Towards an EFT for warm QCD

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[Phys.Rev. D97 (2018), 036025] with Sourendu Gupta, and ongoing work

Introduction and motivation

QCD crossover

- Lattice QCD is the only rigorous technique we know to compute the thermodynamics of QCD in the crossover region
- We know quantitatively from Lattice calculations that for $2+1$ flavor, the transition from hadronic matter at low T to the QGP at high T is a crossover around $145-165$MeV *[Brookhaven/HotQCD, TIFR, Wuppertal-Budapest, Bielefeld, collaborations]*
- But it is challenging to compute transport properties on the lattice
- Finite μ is also challenging but significant progress made. For eg. *[Datta, Gavai, Gupta (TIFR group); HOTQCD; Bielefeld group]*

Simpler theory for long range correlations?

- The chiral condensate $\langle\bar{\psi}\psi\rangle \to 0$ in the chiral limit at the critical temperature T_c. For finite quark mass, m_q, the condensate drops rapidly near the crossover temperature T_{co}
- If a quark description valid near the crossover then this implies that the quarks are light near T_{co}
- For finite m_q, there could be other light degrees of freedom. We assume here that there are none

The NJL model

- Can one write a simpler effective model that captures the correlations on length scales larger than $1/T$?
- NJL is a simple, and widely studied EFT model that captures the physics of the chiral crossover (*[Nambu, Jona-Lasinio (1961)]*)
- It can be justified on the assumption that quarks are light degree of freedom near the crossover
- It captures qualitative features like a rapid rise in the pressure and free energy near the crossover

The NJL model

- The parameters of the model are fixed by using the vacuum properties for example π mass, and π decay constant in vacuum where it is not justified
- The interaction between quarks is typically taken to be of a very specific form

$$\mathcal{L} = \lambda[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma^5 t^a \psi)^2]$$

- Since the NJL model is not valid beyond energies of the order of T, it is not appropriate to use it to calculate pressure, energy density etc.
- From this point of view more natural to compare correlation functions on length scales larger than $1/T$

Additional fields

- Additional light fields can be introduced (eg. PNJL model *(See Rajarshi's talk)*)
- Taking the EFT the point of view all terms consistent with symmetries upto a certain order should be added
- Not appropriate to match thermodynamic properties but long distance properties
- Matching should be done near the crossover

Formalism

The Euclidean action

- $$\mathcal{L} = d^0 T_0^4 + \overline{\psi}\partial\!\!\!/_4\psi - \mu\overline{\psi}\gamma_4\psi + d^4\overline{\psi}\partial\!\!\!/_i\psi + d^3 T_0\overline{\psi}\psi + \mathcal{L}_6$$

- $$\begin{aligned}
\mathcal{L}_6 = &+\frac{d^{61}}{T_0^2}\left[(\overline{\psi}\psi)^2 + (\overline{\psi}i\gamma^5 t^a\psi)^2\right] + \frac{d^{62}}{T_0^2}\left[(\overline{\psi}t^a\psi)^2 + (\overline{\psi}i\gamma^5\psi)^2\right]\\
&+\frac{d^{63}}{T_0^2}(\overline{\psi}\gamma_4\psi)^2 + \frac{d^{64}}{T_0^2}(\overline{\psi}i\gamma_i\psi)^2 + \frac{d^{65}}{T_0^2}(\overline{\psi}\gamma_5\gamma_4\psi)^2 + \frac{d^{66}}{T_0^2}(\overline{\psi}i\gamma_5\gamma_i\psi)^2\\
&+\frac{d^{67}}{T_0^2}\left[(\overline{\psi}\gamma_4 t^a\psi)^2 + (\overline{\psi}\gamma_5\gamma_4 t^a\psi)^2\right] + \frac{d^{68}}{T_0^2}\left[(\overline{\psi}\gamma^i t^a\psi)^2 + (\overline{\psi}\gamma^5\gamma^i t^a\psi)^2\right]\\
&+\frac{d^{69}}{T_0^2}\left[(\overline{\psi}i\Sigma_{i4}\psi)^2 + (\overline{\psi}i\gamma^5\Sigma_{ij}t^a\psi)^2\right] + \frac{d^{60}}{T_0^2}\left[(\overline{\psi}i\Sigma_{i4}t^a\psi)^2 + (\overline{\psi}\Sigma_{ij}\psi)^2\right]\\
&+\mathcal{O}\left(\frac{1}{T_0^5}(\overline{\psi}\psi)^3\right),
\end{aligned}$$

