Unravelling the Flavour Physics (of quarks)

DHEP Journal Club

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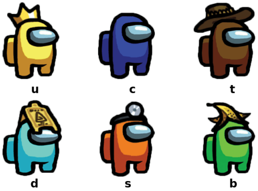


Outline

Flavor in SM

- Flavour in the SM
- Quark Model History
- The CKM matrix

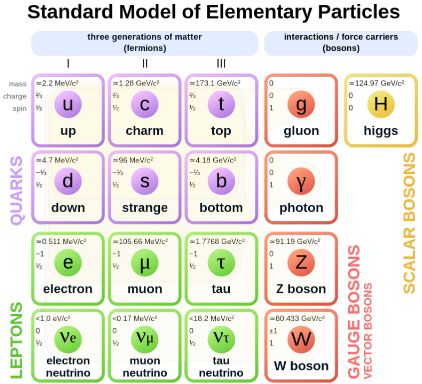
Mixing and CP violation

- Neutral Meson Mixing (no CPV)
- CP violation

Flavour in the SM

Flavour and Colour

Just as ice cream has both color and flavor so do quarks. - Murray Gell-Mann



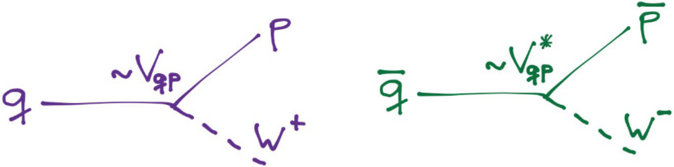
Flavour in the SM

- CKM matrix transforms the mass eigenstate basis to the flavour eigenstate basis
  - and brings with it a rich variety of observable phenomena

mass eigenstates $\neq$ weak eigenstates

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix} \quad (13)$$

- The up-type quark to down-type quark transition probability proportional to the squared magnitude of the CKM matrix elements, $|V_{ij}|^2$



$$\frac{g}{\sqrt{2}} \bar{u}_{Li} V_{ij} \gamma_\mu W^{\mu+} d_{Lj}$$

Isospin

- What's the difference between a proton (p) and a neutron (n^0)?
  - They have similar masses
  - They have a similar strong coupling
  - Just have a different charge
- In 1932 Heisenberg proposed that (p, n^0) are members of an isospin doublet
  - Can be treated as the same particle with different isospin projections

$$p : (I, I_z) = (1/2, +1/2), \quad n : (I, I_z) = (1/2, -1/2)$$

  - The pions can be arranged as an isospin triplet

$$\pi^+ : (I, I_z) = (1, +1), \quad \pi^0 : (I, I_z) = (1, 0), \quad \pi^- : (I, I_z) = (1, -1)$$

  - Isospin is conserved in strong interactions
  - Isospin is violated in weak interactions
- We now know this is not the correct model (it's not an exact symmetry) but it's still a very useful concept
  - It works because $m_u \sim m_d < \Lambda_{\rm QCD}$ and can be used to predict interaction rates:

$$\sigma(p + p \to d + \pi^+) : \sigma(p + n \to d + \pi^0) = 2 : 1$$

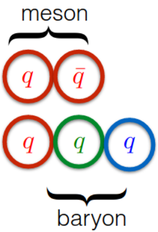
can you explain this 2:1 ratio?

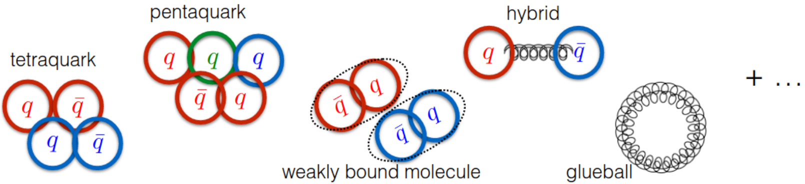
The Quark Model

- Many new particles (a "zoo") discovered in the 60s
- Gell-Mann, Nishijima and Ne'eman introduced the quark "model" (u, d, s) which could elegantly categorise them (the "eight-fold way" - flavour SU(3) symmetry)
- Gell-Mann and Pais
  - Strangeness conserved in strong interactions (production)
  - Strangeness violated in weak interactions (decay)

The Quark Model

- Can only make colour neutral objects
  - Quark anti-quark mesons ($q\bar{q}$) or three quark baryons (qqq). Nearly all known states fall into one of these two categories
  - Can also build colour neutral states containing more quarks (e.g. 4 or 5 quark states). Only quite recently confirmed (and still not entirely understood).





Cabibo angle

- Compare rates of:

  $s \to u$: $K^+ \to \mu^+ \nu_\mu$ $(\Lambda^0 \to p\pi^-,\ \Sigma^+ \to n e^+ \nu_e)$

  $d \to u$: $\pi^+ \to \mu^+ \nu_\mu$ $(n \to p e^+ \nu_e)$

  - Apparent that $s \to u$ transitions are suppressed by a factor ~ 20
- Cabibbo (1963) suggested that "down-type" is some ad-mixture of d and s
  - The first suggestion of quark mixing
  - Physical state is an admixture of flavour states

$$\begin{pmatrix} u \\ d' \end{pmatrix} = \begin{pmatrix} u \\ d\cos(\theta_C) + s\sin(\theta_C) \end{pmatrix} \tag{14}$$

- The mixing angle is determined experimentally to be $\sin(\theta_C) = 0.22$.

GIM mechanism

- Cabibbo's solution opened up a new experimental problem
  - $K^+ \to \mu^+ \nu_\mu$ had been seen but not $K^0_L \to \mu^+ \mu^-$
    - $\mathcal{B}(K^0_L \to \mu^+ \mu^-) \approx 7 \times 10^{-9}$
    - $\mathcal{B}(K^0_L \to e^+ e^-) \approx 1 \times 10^{-11}$
  - $K^+ \to \pi^0 \mu^+ \nu_\mu$ had been seen but not $K^0_L \to \pi^0 \mu^+ \mu^-$
    - $\mathcal{B}(K^0_L \to \pi^0 \mu^+ \mu^-) \approx 1 \times 10^{-10}$
- If the doublet of the weak interaction is the one Cabibbo suggested, Eq. (14), then one can have neutral currents

$$J^0_\mu = \bar{d}' \gamma_\mu (1 - \gamma_5) d' \tag{15}$$

which introduces tree level FCNCs (which we don't see)

