The Fluid-Gravity Correspondence

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Introduction

- In this talk I describe a precise 'duality' between two of the best studied nonlinear partial differential equations in physics, namely the Navier Stokes equations of hydrodynamics and Einstein's equations of general relativity.
- The study of this 'fluid gravity correspondence' has led to the realization that the 'standard' equations of charged relativistic hydrodynamics (see e.g. Landau and Lifshitz) are incomplete in certain contexts, and has stimulated investigations that have remedied this lacuna.
- The so called 'Fluid Gravity Correspondence' has also focussed attention on the lack of a satisfactory systematic framework for hydrodynamical equations in general, and has led to work that aims at filling this gap.

Relativistic Fluid Dynamics

- I start this talk with a quick review of the structure of the equations of hydrodynamics. In the simplest context the variables of hydrodynamics are local values of thermodynamical fields (e.g. $T(x)$, $\mu(x)$, $u^\mu(x)$)
- The equations of hydrodynamics are conservation laws

$$\begin{aligned} \partial_\mu T^{\mu\nu} &= F^{\mu\nu} J_\nu \\ \partial_\mu J^\mu &= cF \wedge F \end{aligned} \tag{1}$$

  with currents expressed as functions of thermodynamical fields.
- The equations that express conserved currents in terms of thermodynamical fields are called constitutive relations. Constitutive relations are specified order by order in a derivative expansion.

Local Equilibriation

- As we have seen, the equations of hydrodynamics are simply conservation laws that follow from symmetries; these equations are exact in any quantum field theory.
- However $\partial^\mu T_{\mu\nu} = 0$ are d equations for $\frac{d(d+1)}{2} - 1$ variables. In $d = 4$, for example, this is 4 equations for 10 variables. These equations constrain dynamics but do not, by themselves, constitute a well defined initial value problem for the stress tensor.
- The key assumption of hydrodynamics is that the condition of local equilibriation determines all 10 components of the stress tensor in terms of 4 independent fields, the energy density and the local velocity. Taken together with a particular set of constitutive relations, the conservation laws specify a well defined initial value problem for the stress tensor and other conserved charges.

Constitutive Relations

- Constitutive relations for any given field theory may, in principal, be deduced by a sufficiently detailed dynamical study of the theory.
- In strongly coupled quantum field theories, however, this derivation of constitutive relations from first principles appears difficult to actually impelement even by numerical methods (the one exception to this statement uses holography: see below). For most practical pruposes hydrodynamics is best regarded as an autonomous effective field theory.
- The equations of motion of this effective field theory are specified once we have the fluid constitutive relations.

Constitutive relations

At zero order in the derivative expansion the fluid constitutive relations are thermodyanamically determined;

$$\begin{aligned} T^{\mu\nu} &= (\rho + P)u^\mu u^\nu + P\eta^{\mu\nu} + \pi^{\mu\nu} \\ J^\mu &= qu^\mu + J^\mu_{diss} \end{aligned} \tag{2}$$

At higher orders in the derivative expansion the constitutive relationship is corrected to

$$\begin{aligned} T^{\mu\nu} &= (\rho + P)u^\mu u^\nu + P\eta^{\mu\nu} + \pi^{\mu\nu} \\ J^\mu &= qu^\mu + J^\mu_{diss} \end{aligned} \tag{3}$$

Here $\pi^{\mu\nu}$ and J^μ_{diss} represent terms of first or higher order in derivatives of the velocity, energy density and conserved charges. At any order in the derivative expansion there is a finite dimensional basis for all such terms. These basis tensors can then be multiplied by arbitrary functions of the energy density and conserved charges.

Constitutive Relations for charge fluids at first order

- The total number of (onshell independent) tensors in the constitutive relations at first order for a charged fluid turns out to be 5 (parity even) structures and 2 parity odd structures.
- This observation may suggest that the equations of hydrodynamics at first order are parameterized by $5+2$ undetermined ‘transport coefficients’. This is actually an overcount, as we will see in much more detail below.

- It has long been argued that $U(N)$ gauge theories reduce to effectively classical systems in the t' Hooft large N limit.
- $\mathcal{N} = 4$ Yang Mills theories are $U(N)$ gauge theories. These theories are conformally invariant; they define a line of fixed points labeled by a continuous coupling constant λ. In 1997 Maldacena identified the corresponding large N classical systems.
- While the classical equations identified by Maldacena are unfamiliar (and appear complicated) at finite λ, they simplify dramatically at large λ. In this limit they reduce to the equations of Einstein (IIB super) gravity on spacetimes that asymptote to $AdS_5 \times S^5$.