- There are no dimension 5 terms (for eg. $\bar{\psi}(\partial)^2\psi$) consistent with the $SU(2)_A$ symmetry
- Dimension 6 terms with derivatives in the mean field approximation $\bar{\psi}(\partial\!\!\!/)^3\psi$ have also been listed but don't play a role in our calculation. This is because we make a mean field approximation

Symmetry constraints

- Time and space distinguished: $SO(3,1) \to SO(3)$. For example, the kinetic term is

$$\overline{\psi}\partial\!\!\!/_4\psi + d^4\overline{\psi}\partial\!\!\!/_i\psi$$

- Similarly, all vector interaction terms can have different spatial and temporal coefficients
- All interaction terms with chiral symmetry written down upto dimension 6

Parameters of the theory

- Take the energy cutoff to be of the order of T or slightly larger. We will use dim-reg with a renormalization scale $M \sim \pi T$
- T_0 sets the scale of the overall problem
- $m_q = d^3 T_0$ acts as the bare quark mass, but is not fitted to π mass at $T = 0$
- Seems hopeless, 12 unknown parameters

Mean field approximation

- But sectors of observables with only specific linear combinations of d's emerge
- For example, in the mean field approximation

$$\bar{\psi}_\alpha \psi_\beta \to \delta_{\alpha\beta} \langle \bar{\psi}\psi \rangle$$

-

$$\mathcal{L}_{\mathrm{MFT}} = -\mathcal{N}\frac{T_0^2}{4\lambda}\Sigma^2 + \overline{\psi}\partial\!\!\!/_4\psi - \mu\overline{\psi}\gamma_4\psi + d^4\overline{\psi}\partial\!\!\!/_i\psi + m_q\overline{\psi}\psi + d^0 T_0^4$$

- Including all the Fierz transformations ($\mathcal{N} = 12$ for 2 flavor),

$$\lambda = (\mathcal{N}+2)d^{61} - 2d^{62} - d^{63} + 3d^{64} + d^{65} - 3d^{66} + d^{67} - d^{68} + 3d^{69} - 3d^{60}$$

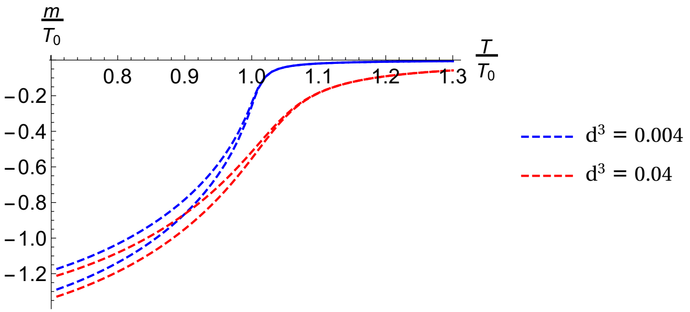
- $m = m_q + \Sigma$

Parameters of the theory

- T_c is the value for the critical point in the chiral limit. Take the scale setting parameter $T_0 = T_c$
- $\frac{(d^4)^3}{\lambda} = \frac{1}{12}$
- Observables will be fit at one point below T_c
- Parameters $m_q = d^3 T_0$, d^4
- M is the renormalization scale in the $\overline{MS}$ scheme
- $$-\Omega = \frac{\mathcal{N} T_0^2 \Sigma^2}{4\lambda} + \frac{\mathcal{N} m^4}{64\pi^2 (d^4)^3} \left[\log\left(\frac{m^2}{(d^4)^2 M^2} \right) - \frac{3}{2} \right] + \frac{\mathcal{N} T}{2\pi^2 (d^4)^3} \int_0^\infty dp p^2 \log\left[1 + \exp\left(-\frac{E}{T} \right) \right]$$

