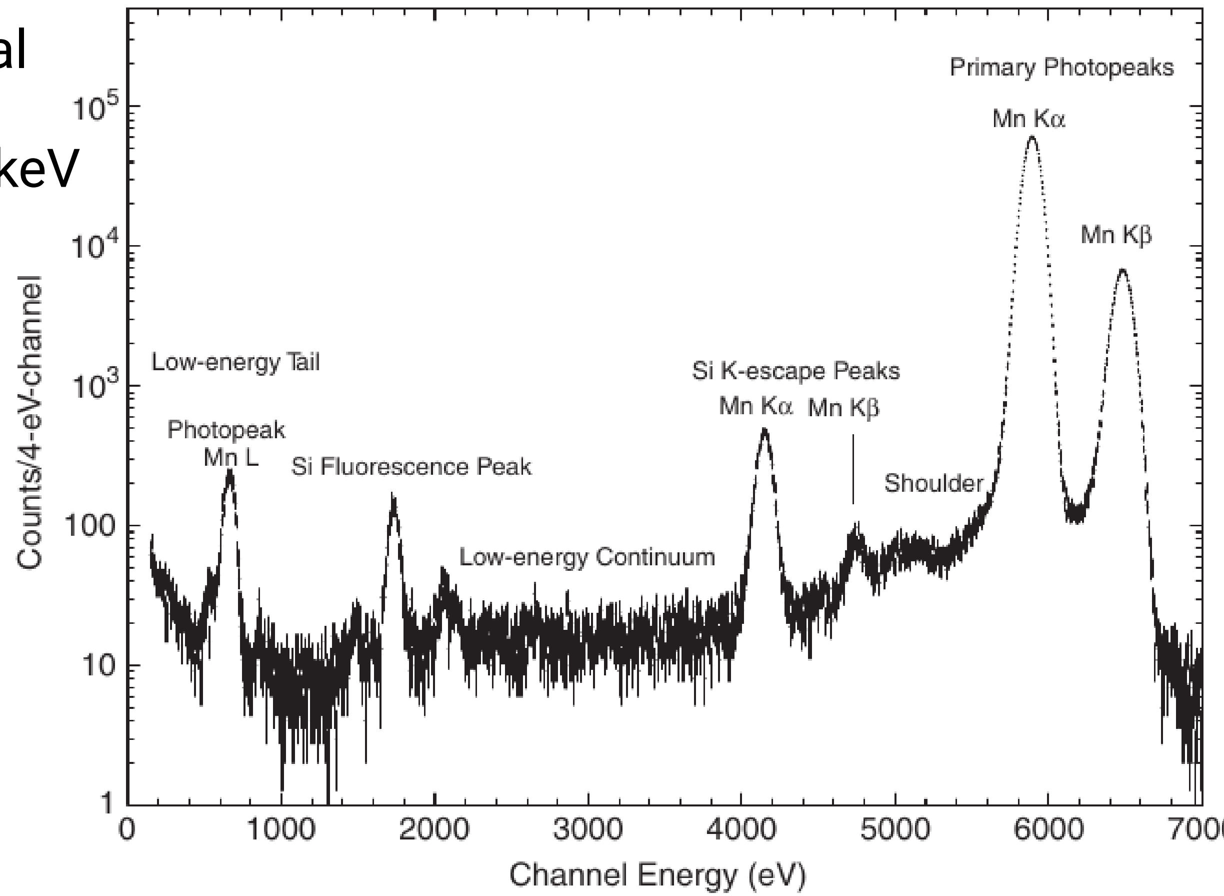
Remaining structures instrumental

Si K-edge = 1.78 keV, Si Kα = 1.74keV

Every photon E > Si K-edge can produce Si Kα

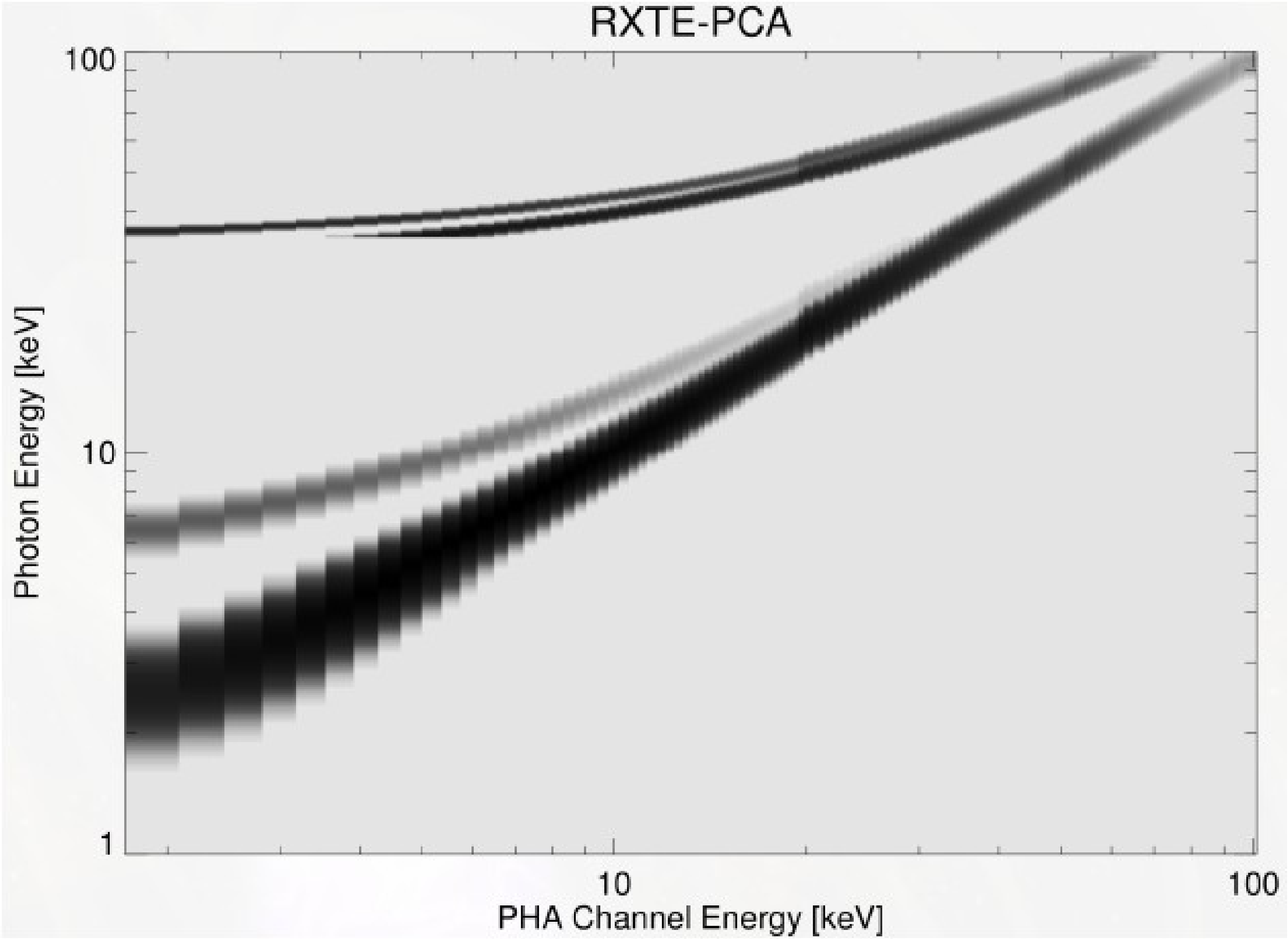
Remaining energy of the X-ray photon
$E_X - E_{k\alpha} = E_{escape}$
detected as escape peak

Si Kα line is also detected from the instrument as they travel to far away locations in the detector



Low energy continuum – incomplete charge collection

Response matrix of RXTE/PCA



Secondary peaks : Escape peaks caused by Xe Kβ and Xe Lα lines

Spectral Analysis

- Most X-ray spectra are of **moderate resolution** (e. g. XMM-Newton EPIC/Chandra ACIS).
- Lines and continuum shape both provide important physical information.
- Therefore, X-ray spectral analysis involves a simultaneous analysis of the entire spectrum rather than an attempt to measure individual line strengths.