Thermal equilibrium from gravity

- The gravitational description of field theory dynamics applies only at very strong field theory coupling.
- Even at strong coupling, however, we have some qualitative expectations of local QFTs. For instance they are expected to equilibriate at every finite temperature.
- What is the gravitational description of this thermal state? Answer (Witten): an asymptotically *AdS* black brane.
- This answer is universal in the following sense. Every 2 derivative theory of gravity interacting with other fields of spin ≤ 2 admits a consistent truncation to Einstein's equations with a negative cosmological constant. Black brane solutions lie in this universal sector.

Black Branes

- In appropriately chosen units, Einstein's equations with a negative cosmological constant in $d+1$ dimensions are

$$R_{MN} - \frac{R}{2} g_{MN} = \frac{d(d-1)}{2} g_{MN} : \quad : M, N = 1 \dots d+1$$

- The black brane at temperature T and velocity u_μ are a d parameter set of exact solutions of these equations

$$ds^2 = \frac{dr^2}{r^2 f(r)} + r^2 \mathcal{P}_{\mu\nu} dx^\mu dx^\nu - r^2 f(r) u_\mu u_\nu dx^\mu dx^\nu$$

$$f(r) = 1 - \left(\frac{4\pi T}{d\ r}\right)^d ; \quad \mathcal{P}_{\mu\nu} = g_{\mu\nu} + u_\mu u_\nu$$

- These solutions have a horizon at $r = \frac{4\pi T}{d}$. The thermal nature of these solutions follows from well known properties of event horizons.

Linearization about black branes

- Einstein's equations allow us to study deviations from thermal equilibrium. Natural first question: what is the spectrum of linearized fluctuations about thermal equilibrium?.
- If we impose the requirement of regularity of the future horizon, the anwer is given by gravitational 'quasinormal modes'. Discrete infinity of such modes labeled by integers. For the n^{th} mode $\omega = \omega_n(k)$. Frequency complex corresponding to decay.
- It follows from conformal invariance that $\omega_n(0) = \frac{f(n)}{T}$. $f(n) \neq 0$ except for the 4 Goldstone modes corresponding to variations of T and u_μ. Infact Policastro Starinets and Son demonstrated that the dispersion relation for these Goldstone modes at small k takes the form predicted by fluid dynamics -(shear and sound waves) provided $\frac{\eta}{s} = \frac{1}{4\pi}$

Hydrodynamics from gravity?

- So we now have a first hint that black branes mimic the behaviour of thermal QFTs for dynamical, not just static purposes. However this check worked at the linear level; can we take it further?
- As we have explained above, we expect the full nonlinear effective dynamics of any locally equilibriated field theory to be given by the equations of hydrodynamics, provided all variations are slow (compared to a dynamical relaxation time).
- Thus the AdS/CFT correspondence appears to imply that the equations of asymptotically *AdS* gravity reduce to (relativistic generalizations of) the Navier Stokes equations at the *full nonlinear level* in an appropriate long distance expansion. Is this exciting suggestion true?

Local temperatures and velocities

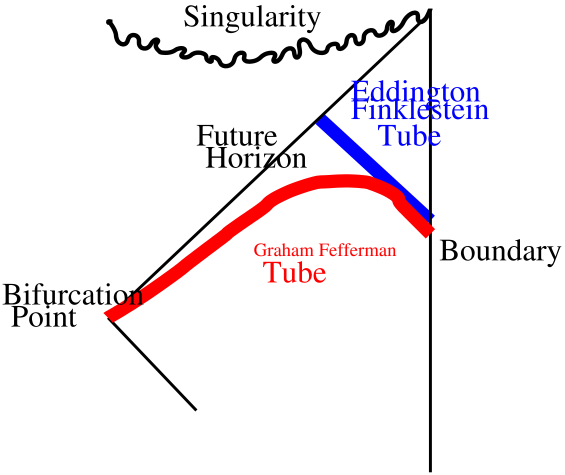
- We look for solutions that 'locally' approximate black branes but with space varying velocities and temperatures. More precisely we search for bulk solution tubewise approximated by black branes. But along which tubes?
- Naive guess: lines of constant x^μ in Schwarschild (Graham Fefferman) coordinates, i.e. metric approximately

$$ds^2 = \frac{dr^2}{r^2 f(r)} + r^2 \mathcal{P}_{\mu\nu}(x) dx^\mu(x) dx^\nu(x) - r^2 f(r) u_\mu u_\nu dx^\mu dx^\nu$$

$$f(r) = 1 - \left(\frac{4\pi T(x)}{d\ r}\right)^d ; \quad \mathcal{P}_{\mu\nu} = g_{\mu\nu}(x) + u_\mu(x) u_\nu(x)$$