Order parameter

- By minimizing the free energy we can find the order parameter m
- In the plot the width is associated with varying $M \in (1.25\pi T_0, 1.75\pi T_0)$



Current correlations and screening masses

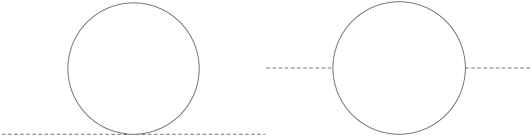
- Long distance behavior of the correlations of currents (for eg. $A^{a\mu} = \bar{\psi}\gamma^{\mu}\gamma^{5}\frac{t^{a}}{2}\psi$) can be used to extract the screening masses of various channels
- We first focus on the axial vector correlations in Euclidean field theory so that we can match to lattice data

Fluctuations of the order parameter

- In mean field $\psi_\alpha \bar{\psi}_\beta \to \frac{1}{\mathcal{N}} \langle \psi_\alpha \bar{\psi}_\alpha \rangle \delta_{\alpha\beta}$
- Fluctuations $\psi \to e^{i\pi^a \tau^a \gamma^5/(2f)} \psi$, $\bar{\psi} \to \bar{\psi} e^{i\pi^a \tau^a \gamma^5/(2f)}$
- This includes the π's in the Hubbard-Stratonovich transformation
- Therefore, $\psi_\alpha \bar{\psi}_\beta \to e_{\beta'\beta}^{i\pi^a \tau^a \gamma^5/(2f)} \langle \psi_\beta \bar{\psi}_\alpha \rangle e_{\alpha\alpha'}^{i\pi^a \tau^a \gamma^5/(2f)}$
- At very long wavelengths an effective lagrangian for the π's is applicable
- $\mathcal{L}_f = \frac{c^2 T_0^2}{2} \pi^2 + \frac{1}{2} (\partial_0 \pi)^2 + \frac{c^4}{2} (\nabla \pi)^2 + \frac{c^{41}}{8} \pi^4 + \cdot\cdot$

π lagrangian

- We start with the two point function
- $\mathcal{L}_f = \frac{c^2 T_0^2}{2}\pi^2 + \frac{1}{2}(\partial_0\pi)^2 + \frac{c^4}{2}(\nabla\pi)^2$



Correlation functions

- Correlations of currents related to π properties
- Two illustrative examples
- $\lim_{q^4\to 0}\int d^4x e^{iqx}\langle P^a(x)P^b(0)\rangle = (\frac{f}{2m_q})^2 c^4 \frac{\delta^{ab}\mathbf{q}^4}{\mathbf{q}^2+M_\pi^2}$
- $\lim_{q^4\to 0}\int d^4x e^{iqx}\langle J_5^{ai}(x)J_5^{bi}(0)\rangle = ((2f)^2)c^4 \frac{\delta^{ab}\mathbf{q}^2}{\mathbf{q}^2+M_\pi^2}$
- $M_\pi^2 = c^2 T_0^2/c^4$ related to the screening length
- Static $\pi-\pi$ correlator decays as $\sim e^{-M_\pi r}$
- $u = \sqrt{c^4}$ is the π "speed"
- From a combination of the static correlators one can extract f, c^4, M_π
- *[Brandt, Francis, Meyer, Robaina (2014)]*

Correlation functions

- A finite temperature generalization of GOR relation is satisfied
- $c^2 T_0^2 = -\frac{\mathcal{N} m_q \langle \bar{\psi}\psi \rangle}{f^2}$
- *[Son, Stephanov (2002)]*
- We can compute f, c^4, M_π in the EFT model and compare to the lattice data
- Because of approximate chiral symmetry, can show that the same combination of d^6's determine π properties

