The many uses of fermions with exact chiral symmetry on the lattice

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Why the chiral fermions are so interesting?

- The chirality of fermions play a crucial role in myriad of physical processes from the early universe to material science.
  Exotic transport properties like Chiral Magnetic effect!

  [Kharzeev, McLerran, Warringa, 07]
- In QCD, it is responsible for very light pions
- Quantum anomalies in the chiral sector of QCD decide also the order of the phase transition. [Pisarski & Wilczek, 83]
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- Defining chiral non-Abelian Gauge theories on the lattice is notoriously difficult. Still unsolved!
- Even formulating vector theories like QCD with fermions having exact chiral symmetry on the lattice was a challenging problem.
- It took 20 yrs since the discovery of Wilson fermions to develop domain wall [Kaplan, 92] and overlap [Narayanan & Neuberger, 93] fermions.
- Rajiv was deeply worried about how to incorporate μ_B in a manner that DW/OV fermions maintain their exact chiral properties on the lattice.
- Through couple of years of struggles and new revelations we now know how to do it! [Gavai & S.S., Phys. Lett. B. 716, 2012, Narayanan and S.S. JHEP 2011.]
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- Overlap fermions has an index theorem even on a finite lattice → it's zero modes can track the topological constituents of QCD.
- Using them to probe the topology in gauge theories revealed onset of a dilute gas of instantons quite early $\sim 1.1\ T_d$!

  [Edwards, Heller & Narayanan, 98, 99, Gavai & Gupta 02]
- For QCD, though the zero and near-zero modes survive quite longer into the chiral-symmetry restored phase.

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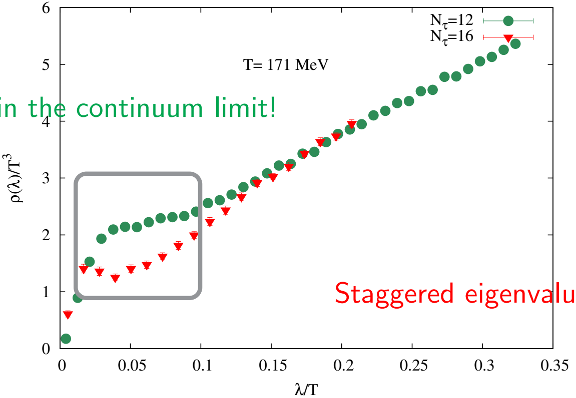
Microscopics of QCD!



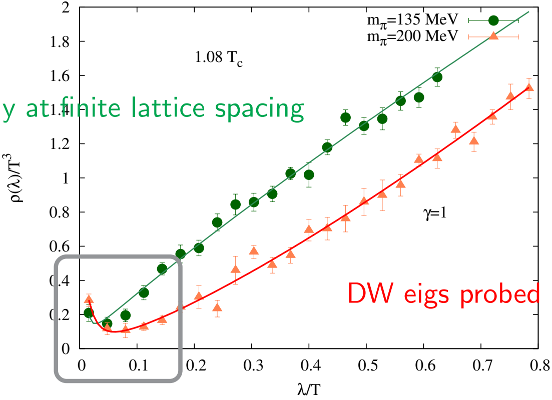
Peak appears in the continuum limit!

Staggered eigenvalues@HotQCD

Absence of chiral symmetry and index washes out minute structures!
Recovered only in the continuum! @HotQCD, 16

Microscopics of QCD!

Peak appears already at finite lattice spacing



DW eigs probed by overlap

A small peak of near-zero modes observed above T_c

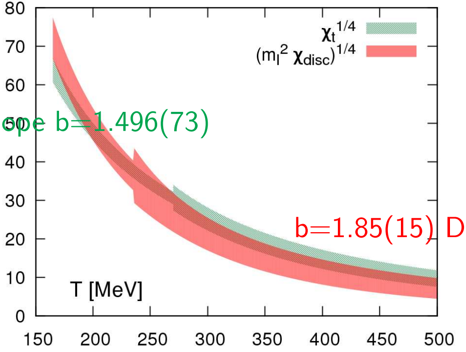
[V. Dick, F. Karsch, E. Laermann, S. Mukherjee, S.S., 16].