- Does not seem useful. Appears to be a bad starting point for perturbation theory. Also has several interpretative difficulties.

Penrose diagram



Ingoing Coordinates

- Causality suggests the use of tubes centered around ingoing null geodesics. In particular we try

$$ds^2 = g^{(0)}_{MN} dx^M dx^N = -2u_\mu(x) dx^\mu dr + r^2 \mathcal{P}_{\mu\nu}(x) dx^\mu dx^\nu \\ - r^2 f(r, T(x)) u_\mu(x) u_\nu(x) dx^\mu dx^\nu$$

- Metric generally regular but not solution to Einstein's equations. However solves equations for constant $u^\mu, T, g_{\mu\nu}$. Consequntly appropriate starting point for a perturbative soln of equations in the parameter $\epsilon(x)$.

Long wavelength expansion

- That is we set

$$g_{MN} = g^{(0)}_{MN}(\epsilon x) + \epsilon g^{(1)}_{MN}(\epsilon x) + \epsilon^2 g^{(2)}_{MN}(\epsilon x) \ldots$$

  and attempt to solve for $g^{(n)}_{MN}$ order by order in ϵ.
- Perturbation expansion surprisingly simple to implement. Nonlinear partial differential equation $\rightarrow \frac{d(d+1)}{2}$ ordinary differential equations, in the variable r at each order and each boundary point.

Hydrodynamics from gravity

- It turns out that all equations can be solved analytically (and rather simply). Upon solving the equations we find that the perturbative procedure spelt out above can be implimented at n^{th} order *only* when an integrability condition of the form $\partial_\mu T^{\mu\nu}_{n-1}(u^\mu(x), T(x))$ where $T^{\mu\nu}_{n-1}(u^\mu(x), T(x))$ is a specific function of temperature and velocities and their spacetime derivatives to $(n-1)^{th}$ order. This function is determined directly from Einstein's equations by the perturbative procedure.
- For every $u^\mu(x)$ and $T(x)$ that satisfies this Fluid Dynamical equation we have a solution to Einstein's equations. The map from fluid dynamics to gravity is locally invertable assuming regularity of the future event horizon.

Explicit Results at second order

We have explicitly implemented our perturbation theory to second order.

$$
\begin{aligned}
ds^2 &= -2u_\mu dx^\mu \left(dr + r\, A_\nu dx^\nu\right) + r^2 g_{\mu\nu} dx^\mu dx^\nu \\
&- \Bigg[\omega_\mu{}^\lambda \omega_{\lambda\nu} + \frac{1}{d-2}\mathcal{D}_\lambda \omega^\lambda{}_{(\mu} u_{\nu)} - \frac{1}{d-2}\mathcal{D}_\lambda \sigma^\lambda{}_{(\mu} u_{\nu)} \\
&+ \frac{\mathcal{R}}{(d-1)(d-2)} u_\mu u_\nu\Bigg] dx^\mu dx^\nu \\
&+ \frac{1}{(br)^d}\left(r^2 - \frac{1}{2}\omega_{\alpha\beta}\omega^{\alpha\beta}\right) u_\mu u_\nu dx^\mu dx^\nu \\
&+ 2(br)^2 F(br)\left[\frac{1}{b}\sigma_{\mu\nu} + F(br)\sigma_\mu{}^\lambda \sigma_{\lambda\nu}\right] dx^\mu dx^\nu \ldots
\end{aligned}
$$

