Statistical significances and projections for proton decay experiments

Prudhvi N. Bhattiprolu

prudhvib@umich.edu

University of Michigan

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Based on 2009.07249 and 2210.07735 with

Stephen P. Martin and James D. Wells

1. Protons are probably not forever

2. Statistics for discovery and exclusion
   - Basic definitions
   - Single-channel counting experiments
   - Multi-channel counting experiments
   - Background uncertainty and other nuisance parameters

3. Application to proton decay

All is not well with the Standard Model

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- Matter-antimatter asymmetry (more on next slide)

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- Matter-antimatter asymmetry (more on next slide)
- Dark matter abundance
- Neutrino masses
- The cosmological constant problem
- ...

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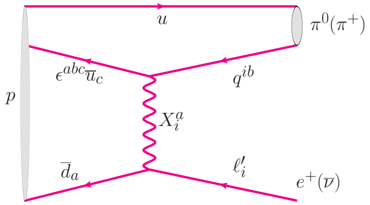
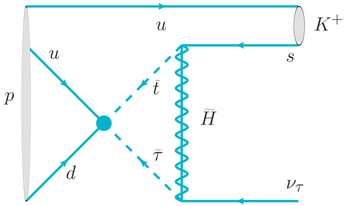
- Grand unified theories predict B violation and can lead to proton decay ($\Delta B = 1$).
- Leading decay modes in non-SUSY and SUSY GUTs:





[2203.08771]

Searches for proton decay

The current strongest limits on proton partial lifetime are from Super-Kamiokande:

$$\tau_p/\text{Br}(p \to \overline{\nu}K^+) > 5.9 \times 10^{33} \text{ years (90\% CL; 2014)}$$
$$\tau_p/\text{Br}(p \to e^+\pi^0) > 2.4 \times 10^{34} \text{ years (90\% CL; 2020)}$$

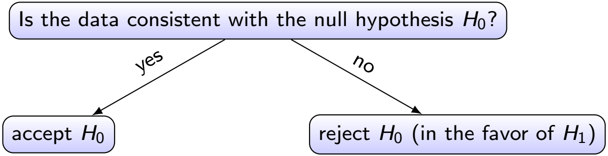
Many proposed neutrino detectors can also search for proton decay:

- Deep Underground Neutrino Detector (DUNE) in the US
- Hyper-Kamiokande (Hyper-K) in Japan
- Jiangmen Underground Neutrino Observatory (JUNO) in China

Later we will study the statistical significance of proton decay at these experiments.

Hypothesis testing

To assess the sensitivity of an experiment, one defines a null hypothesis H_0, which is compared to an alternative hypothesis H_1.



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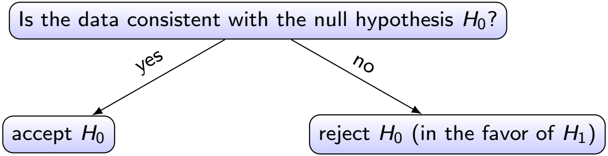

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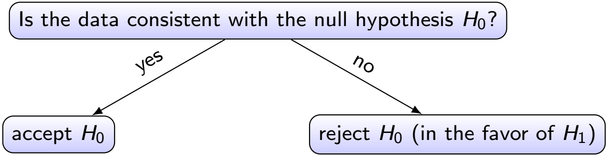

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In particle physics searches, p-value reported as a significance Z:

$$Z = \sqrt{2}\,\mathrm{erfc}^{-1}(2p)$$

Exclusion

Null hypothesis is signal+background $H_0 = H_{s+b}$; Signal absent in data $H_1 = H_b$

Discovery

Null hypothesis is background-only $H_0 = H_b$; Signal present in data $H_1 = H_{s+b}$

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Consider a test-statistic Q (larger Q is more signal-like). For outcome Q_{obs}:

$$\begin{aligned} p_{\text{excl}} &= P(Q \leq Q_{\text{obs}} | H_{s+b}) \\ p_{\text{disc}} &= P(Q \geq Q_{\text{obs}} | H_b) \end{aligned}$$

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Small p-values considered as an evidence against H_0:

90% or 95% exclusion:	$p_{\text{excl}} < 0.1$ or 0.05
	$Z_{\text{excl}} > 1.282$ or 1.645
3σ evidence or 5σ discovery:	$Z_{\text{disc}} > 3$ or 5
	$p_{\text{disc}} < 0.001350$ or 2.867×10^{-7}

Single Poisson channel

Consider a search for new physics signal, assuming Poisson statistics, with

$$\begin{aligned} s &= \text{mean signal events} \\ b &= \text{mean background events} \end{aligned}$$

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Poisson probability to observe k events:

$$P(k|\mu) = e^{-\mu} \mu^k / k!$$

For an outcome of n observed events,

p-value for exclusion:

$$p_{\text{excl}}(n, b, s) = \sum_{k=0}^{n} P(k|s+b) = \frac{\Gamma(n+1, s+b)}{\Gamma(n+1)}$$

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Here, $\Gamma(x)$, $\Gamma(x, y)$, and $\gamma(x, y)$ are the ordinary, upper incomplete, and lower incomplete gamma functions.

The CL_s technique

A *modified* frequentist measure for exclusion is commonly used at LHC searches (starting from h searches at LEP) [Zech 1989, Read 2002]:

$$\mathrm{CL}_s(Q_{\mathrm{obs}}) = \frac{P(Q \le Q_{\mathrm{obs}} | H_{s+b})}{P(Q \le Q_{\mathrm{obs}} | H_b)}$$

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- Avoids reporting exclusion when experiment not sensitive (e.g. $n < b$ in a single Poisson channel)
- Used in place of p_{excl} although CL_s is not a p-value or probability
- More conservative than p_{excl}

For a simple experiment that counts n events:

$$\mathrm{CL}_s(n, b, s) \quad = \quad \frac{p_{\mathrm{excl}}(n, b, s)}{p_{\mathrm{excl}}(n, b, 0)} = \frac{\Gamma(n+1, s+b)}{\Gamma(n+1, b)}$$

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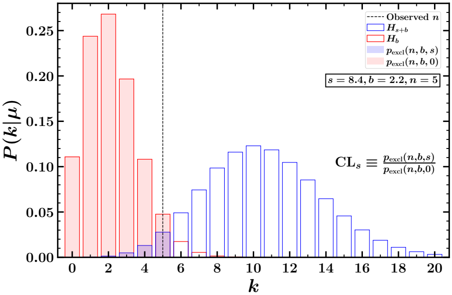

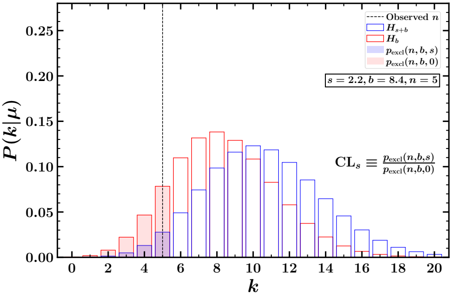

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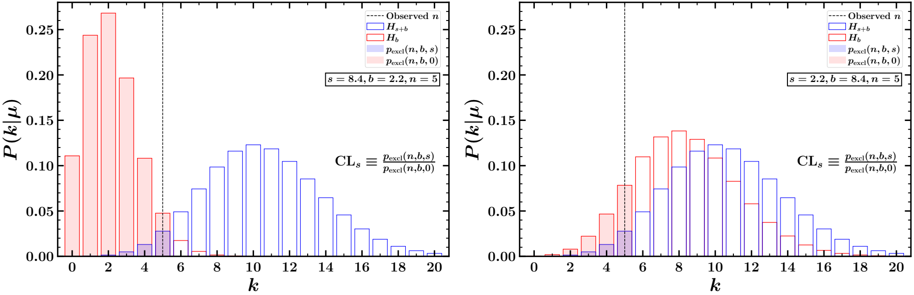
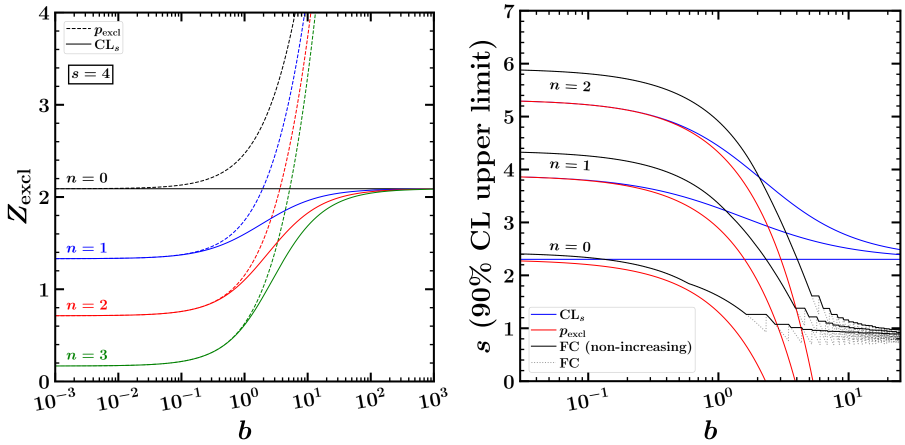


- Little (Large) overlap between H_{s+b} and H_b in the left (right) plot
- Since $s + b$ is same in both cases, $p_{\mathrm{excl}} = 0.0475$ ($> 95\%$ excl.) is the same!
- However $\mathrm{CL}_s = 0.0487$ (left; $> 95\%$ excl.) and 0.3022 (right; no excl.)