What do topological fluctuations tell us?

- Topological susceptibility $\chi_t = T < Q^2 > /V$ measures the topological fluctuations of QCD vacuum.
- Characterizing, $\chi_t^{1/4}(T) = (c_0 + c_2 \cdot a^2) \cdot (T_c/T)^{b+..}$

[Petreczky, Schadler, S.S., PLB 16].

Abrupt change in slope b=1.496(73)

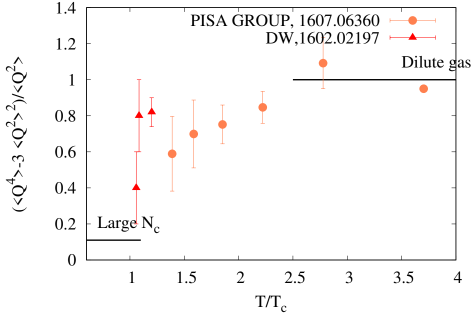


b=1.85(15) Dilute Instanton gas

[See also Borsanyi et. al, Nature 16, C. Bonati et. al., 16, F. Burger et. al, 18]

Higher moments of topological fluctuations

- A better observable: $$\frac{< Q^4 > -3 < Q^2 >^2}{< Q^2 >}$$
- At $T = 0$ QCD consistent with χ_{PT} prediction of χ_t [Villadoro et. al, 15].
- Departure from χ_{PT} expectations but a slow rise towards DIG $\gtrsim T_c \rightarrow$ residual interactions between instantons or different topological d.o.f?



Understanding the topological constituents?

- Instantons were shown to cause color-confinement in 3D [Polyakov, 77].
- In 4D the potential is not long-ranged to ensure confinement.
- Interacting instantons explains many properties related to chiral symmetry breaking. [Shuryak, 82, Shuryak & Schaefer 96].
- Why are confinement and chiral symmetry breaking so intimately connected in QCD?
- At finite T, instantons characterized by the holonomy and Q. [Gross, Pisarski, Yaffe, 83].
- Immediately above T_c, a range of temperature where the Polyakov loop has non-trivial eigenvalues.

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Topology at finite temperature

- Instantons with a non-trivial holonomy $\rightarrow$ dyons with non-trivial color electric $+$magnetic charges. [Kraan & van Baal, 98, Lee & Lu, 98].
- For $SU(N_c)$ color there are N_c such dyons.
- Topological charge $= 1/N_c$ of the host-instanton.
- Dyons can directly interact with the holonomy potential. It can drive towards the confining values? [Diakonov, 2006]
- Just above T_c there is a region where the holonomy is still non-trivial!
- Do dyons really exist ? Yes several evidences!

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- Can we identify different species of dyons in the hot QCD medium.
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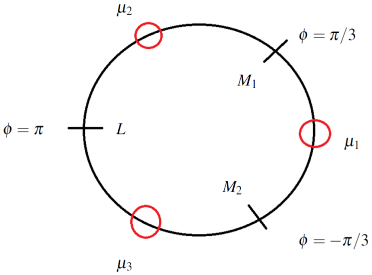
Dyon-zero modes in SU(3)

- Holonomy $L = \frac{1}{3} Tr e^{\mathrm{idiag}(\mu_1,\mu_2,\mu_3)}$ $\rightarrow$ the *ith* dyon action is characterized by $\mu_{i+1} - \mu_i$.
- The zero mode of Dirac operator with b.c $\psi(t+\beta) = \mathrm{e}^{\mathrm{i}\phi}\psi(\mathrm{t})$ have a normalizable solution for ith-dyon background if $\phi \, \epsilon \, [\mu_{i+1} - \mu_i]$



Dyon zero modes in SU(3)