Explicit Results at second order

$$
\begin{aligned}
&-2(br)^2\frac{\sigma_{\alpha\beta}\sigma^{\alpha\beta}}{d-1}P_{\mu\nu}K_1(br)-\frac{u_\mu u_\nu}{(br)^{d-2}}\frac{\sigma_{\alpha\beta}\sigma^{\alpha\beta}}{(d-1)}K_2(br)\\
&+\frac{2\,L(br)}{(br)^{d-2}}\left[P^\lambda_\mu\mathcal{D}_\alpha\sigma^\alpha{}_\lambda u_\nu+P^\lambda_\nu\mathcal{D}_\alpha\sigma_\lambda{}^\alpha u_\mu\right]dx^\mu dx^\nu\\
&-2(br)^2H_1(br)\left[u^\lambda\mathcal{D}_\lambda\sigma_{\mu\nu}+\sigma_\mu{}^\lambda\sigma_{\lambda\nu}-\frac{\sigma_{\alpha\beta}\sigma^{\alpha\beta}}{d-1}P_{\mu\nu}\right.\\
&\qquad\left.+C_{\mu\alpha\nu\beta}u^\alpha u^\beta\right]dx^\mu dx^\nu\\
&+2(br)^2H_2(br)\left[u^\lambda\mathcal{D}_\lambda\sigma_{\mu\nu}+\omega_\mu{}^\lambda\sigma_{\lambda\nu}-\sigma_\mu{}^\lambda\omega_{\lambda\nu}\right]dx^\mu dx^\nu
\end{aligned}
$$

Explicit results at second order

Where

$$F(br) \equiv \int_{br}^{\infty} \frac{y^{d-1}-1}{y(y^d-1)} dy \; ; \; L(br) \equiv \int_{br}^{\infty} \xi^{d-1} d\xi \int_{\xi}^{\infty} dy \; \frac{y-1}{y^3(y^d-1)}$$

$$H_2(br) \equiv \int_{br}^{\infty} \frac{d\xi}{\xi(\xi^d-1)} \int_1^{\xi} y^{d-3} dy \left[1 + (d-1) y F(y) + 2 y^2 F'(y) \right]$$

$$K_1(br) \equiv \int_{br}^{\infty} \frac{d\xi}{\xi^2} \int_{\xi}^{\infty} dy \; y^2 F'(y)^2 \; ; \; H_1(br) \equiv \int_{br}^{\infty} \frac{y^{d-2}-1}{y(y^d-1)} dy$$

$$K_2(br) \equiv \int_{br}^{\infty} \frac{d\xi}{\xi^2} \Big[1 - \xi(\xi-1) F'(\xi) - 2(d-1)\xi^{d-1} + \left(2(d-1)\xi^d - (d-2) \right) \int_{\xi}^{\infty} dy \; y^2 F'(y)^2 \Big]$$

Second order boundary stress tensor

The dual stress tensor corresponding to this metric is given by $(4\pi T = b^{-1} d)$

$$\begin{aligned} T_{\mu\nu} &= p\left(g_{\mu\nu} + d u_\mu u_\nu\right) \\ &\quad - 2\eta\left[\sigma_{\mu\nu} - \tau_\pi u^\lambda \mathcal{D}_\lambda \sigma_{\mu\nu} - \tau_\omega\left(\sigma_\mu{}^\lambda \omega_{\lambda\nu} - \omega_\mu{}^\lambda \sigma_{\lambda\nu}\right)\right] \\ &\quad + \xi_\sigma\left[\sigma_\mu{}^\lambda \sigma_{\lambda\nu} - \frac{\sigma_{\alpha\beta}\sigma^{\alpha\beta}}{d-1} P_{\mu\nu}\right] + \xi_C C_{\mu\alpha\nu\beta} u^\alpha u^\beta \\ p &= \frac{1}{16\pi G_{d+1} b^d} \qquad ; \qquad \eta = \frac{s}{4\pi} = \frac{1}{16\pi G_{d+1} b^{d-1}} \\ \tau_\pi &= (1 - H_1(1))b \qquad ; \qquad \tau_\omega = H_1(1)b \qquad ; \qquad \xi_\sigma = \xi_C = 2\eta b \end{aligned}$$

Gravitational constitutive relations

- Note that gravity reduces to fluid dynamics with particular (holographically determined) values for dissipative parameters. As we have seen the schematic form of the fluid stress tensor is

$$T_{\mu\nu} = aT^d(g_{\mu\nu} + du_\mu u_\nu) + bT^{d-1}\sigma_{\mu\nu} + T^{d-2}\sum_{i=1}^{5} c_i S^i_{\mu\nu}$$

- a is a thermodynamic parameter. b is related to the viscosity: we find $\eta/s = 1/(4\pi)$. c_i coefficients of the five traceless symmetric Weyl covariant two derivative tensors are second order transport coefficients. Value disagree with the predictions of the Israel Stewart formalism.
- Recall that results universal. Should yield correct order of magnitude estimate of transport coefficients in any strongly coupled CFT.