- Glashow, Iliopoulos and Maiani (1970) provided a solution by adding a second doublet

$$\begin{pmatrix} c \\ s' \end{pmatrix} = \begin{pmatrix} c \\ -d \sin(\theta_C) + s \cos(\theta_C) \end{pmatrix} \tag{16}$$

  - This exactly cancels the term above, Eq. (15)
  - Thus FCNC contributions are suppressed via loops

GIM suppression

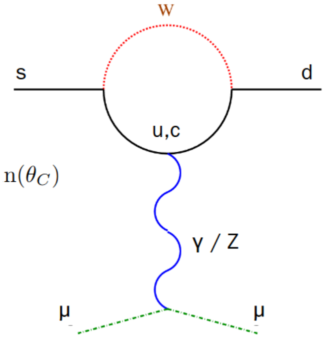
- Consider the $s \to d$ transition required for $K^0_L \to \mu^+\mu^-$
- Given that $m_u, m_c \ll m_W$

$$\mathcal{A} \approx V_{us}V^*_{ud} + V_{cs}V^*_{cd}$$
$$= \sin(\theta_C)\cos(\theta_C) - \cos(\theta_C)\sin(\theta_C)$$
$$= 0$$

- Indeed 2×2 unitarity implies that

$$V_{us}V^*_{ud} + V_{cs}V^*_{cd} = 0$$

- **Predicts the existence of the charm quark:**
  - Kaon mixing
  - Low branching fractions for FCNC decays

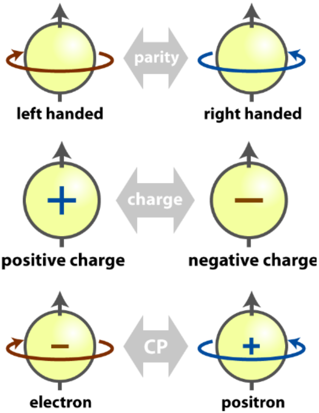


Parity violation

- Two decays were found for charged strange mesons
  - $\theta \to \pi^+\pi^0$
  - $\tau \to \pi^+\pi^-\pi^+$
- The $\theta - \tau$ puzzle
  - Masses and lifetimes of θ and τ are the same
  - But 2π and 3π final states have the opposite parity
- The resolution is that θ and τ are the same particle, K^+, and parity is violated in the decay

C and P

- Prior to 1956 it was thought that the laws of physics were invariant under parity, P, (*i.e.* a mirrored reflection)
  - Shown to be violated in β decays of Co-60 by C. S. Wu (following an idea by T. D. Lee and C. N. Yang)
- Now known that parity, P, is maximally violated in weak decays
  - There are no right-handed neutrinos
- Charge, C, is also maximally violated in weak decays
  - There is no left-handed anti-neutrino
- The product CP is conserved (Landau 1957) and distinguishes absolutely between matter and antimatter
- The product CPT is conserved in any Lorentz invariant gauge field theory



Neutral Kaon mixing

- Ignoring CP-violation, in the neutral kaon system the two physical (mass/lifetime) states are admixtures of the strangeness (flavour) states

$$|K_1\rangle = \frac{|K^0\rangle - |\overline{K}^0\rangle}{\sqrt{2}} \quad \text{and} \quad |\mathrm{K}_2\rangle = \frac{|\mathrm{K}^0\rangle + |\overline{\mathrm{K}}^0\rangle}{\sqrt{2}} \tag{17}$$

under parity, P, and charge conjugation, C, the flavour states transform as

$$\mathcal{P}|K^0\rangle = -|K^0\rangle, \quad \mathcal{C}|K^0\rangle = |\overline{K}^0\rangle \quad \text{and} \quad \mathcal{CP}|K^0\rangle = -|\overline{K}^0\rangle. \tag{18}$$

- For the physical states

$$\mathcal{P}|K_{1,2}\rangle = -|K_{1,2}\rangle, \quad \mathcal{C}|K_{1,2}\rangle = \mp|K_{1,2}\rangle \quad \text{and} \quad \mathcal{CP}|K_{1,2}\rangle = \pm|K_{1,2}\rangle. \tag{19}$$

i.e. they are eigenstates of P, C and CP as well.

- What does this tell us about their decays?
  - $\pi^+\pi^-$ has $P = +1$, $C = +1$, $CP = +1$ - shorter lived $K_1 = K^0_S$
  - $\pi^+\pi^-\pi^0$ has $P = -1$, $C = +1$, $CP = -1$ - longer lived $K_2 = K^0_L$
- If CP is preserved K^0_L decay to two pions should be forbidden

Parameters of the CKM matrix

- 3×3 complex matrix
  - 18 parameters
- Unitary
  - 9 parameters (3 mixing angles, 6 complex phases)
- Quark fields absorb 5 of these (unobservable) phases
- Left with:
  - 3 mixing angles $(\theta_{12}, \theta_{23}, \theta 13)$
  - one complex phase (δ) which gives rise to CP-violation in the SM

The CKM Matrix

$$V_{\rm CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

- A highly predictive theory

Parameters of the CKM matrix

- Absorbing quark phases can be done because under a quark phase transformation

$$u_L^i \to e^{i\phi_u^i} u_L^i, \quad d_L^i \to e^{i\phi_d^i} d_L^i \tag{20}$$

and a simultaneous rephasing of the CKM matrix ($V_{jk} \to e^{i(\phi_j - \phi_k)} V_{jk}$)

$$V_{\rm CKM} \to \begin{pmatrix} e^{i\phi_u} & & \\ & e^{i\phi_c} & \\ & & e^{i\phi_t} \end{pmatrix} \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} e^{i\phi_d} & & \\ & e^{i\phi_s} & \\ & & e^{i\phi_b} \end{pmatrix} \tag{21}$$

the charged current $J^\mu = \bar{u}_{Li} V_{ij} \gamma^\mu d_{Lj}$ is left invariant

- So all additional quark phases are rephased to be relative to just one

Degrees of freedom in an N generation CKM matrix

Number of generations	**2**	**3**	N
Number of real parameters	4	9	N^2
Number of imaginary parameters	4	9	N^2
Number of constraints ($VV^\dagger = \mathbb{1}$)	-4	-9	$-N^2$
Number of relative quark phases	-3	-5	$-(2N-1)$
Total degrees of freedom	1	4	$(N-1)^2$
Number of Euler angles	1	3	$N(N-1)/2$
Number of CP phases	0	1	$(N-1)(N-2)/2$