The density at any spacetime point x is:

$$\rho(x) = -\frac{1}{4\pi^2}\partial_\mu^2 f_x(\phi, \phi) \ ,$$

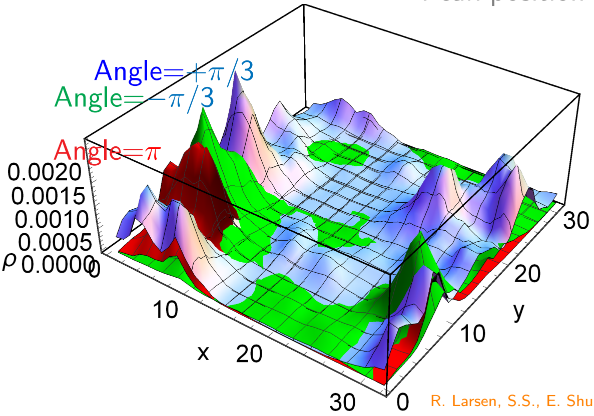
where

$$\left[\left(\frac{1}{i}\partial_\phi - \tau\right)^2 + r^2(x,\phi) + \sum_{m=1}^{3} \delta(\phi - \mu_m)\frac{|x_m - x_{m+1}|}{2\pi}\right] f_x(\phi, \phi') = \delta(\phi - \phi')$$

- distances between center of the m-th and $(m+1)$-th dyon given as $x_m - x_{m+1}$ where $m = 1, 2, 3$.
- $r^2(x,\phi) = r_m^2(x), \phi \in [\mu_m, \mu_{m+1}]$ is the distance between the observation point x and the center of the m-th dyon.

Snapshot of QCD vacuum at $\sim 1.1\, T_c$

Peak position shifts with angle $\rightarrow$ dyons



R. Larsen, S.S., E. Shuryak, Phys. Lett. B. 794, 2019, and in prep.

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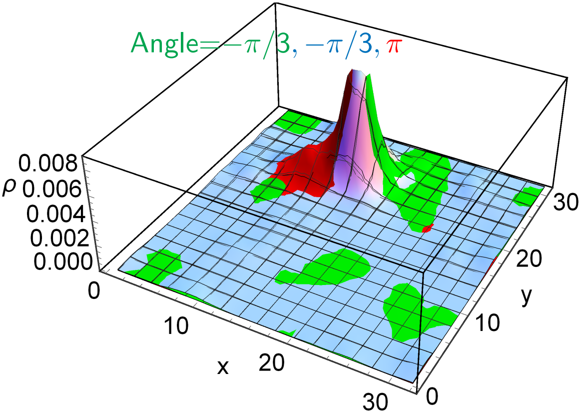
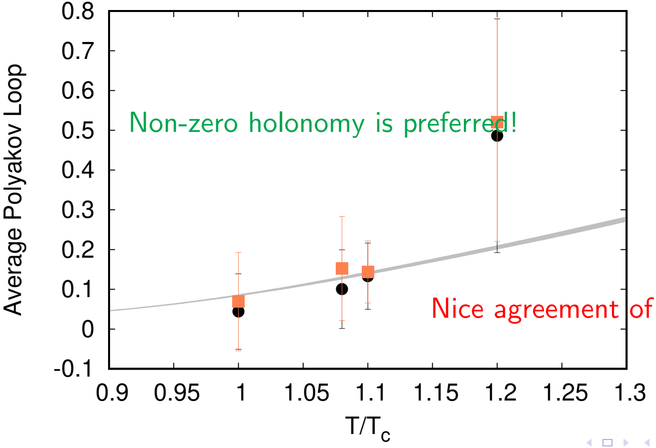


The fermion zero modes insensitive to temporal periodicity phase → Dyon or caloron? R. Larsen, S.S., E. Shuryak, Phys. Lett. B. 794, 2019, and in prep..

