The mass of the QCD axion

Kalman Szabo
Forschungszentrum Jülich & University of Wuppertal

Borsanyi, Fodor, Guenther, Kampert, Katz, Kawanai, Kovacs, Mages, Pasztor, Pittler, Redondo, Ringwald

Calculation of the axion mass based on high-temperature lattice QCD

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Outline

1. Axions

2. Instantons

Axions

Strong CP problem

Most general $SU(3)$ symmetric Lagrangian

$$\boxed{L = L_{QCD} + \theta \cdot G\widetilde{G}}$$

θ could be the source of P, CP violation.

It isn't. From nEDM experiments $\rightarrow \theta < 10^{-10}$

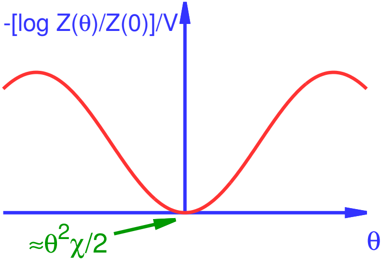
Why?

θ dependence of QCD

Calculate the Feynman path integral!

$$Z(\theta) = \int [dG][d\psi^\dagger][d\psi] \exp\left(i \int d^4x \, (L_{QCD} + \theta G\widetilde{G}) \right)$$

Has a minimum at $\theta = 0$!



$Q = \int G\widetilde{G}$ topological charge, $\chi \equiv \frac{\langle Q^2 \rangle}{V}$ top. susceptibility

A solution by Peccei-Quinn '77

Turn the parameter into

a dynamical field!

figs/thetapot/plot.gif

$$L_{QCD} + \theta \cdot G\widetilde{G} + \frac{1}{2} f_a^2 \cdot (\partial_\mu \theta)^2 + V(\theta, \partial_\mu \theta)$$

with $V(\theta, \partial_\mu \theta)$ such, that minimum stays at $\theta = 0$.

PQ: Spontaneously broken global $U(1)_{PQ}$ at scale f_a.
θ= Goldstone mode. Only derivative couplings $V = V(\partial\theta)$.

→ axion pseudo-Goldstone $m_a^2 = \chi / f_a^2$ [Weinberg,Wilczek]

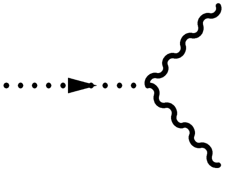
The QCD axion [Weinberg,Wilczek]

Pseudo-Goldstone boson with mass $m_a^2 = \chi / f_a^2$

Couplings? Model dependent.

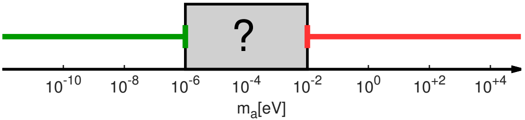
Introduce a $U(1)_{PQ}$ field φ in Mexican-hat plus

$\rightarrow$ heavy quark Q [KSVZ] $\rightarrow$ two Higgs H_u, H_d [DFSZ]



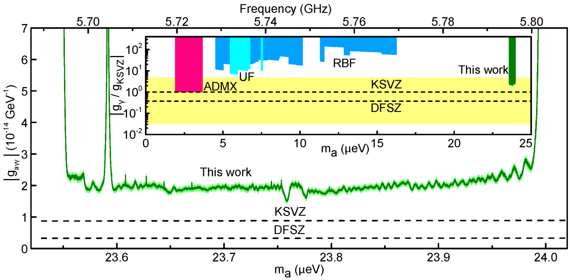
Smaller mass more elusive

The axion mass window



Can't be too large → would have "seen" it, since coupling $\sim m_a$

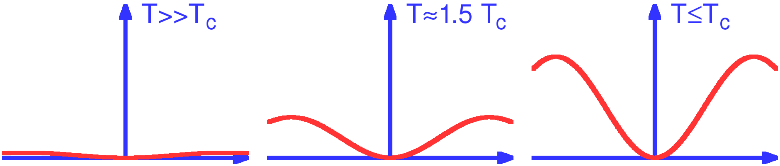
[HAYSTAC '16]



Can't be too small → too much of them . . .

Axions from early Universe [Preskill,Wilczek,Wise, . . . '83]

Potential becomes flat at QCD transition ($T_c \approx 150$MeV)



Axion field equation in expanding Universe:

$$\frac{d^2\theta}{dt^2} + 3H(T)\frac{d\theta}{dt} + \chi(T)/f_a^2 \sin\theta = 0$$

PQ breaking at $T \sim f_a \gtrsim 10^{11}$GeV then decoupling $\rightarrow$ initial angle θ_0. As universe cools H-friction decreases, potential increases, axion rolls down and starts oscillating.

Acts as (cold) dark matter!

Constraining m_a from dark matter

Number of axions $\Omega_a(m_a)$: smaller m_a gives larger Ω_a.