- At small b, $p_{excl} \simeq \mathrm{CL}_s$
- With p_{excl}, one can claim absurdly large Z for large b ($n \ll b$)!

Interestingly, CL_s for a single Poisson channel can also be obtained using Bayes' theorem with a flat prior for the signal [Helene 1982]:

$$\mathrm{CL}_{\mathrm{excl}}(n, b, s) \;=\; \frac{\displaystyle\int_s^\infty ds'\, \mathcal{L}(s'|n, b)}{\displaystyle\int_0^\infty ds'\, \mathcal{L}(s'|n, b)} = \frac{\displaystyle\int_s^\infty ds'\, e^{-(s'+b)}\,(s'+b)^n}{\displaystyle\int_0^\infty ds'\, e^{-(s'+b)}\,(s'+b)^n}$$

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Later, I will argue for a generalization based on $\mathrm{CL_{excl}}$ rather than CL_s for counting experiments with N independent search channels.

Bayes factors for discovery

An approach to discovery significance was proposed using Bayes factors [Berger 2008]:

$$B_{01} = \frac{P(Q_{\text{obs}}|H_b)}{\int_0^\infty ds'\, \pi(s')\, P(Q_{\text{obs}}|H_{s'+b})}$$

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and propose to use CL_{disc} in place of p to obtain a discovery significance.

- Avoids claiming discovery when experiment not sensitive to signal model
- More conservative than p_{disc}
- Very similar to the usage of CL_s for exclusion

For a single-channel counting experiment with n observed counts:

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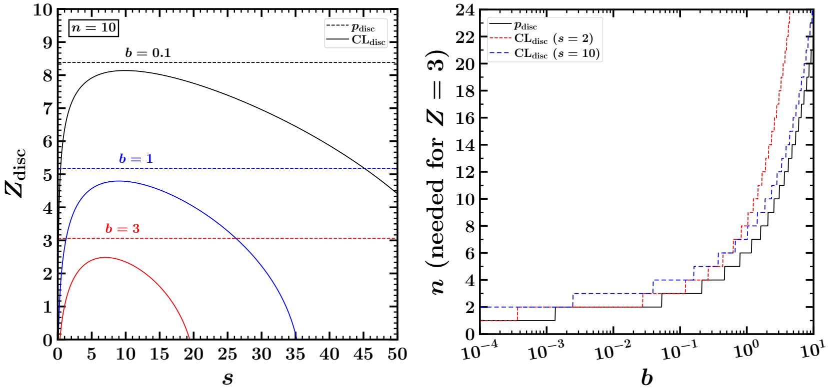

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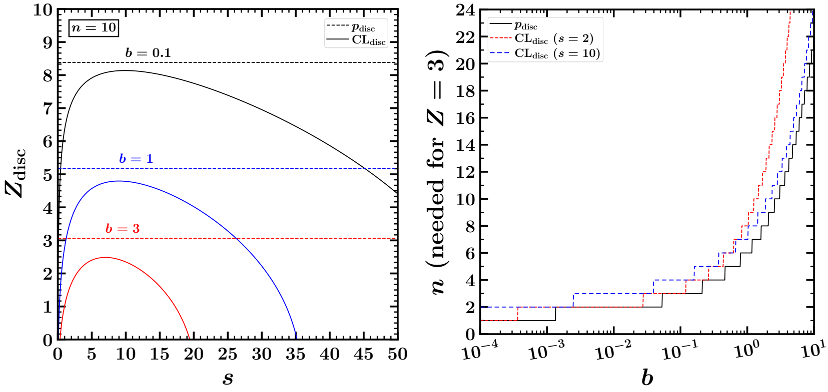

- p_{disc} independent of s; $Z(\mathrm{CL}_{\mathrm{disc}})$ maximized at $s = n - b$ (left plot)
- $\mathrm{CL}_{\mathrm{disc}}$ is more conservative than p_{disc}

How to project future sensitivities at counting experiments?

A very naive estimate: $Z_{\text{disc}} \sim Z_{\text{excl}} \sim \frac{s}{\sqrt{b}}$ (valid only for very large b).

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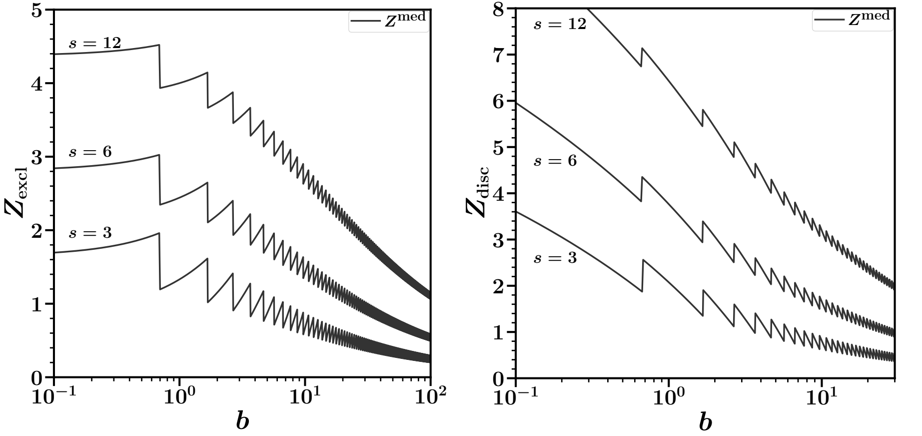
A common prescription:

- Step 1: simulate many pseudo-experiments with data generated under H_1.
  Outcome: $n_1, n_2, n_3, \ldots$
- Step 2: calculate the p-value for each pseudo-experiment with respect to H_0.
  Corresponding p-values: $p_1, p_2, p_3, \ldots$
- Step 3: synthesize the results into a significance estimate Z_{disc} or Z_{excl}.
  Challenge: many possible significance measures:
  - Median expected significance Z^{med}: Median$\{Z(p_1), Z(p_2), Z(p_3), \ldots\}$ (commonly used)

The "sawtooth" problem for median expected significances



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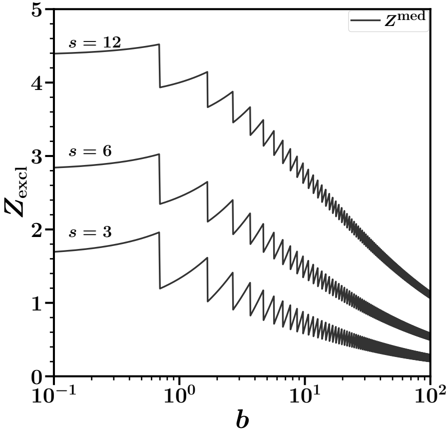

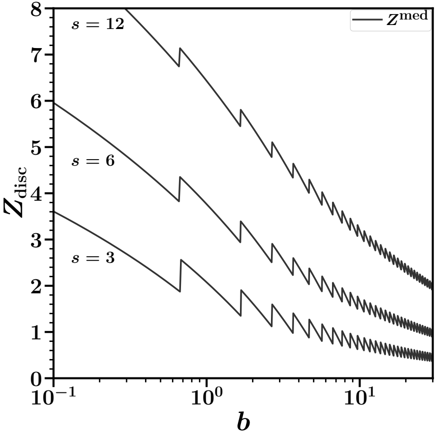

- Serious flaw: Z can **decrease** for **decrease** in b!
- Reason: median gets "stuck"
- Much worse for exclusion
- Result exactly reproducible

“Asimov” approximation for median expected significance

Based on the likelihood ratio method used in gamma-ray astronomy [Li-Ma 1983], Cowan Cranmer Gross Vitells 1007.1727 derived an approximation for median expected discovery significance:

$$Z_{\text{disc}}^{\text{CCGV}} = \sqrt{2\left[(s+b)\ln(1+s/b) - s\right]}$$

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At very large b, $Z_{\text{disc}}^{\text{CCGV}} \sim Z_{\text{excl}}^{\text{KM}} \sim \frac{s}{\sqrt{b}}$

Our proposal: exact Asimov criterion

Replace the observed n in p-values by its expected mean (a non-integer in general):

$$\begin{aligned} \langle n_{\text{disc}} \rangle &= s + b \\ \langle n_{\text{excl}} \rangle &= b \end{aligned}$$

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Plug in to obtain the expected p-values:

$$\begin{aligned} p_{\text{disc}}^{\text{Asimov}} &= \frac{\gamma(s+b, b)}{\Gamma(s+b)} \\ p_{\text{excl}}^{\text{Asimov}} &= \frac{\Gamma(b+1, s+b)}{\Gamma(b+1)} \end{aligned}$$

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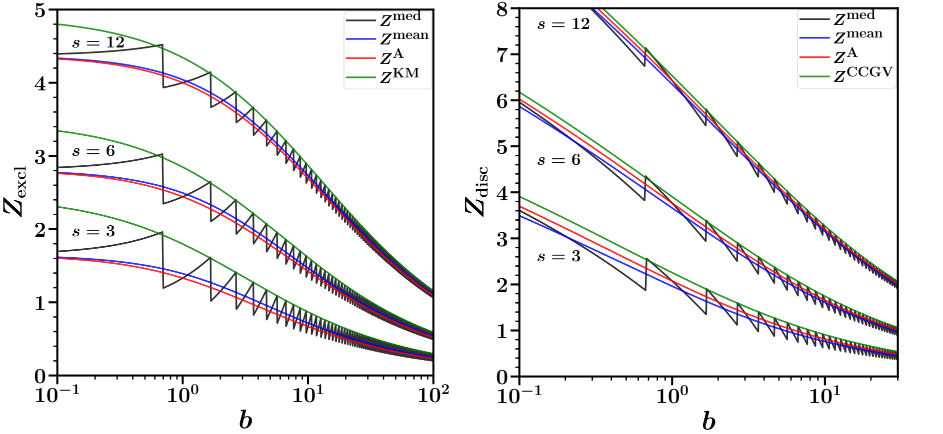
- More conservative results than CCGV and KM.
- Easy to compute, no pseudo-experiments required.