Background

- Background are events produced in a detector which are not associated with the astrophysical source of Interest.
- Background can be severe.

$$D(I) = T \int R(I,E)\, A(E)\, f(E)\, dE + B(I)$$

How to deal with Background?

$$D(I) = T \int R(I,E)\, A(E)\, f(E)\, dE + B(I)$$

One can include background in the model but this is complicated and is not usually used.

- The usual method is to extract a spectrum from another part of the image or another observation and the subtract from the source(+background) spectrum.

$$C(I)=[D(I)-B(I)]/T=\int R(I,E)\, A(E)\, f(E$$

Source spectrum from the observed count spectrum

$$D(I) = T\int R(I,E)\ A(E)\ f(E)\ dE + B(I)$$

$$C(I) = \int R(I,E)\ A(E)\ f(E) dE$$

In matrix form: $C_i = \sum R_{ij}\ A_j\ f_j$

The obvious tempting solution is to calculate the inverse of R_{ij}, pre-multiply both sides and rearrange :

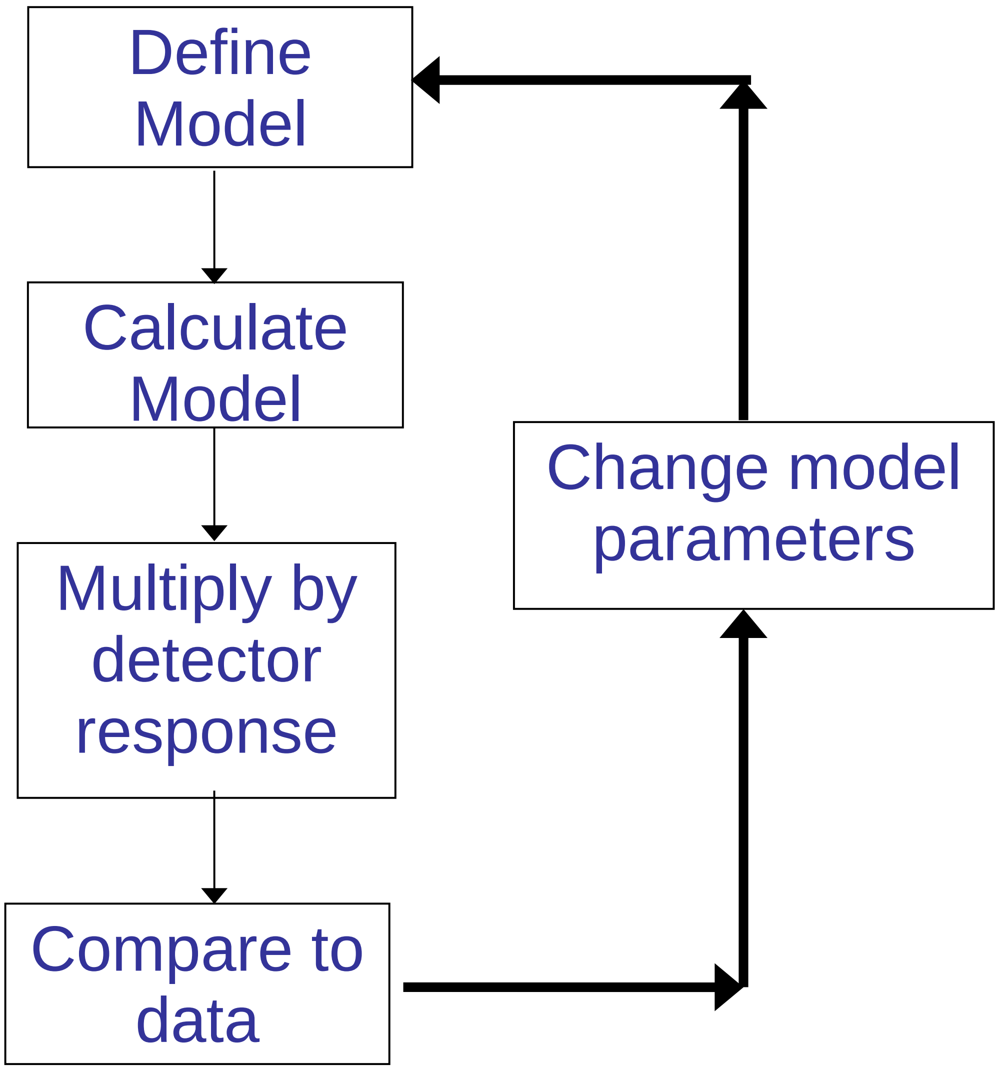
$$(1/A_j) \sum (R_{ij})-1C_i = f_j$$

This does not work! The f_j derived in this way are very sensitive to slight changes in the data C_i.

Forward-fitting

The standard method of analyzing X-ray spectra is “forward-fitting”.

- Calculate a model spectrum.
- Multiply the result by an instrumental response matrix (R(I,E)*A(E)).
- Compare the result with the actual observed data by calculating some statistic.
- Modify the model spectrum and repeat till the best value of the statistic is obtained.



Spectral fitting software packages

- **XSPEC** - General spectral fitting program with many models available (part of HEASOFT software). Most people use XSPEC *(Available in IUCAA computer lab)*.
- **ISIS** – Similar to XSPEC, good scripting capability with slang, XSPEC model library available.
- **Sherpa** – part of CIAO, XSPEC model library available, good scripting capability.
- **SPEX** - Spectral fitting program specializing in collisional plasmas and high resolution spectroscopy.

Model

XSPEC model library consists of individual model components. There are two basic types