$\Omega_a \leqslant \Omega_{DM}$ lower bound on m_a.

Assuming all DM is axion $\rightarrow$ prediction for m_a.

We need:

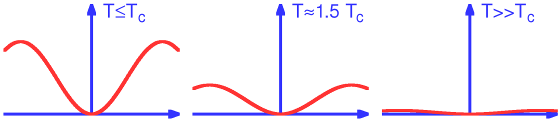
1. Axion potential $\chi(T)$

2. Hubble rate H(T) $\rightarrow$ equation of state $\epsilon(T), p(T)$

Instantons from the lattice

Topological susceptibility at $T > 0$

$\chi(T) = \frac{\langle Q^2 \rangle}{V} \sim$ fraction of gauge field configurations with non-trivial topology (Q)



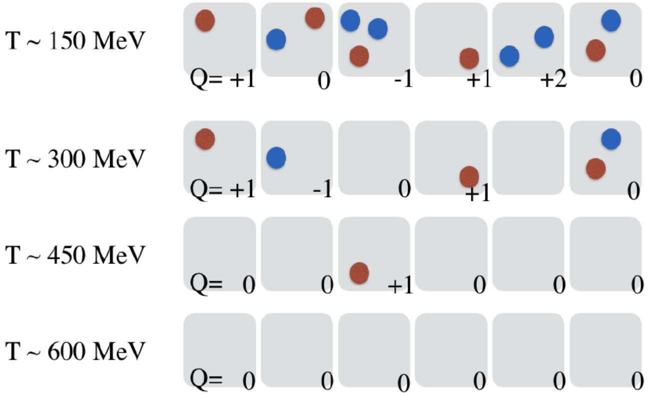
Strong suppression for high temperatures:

1. path integral weight $\exp(-S_Q/g^2)$ with $g(T) \to 0$
2. fermion index theorem $\det(D+m) \sim m^{|Q|}$

Signal is small $\to$ challenges:

large statistical error and large lattice artefacts

$\chi(T)$ from standard approach

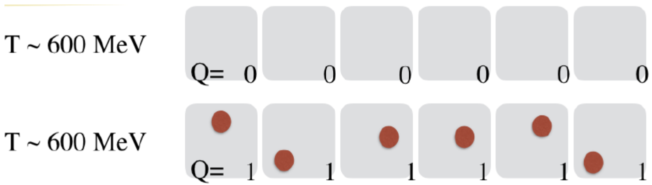


Simulate for centuries to get the first $Q > 0$ configuration!

$\chi(T)$ from fixed Q integral

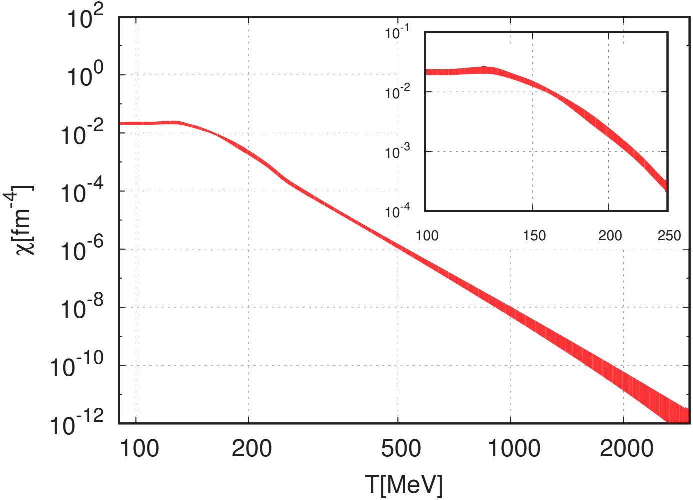
Determine slope instead of susceptibility:
see also in [Frison et al '16]

$$-\frac{d\log\chi}{d\log T} = b = 4 + \frac{d\beta}{dT}\langle S_g\rangle_{1-0} + \sum_f \frac{dm_f}{dT} m_f \langle\overline{\psi}\psi\rangle_{1-0}$$