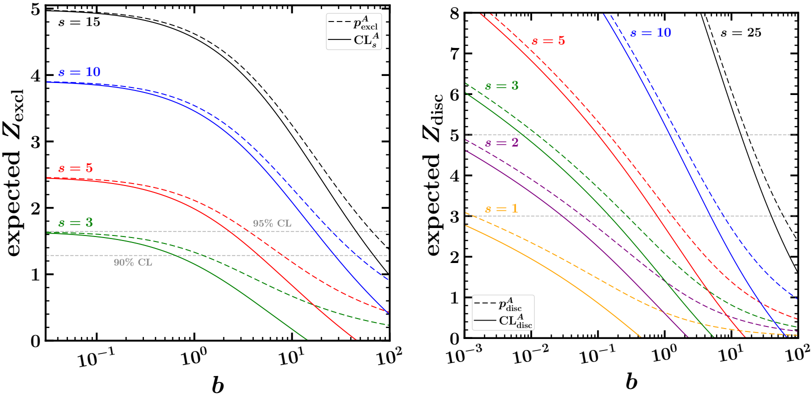
Comparing various measures for expected significances



- Z^{A} decreases monotonically with b
- Z^{A} more conservative than $Z^{\mathrm{CCGV}}_{\mathrm{disc}}$ or $Z^{\mathrm{KM}}_{\mathrm{excl}}$
- Z^{mean}, Z^{A} are both reasonable, but Z^{A} is easier to compute.

The sawtooth problem also occurs for median expected CL_s and CL_{disc}, and can be avoided by using the exact Asimov criterion:

$$\mathrm{CL}_s^A = \mathrm{CL}_{\mathrm{excl}}^A = \Gamma(b+1, s+b)/\Gamma(b+1, b)$$
$$\mathrm{CL}_{\mathrm{disc}}^A = e^s/(1+s/b)^{s+b}$$



- CL_s^A and $\mathrm{CL}_{\mathrm{disc}}^A$ are more conservative than p_{excl} and p_{disc}

Multi-channel counting experiments

Consider a counting experiment with N independent channels. For each channel $i = 1, \ldots, N$,

$$\begin{aligned} s_i &= \text{mean signal events} \\ b_i &= \text{mean background events} \end{aligned}$$

governed by Poisson statistics.

† Other choices (e.g. PLR) that are more complicated but give very similar (and often identical) results.

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(Modified) Frequentist measures:

- Define a test-statistic Q. A simple choice†:

$$Q = \ln(q) \quad \text{with} \quad q(\vec{n}, \vec{b}, \vec{s}) = \prod_{i=1}^{N} \frac{P(n_i|s_i + b_i)}{P(n_i|b_i)}$$

  such that larger Q is more signal-like
- Given an observation $\{n_i\}$, compute p_{excl} (or CL_s) and p_{disc} by imposing $Q \leq Q(\vec{n})$ (background-like) and $Q \geq Q(\vec{n})$ (signal-like)

†Other choices (e.g. PLR) that are more complicated but give very similar (and often identical) results.

Bayesian-motivated measures:

Experimental outcome: $\{n_i\}$ observed counts in each of the channels

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$$\mathrm{CL}_{\mathrm{excl}}(\vec{n}, \vec{b}, \vec{s}) = \frac{\displaystyle\int_s^\infty ds' \prod_{i=1}^{N} P(n_i | r_i s' + b_i)}{\displaystyle\int_0^\infty ds' \prod_{i=1}^{N} P(n_i | r_i s' + b_i)} \quad \text{with} \quad s = \sum_{i=1}^{N} s_i \text{ and } r_i = s_i / s$$

Note: Unlike the special case of single Poisson channel, $\mathrm{CL}_{\mathrm{excl}} \neq \mathrm{CL}_s$ in general

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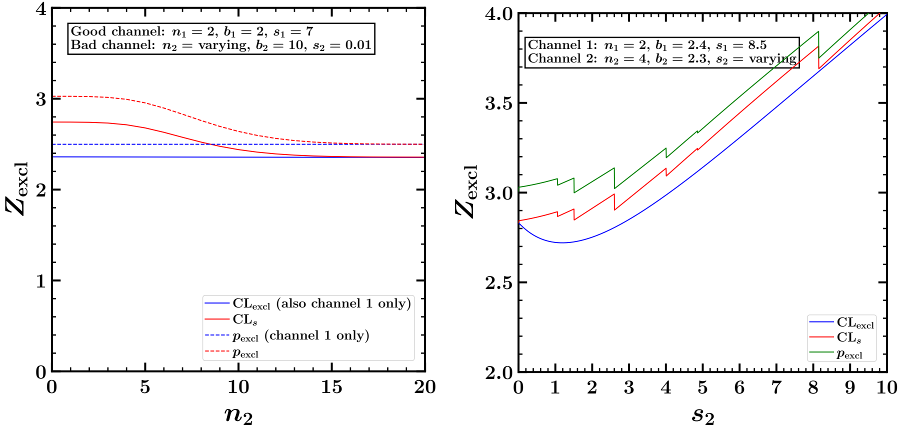
For discovery:

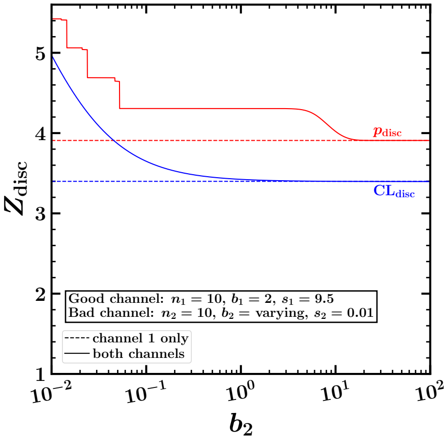
$$\mathrm{CL}_{\mathrm{disc}}(\vec{n},\vec{b},\vec{s}) = \prod_{i=1}^{N} \frac{P(n_i|b_i)}{P(n_i|s_i + b_i)}$$

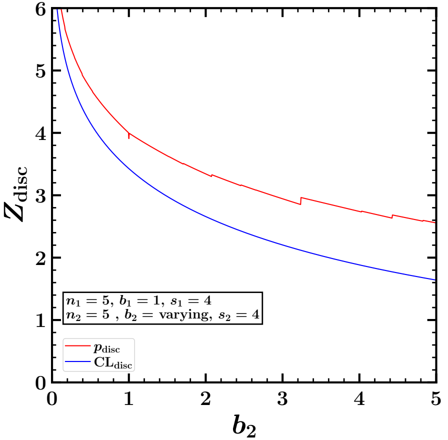
Exclusion for multi-channel experiments (Examples)



- Left: adding a channel with $s/b \ll 1$ can counter-intuitively increase $Z(p_{\mathrm{excl}})$ and $Z(\mathrm{CL}_s)$!
- Right: p_{excl} and CL_s exhibit discontinuities as s_2 is varied
- Bayesian measure $\mathrm{CL_{excl}}$ behaves as intuitively expected and is conservative

Discovery for multi-channel experiments (Examples)

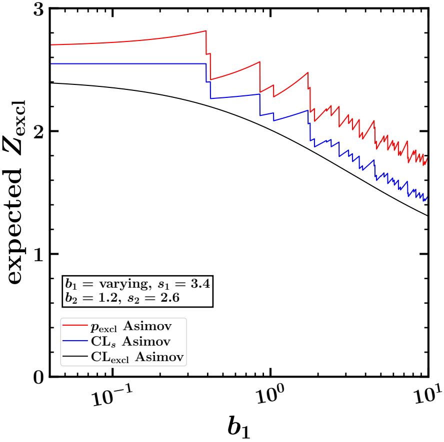


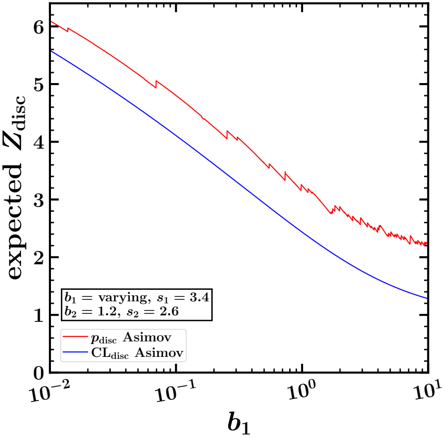


- p_{disc} is sensitive to a non-informative channel and has discontinuities
- Bayesian measure CL_{disc} behaves sensibly and is conservative

Expected sensitivities for multiple channels (Examples)

One can define Asimov results by replacing n_i by b_i (excl.) or $s_i + b_i$ (disc.)