- additive component (an emission component e.g. blackbody, Gaussian line)
- Multiplicative component (something which modifies the spectrum e.g. Absorption).

Model = M1 * M2 * (A1 + A2 + M3*A3)

```
XSPEC12>model ?
 Additive Models:
apec       bapec      bbody      bbodyrad    bexrav     bexriv     bkn2pow    bknpower    bmc        bremss
    bvapec      c6mekl     c6pmekl     c6pvmkl     c6vmekl     cemekl      cevmkl      cflow       ompLS
compPS     compST     compTT     compbb     cutoffpl    disk       diskbb     diskir     diskline    diskm
 disko      diskpbb     diskpn      equil       expdec      ezdiskbb    gaussian    gnei  grad    grbm        kerrbb
  kerrd       kerrdisk     laor        laor2       lorentz     meka        mekal       mkcflow     nei         npshock
nsa        nsagrav   nsatmos    nsmax      nteea      nthComp    pegpwrlw    pexrav     pexriv     plcabs
posm       powerlaw   pshock     raymond    redge      refsch     sedov      smaug      srcut      sresc      step
vapec      vbremss vequil     vgnei      vmcflow    vmeka      vmekal     vnei       vnpshock    vpshock    vraymond
vsedov     zbbody     zbremss    zgauss     zpowerlw

 Multiplicative Models:
SSS_ice     TBabs      TBgrain     TBvarabs    absori     acisabs     cabs       constant    cyclabs     dust       edge
expabs      expfac      gabs       highecut    hrefl      notch      pcfabs     phabs      plabs      pwab       redden
smedge     spexpcut    spline     swind1     uvred      varabs     vphabs     wabs       wndabs     xion       zTBabs
zdust      zedge      zhighect    zpcfabs     zphabs     zredden     zsmdust     zvarabs     zvfeabs     zvphabs     zwabs
zwndabs     zxipcf

 Convolution Models:
cflux      gsmooth     kdblur      kdblur2     kerrconv    lsmooth     partcov     rdblur      reflect     simpl
 Mixing Models:
ascac      projct      recorn      suzpsf      xmmpsf

Pile-up Models: pileup
```