finally perform an integral $\chi(T) = -\int d\log T\; b(T)$

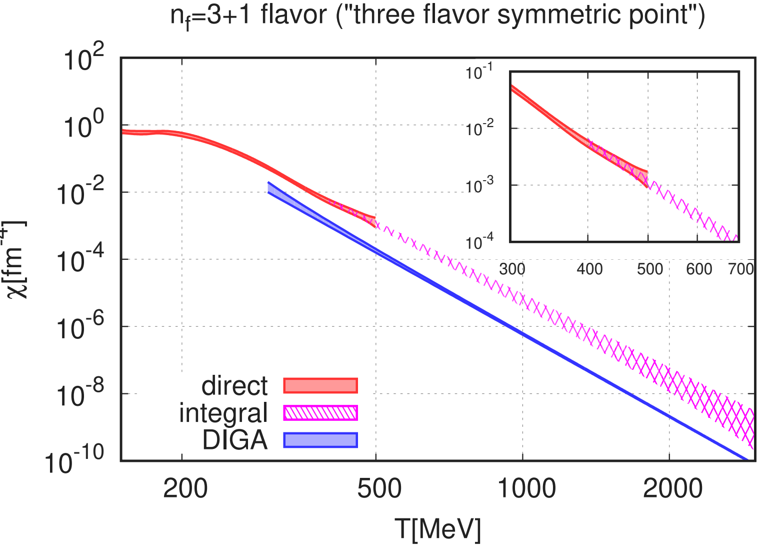
Topological susceptibility



Exponent consistent with Dilute Instanton Gas Approximation (−8), prefactor is 5x larger.

DIGA [Gross,Pisarski,Yaffe '81]

$$f(\theta) = \chi_{1loop}(T) \cdot (1 - \cos\theta)$$



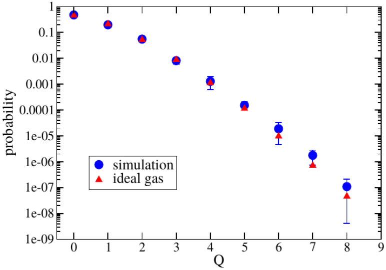
Ideal gas approximation

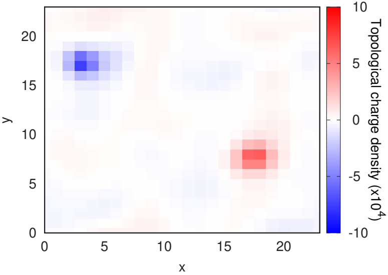
independent objects carrying ± 1 topological charge $\rightarrow$

$$f(\theta) = \chi(T) \cdot (1 - \cos\theta)$$

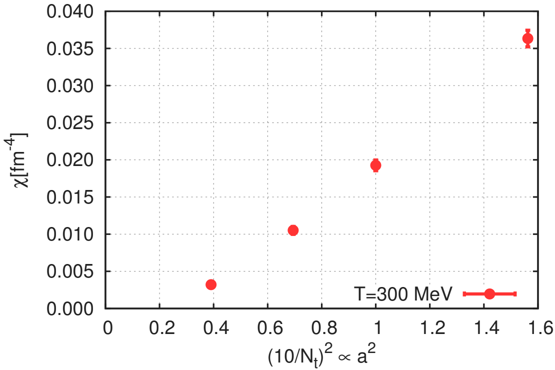
seems valid above $T \gtrsim T_c$

eg. T=180 MeV physical point $L = 6.6$ fm:





Difficult continuum extrapolation



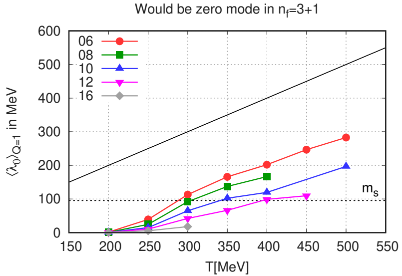
Non-chiral fermions have no exact fermion zero modes

$\det(D+m) \sim (m+\lambda_0)^{|Q|}$ with $\lambda_0 \neq 0$ on the lattice

$\rightarrow$ Too large χ, too small slope!

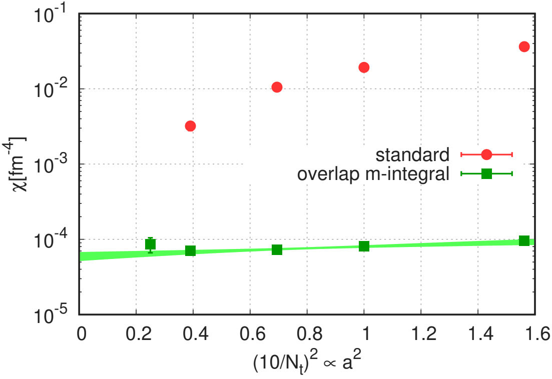
Continuum instanton and zero mode

Lattice instanton and zero mode



Continuum extrapolation

Doing full simulation with chiral fermions is too expensive.