Event Horizons

- Our solutions are singular at $r = 0$. Quite remarkably it is possible (under certain conditions) to demonstrate that these solutions have event horizons and to explicitly determine the event horizon manifold order by order in the derivative expansion. This horizon shields the $r = 0$ singularity from the boundary.
- Our control over the event horizon, together with the classic area increase theorem of general relativity, can be used to derive an 'entropy current' for our fluid flows that is local and has positive divergence.

Entropy Current at second order

Explicitly this entropy current is given to second order by

$$4\,G_{d+1}\,b^{d-1}\,J_S^\mu = [\,1 + b^2\,(\,A_1\,\sigma^{\alpha\beta}\,\sigma_{\alpha\beta} + A_2\,\omega^{\alpha\beta}\,\omega_{\alpha\beta} + A_3\,\mathcal{R}\,)\,]\,u^\mu + b^2\,[\,B_1\,\mathcal{D}_\lambda\,\sigma^{\mu\lambda} + B_2\,\mathcal{D}_\lambda\,\omega^{\mu\lambda}\,]$$

where

$$A_1 = \frac{2}{d^2}(d+2) - \frac{K_1(1)d + K_2(1)}{d}\,, \qquad A_2 = -\frac{1}{2d}, \qquad B_2 = \frac{1}{d-2}$$

$$B_1 = -2A_3 = \frac{2}{d(d-2)}$$

- Our gravitational discussion has so far focussed on uncharged fluids. In order to study the gravitational dual of charged fluid flows we need to study bulk equations with a Maxwell field in addition to the bulk metric.
- The addition of a bulk Maxwell field endows the boundary theory with a conserved global charge. Equilibrium states in such a system are labeled by a charge density together with the energy density and velocity; in the bulk these equilibrium configurations are given by charged AdS-Reisnner Nordstorm black branes.

Charged hydrodynamics

- Following the general procedure described in this talk, it has been established that the AdS Einstein -Maxwell equations reduce, in an appropriate long wavelength limit, to the equations of charged relativistic hydrodynamics.
- The procedure yields expressions for the stress tensor and charge currents as a function of local temperatures, velocities and chemical potentials.
- We find a surprise here even at first order in the derivative expansion. In addition to the usual diffusive currents, in $d = 4$ we find a term in the charge current proportional to $\epsilon_{\mu\nu\rho\sigma}\omega^{\nu\rho}u^{\sigma}$. This is important because this term was ignored by Landau and Lifshitz and perhaps all authors subsequently. The fact that such a term is allowed in the equations of hydrodynamics was unexpected, and we now turn to explain it.

Hydrodynamics as an effective field theory

- How were the Landau Lifshitz constitutive relations obtained?
- Of course Landau Lifshitz worked in the derivative expansion imposing the constraints of symmetry. In the case of charged relativistic hydrodynamics at first order, however, this procedure allows for 7=5+2 first order constitutive parameters. The equations listed by Landau Lifshitz have three parameters (bulk viscosity, shear viscosity and conductivity).
- The cut from 7 to 3 followed from a requirement imposted by Landau and Lifshitz; the equations of hyrdodynamics must be compatable with the existence of an entropy current whose divergence is positive at every point in every allowed fluid flow. Physical intuition was used to guess the form of this entropy current at first order; the requirement of positivity of divergence of this entropy current then set 4 of the 7 symmetry allowed parameters to zero.

The Entropy positivity constraint

- As we have explained above, the gravitational results disagree with those of Landau Lifshitz. This is puzzling as the existence of a point wise positive divergence entropy current may be proved directly within gravity. How is all this consistent?
- Turns out that the Landau-Lifshitz guess for the form of the entropy current was not always correct. A good way to implement the Landau criterion is to make no a priori assumptions about the form of the entropy current, but allow it to be the most general allowed by symmetry and simultaneously constrain the form of the entropy current and constitutive relations by the requirement of positive divergence in an *arbitrary* consistent background

Local positivity of entropy production

- This principle turns out to be surprisingly constraining. At first order in ordinary relativistic charged fluid dynamics it sets two of the five symmetry allowed constitutive parameters to zero, and determines two others as a function of the anomaly coefficient. The last subtlety was missed by Landau and Lifshitz (who set those coefficients to zero) and gives modified constitutive relations in agreement with holographic computations.
- At second order in uncharged fluid dynamics it kills 5 out of the 15 parameters . At first order in parity invariant superfluid dynamics it kills 26 of the 47 parameters. Systematic principle to generate 'allowed' constitutive relations.