- Exact Asimov Bayesian-motivated CL$_{\rm excl}$, CL$_{\rm disc}$ are perfectly straightforward to obtain, behave sensibly and do not suffer from "sawtooth" problems

Background uncertainty and other nuisance parameters

E.g., can map the uncertain background case to the "on-off problem" from gamma-ray astronomy:

- Signal-on region: measurement of n counts
- Signal-off region: measurement of m counts in a background-only region

Assume $\tau =$ ratio of background means in "off" and "on" regions is known.

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And, the probability density of b using Bayes' theorem:

$$f(b) \;=\; \underbrace{f(b|\hat{b}, \Delta_b)}_{\text{posterior}} = \underbrace{\tau}_{\text{normalization}} \underbrace{e^{-\tau b}(\tau b)^m/m!}_{\text{Poisson likelihood}} \underbrace{1}_{\text{prior}}$$

such that $\int_0^\infty db\, f(b) = 1$.

Probability to observe n events, given $s, \hat{b}, \Delta_{\hat{b}}$: (averaging over all possible b)

$$\Delta P(n, \hat{b}, \Delta_b, s) \quad = \quad \int_0^\infty db\, f(b|\hat{b}, \Delta_b) \quad \underbrace{P(n|s+b)}_{\text{Poisson probability}}$$

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We can extend the definitions of $p_{\rm excl}$, $\mathrm{CL}_s = \mathrm{CL}_{\rm excl}$, $p_{\rm disc}$, $\mathrm{CL}_{\rm disc}$ for a single Poisson channel by simply replacing $P(n|s+b)$ with $\Delta P(n, \hat{b}, \Delta_b, s)$, e.g.

$$p_{\rm excl}(n, \hat{b}, \Delta_b, s) = \sum_{k=0}^{n} \Delta P(k, \hat{b}, \Delta_b, s)$$

$$\mathrm{CL}_{\rm excl}(n, \hat{b}, \Delta_b, s) = \frac{\int_s^\infty ds'\, \Delta P(n, \hat{b}, \Delta_b, s')}{\int_0^\infty ds'\, \Delta P(n, \hat{b}, \Delta_b, s')} = \frac{p_{\rm excl}(n, \hat{b}, \Delta_b, s)}{p_{\rm excl}(n, \hat{b}, \Delta_b, 0)}$$

$$\mathrm{CL}_{\rm disc}(n, \hat{b}, \Delta_b, s) = \frac{\Delta P(n, \hat{b}, \Delta_b, 0)}{\Delta P(n, \hat{b}, \Delta_b, s)}$$

More generally, for any probability distributions $f(\vec{b})$ and $g(\nu)$ for the background and other nuisance parameters, one can marginalize (integrate) over b_i and ν.

$$
\begin{aligned}
\mathrm{CL}_{\mathrm{excl}} &= \frac{1}{D} \int d\nu\, g(\nu) \int d\vec{b}\, f(\vec{b}) \int_s^\infty ds' \prod_{i=1}^{N} P(n_i | r_i s' + b_i) \\
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\end{aligned}
$$

For exact Asimov expectations, for n_i we plug in the mean values:

$$
\begin{aligned}
\langle n_{i,\mathrm{excl}} \rangle &= \int d\nu\, g(\nu) \int_0^\infty d\vec{b}\, f(\vec{b}) \sum_{n_i=0}^{\infty} n_i P(n_i|b_i) \\
\langle n_{i,\mathrm{disc}} \rangle &= \int d\nu\, g(\nu) \int_0^\infty d\vec{b}\, f(\vec{b}) \sum_{n_i=0}^{\infty} n_i P(n_i|s_i + b_i)
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Our choice of statistical measures

Consider the application of CL_{excl} and CL_{disc} for proton decay experiments to:

- obtain current lower limits on τ_p in $p \to \overline{\nu} K^+$ and $p \to e^+ \pi^0$ at various CL based on SuperK's data (generalizing the 90% CL published limits)
- project exclusion and discovery reaches at DUNE, JUNO, and HyperK using the exact Asimov criterion

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The usage of these Bayesian-motivated measures is ideal, as they:

- guard against false exclusion/discovery and are more conservative than frequentist methods
- are well-behaved in multi-channel counting experiments and immune to "bad" channels
- easily can include uncertainties in the backgrounds and signal efficiencies
- can easily yield the exact Asimov expectations that do not suffer from discontinuities

Proton decay experiments with single search channel

(e.g. for preliminary estimates at DUNE and JUNO)

Number of decays in a specific decay channel ($p \to \overline{\nu} K^+$ or $e^+\pi^0$):

$$\Delta N = \underbrace{N_p}_{\text{protons/kton}} \times \underbrace{\Gamma}_{\text{partial width}} \times \underbrace{\lambda}_{\text{exposure in kton-yrs}}$$

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The observed limit or expected reach on τ_p:

$$\tau_p = 1/\Gamma = N_p \epsilon \lambda / s$$

where s is the number of signal events that gives

- $\mathrm{CL}^{(A)}_{\mathrm{excl}} = \alpha$ for (expected) exclusion at confidence level $1 - \alpha$
- $\mathrm{CL}^{A}_{\mathrm{disc}} = \frac{1}{2}\mathrm{erfc}(Z/\sqrt{2})$ for expected discovery at significance Z

Proton decay experiments with N search channels

(e.g. SuperK, HyperK with uncertain b_i, ϵ_i)

For the observed exclusion limit at $\text{CL} = 1 - \alpha$, we solve for Γ from:

$$\alpha = \underbrace{\frac{1}{D}}_{\text{normalization}} \int_{\Gamma}^{\infty} d\Gamma' \prod_{i=1}^{N} \int_0^1 d\epsilon_i \underbrace{g(\epsilon_i)}_{\text{Gaussian}} \int_0^{\infty} db_i \underbrace{f(b_i)}_{\text{on-off}} \underbrace{P(n_i | s_i' + b_i)}_{\text{Poisson likelihood}}$$

and translate that to a lower limit on $\tau_p = 1/\Gamma$.

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For exact Asimov exclusion reach, replace n_i by $\langle b_i \rangle$:

$$\langle b_i \rangle = \int_0^{\infty} db_i \, f(b_i) \, b_i$$

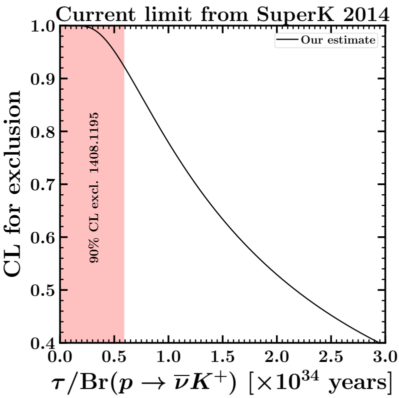
And, for exact Asimov discovery reach, we use:

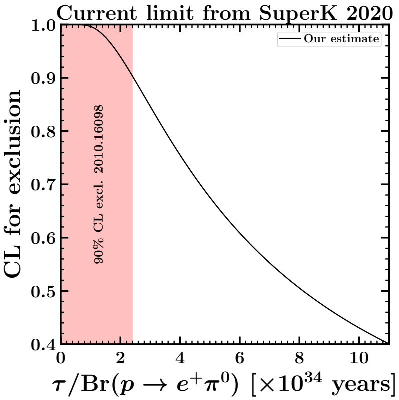
$$\frac{1}{2}\mathrm{erfc}\left(\frac{Z}{\sqrt{2}}\right) = \frac{\prod_i \int_0^\infty db_i\, f(b_i)\, P(\langle n_i\rangle | b_i)}{\prod_i \int_0^1 d\epsilon_i\, g(\epsilon_i)\, \int_0^\infty db_i\, f(b_i)\, P(\langle n_i\rangle | s_i + b_i)}$$

where $s_i = N_p \lambda_i \epsilon_i \Gamma$ and $\langle n_i\rangle = \langle s_i\rangle + \langle b_i\rangle$, with

$$\begin{aligned}
\langle s_i\rangle &= \Gamma N_p \lambda_i \int_0^1 d\epsilon_i\, g(\epsilon_i)\, \epsilon_i \\
\langle b_i\rangle &= \int_0^\infty db_i\, f(b_i)\, b_i
\end{aligned}$$

Generalizing SuperK limits [1408.1195 and 2010.16098]





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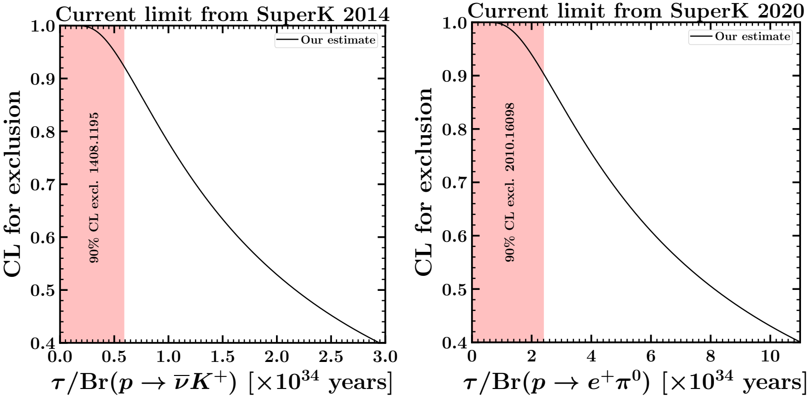