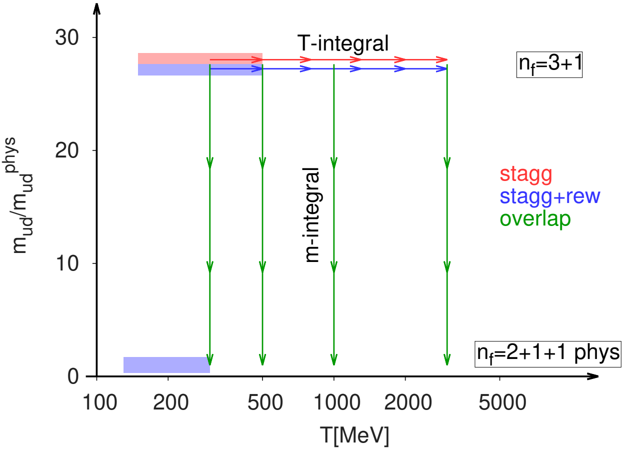


1. Simulate at large mass ($30 \cdot m_{ud}^{phys}$), continuum extrapolation behaves much better.

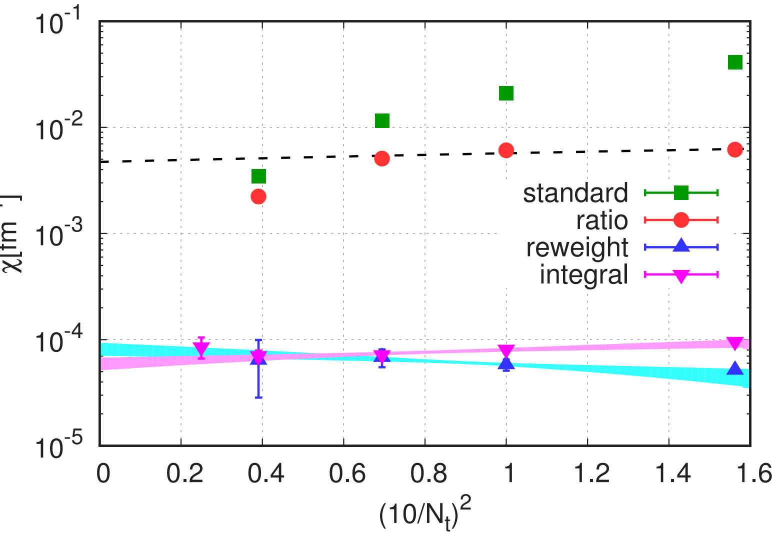
2. Calculate difference to m_{ud}^{phys} by integrating in m using fermion with exact chiral symmetry.

Map of simulations

- 4-stout staggered $n_f = 3 + 1$
- 4-stout staggered with fixed top.
- 4-stout staggered $n_f = 2 + 1 + 1$
- dynamical overlap $n_f = 2 + 1$ with fixed top.

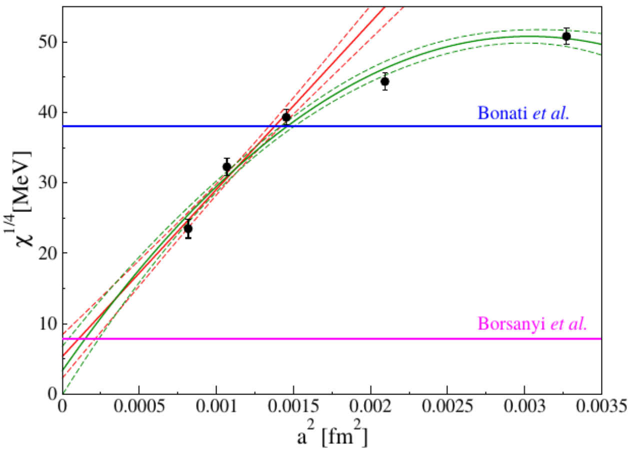


Continuum extrapolation at T=300 MeV



Comparison with others [Bonati et al '16 '18]

Continuum extrapolation at T=430 MeV:

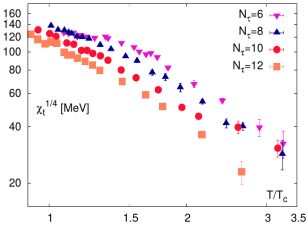


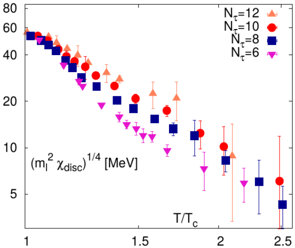
Comparison with others [Petreczky, Sharma '16]

$\chi(T)$ from HISQ fermions

Use two different definitions for topological charge (gluonic and fermionic).

Both have sizeable discretization errors but approach the continuum limit from different directions.



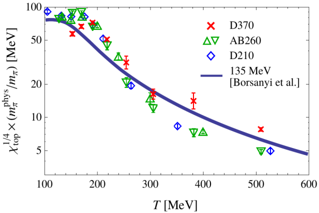


Comparison with others

[Petreczky, Sharma '16] "the dependence is found to be consistent with dilute instanton gas approximation"

[Taniguchi et al '17] "a decrease in T which is consistent with the predicted $\chi(T) \propto T^{-8}$"

[Lombardo et al '18]: "with an exponent close to the one predicted by the DIGA"



[Bonati et al '18] "The continuum extrapolation is in agreeement with previous lattice determinations"

The simplest estimate

Assuming

1. all DM is axion $\Omega_{DM} = \Omega_a(m_a)$
2. axion field is spatially constant in very large domains
3. there are many domains with random initial value of the field (θ_0)

Evolution equations are simple to solve.