CKM parameterisations

- The standard form is to express the CKM matrix in terms of three rotation matrices and one CP-violating phase (δ)

$$V_{\rm CKM} = \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}}_{\text{2nd and 3rd gen. mixing}} \underbrace{\begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{+i\delta} & 0 & c_{13} \end{pmatrix}}_{\text{1st and 3rd gen. mixing + CPV phase}} \underbrace{\begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{\text{1st and 2nd gen. mixing}} \quad (22)$$

$$= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{-i\delta} & -c_{13}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \quad (23)$$

where

$$c_{ij} = \cos(\theta_{ij}) \quad \text{and} \quad s_{ij} = \sin(\theta_{ij})$$

CKM parameterisations

- Emprically $s_{12} \sim 0.2,\ s_{23} \sim 0.04,\ s_{13} \sim 0.004$
- CKM matrix exhibits a very clear hierarchy
- The so-called Wolfenstein parameterisation exploits this
- Expand in powers of $\lambda = \sin(\theta_{12})$
- Use four real parameters which are all $\sim O(1),\ (A, \lambda, \rho, \eta)$

The CKM Wolfenstein parameterisation

$$V_{\mathrm{CKM}} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4) \quad (24)$$

- The CKM matrix is almost diagonal
  - Provides strong constraints on NP models in the flavour sector
- Have seen already that quark masses also exhibit a clear hierarchy
- The flavour hierarchy problem
  - Where does this structure come from?

CKM Unitarity Constraints

- The unitary nature of the CKM matrix provides several constraints, $VV^\dagger = \mathbb{1}$
- The ones for off-diagonal elements consist of three complex numbers summing to 0
  - Hence why these are often represented as triangles in the real / imaginary plane (see next slide)

Constraints along diagonal

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$$
$$|V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2 = 1$$
$$|V_{td}|^2 + |V_{ts}|^2 + |V_{tb}|^2 = 1$$

$$|V_{ud}|^2 + |V_{cd}|^2 + |V_{td}|^2 = 1$$
$$|V_{us}|^2 + |V_{cs}|^2 + |V_{ts}|^2 = 1$$
$$|V_{ub}|^2 + |V_{cb}|^2 + |V_{tb}|^2 = 1$$

Constraints off-diagonal

$$V_{ud}V_{us}^* + V_{cd}V_{cs}^* + V_{td}V_{ts}^* = 0$$
$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$$
$$V_{us}V_{ub}^* + V_{cs}V_{cb}^* + V_{ts}V_{tb}^* = 0$$

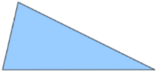
$$V_{ud}V_{cd}^* + V_{us}V_{cs}^* + V_{ub}V_{cb}^* = 0$$
$$V_{ud}V_{td}^* + V_{us}V_{ts}^* + V_{ub}V_{tb}^* = 0$$
$$V_{cd}V_{td}^* + V_{cs}V_{ts}^* + V_{cb}V_{tb}^* = 0$$

CKM Unitarity Triangles and the Jarlskog Invariant

- The off-diagonal constraints can be represented as triangles in the complex plane

$$V_{ud}V_{us}^{*} + V_{cd}V_{cs}^{*} + V_{td}V_{ts}^{*} = 0$$
$$\lambda \quad + \quad \lambda \quad + \quad \lambda^5$$

$$V_{ud}V_{ub}^{*} + V_{cd}V_{cb}^{*} + V_{td}V_{tb}^{*} = 0$$
$$\lambda^3 \quad + \quad \lambda^3 \quad + \quad \lambda^3$$



$$V_{us}V_{ub}^{*} + V_{cs}V_{cb}^{*} + V_{ts}V_{tb}^{*} = 0$$
$$\lambda^4 \quad + \quad \lambda^2 \quad + \quad \lambda^2$$

- All the triangles have the equivalent area (known as the Jarlskog invariant), $J/2$
- J is a phase convention independent measure of CP-violation in the quark sector

$$|J| = \mathcal{I}m(V_{ij}V_{kl}V_{kj}^{*}V_{il}^{*}) \quad \text{for } i \neq k \text{ and } j \neq k \tag{25}$$

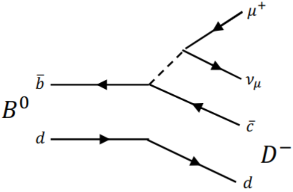
- In the standard notation

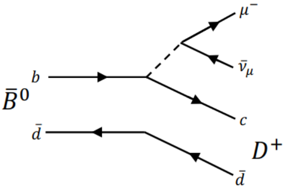
$$J = c_{12}c_{13}^{2}c_{23}s_{12}s_{23}s_{13}\sin(\delta) \tag{26}$$

- The small size of the Euler angles means J (and CP-violation) is small in the SM

Neutral Meson Mixing

- In 1987 the ARGUS experiment observed coherently produced $B^0 - \overline{B}^0$ pairs and observed them decaying to same sign leptons
- How is this possible?
  - Semileptonic decays "tag" the flavour of the initial state

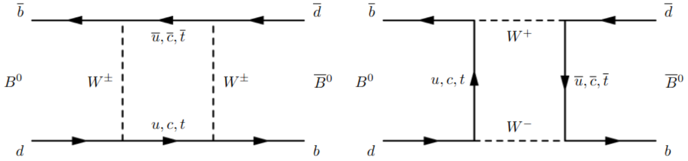




- The only explanation is that B^0–$\overline{B}^0$ can oscillate
- Rate of mixing is large → top quark must be heavy

Neutral Meson Mixing

- In the SM occurs via box diagrams involving a charged current ($W^{\pm}$) interaction
- Weak eigenstates are not the same as the physical mass eigenstates
  - The particle and antiparticle flavour states (via CPT theorem) have equal and opposite charge, identical mass and identical lifetimes
  - But the mixed states (*i.e.* the physical B_L^0 and B_H^0) can have $\Delta m, \Delta\Gamma \neq 0$



- In the SM we have four possible neutral meson states
  - K^0, D^0, B^0, B_s^0 (mixing has been observed in all four)
  - Although they all have rather different properties (as we will see in a second)

Coupled meson systems

- A single particle system evolves according to the time-dependent Schrödinger equation

$$i\frac{\partial}{\partial t}|X(t)\rangle = \mathcal{H}|X(t)\rangle = \left(M - i\frac{\Gamma}{2}\right)|M(t)\rangle \quad (3)$$