Interesting behaviour in the chiral limit

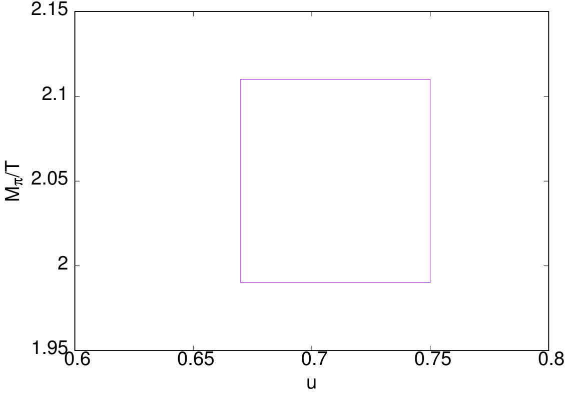
- $m_q \to 0$ implies $M_\pi \to 0$. Well known from the Goldstone theorem
- Interesting behaviour of c_4 at T_c in the chiral limit:

$$c^4 \propto \int \frac{p^2 dp}{1+\exp(p/T)}[\frac{2}{p} - \frac{1}{T(1+\exp(p/T))}] = 0$$

Results

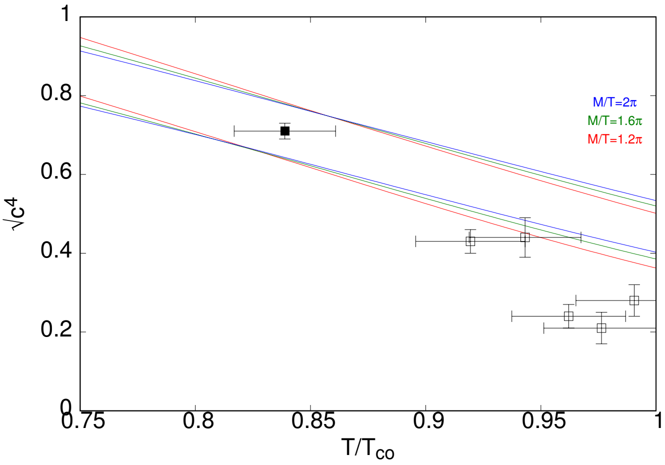
Inputs

- Matching u and M_π at $T = 0.84\,T_{co}$
- Error in T associated with $T_{co} = 211(5)$MeV
- Input from *[Brandt, Francis, Meyer, Robaina (2014)]* (figure below). Heavy π
- Fitted values $d^3 = 0.57\left[\pm 6(\mathrm{input})\right]\left[\pm 3(\mathrm{scale})\right]\left[\pm 3(\mathrm{T})\right]$, $d^4 = 1.20\left[\pm 6(\mathrm{input})\right]\left[\pm 4(\mathrm{scale})\right]\left[\pm (4)\mathrm{T}\right]$