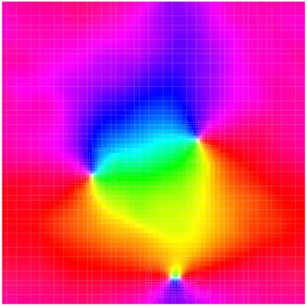
$$\rightarrow m_a = 28(1)\mu\text{eV}$$

Howto improve: take into account spatial dependence $\theta(\vec{x})$ and take θ_0 from PQ transition

Axion strings

Axion strings [Vilenkin,Everett]

θ_0 can be undefined $\equiv$ axion string.



What is their effect on axion production? Vastly different estimates.

Proper way: classical field theory simulation,

but extreme demanding: f_a, H differ by factor 10^{30}!

Heavy string simulation [Moore, Klaer '17]

Problem: coarse lattice does not resolve string core $\rightarrow$ too small string tension.

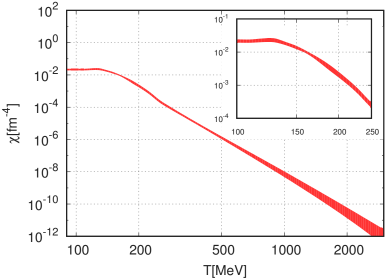
Idea: make string cores artifically heavier, while not changing long distance properties. Attach a local string to each global string.

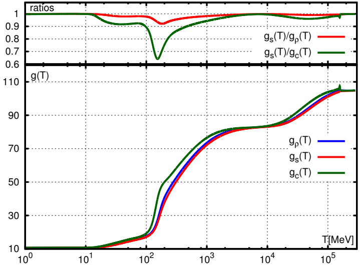
Surprise: less axions in the presence of strings.

$$\rightarrow m_a = 26.2(3.4)\mu\mathrm{eV}$$

Summary

Lattice QCD has made a good progress in calculating the necessary inputs for axion cosmology.





Several algorithmic developments were necessary.

Still not calculated: axion potential beyond leading order b_2

Still not well understood: global string dynamics, simulations with large string tension is already possible

On good way to a solid theory prediction!

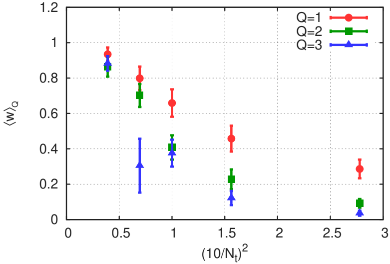
Backup

Reweighting

Problem: In continuum weight is m, on the lattice $m + \lambda_0[U]$.

Solution: change weight of configuration by $w[U] \equiv \frac{m}{m+\lambda_0[U]}$

$\langle w \rangle_Q$ must approach 1 in the continuum limit.



Improves the observable without changing the action.

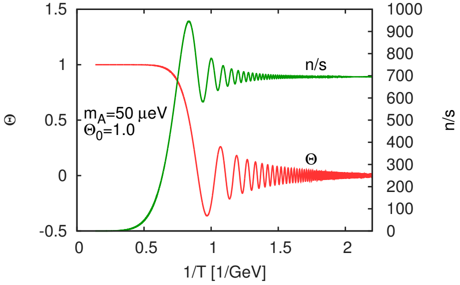
$$\chi = \frac{\sum Q^2 \; Z_Q}{\sum Z_Q} \rightarrow \chi_{\text{rew}} = \frac{\sum Q^2 \; \langle w \rangle_Q \; Z_Q}{\sum \langle w \rangle_Q \; Z_Q}$$

Number of axions

Energy density: $\epsilon_a(t) = \frac{1}{2} f_a^2 \dot{\theta}^2 + \chi(t)(1 - \cos\theta)$

Number density: $n_a(t) = \frac{\epsilon_a(t)}{m_a(t)}$

Number density normalized by total entropy density converges to a constant.



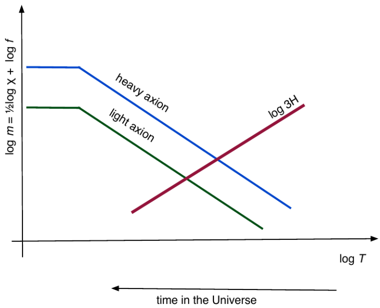
Axion energy density today:

$$\Omega_a \equiv \epsilon_{a,\text{today}} = n_{a,\text{today}}\ m_a = \frac{n_{a,\text{today}}}{s_\text{today}}\ s_\text{today}\ m_a \approx \frac{n_a(t)}{s(t)}\ s_\text{today}\ m_a$$