- For neutral mesons, mixing leads to a coupled system

$$i\frac{\partial}{\partial t}\begin{pmatrix}|B^0\rangle\\|\overline{B}^0\rangle\end{pmatrix} = \mathcal{H}\begin{pmatrix}|B^0\rangle\\|\overline{B}^0\rangle\end{pmatrix} = \left(\boldsymbol{M} - i\frac{\boldsymbol{\Gamma}}{2}\right)\begin{pmatrix}|B^0\rangle\\|\overline{B}^0\rangle\end{pmatrix} \quad (4)$$

$$= \begin{pmatrix}M_{11} - i\Gamma_{11}/2 & M_{12} - i\Gamma_{12}/2\\ M_{12}^* - i\Gamma_{12}^*/2 & M_{22} - i\Gamma_{22}/2\end{pmatrix}\begin{pmatrix}|B^0\rangle\\|\overline{B}^0\rangle\end{pmatrix} \quad (5)$$

where

$$M_{12} = \frac{1}{2M}\mathcal{A}(B^0 \to \overline{B}^0) = \langle\overline{B}^0|\mathcal{H}(\Delta B = 2)|B^0\rangle \quad (6)$$

Coupled meson systems

- To start with we will neglect CP-violation in mixing (approximately the case for all four neutral meson species)
- Neglecting CP-violation, the physical states are an equal mixture of the flavour states

$$|B_L^0\rangle = \frac{|B^0\rangle + |\overline{B}^0\rangle}{2}, \qquad |B_H^0\rangle = \frac{|B^0\rangle - |\overline{B}^0\rangle}{2}$$

with mass and width differences

$$\Delta\Gamma = \Gamma_H - \Gamma_L = 2|\Gamma_{12}|, \qquad \Delta M = M_H - M_L = 2|M_{12}|$$

so that the physical system evolves as

$$i\frac{\partial}{\partial t}\begin{pmatrix}|B_L^0\rangle \\ |B_H^0\rangle\end{pmatrix} = \mathcal{H}\begin{pmatrix}|B_L^0\rangle \\ |B_H^0\rangle\end{pmatrix} = \left(\boldsymbol{M} - i\frac{\boldsymbol{\Gamma}}{2}\right)\begin{pmatrix}|B_L^0\rangle \\ |B_H^0\rangle\end{pmatrix} \tag{7}$$

$$= \begin{pmatrix}M_L - i\Gamma_L/2 & 0 \\ 0 & M_H - i\Gamma_H/2\end{pmatrix}\begin{pmatrix}|B_L^0\rangle \\ |B_H^0\rangle\end{pmatrix} \tag{8}$$

Time evolution

- Solving the Schrödinger equation gives the time evolution of a pure state $|B^0\rangle$ or $|\overline{B}^0\rangle$ at time $t = 0$

$$|B^0(t)\rangle = g_+(t)|B^0\rangle + \frac{q}{p}g_-(t)|\overline{B}^0\rangle$$
$$|\overline{B}^0(t)\rangle = g_+(t)|\overline{B}^0\rangle + \frac{p}{q}g_-(t)|B^0\rangle \quad (9)$$

where

$$g_+(t) = e^{-iMt}e^{-\Gamma t/2}\left[\quad\cosh\left(\frac{\Delta\Gamma t}{4}\right)\cos\left(\frac{\Delta m t}{2}\right) - i\sinh\left(\frac{\Delta\Gamma t}{4}\right)\sin\left(\frac{\Delta m t}{2}\right)\right]$$
$$g_-(t) = e^{-iMt}e^{-\Gamma t/2}\left[-\sinh\left(\frac{\Delta\Gamma t}{4}\right)\cos\left(\frac{\Delta m t}{2}\right) + i\cosh\left(\frac{\Delta\Gamma t}{4}\right)\sin\left(\frac{\Delta m t}{2}\right)\right] \quad (10)$$

and $M = (M_L + M_H)/2$ and $\Gamma = (\Gamma_L + \Gamma_H)/2$

- No CP-violation in mixing means that $|p/q| = 1$ (and thus we have equal admixtures)

Time evolution

- Using Eq. (10) flavour remains unchanged (+) or will oscillate (−) with probability

$$|g_\pm(t)|^2 = \frac{e^{-\Gamma t}}{2}\left[\cosh\left(\frac{\Delta\Gamma t}{2}\right) \pm \cos(\Delta m t)\right] \tag{11}$$

- With no CP violation in the mixing, the time-integrated mixing probability is

$$\frac{\int |g_-(t)|^2 dt}{\int |g_-(t)|^2 dt + \int |g_+(t)|^2 dt} = \frac{x^2+y^2}{2(x^2+1)} \tag{12}$$

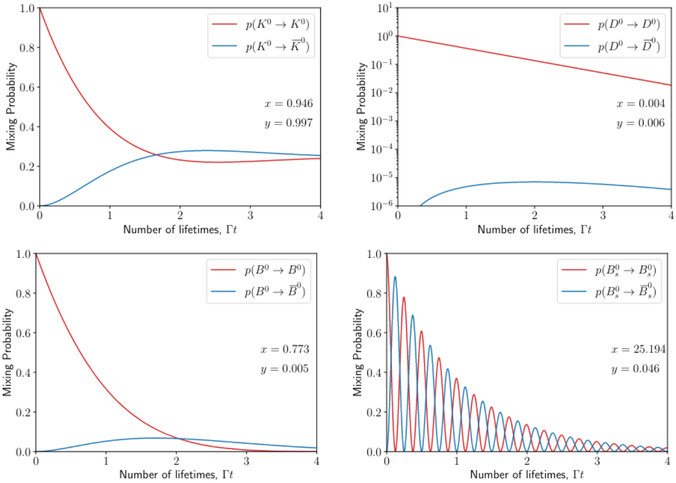
where

$$x = \frac{\Delta m}{\Gamma} \quad \text{and} \quad y = \frac{\Delta\Gamma}{2\Gamma} \tag{13}$$

- The four different neutral meson species which mix have very different values of (x, y) and therefore very different looking time evolution properties