Decay mode	SuperK's 90% CL limit	Our 90% CL limit	Our 68% CL limit
$p \to \overline{\nu} K^+$	5.9×10^{33} years	6.6×10^{33} years	1.3×10^{34} years
$p \to e^+ \pi^0$	2.4×10^{34} years	2.4×10^{34} years	4.9×10^{34} years

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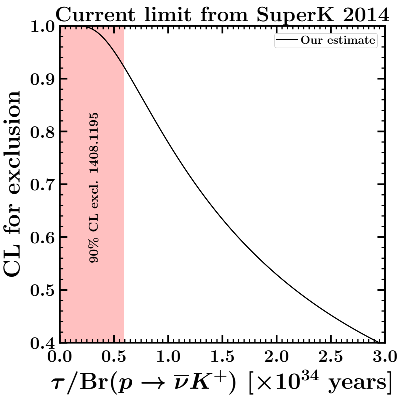

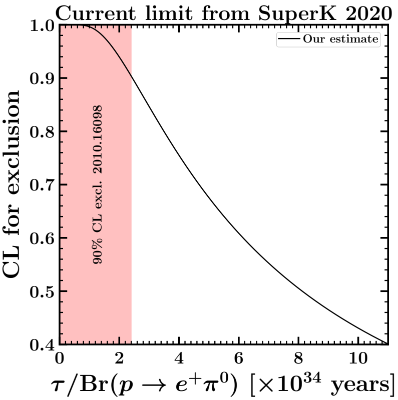

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Actually, our 90% CL limit for $\overline{\nu} K^+$ agrees perfectly with a later unpublished limit by SuperK based on same data Takhistov 1605.03235

Projections for $p \to \overline{\nu} K^+$ at DUNE[†]

40 kton liquid Argon detector that is more sensitive to $p \to \overline{\nu} K^+$ mode

[†] Based on the signal efficiencies and background rates from DUNE 1512.06148, Alt Thesis 2020, Alt ICHEP 2020, Alt Radics Rubbia 2010.06552, DUNE 2002.03005

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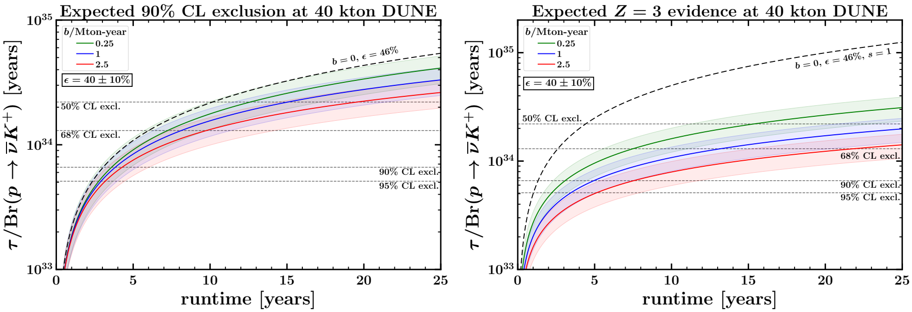

- The long dashed black line shows the idealized optimistic case of $b = 0$
- We require $s \geq 1$ to claim a discovery with $b = 0$

†Based on the signal efficiencies and background rates from DUNE 1512.06148, Alt Thesis 2020, Alt ICHEP 2020, Alt Radics Rubbia 2010.06552, DUNE 2002.03005

We can also find the required runtime at DUNE for an expected exclusion/discovery from

$$\Delta t = \frac{s\tau_p}{N_p N_{\mathrm{kton}}\epsilon}$$

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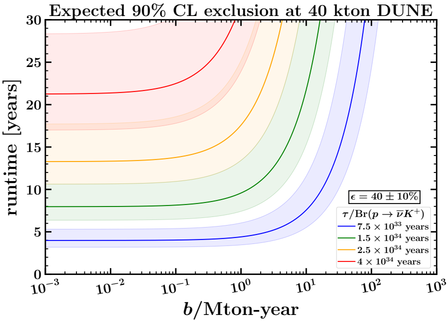

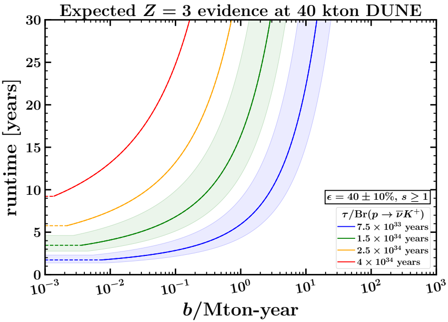

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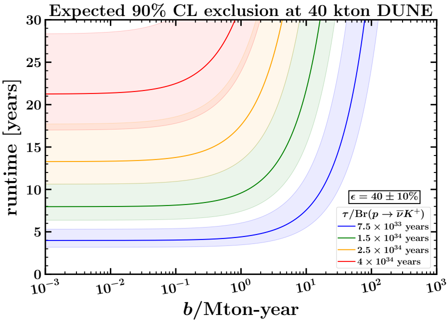

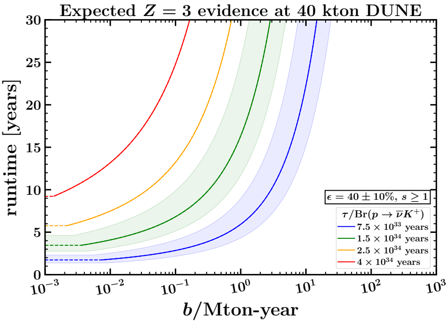

- If background rate increases, the required runtime increases more steeply for discovery than for exclusion
- Right: horizontal dashed lines due to $s \geq 1$ to claim discovery at small b

Projections for $p \to \overline{\nu} K^+$ at JUNO[†]

20 kton liquid scintillator detector that is sensitive to $p \to \overline{\nu} K^+$ mode

[†] Based on signal efficiencies and background rates from 1507.05613 and 2104.02565

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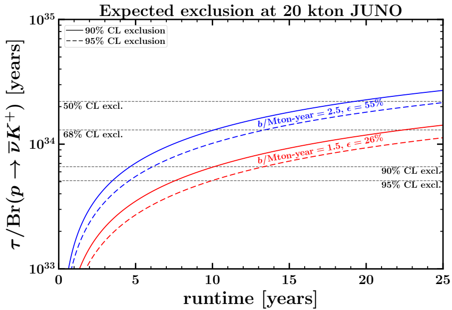

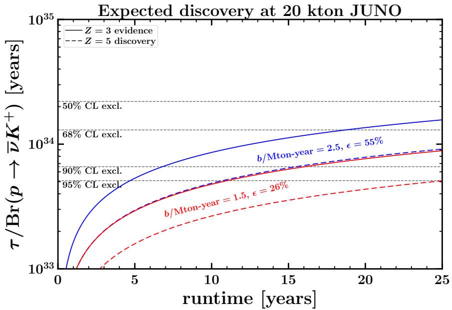

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Projections for $p \to \overline{\nu} K^+$ at Hyper-Kamiokande[†]

186 kton water Cerenkov detector that is more sensitive to $p \to e^+ \pi^0$ mode

[†] Based on signal efficiencies and background rates for multiple independent search channels from 1805.04163.

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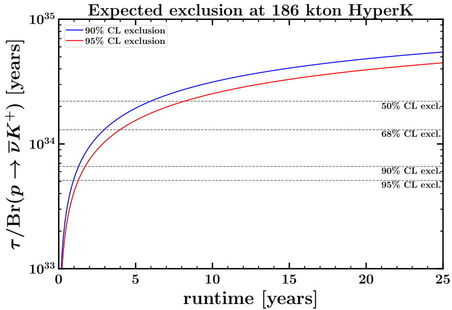

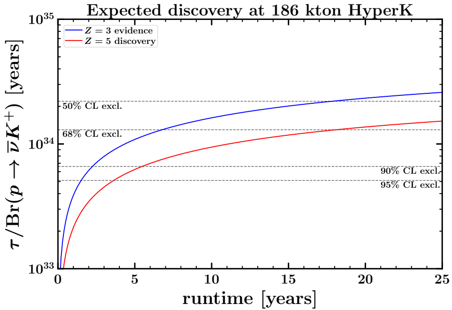

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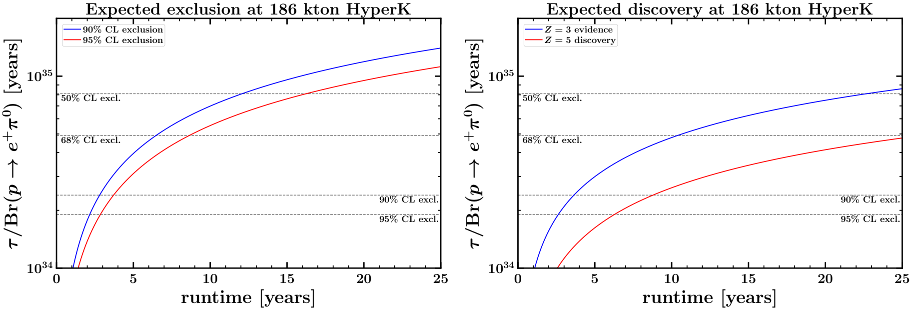
Projections for $p \to e^+\pi^0$ at Hyper-Kamiokande†