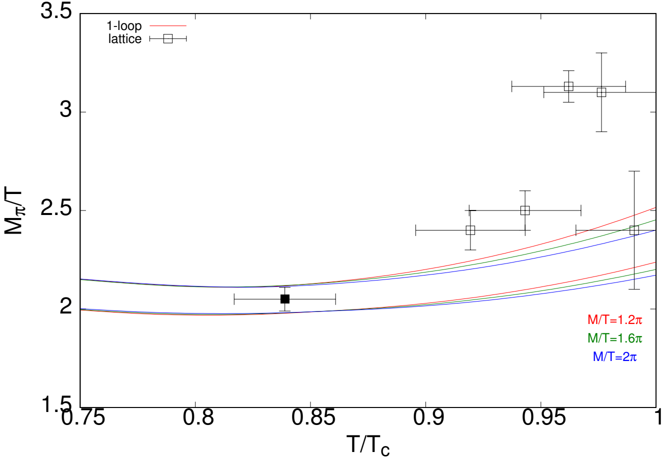
u

- Pion velocity



M_π

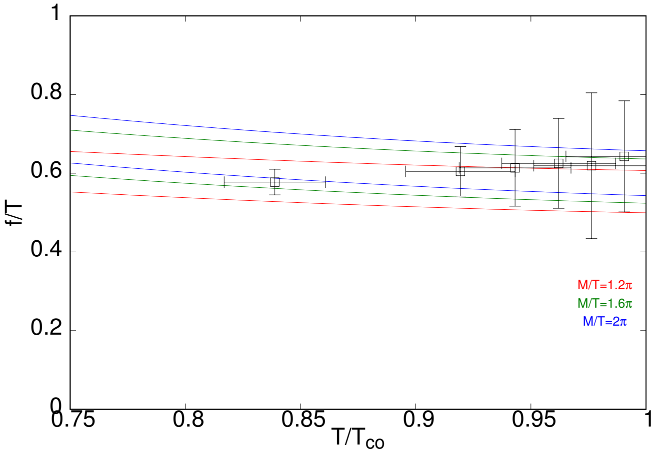
- Pion Debye screening mass



- Also see *[Ishii et. al. (2013); S Cheng, S Datta et. al. (2011)]*

f

- Pion constant f
- An independent prediction

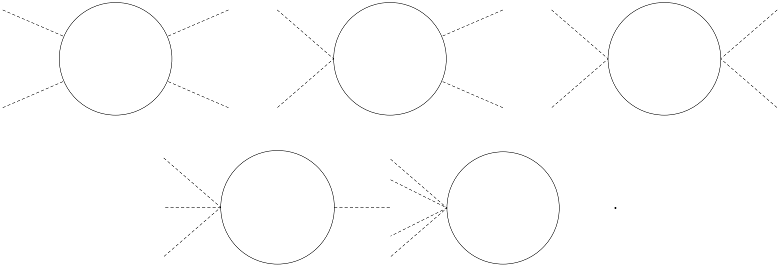


T_c

- The peak of the chiral susceptibility in the EFT model occurs at $T_{co} = 1.24\, T_c$
- Taking $T_{co} = 211(5)$, we get $T_c = 170 \pm 6$
- Larger than the value of T_c from the lattice for $2+1$ flavors
- However for 2 flavors this agrees with the lattice prediction *[Brandt et. al. (2013)]*

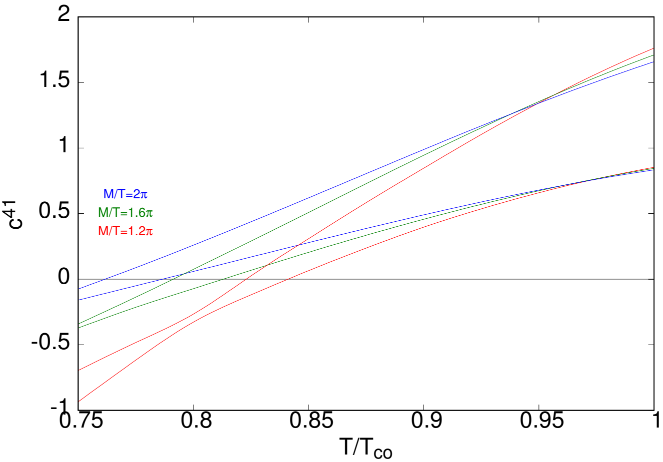
π four point function

- $\mathcal{L}_4 = \frac{c^{41}}{8}\pi^4$



c_{41}

- Pion four point function



Towards finite μ

- If we use the standard modification $H \to H - \mu N$
- In dim-reg an interesting result that $T_c(\mu)^2 + \frac{3}{\pi^2}\mu^2 = T_0^2$ in the chiral limit
- In particular, implies that for small μ, $T_c(\mu) = T_c(0) - \frac{1}{2}\kappa\frac{\mu^2}{T_c(0)} + \mathcal{O}(\mu^3)$
- $T_c(0)\kappa = \frac{3}{\pi^2}$
- Thus the mean field prediction is roughly $5 - 10$ times the lattice prediction for $2+1$ flavors *[Bielefeld, HotQCD, collaborations]*
- Several corrections in the EFT required at finite μ