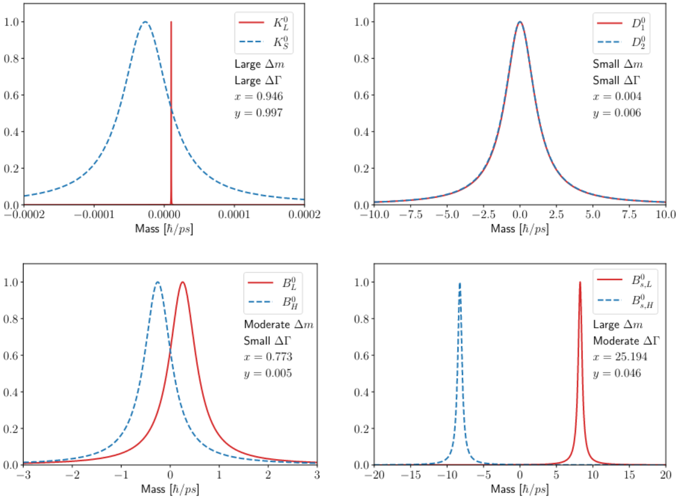
Neutral Meson Mixing

$$|g_\pm(t)|^2 = \frac{e^{-\Gamma t}}{2}\left[\cosh\left(\frac{\Delta\Gamma t}{2}\right) \pm \cos(\Delta m t)\right] \quad (14)$$



Neutral Meson Mixing

- Mass and width differences of the neutral meson mixing systems



Measuring CP violation

1. Need at least two interfering amplitudes
2. Need two phase differences between them
   - One CP conserving ("strong") phase difference (δ)
   - One CP violating ("weak") phase difference (ϕ)

- If there is only a single path to a final state, f, then we cannot get direct CP violation
- If there is only one path we can write the amplitudes for decay as

$$\mathcal{A}(B \to f) = A_1 e^{i(\delta_1 + \phi_1)}$$
$$\mathcal{A}(\bar{B} \to \bar{f}) = A_1 e^{i(\delta_1 - \phi_1)}$$

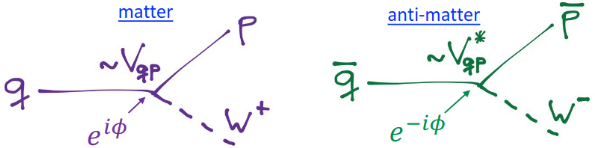
- Which gives an asymmetry of

$$\mathcal{A}_{CP} = \frac{|\mathcal{A}(\bar{B} \to \bar{f})|^2 - |\mathcal{A}(B \to f)|^2}{|\mathcal{A}(\bar{B} \to \bar{f})|^2 + |\mathcal{A}(B \to f)|^2} = 0 \tag{17}$$

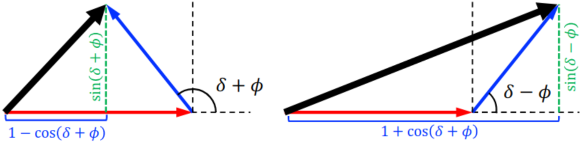
- In order to observe CP-violation we need a second amplitude.
- This is often realised by having interefering tree and penguin amplitudes

Measuring CP violation

- We measure quark couplings which have a complex phase
- This is only visible when there are two amplitudes



- Below we represent two amplitudes (red and blue) with the same magnitude $= 1$
  - The strong phase difference is, $\delta = \pi/2$
  - The weak phase difference is, $\phi = \pi/4$



$\Gamma(B \to f) = |A_1 + A_2 e^{i(\delta+\phi)}|^2$ $\qquad$ $\Gamma(\bar{B} \to \bar{f}) = |A_1 + A_2 e^{i(\delta-\phi)}|^2$

Measuring (direct) CP violation

- Introducing the second amplitude we now have

$$\mathcal{A}(B \to f) = A_1 e^{i(\delta_1+\phi_1)} + A_2 e^{i(\delta_2+\phi_2)} \quad (18)$$

$$\mathcal{A}(\bar{B} \to \bar{f}) = A_1 e^{i(\delta_1-\phi_1)} + A_2 e^{i(\delta_2-\phi_2)} \quad (19)$$

- Which gives an asymmetry of

$$\mathcal{A}_{CP} = \frac{|\mathcal{A}(\overline{B} \to \bar{f})|^2 - |\mathcal{A}(B \to f)|^2}{|\mathcal{A}(\overline{B} \to \bar{f})|^2 + |\mathcal{A}(B \to f)|^2} \quad (20)$$

$$= \frac{4A_1A_2\sin(\delta_1-\delta_2)\sin(\phi_1-\phi_2)}{2A_1^2 + 2A_2^2 + 4A_1A_2\cos(\delta_1-\delta_2)\cos(\phi_1-\phi_2)} \quad (21)$$

$$= \frac{2r\sin(\delta)\sin(\phi)}{1 + r^2 + 2r\cos(\delta)\cos(\phi)} \quad (22)$$

where $r = A_1/A_2$, $\delta = \delta_1 - \delta_2$ and $\phi = \phi_1 - \phi_2$

- This is only non-zero if the amplitudes have **different** weak **and** strong phases
- This is CP-violation in decay (often called "direct" CP violation).
  - This is the only possible route of CP violation for a charged initial state
  - We will see now that for a neutral initial state there are other ways of realising CP violation

Classification of CP violation

- First let's consider a generalised form of a neutral meson, X^0, decaying to a final state, f
- There are four possible amplitudes to consider

$$A_f = \langle f | X^0 \rangle \qquad \bar{A}_f = \langle f | \bar{X}^0 \rangle$$
$$A_{\bar{f}} = \langle \bar{f} | X^0 \rangle \qquad \bar{A}_{\bar{f}} = \langle \bar{f} | \bar{X}^0 \rangle$$

- Define a complex parameter, λ_f (**not** the Wolfenstein parameter, λ)

$$\lambda_f = \frac{q}{p}\frac{\bar{A}_f}{A_f}, \qquad \bar{\lambda}_f = \frac{1}{\lambda_f}, \qquad \lambda_{\bar{f}} = \frac{q}{p}\frac{\bar{A}_{\bar{f}}}{A_{\bar{f}}}, \qquad \bar{\lambda}_{\bar{f}} = \frac{1}{\lambda_{\bar{f}}}$$

Classification of CP violation

Can realise CP violation in three ways:

1. CP violation in decay
   - For a charged initial state this is only the type possible

$$\Gamma(X^0 \to f) \neq \Gamma(\bar{X}^0 \to \bar{f}) \Longrightarrow \boxed{\left|\frac{\bar{A}_{\bar{f}}}{A_f}\right| \neq 1} \tag{23}$$

2. CP violation in mixing

$$\Gamma(X^0 \to \bar{X}^0) \neq \Gamma(\bar{X}^0 \to X^0) \Longrightarrow \boxed{\left|\frac{p}{q}\right| \neq 1} \tag{24}$$

3. CP violation in the interference between mixing and decay

$$\Gamma(X^0 \to f) \neq \Gamma(X^0 \to \bar{X}^0 \to f) \Longrightarrow \arg(\lambda_f) = \boxed{\arg\left(\frac{q}{p}\frac{\bar{A}_f}{A_f}\right) \neq 0} \tag{25}$$