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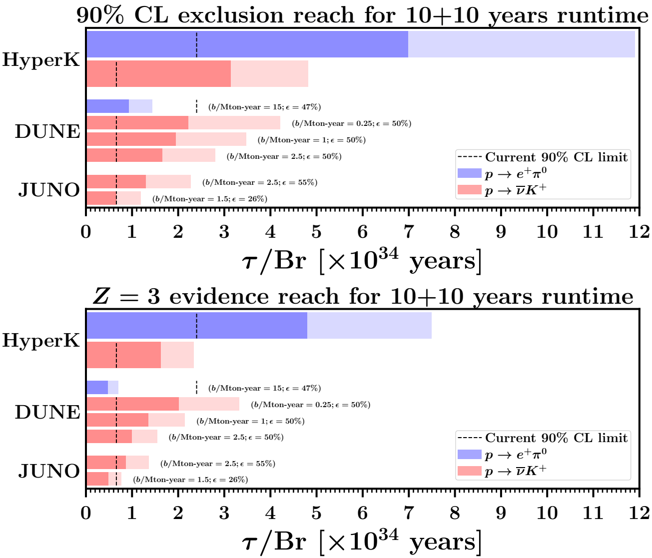
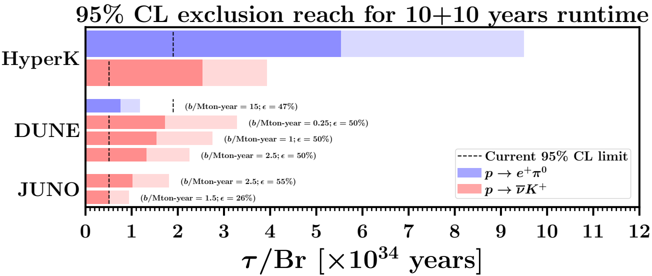
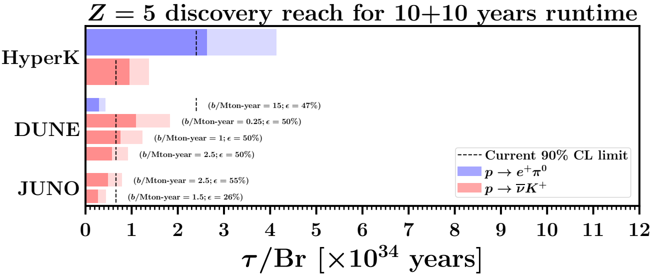
Expected reaches for 10+10 years runtime




95% CL exclusion reach for 10+10 years runtime
HyperK
DUNE
JUNO
(b/Mton-year = 15; ϵ = 47%)
(b/Mton-year = 0.25; ϵ = 50%)
(b/Mton-year = 1; ϵ = 50%)
(b/Mton-year = 2.5; ϵ = 50%)
(b/Mton-year = 2.5; ϵ = 55%)
(b/Mton-year = 1.5; ϵ = 26%)
Current 95% CL limit
$p \to e^+\pi^0$
$p \to \overline{\nu}K^+$
0 1 2 3 4 5 6 7 8 9 10 11 12
τ/Br [$\times 10^{34}$ years]


$Z = 5$ discovery reach for 10+10 years runtime
HyperK
DUNE
JUNO
(b/Mton-year = 15; ϵ = 47%)
(b/Mton-year = 0.25; ϵ = 50%)
(b/Mton-year = 1; ϵ = 50%)
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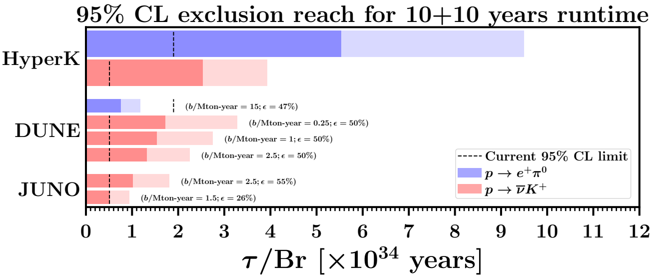
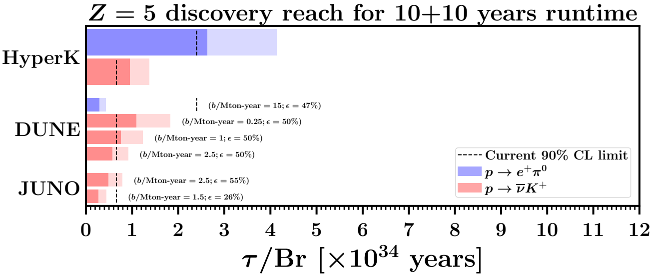
(b/Mton-year = 2.5; ϵ = 55%)
(b/Mton-year = 1.5; ϵ = 26%)
Current 90% CL limit
$p \to e^+\pi^0$
$p \to \overline{\nu}K^+$
0 1 2 3 4 5 6 7 8 9 10 11 12
τ/Br [$\times 10^{34}$ years]





- Can probe a significant fraction of the parameter space of various presently viable GUTs
- Prospects for a definitive $Z = 5$ discovery are particularly modest

Conclusion

- We pointed out various flaws associated with frequentist statistical measures for multi-channel counting experiments
- We argued in favor of conservative Bayesian-motivated statistical measures $\mathrm{CL}_{\mathrm{excl}}$ and $\mathrm{CL}_{\mathrm{disc}}$
- We advocate for the standard use of exact Asimov criterion to project sensitivities
- We applied these methods to study statistical significances for proton decay experiments

Conclusion

- We pointed out various flaws associated with frequentist statistical measures for multi-channel counting experiments
- We argued in favor of conservative Bayesian-motivated statistical measures $\mathrm{CL}_{\mathrm{excl}}$ and $\mathrm{CL}_{\mathrm{disc}}$
- We advocate for the standard use of exact Asimov criterion to project sensitivities
- We applied these methods to study statistical significances for proton decay experiments
- Easy-to-use Python (+ backend `C++` code) package ZSTATS v2.0 available on Github

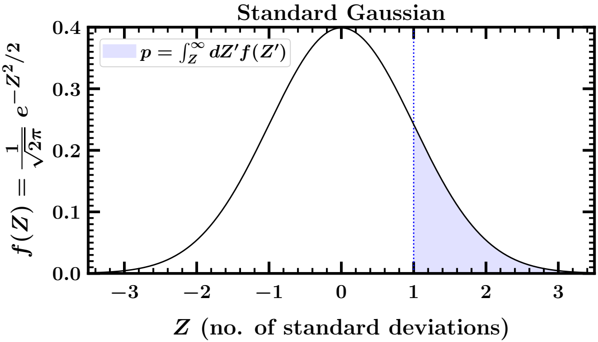
  includes code snippets that generate the data for all the plots

BACKUP SLIDES

Converting p to Z and back

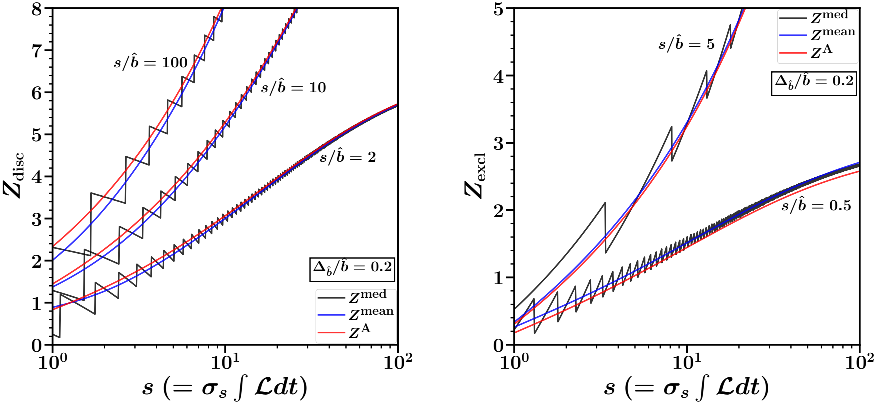
One-sided p-value in terms of the significance Z:

$$Z = \sqrt{2}\,\mathrm{erfc}^{-1}(2p)$$



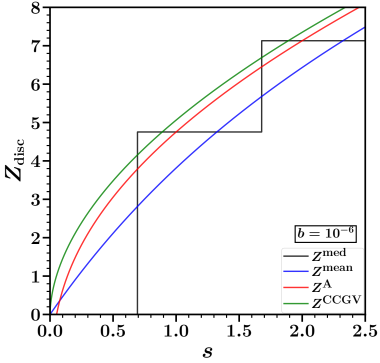
Uncertain background case: Comparing various measures

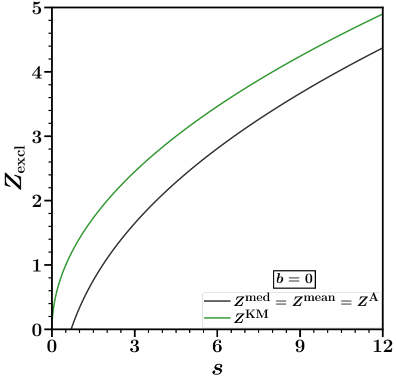
Taking s and $\hat{b}$ to be proportional to $\int \mathcal{L}dt$ (temporal progress of the experiment). Assume $\Delta_{\hat{b}}/\hat{b} = 0.2$.