P_π: a qualitative comment

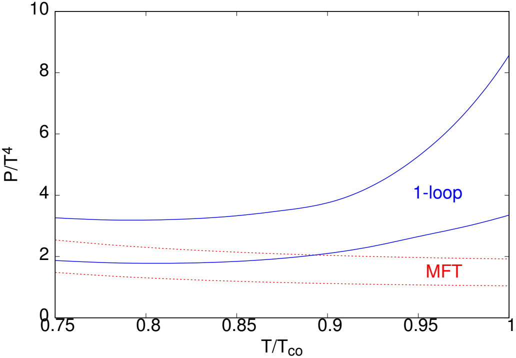
- Pressure of the π

$$P_\pi = -\frac{3(c^2T_0^2)^2}{64\pi^2(c^4)^{(3/2)}}[\log(\frac{c^2T_0^2}{c^4M^2}) - \frac{3}{2}]$$
$$-3T\int \frac{d^3\mathbf{p}}{(2\pi)^3}\log(1-e^{E^\pi/T})$$

- $E^\pi = \sqrt{c^4\mathbf{p}^2 + c^2T_0^2}$
- If c^4 is small the pressure is large. Energetic cost is small

P_π

- Rise in the pressure of the π because of the thermal piece
$$-3T\int\frac{d^3\mathbf{p}}{(2\pi)^3}\log(1-e^{E^\pi/T}) \tag{1}$$
as u decreases
- Disclaimer: Not rigorous; a curiousity



Real time dynamics

Real time dynamics

- One interesting application of the formalism is to compute real time quantities in the small frequency and small momentum limit
- A plausible assumption is that this can be obtained from the analytic continuation of the fermionic lagrangian
- The main change in analysis is that instead of the imaginary time propagator, we use the real time propagator for the fermions

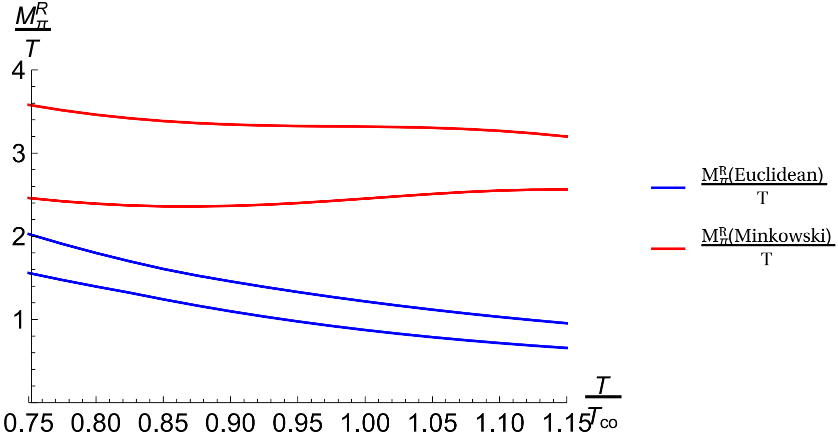
$$[\frac{i}{i\not{p} - m + i\epsilon} - 2\pi\delta(p^2 + m^2) n_F(p^0)(i\not{p} + m)]$$

- $n_F(E) = \frac{1}{\exp(E/T)+1}$
- Note that d^4 is hidden in the definitions, $\not{p} = -p^0\gamma^0 + d^4 p^i \gamma^i$