- We just saw an example of CP violation in decay
- Let's extend our formalism of neutral mixing, Eqs. (9–13), to include CP violation

Neutral Meson Mixing with CP violatio

- Allowing for CP violation, $M_{12} \neq M_{12}^*$ and $\Gamma_{12} \neq \Gamma_{12}^*$
- The physical states can now be unequal mixtures of the weak states

$$\begin{aligned} |B_L^0\rangle &= p|B^0\rangle + q|\overline{B}^0\rangle \\ |B_H^0\rangle &= p|B^0\rangle - q|\overline{B}^0\rangle \end{aligned} \tag{26}$$

where

$$|p|^2 + |q|^2 = 1$$

- The states now have mass and width differences

$$|\Delta\Gamma| \approx 2|\Gamma_{12}|\cos(\phi), \quad |\Delta M| \approx 2|M_{12}|, \quad \phi = \arg(-M_{12}/\Gamma_{12}) \tag{27}$$

- We'll see some examples of this later
- Now to equip ourselves with the formalism for a generalised meson decay

Generalized Meson Decay Formalism

The probability that state X^0 at time t decays to f at time t

- contains terms for CPV in decay, mixing and the interference between the two

$$\Gamma_{X^0\to f}(t) = \boxed{|A_f|^2} \left(\boxed{|g_+(t)|^2} + \boxed{|\lambda_f|^2} \boxed{|g_-(t)|^2} + \boxed{2\mathcal{R}e\left[\lambda_f g_+^*(t) g_-(t)\right]} \right) \quad (28)$$

$$\Gamma_{X^0\to \bar{f}}(t) = \boxed{|\bar{A}_{\bar{f}}|^2} \boxed{\left|\frac{q}{p}\right|^2} \left(\boxed{|g_-(t)|^2} + \boxed{|\lambda_{\bar{f}}|^2} \boxed{|g_+(t)|^2} + \boxed{2\mathcal{R}e\left[\lambda_{\bar{f}} g_+(t) g_-^*(t)\right]} \right) \quad (29)$$

$$\Gamma_{\overline{X}^0\to f}(t) = \boxed{|A_f|^2} \boxed{\left|\frac{p}{q}\right|^2} \left(\boxed{|g_-(t)|^2} + \boxed{|\lambda_f|^2} \boxed{|g_+(t)|^2} + \boxed{2\mathcal{R}e\left[\lambda_f g_+(t) g_-^*(t)\right]} \right) \quad (30)$$

$$\Gamma_{\overline{X}^0\to \bar{f}}(t) = \boxed{|\bar{A}_{\bar{f}}|^2} \left(\boxed{|g_+(t)|^2} + \boxed{|\lambda_{\bar{f}}|^2} \boxed{|g_-(t)|^2} + \boxed{2\mathcal{R}e\left[\lambda_{\bar{f}} g_+^*(t) g_-(t)\right]} \right) \quad (31)$$

where the mixing probabilities are as before

$$|g_\pm(t)|^2 = \frac{e^{-\Gamma t}}{2}\left[\cosh\left(\frac{\Delta\Gamma t}{2}\right) \pm \cos(\Delta m t)\right] \quad (32)$$

$$g_+^* g_-^{(*)} = \frac{e^{-\Gamma t}}{2}\left[\sinh\left(\frac{\Delta\Gamma t}{2}\right) \pm i \sin(\Delta m t)\right] \quad (33)$$

Generalized Meson Decay Formalism

- From the above we get the "master equations" for neutral meson decay

$$\Gamma_{X^0\to f}(t) = |A_f|^2 \qquad (1+|\lambda_f|^2)\frac{e^{-\Gamma t}}{2}\Big[\cosh(\tfrac{1}{2}\Delta\Gamma t) + C_f\cos(\Delta m t) + D_f\sinh(\tfrac{1}{2}\Delta\Gamma t) - S_f\sin(\Delta m t)\Big] \quad (34)$$

$$\Gamma_{\overline{X}^0\to f}(t) = |A_f|^2\left|\frac{p}{q}\right|^2(1+|\lambda_f|^2)\frac{e^{-\Gamma t}}{2}\Big[\cosh(\tfrac{1}{2}\Delta\Gamma t) - C_f\cos(\Delta m t) + D_f\sinh(\tfrac{1}{2}\Delta\Gamma t) + S_f\sin(\Delta m t)\Big] \quad (35)$$

where

$$C_f = \frac{1-|\lambda_f|^2}{1+|\lambda_f|^2}, \quad D_f = \frac{2\mathcal{R}e(\lambda_f)}{1+|\lambda_f|^2}, \quad S_f = \frac{2\mathcal{I}m(\lambda_f)}{1+|\lambda_f|^2} \quad (36)$$

- and equivalents for the CP conjugate final state $\bar{f}$
- **The time-dependent CP asymmetry is** (for non-CP-eigenstates there are two)

$$\mathcal{A}_{CP}(t) = \frac{\Gamma_{X^0\to f}(t) - \Gamma_{\overline{X}^0\to f}(t)}{\Gamma_{X^0\to f}(t) + \Gamma_{\overline{X}^0\to f}(t)} = \boxed{\frac{2C_f\cos(\Delta m t) - 2S_f\sin(\Delta m t)}{2\cosh(\frac{1}{2}\Delta\Gamma t) + 2D_f\sinh(\frac{1}{2}\Delta\Gamma t)}} \quad (37)$$

Specific Meson Decay Formalism

- In the B^0 system $\Delta\Gamma \sim 0$

$$\Gamma_{X^0\to f}(t) = |A_f|^2 \quad (1+|\lambda_f|^2)\frac{e^{-i\Gamma t}}{2}\left[+ C_f\cos(\Delta m t) - S_f\sin(\Delta m t)\right] \tag{38}$$

$$\Gamma_{\overline{X}^0\to f}(t) = |A_f|^2 \left|\frac{p}{q}\right|(1+|\lambda_f|^2)\frac{e^{-i\Gamma t}}{2}\left[- C_f\cos(\Delta m t) + S_f\sin(\Delta m t)\right] \tag{39}$$

- **The time-dependent CP asymmetry is**

$$\mathcal{A}_{CP}(t) = \frac{\Gamma_{X^0\to f}(t) - \Gamma_{\overline{X}^0\to f}(t)}{\Gamma_{X^0\to f}(t) + \Gamma_{\overline{X}^0\to f}(t)} = \boxed{C_f\cos(\Delta m t) - S_f\sin(\Delta m t)} \tag{40}$$