The lighter the more

Looks paradox, since $\boxed{\epsilon_0 = m_0 \cdot n_0}$

- the lighter, the later it oscillates $3H = m = m_0\sqrt{\chi}$



- later Hubble-dilution is smaller (T^3)
- later energy density ($\chi = T^{-b}$) is larger

Densities at oscillation (T) and today (0):

$$\epsilon = \chi, \qquad n = \frac{\epsilon}{m} = \frac{\chi}{m_0\sqrt{\chi}} = \frac{H}{m_0^2} = \frac{T^2}{m_0^2}, \qquad s = T^3, \qquad n_0 = \frac{n}{s} s_0 = \frac{1}{m_0^2 T}$$

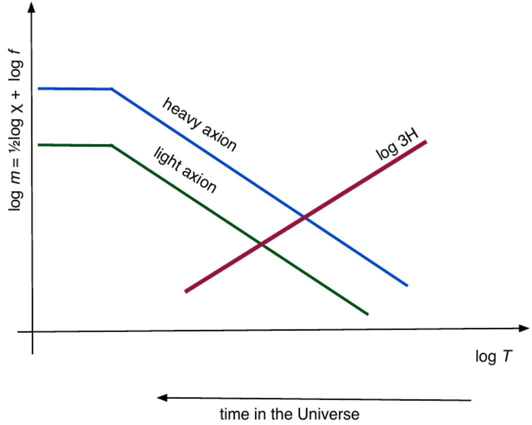
$$\longrightarrow \boxed{\epsilon_0 = \frac{1}{m_0 T}}$$

Lighter mass more axions

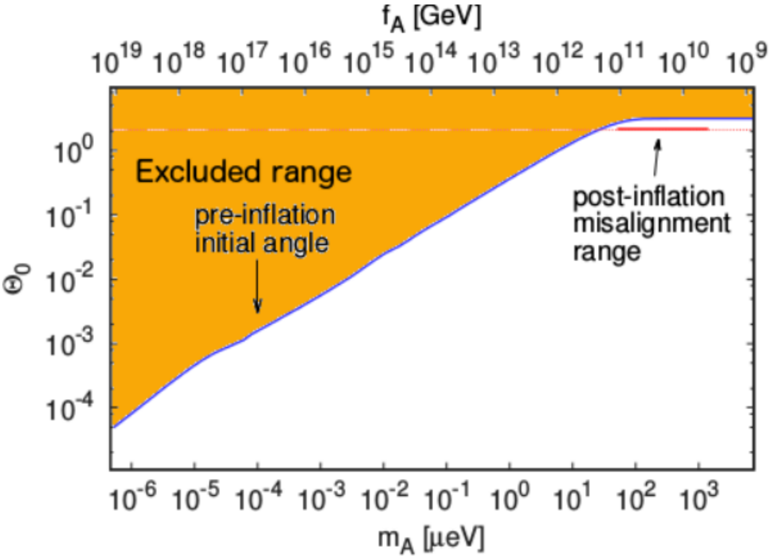
Have to solve

$$\frac{d^2\theta}{dt^2} + 3H(T)\frac{d\theta}{dt} + \frac{\chi(T)}{f_a^2}\sin\theta = 0$$

Rolling starts when $3H(T) \approx \sqrt{\chi(T)}/f_a$

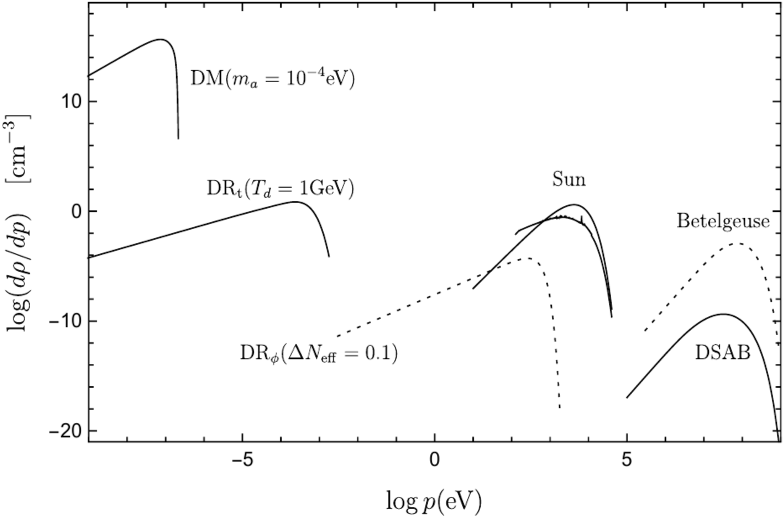


Axion mass and initial angle



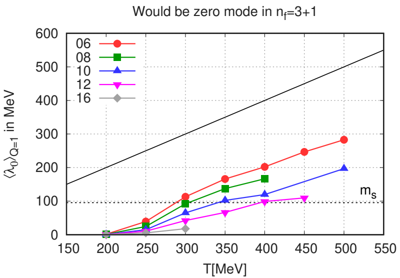
Sources of axions

[Irastorza,Redondo '18]