Comparing with the semiclassical theory

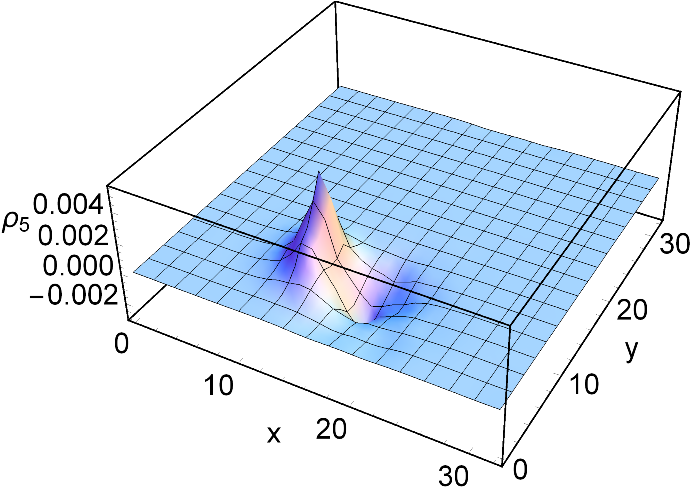
- Analytic solutions of the dyons are known [Kraan & van Baal, Lee and Lu, 98].
- Choose an initial value of Polyakov loop and locate diff. dyons at the positions of the lattice zero modes → Fit it to analytic profiles assuming weakly interacting ensemble R. Larsen, S.S., E. Shuryak, Phys. Lett. B. , and in prep.



Nice agreement of fit results to QCD

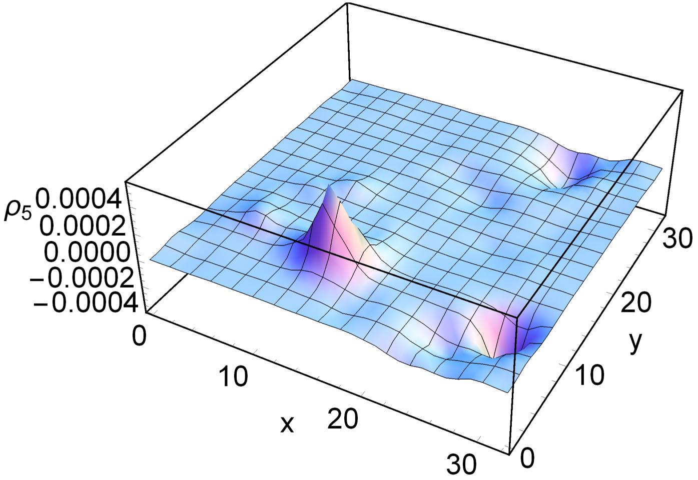
What do the near-zero modes tell us?

- *L*-dyon-pairs are rarer and only those which are near-by appear at high *T* as expected.



What do the near-zero modes tell us?

- M-dyons appear for all separations!