Real time dynamics

- For example let us now consider $\langle J_5^{ai}(x) J_5^{ai}(0)\rangle$ with x in Minkowski space
- Using $J_5^{ai} \propto f\partial_i \pi^a$ we obtain the following
- At one loop order the diagrams are the same with the only difference now that we need the real time propagators for the fermions
- The π propagation in real time formalism $\int d^4x e^{iqx} \langle \pi^a(x)\pi^b(0)\rangle|_{\text{fermionic continuation}} = \frac{i\delta^{ab}}{A(q^0)^2 - B\mathbf{q}^2 - C}$
- Compare to the rotation to imaginary time $\int d^4x e^{iqx} \langle \pi^a(x)\pi^b(0)\rangle|_{\pi\text{continuation}} = \frac{i\delta^{ab}}{(q^0)^2 - c_4\mathbf{q}^2 - c_2 T_0^2}$
- $M_\pi^P = \sqrt{\frac{C}{A}}$ (This is what Sourendu called the kinetic mass)
- Subtlety related to order of limits: can not use the static limit where $q^0 \to 0$ first
- Preliminary results *[Ongoing with S. Gupta]*

Pole mass of π



Salient features

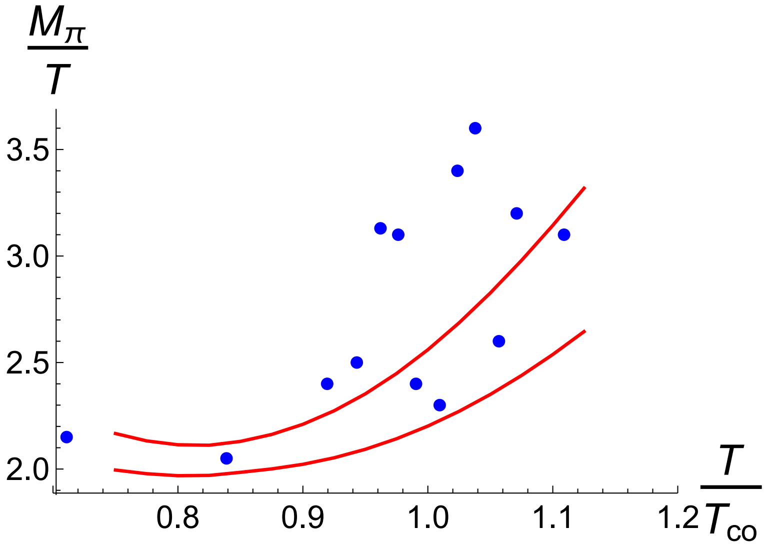
- The pole mass differs in the static and the dynamic limit
- The dynamic limit is relevant for transport properties like conductivity, where $\lim_{q\to 0}$ is taken before $\lim_{\omega\to 0}$
- At one loop order there is no damping at small q. One needs to go to three loops (in the fermions) to obtain π damping

Conclusions

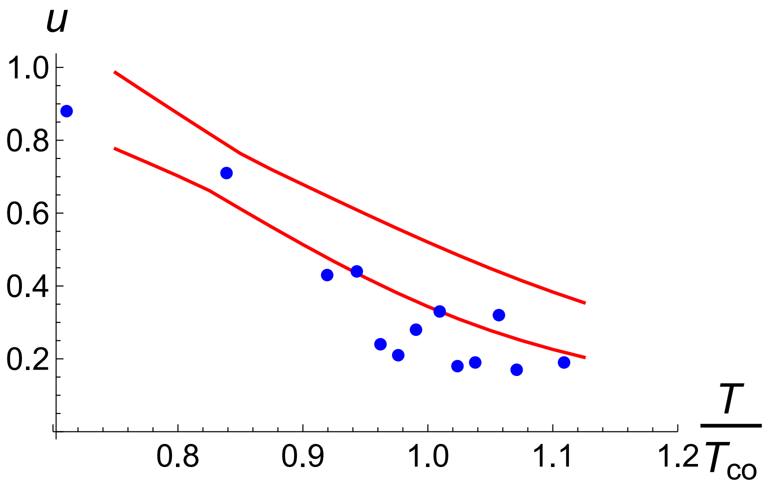
- The EFT based approach can be used to calculate long distance correlation functions in both Euclidean and Minkowski space
- In particular we analyze the modification of the π properties near the crossover
- Qualitatively, note that the medium modification of the properties of hadrons (π), in particular the reduction of the "speed" u just below T_c
- Can be used to calculate dynamical properties