Continuum instanton and zero mode

Lattice instanton and zero mode



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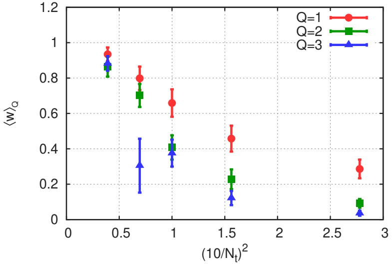

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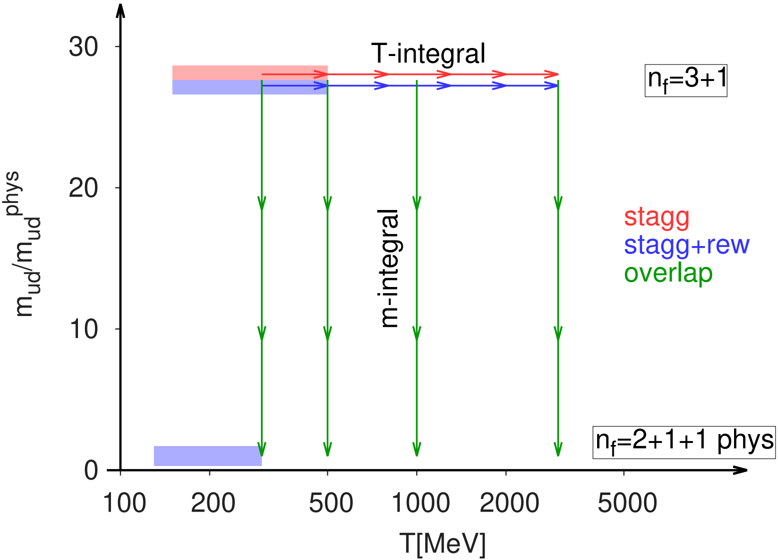
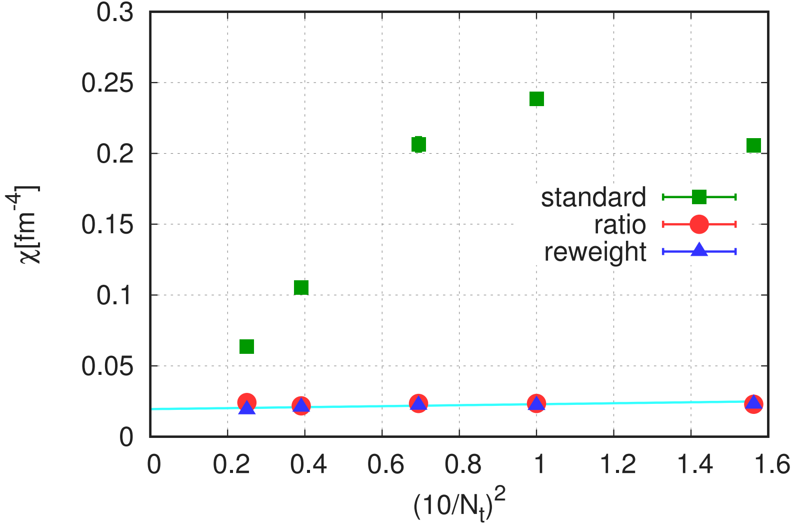
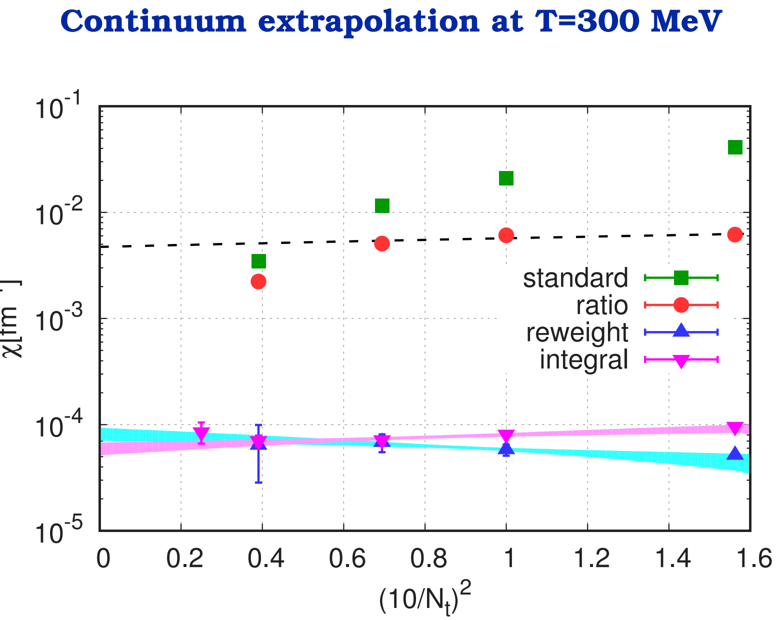
Map of simulations