Specific Meson Decay Formalism

- In the D^0 system Δm and $\Delta\Gamma$ are both small

$$\Gamma_{X^0\to f}(t) = |A_f|^2 \quad (1+|\lambda_f|^2)\frac{e^{-i\Gamma t}}{2}\left[1 \qquad + C_f \right.$$
$$\left. + D_f \tfrac{1}{2}\Delta\Gamma t \qquad - S_f \Delta m t \qquad \right] \quad (41)$$

$$\Gamma_{\overline{X}^0\to f}(t) = |A_f|^2 \left|\frac{p}{q}\right|(1+|\lambda_f|^2)\frac{e^{-i\Gamma t}}{2}\left[1 \qquad - C_f \right.$$
$$\left. + D_f \tfrac{1}{2}\Delta\Gamma t \qquad + S_f \Delta m t \qquad \right] \quad (42)$$

- **The time-dependent CP asymmetry is**

$$\mathcal{A}_{CP}(t) = \frac{\Gamma_{X^0\to f}(t) - \Gamma_{\overline{X}^0\to f}(t)}{\Gamma_{X^0\to f}(t) + \Gamma_{\overline{X}^0\to f}(t)} = \boxed{\frac{C_f - S_f\Delta m t}{1+\frac{1}{2}D_f\Delta\Gamma t}} \quad (43)$$

Specific Meson Decay Formalism

- With no tagging of flavour we see no asymmetry (just get the sum)

$$\Gamma_{X^0\to f}(t) = |A_f|^2 \quad (1+|\lambda_f|^2)\frac{e^{-i\Gamma t}}{2}\Big[\cosh(\tfrac{1}{2}\Delta\Gamma t) + D_f \sinh(\tfrac{1}{2}\Delta\Gamma t)\Big] \tag{44}$$

$$\Gamma_{\overline{X}^0\to f}(t) = |A_f|^2 \left|\frac{p}{q}\right|(1+|\lambda_f|^2)\frac{e^{-i\Gamma t}}{2}\Big[\cosh(\tfrac{1}{2}\Delta\Gamma t) + D_f \sinh(\tfrac{1}{2}\Delta\Gamma t)\Big] \tag{45}$$

- **The time-dependent CP asymmetry is**

$$\mathcal{A}_{CP}(t) = \frac{\Gamma_{X^0\to f}(t) - \Gamma_{\overline{X}^0\to f}(t)}{\Gamma_{X^0\to f}(t) + \Gamma_{\overline{X}^0\to f}(t)} = \boxed{0} \tag{46}$$

CP violation status

	K^0	K^+	Λ^0	D^0	D^+	D_s^+	Λ_c^+	B^0	B^+	B_s^0	Λ_b^0
CP violation in mixing	✓✓	-	-	✗	-	-	-	✗	-	✗	-
CP violation in interference	✓	-	-	✗	-	-	-	✓✓	-	✓✓	-
CP violation in decay	✓	✗	✗	✓✓	✗	✗	✗	✓✓	✓✓	✓	✓

KEY:

✓✓	Strong evidence $(> 5\sigma)$
✓	Some evidence $(> 3\sigma)$
✗	Not seen
-	Not possible

- We discussed a myriad of topics under the umbrella of Flavor physics.
- The next talk will be focused on metrology of CKM parameters, EFTs and FCNCs.

Flavour in the SM

A brief theoretical interlude which we will flesh out with some history afterwards

- Particle physics can be described to excellent precision by a relatively straightforward and very beautiful theory (we all know and love the SM):

$$\mathcal{L}_{\rm SM} = \mathcal{L}_{\rm Gauge}(A_a, \psi_i) + \mathcal{L}_{\rm Higgs}(\phi, A_a, \psi_i) \tag{1}$$

- It contains:
  - Gauge terms that deal with the free fields and their interactions via the strong and electroweak interactions
  - Higgs terms that give rise to the masses of the SM fermions and weak bosons

Flavour in the SM

- The Gauge part of the Lagrangian is well verified

$$\mathcal{L}_{\text{Gauge}} = \sum_j i\bar{\psi}_j \not{D}\psi_j - \sum_a \frac{1}{4g_a^2} F^a_{\mu\nu} F^{\mu\nu,a} \tag{2}$$

- Parity is violated by electroweak interactions
- Fields are arranged as left-handed doublets and right-handed singlets

$$\psi = \boxed{Q_L, u_R, d_R, c_R, s_R, t_R, b_R} \text{ quarks} \tag{3}$$

$$\boxed{L_L, e_R, \mu_R, \tau_R} \text{ leptons} \tag{4}$$

with

$$Q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix}, \begin{pmatrix} c_L \\ s_L \end{pmatrix}, \begin{pmatrix} t_L \\ b_L \end{pmatrix} \text{ and } L_L = \begin{pmatrix} e_L \\ \nu_{eL} \end{pmatrix}, \begin{pmatrix} \mu_L \\ \nu_{\mu L} \end{pmatrix}, \begin{pmatrix} \tau_L \\ \nu_{\tau L} \end{pmatrix} \tag{5}$$

- The Lagrangian is invariant under a specific set of symmetry groups: $\mathrm{SU}(3)_c \times \mathrm{SU}(2)_\mathrm{L} \times \mathrm{U}(1)_\mathrm{Y}$

Quark Gauge Couplings

- Without the Higgs we have flavour universal gauge couplings equal for all three generations (huge degeneracy)

$$\mathcal{L}_{\text{quarks}} = \sum_{j}^{3} \underbrace{i\bar{Q}_j \not{D}_Q Q_j}_{\text{left-handed doublets}} \underbrace{+ i\bar{U}_j \not{D}_U U_j + i\bar{D}_j \not{D}_D D_j}_{\text{right-handed singlets}} \quad (6)$$

leptons have been omitted for simplicity

- with the covariant derivatives

$$\begin{aligned} D_{Q,\mu} &= \partial_\mu + i g_s \lambda_\alpha G^\alpha_\mu + i g \sigma_i W^i_\mu + i Y_Q g' B_\mu \\ D_{U,\mu} &= \partial_\mu + i g_s \lambda_\alpha G^\alpha_\mu + i Y_U g' B_\mu \\ D_{D,\mu} &= \partial_\mu + i g_s \lambda_\alpha G^\alpha_\mu + i Y_D g' B_\mu \end{aligned}$$