Backup slides

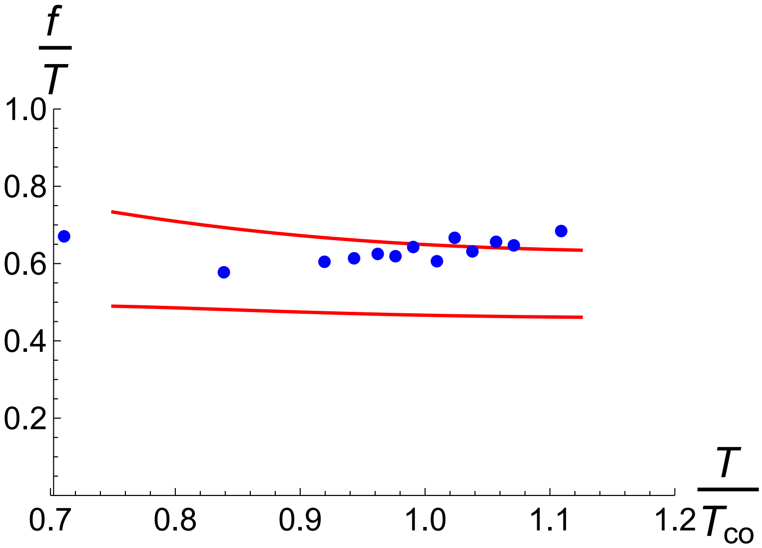
Screening mass of π



Speed of π

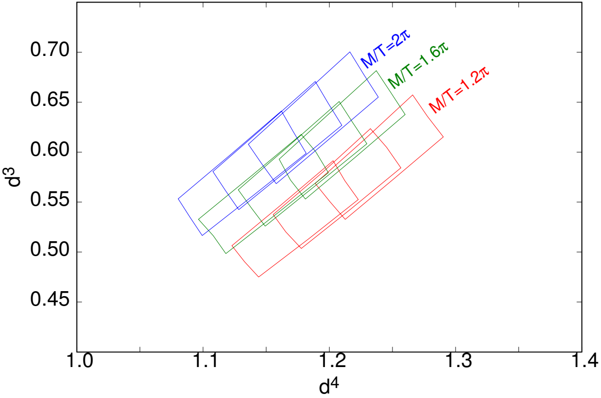


f of π



Outputs

- By fitting u and M_π parameters we obtain the fermionic parameters
- Uncertainty associated with M
- Different boxes associated with varying T_{co} in the error band
- Useful if the fermionic parameters do not vary rapidly with T



Free energy expression

- $$\Omega = -\frac{\mathcal{N} T_0^2 \Sigma^2}{4\lambda} - \mathcal{N} I_0$$
- $$I_0 = \frac{T}{2} \sum_{p^4=(2n+1)\pi T} \int \frac{d^3\mathbf{p}}{(2\pi)^3} \log(\frac{m^2 + \mathbf{p}^2 + (p^4)^2}{T^2})$$
  $$= \int \frac{d^3\mathbf{p}}{(2\pi)^3} \big(E_p + \log[1 + \exp(-E_p/T)]\big)$$
- $E_p = \sqrt{(d^4)^2\mathbf{p}^2 + m^2}$
- $I_0 = \frac{m^4}{64\pi^2(d^4)^3}[-\frac{3}{2} - \log(\frac{(d^4)^2 M^2}{m^2})] + \frac{1}{2\pi^2}\int dp p^2 \log[1 + \exp(-E_p/T)]$