Continuum extrapolation at T=150 MeV

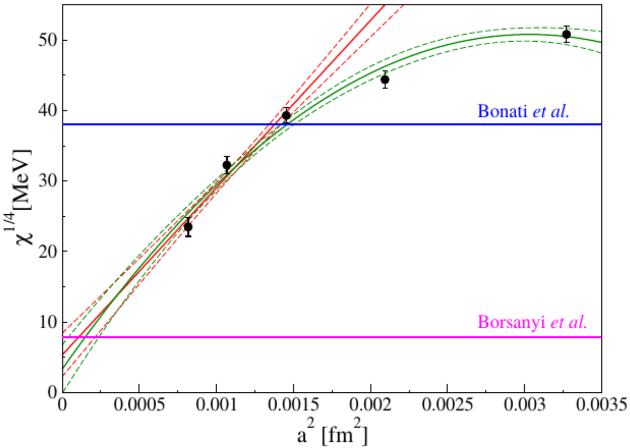




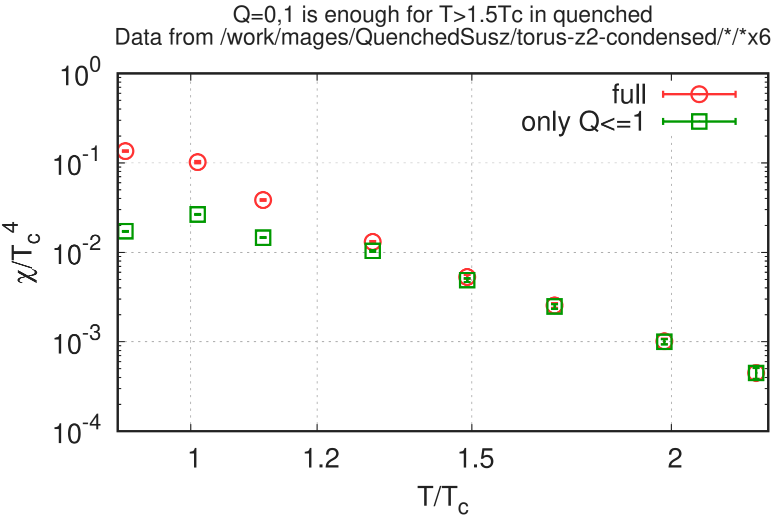
Continuum extrapolation at T=300 MeV
$\chi[\mathrm{fm}^{-4}]$
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
$(10/N_t)^2$
standard
ratio
reweight
integral

Continuum extrapolation at T=430 MeV

[Bonati et al '18]

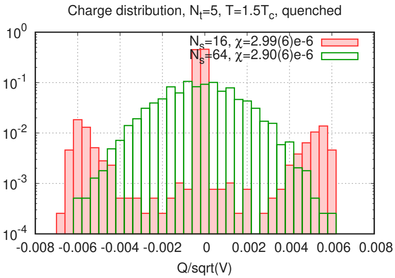


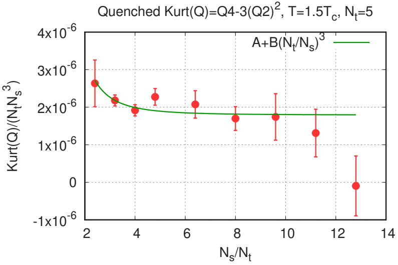
Contribution from $Q = 0, \pm 1$



Volume dependence illustration

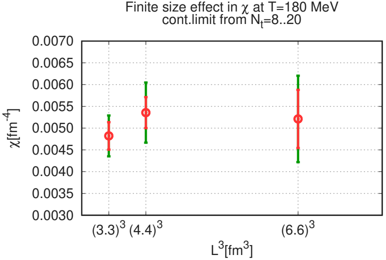
Q distribution depends (extensive quantity)



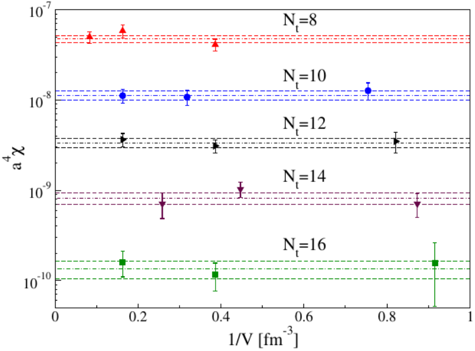


susceptibility, kurtosis (intensive quantitites) not

Volume (in)dependence at the physical point



[WB’16]



[Bonati et al ’18]