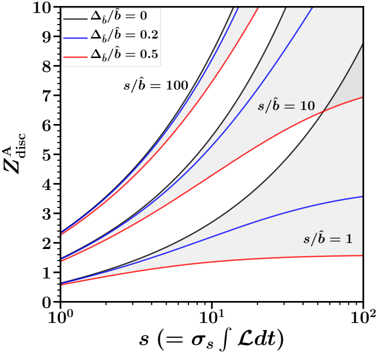
- Z^{med}, again, suffers from the sawtooth behavior
- Z^{A}, Z^{mean} reasonable and monotonic measures

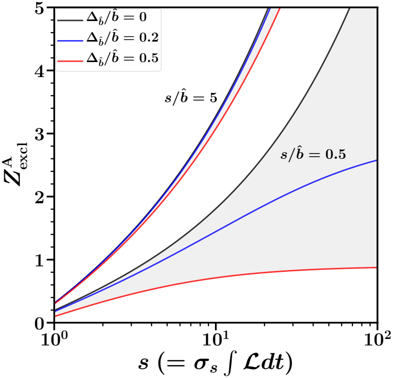
Extreme no background limit





Z^{A}_{disc} and Z^{A}_{excl} for various $\Delta_{\hat{b}}/\hat{b}$





Other alternatives: Mean expected significances

Mean expected significances: (involves simulation of pseudo-experiments)

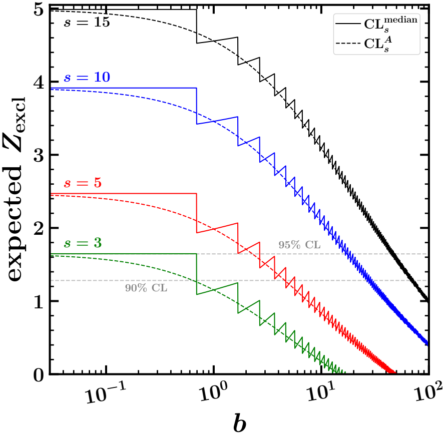
Z^{mean}: Mean$\{Z(p_1), Z(p_2), Z(p_3), \ldots\}$

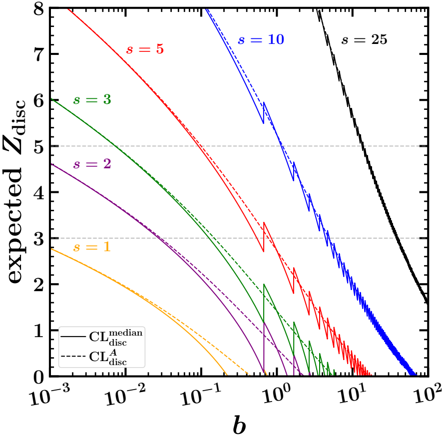
$Z^{p\,\text{mean}}$: Z(Mean$\{p_1, p_2, p_3, \ldots\}$)

Note: $Z^{\text{mean}} \neq Z^{p\,\text{mean}}$ (unlike the case with Z^{med})

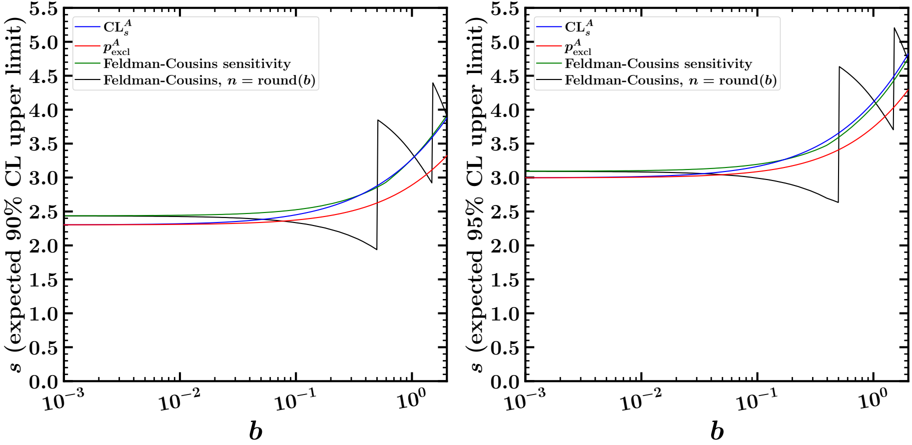
$Z^{p\,\text{mean}}$ is much lower than all others, dominated by unlikely outcomes with large p values. So, not reasonable.

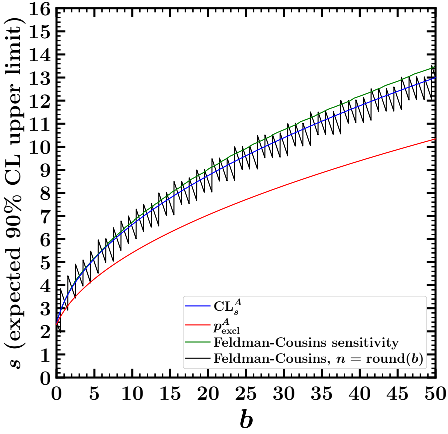
Median CL_{excl} and CL_{disc} for single channel



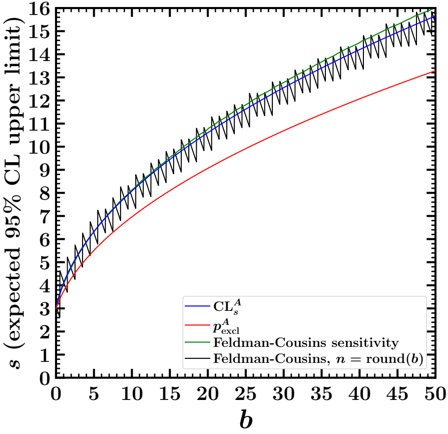


Expected upper limits



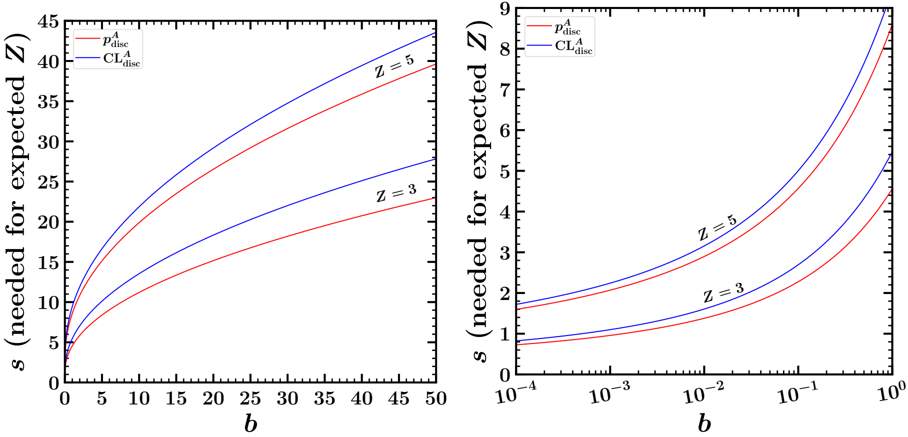


s (expected 90% CL upper limit)
b
CL_s^A
p_{excl}^A
Feldman-Cousins sensitivity
Feldman-Cousins, n = round(b)



s (expected 95% CL upper limit)
b
CL_s^A
p_{excl}^A
Feldman-Cousins sensitivity
Feldman-Cousins, n = round(b)

Signal needed for expected discovery



Zero background limit

Note that the discovery statistics $p_{\rm disc}$ and $\mathrm{CL}_{\rm disc}$ are not well-defined in the strict background-free limit $b \to 0$. Specifically,

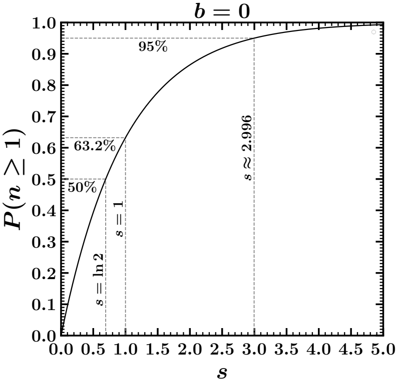
$$
\begin{aligned}
p_{\rm disc}(n, 0) &= \begin{cases} 0 \text{ if } n \neq 0 \\ 1 \text{ if } n = 0, \end{cases} \\
\mathrm{CL}_{\rm disc}(n, 0, s) &= \begin{cases} 0 \text{ if } n \neq 0, s \neq 0 \\ 1 \text{ otherwise.} \end{cases}
\end{aligned}
$$

Since $\langle n_{\rm disc} \rangle = s$ for $b = 0$, the above implies that the exact Asimov expected discovery significances are both infinite, $Z(p^{\rm A}_{\rm disc}) = Z(\mathrm{CL}^{A}_{\rm disc}) = \infty$, for any non-zero s (however small).