and $Y_Q = 1/6$, $Y_U = 2/3$, $Y_D = -1/3$

Yukawa couplings

- In order to realise fermion masses we introduce "Yukawa couplings"
- This is rather ad-hoc. It is necessary to understand the data but is not stable with respect to quantum corrections (the Hierarchy problem).
- By doing this we introduce flavour non-universality via the Yukawa couplings between the Higgs and the quarks

$$\mathcal{L}_{\text{Yukawa}} = \sum_{i,j}^{3} (-\bar{Q}^i_L Y^{ij}_U \tilde{H} u^j_R - \bar{Q}^i_L Y^{ij}_D H d^j_R + h.c.) \tag{7}$$

leptons have been omitted for simplicity

- Replace H by its vacuum expectation value, $\langle H \rangle = (0, \nu)^T$, and we obtain the quark mass terms

$$\sum_{i,j}^{3} (-\bar{u}^i_L m^{ij}_U u^j_R - \bar{d}^i_L m^{ij}_D d^j_R) \tag{8}$$

with the quark mass matrices given by $m_A = \nu Y_A$ with $A = (U, D, L)$

Diagonalising the mass matrices

- Quark mass matrices, m_U, m_D, m_L, are 3×3 complex matrices in "flavour space" with *a priori arbitary values*.
  - We can diagonalise them via a field redefintion

$$u_L = \hat{U}_L u_L^m, \;\; u_R = \hat{U}_R u_R^m, \;\; d_L = \hat{D}_L d_L^m, \;\; d_R = \hat{D}_R d_R^m \tag{9}$$

  - such that in the mass eigenstate basis the matrices are diagonal

$$m_U^{\text{diag}} = \hat{U}_L^\dagger m_U \hat{U}_R, \quad m_D^{\text{diag}} = \hat{D}_L^\dagger m_D \hat{D}_R \tag{10}$$

- The right-handed SU(2) singlet is invariant but recall the left-handed SU(2) doublet gives rise to terms like

$$\frac{g}{\sqrt{2}} \bar{u}_L^i \gamma_\mu W^\mu d_L^i \tag{11}$$

- In the mass basis this then becomes

$$\frac{g}{\sqrt{2}} \bar{u}_L^i \underbrace{\hat{U}_L^{\dagger ij} \hat{D}_L^{jk}}_{\hat{V}_{\text{CKM}}} \gamma_\mu W^\mu d_L^k \tag{12}$$

This combination, $\hat{V}_{\text{CKM}} = \hat{U}_L^{\dagger ij} \hat{D}_L^{jk}$, **is the** physical CKM matrix and generates flavour violating charged current interactions. It is complex and unitary, $VV^\dagger = \mathbb{1}$

Flavour in the SM

- The gauge part of the SM Lagrangian is invariant under U(3) symmetries of the left-handed doublets and right-handed singlets if the fermions are massless

$$\mathcal{L}_{\text{Gauge}} = \sum_j i\bar{\psi}_j \not{D} \psi_j - \sum_a \frac{1}{4g_a^2} F^a_{\mu\nu} F^{\mu\nu,a}$$

- These U(3) symmetries are broken by the Yukawa terms. The only remaining symmetries correspond to lepton number and baryon number conservation
- These are "accidental" symmetries, coming from the particle content, rather than being explicitly imposed

We will return to the CKM matrix and CKM metrology later!

particle zoo

SU(2) flavour mixing

- Four possible combinations from two quarks (u and d)

$$u\overline{u}, d\overline{d}, u\overline{d}, \overline{u}d$$

- Under SU(2) symmetry the π^0 and η states are members of an isospin triplet and singlet respectively

$$\pi^0 = \frac{1}{\sqrt{2}}(u\overline{u} - d\overline{d}), \quad \eta = \frac{1}{\sqrt{2}}(u\overline{u} + d\overline{d})$$

SU(3) flavour mixing

- Introducing the strange quark (under SU(3) symmetry) we now have an octuplet and a singlet

$$\pi^0 = \frac{1}{\sqrt{2}}(u\overline{u} - d\overline{d}), \quad \eta_1 = \frac{1}{\sqrt{3}}(u\overline{u} + d\overline{d} + s\overline{s}), \quad \eta_8 = \frac{1}{\sqrt{6}}(u\overline{u} + d\overline{d} - 2s\overline{s})$$

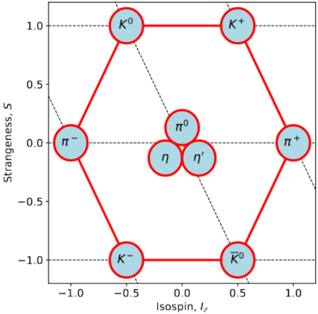
- The physical states involve a further mixing

$$\eta = \eta_1 \cos\theta + \eta_8 \sin\theta, \quad \eta' = -\eta_1 \sin\theta + \eta_8 \cos\theta$$

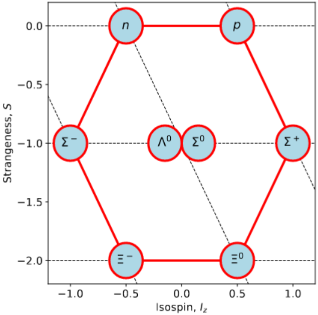
Particle zoo

- Can elegantly categorise states by isospin (up/downess) and strangeness
- Also get the excited states which can be categorised in the same way

Spin-0 Mesons



Spin-1/2 Baryons



Homework

- What is the quark content of these states?
- Do you know the spin-1 (spin-3/2) states?

CKM mechanism

- In 1973 Kobayashi and Maskawa introduce the CKM mechanism to explain CP-violation
- As we will see this requires a third generation of quark and so they predict the existence of b and t quarks

CP Violation in the Renormalizable Theory of Weak Interaction

Makoto Kobayashi, Toshihide Maskawa (Kyoto U.)

Feb 1973 - 6 pages

Prog.Theor.Phys. 49 (1973) 652-657

Also in *Lichtenberg, D. B. (Ed.), Rosen, S. P. (Ed.): Developments In The Quark Theory Of Hadrons, Vol. 1*, 218-223.

DOI: 10.1143/PTP.49.652

KUNS-242

Abstract (Oxford Journals)

In a framework of the renormalizable theory of weak interaction, problems of CP-violation are studied. It is concluded that no realistic models of CP-violation exist in the quartet scheme without introducing any other new fields. Some possible models of CP-violation are also discussed.