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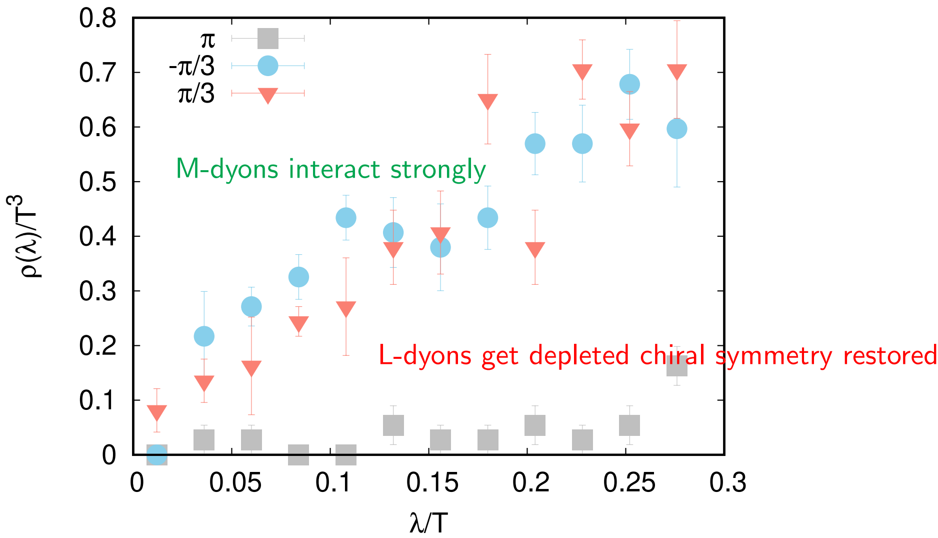
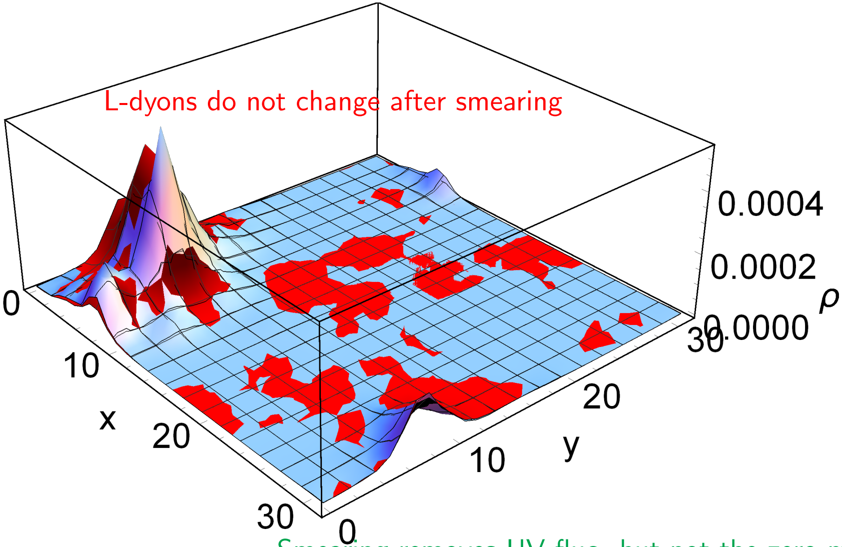
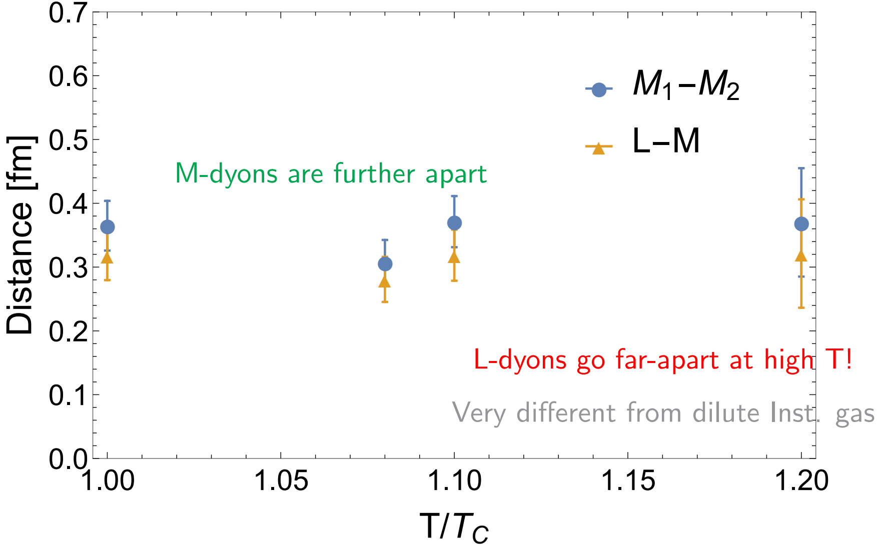


How robust are the zero-modes



Smearing removes UV fluc. but not the zero-mode

Characterizing the dyons



R. Larsen, S.S., E. Shuryak, in prep.

Summary

- We have shown that the use of chiral fermions allow us to unambiguously distinguish between a dyon and a caloron.
- We understand their interactions and can reproduce the holonomy.
- For temperatures just above the crossover transition we find a good agreement with the semi-classical theory of dyons within $10 - 20\%$.
- Need to develop techniques to measure the densities and thermal distributions of different dyon species.
- Understand how the dyon pictures goes over to high-T 3D confining theories

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