In order to be conservative, we can impose an additional requirement that $P(n \geq 1)$ should be greater than some fixed value in order to claim an expected discovery.

$$P(n \geq 1) = \sum_{n=1}^{\infty} P(n|s) = 1 - e^{-s}$$

- $P(n \geq 1) \geq 50\%$: $s \geq \ln 2$ and $n_{\text{disc}}^{\text{median}} \geq 1$
- $P(n \geq 1) \geq 63.2\%$: $s \geq 1$ and $\langle n_{\text{disc}} \rangle = s \geq 1$



Frequentist measures for multi-channel experiments

For an observation $\{n_i\}$,

$$p_{\text{excl}}(\vec{n}, \vec{b}, \vec{s}) = \sum_{\{k_i\}} \prod_{i=1}^{N} P(k_i | s_i + b_i) \quad \text{with} \quad Q(\vec{k}) \leq Q(\vec{n})$$

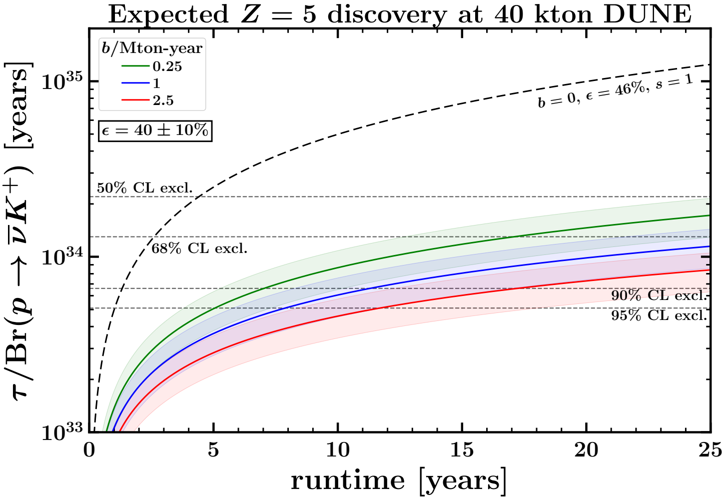
$$\text{CL}_s(\vec{n}, \vec{b}, \vec{s}) = \frac{p_{\text{excl}}(\vec{n}, \vec{b}, \vec{s})}{p_{\text{excl}}(\vec{n}, \vec{b}, 0)}$$

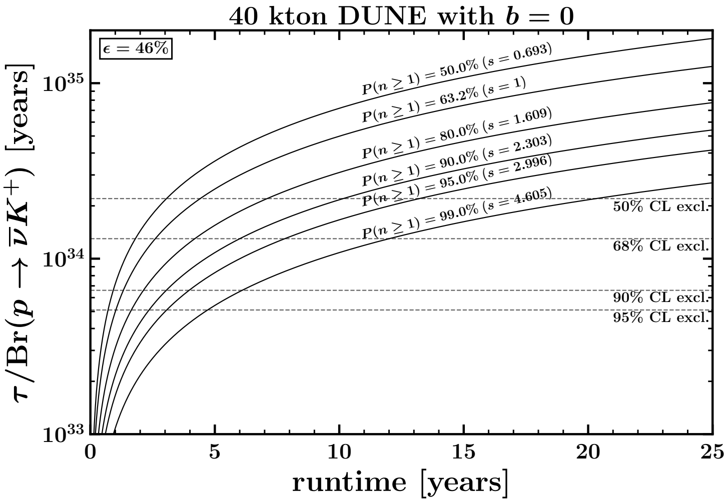
for exclusion, and

$$p_{\text{disc}}(\vec{n}, \vec{b}, \vec{s}) = \sum_{\{k_i\}} \prod_{i=1}^{N} P(k_i | b_i) \quad \text{with} \quad Q(\vec{k}) \geq Q(\vec{n})$$

for discovery.

Discovery at DUNE

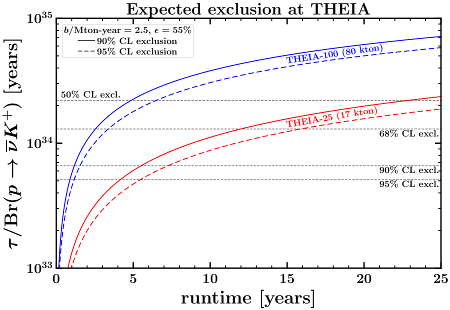


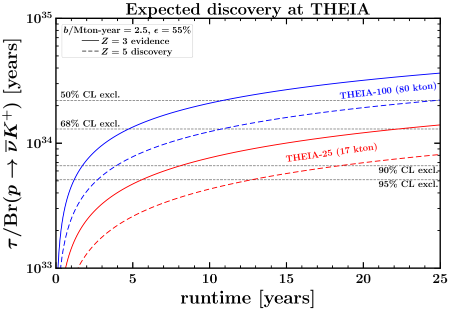


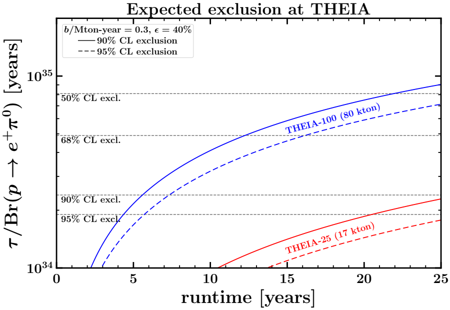
40 kton DUNE with $b = 0$
$\epsilon = 46\%$
$P(n \geq 1) = 50.0\%$ $(s = 0.693)$
$P(n \geq 1) = 63.2\%$ $(s = 1)$
$P(n \geq 1) = 80.0\%$ $(s = 1.609)$
$P(n \geq 1) = 90.0\%$ $(s = 2.303)$
$P(n \geq 1) = 95.0\%$ $(s = 2.996)$
$P(n \geq 1) = 99.0\%$ $(s = 4.605)$
50% CL excl.
68% CL excl.
90% CL excl.
95% CL excl.
$\tau/\mathrm{Br}(p \to \overline{\nu} K^+)$ [years]
10^{35}
10^{34}
10^{33}
0
5
10
15
20
25
runtime [years]

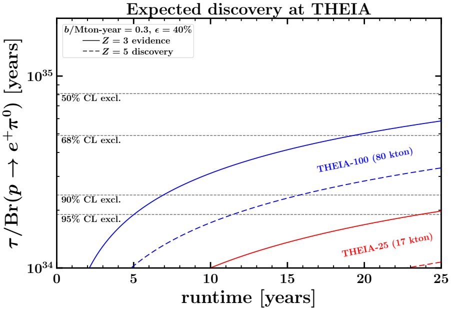
THEIA

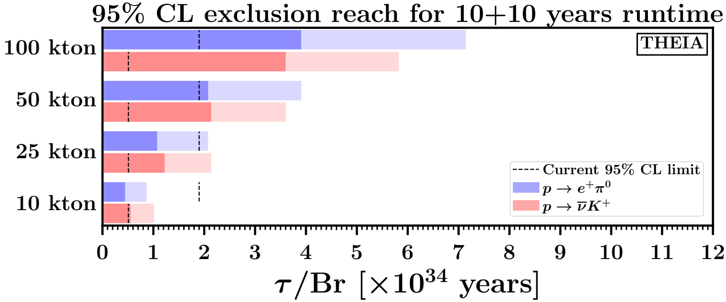
A novel detector concept with water based liquid scintillator, 10% liquid scintillator and 90% water, that can detect and distinguish between Cerenkov and the scintillation light

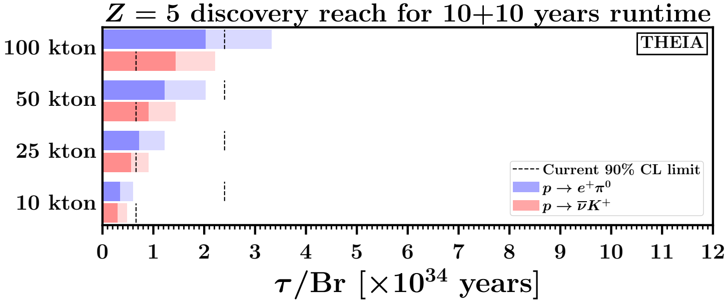





Expected exclusion at THEIA
b/Mton-year = 0.3, ε = 40%
90% CL exclusion
95% CL exclusion
50% CL excl.
68% CL excl.
90% CL excl.
95% CL excl.
THEIA-100 (80 kton)
THEIA-25 (17 kton)
τ/Br(p → e+π0) [years]
runtime [years]


Expected discovery at THEIA
b/Mton-year = 0.3, ε = 40%
Z = 3 evidence
Z = 5 discovery
50% CL excl.
68% CL excl.
90% CL excl.
95% CL excl.
THEIA-100 (80 kton)
THEIA-25 (17 kton)
τ/Br(p → e+π0) [years]
runtime [years]


95% CL exclusion reach for 10+10 years runtime
THEIA
100 kton
50 kton
25 kton
10 kton
Current 95% CL limit
$p \to e^+\pi^0$
$p \to \overline{\nu}K^+$
0 1 2 3 4 5 6 7 8 9 10 11 12
$\tau/\mathrm{Br}\ [\times 10^{34}$ years]


$Z = 5$ discovery reach for 10+10 years runtime
THEIA
100 kton
50 kton
25 kton
10 kton
Current 90% CL limit
$p \to e^+\pi^0$
$p \to \overline{\nu}K^+$
0 1 2 3 4 5 6 7 8 9 10 11 12
$\tau/\mathrm{Br}\ [\times 10^{34}$ years]