Additive Models

Basic additive (emission) models include :

- blackbody
- thermal bremsstrahlung
- power-law
- collisional plasma (raymond, mekal, apec)
- Gaussian or Lorentzian lines

There are many more models available covering specialised topics such as accretion disks, comptonized plasmas, line profile in Kerr geometry or Schwarzschild geometry ....

Multiplicative Models

Multiplicative models include :

- photoelectric absorption due to our Galaxy
- photoelectric absorption due to ionized material
- high energy exponential roll-off.
- ...
- ...

Convolution Models

These are models which take as input the current model and manipulate it in some way. Examples are :

- Smoothing with a Gaussian or Lorentzian function (e.g. velocity broadening)
- Compton reflection
- Kdblur, kdblur2

Finding the best fit

- Finding the best-fit means minimizing the statistic value.

  Generally χ^2 statistic is used.

$$\chi^2 = \Sigma(C(I) - C_p(I))^2/\sigma(I)^2 \; ; \; \sigma(I) = \sqrt{C(I)}$$

- For statistically acceptable fits, the reduced χ^2

$$\chi^2/\nu \approx 1$$

where the degree of freedom,

ν = number of channels – number of model parameters

Issues in spectral analysis

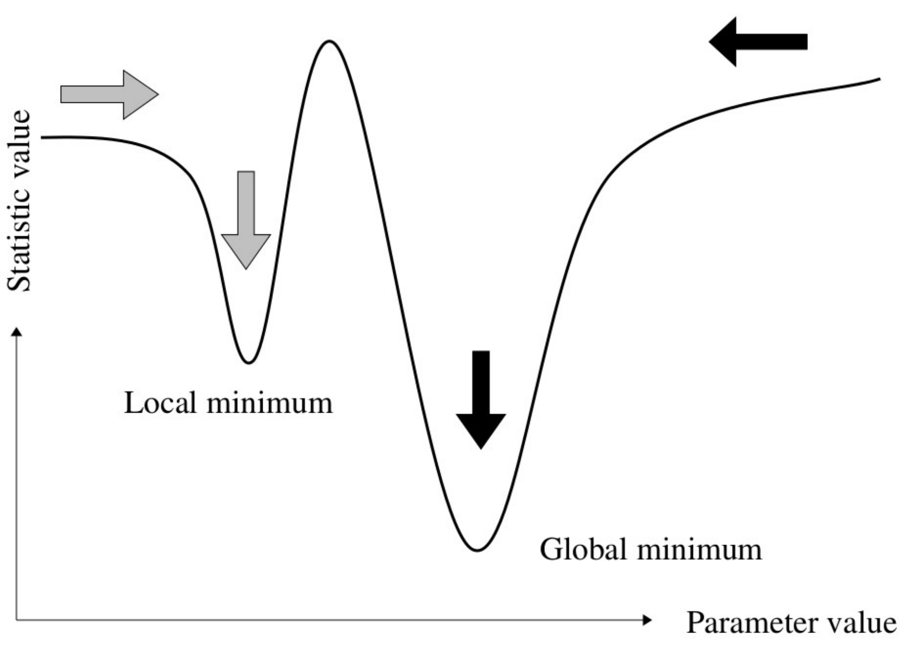
- which model should be fit to the data?

“All models are wrong but some are useful.” - George Box

“If we knew the correct model, we wouldn't have been doing what we are doing!” - Tahir Yaqoob

Some Issues in Spectral fitting:

- Local minima in χ^2 space
- Low count data (group the data to a minimum of 20 counts/channel), so that χ^2 statistic is useful.



Thank You