Constraints from Equilibrium

- Another idea (going back to Einstein), constraints from equilibrium.
- Main idea: consider the field theory on the background

$$ds^2 = -e^{2\sigma(\vec{x})}\left(dt + a_i(\vec{x})dx^i\right)^2 + g_{ij}(\vec{x})dx^i dx^j \quad (4)$$

$$A = A_0(\vec{x})dx^0 + \mathcal{A}_i(\vec{x})dx^i \quad (5)$$

- As this background is stationary, the total H and charge Q of the system are conserved and the partition function

$$Z = \mathrm{Tr} e^{-\frac{H-\mu_0 Q}{T_0}} \quad (6)$$

  is well defined.
- Idea: determine the most general symmetry allowed partition function, order by order in the derivative expansion, and constrain hydrodynamics by matching in equilibrium.

Partition function for charge fluids

- For a charged fluid at first order in the derivative expansion the most general partition function takes the form

$$
\begin{aligned}
W &= \ln Z = W^0 + W^1_{inv} + W^1_{anom} \\
W^0 &= \int \sqrt{g_3}\frac{e^\sigma}{T_0} P\left(T_0 e^{-\sigma}, e^{-\sigma}(A_0+\mu_0)\right) \\
W^1_{inv} &= \frac{C_0}{2}\int AdA + \frac{C_1}{2}\int ada + \frac{C_2}{2}\int Ada \\
W^1_{anom} &= \frac{C}{2}\left(\int \frac{A_0}{3} AdA + \frac{A_0^2}{6} Ada\right)
\end{aligned}
\tag{7}
$$

where A_i

$$A_i = \mathcal{A}_i - A_0 a_i \tag{8}$$

- Note anomaly term proportional to C

Constraints from Equilibrium

- The stress tensor and charge current in equilibrium are obtained by differentiating this partition function w.r.t. the metric and gauge field.
- Within hydrodynamics we expect

$$T(x) = T_0 e^{-\sigma} + T_1(x), \quad \mu(x) = (A_0(x) + \mu_0)e^{-\sigma} + \mu_1$$

$$u^\mu = e^\sigma(1,0,0,0) + u_1^\mu$$

- Plugging these relations into the hydrodynamical constitutive relations we find automatic agreement with the stress tensor and charge current from the partition function at the zero derivative level, provided we identify the function $P(T,\mu)$ as the pressure of the system as a function of its temperature and chemical potential.

Constraints from Equilibrium

- Carrying on to first order it turns out we can determine the corrections to the equilibrium velocities, chemical potentials and temperatures, together with all first order constitutive coefficents that multiply expressions that are nonvanishing in equilibrium in terms of the partition function data
- Yields the same 3 parameter set of equations of charged relativistic hydrodynamics as obtained from the systematic entropy function method. In this language Landau Lifshitz missed the possibility of Chern Simons terms. The exact agreement between the constraints of entropy positivity and equilibriation has been checked in several different contexts.
- Imp question: Does this ageement persist to all orders? Yes! S. Bhattacharya has demonstrated that existence of equilibrium plus the stability of this equilibrium are completely equivalent to the constraints from the local form

Consequences for gravity

- It is possible that Bhattacharyya's construction can be uplifted to gravity. By reversing the fluid gravity map it could lead to the construction of a Wald entropy current for higher derivative gravity, atleast in certain circumstances. We have attempted to implement this programme, so far without success?

Other Constraints on Hydrodynamics

- As explained above, two different physical constraints on hydrodynamics have been shown to be equivalent. Does this impressive achievement suggest that either of these constraints fully exhauste the set of non obvious constraints on the equations of hydrodynamics? No.
- Jensen, Loganaygam and Yarom have obtained further constraints from the principle of Euclidean consistency. Remarkable result, relating mixed current - gravitational anomalies to hydrodynamical transport coefficients. Does not follow directly from entropy positivity.
- Additional constraints from Onsager relations. Full set of constraints on ‘classical’ hydrodynamics? Path integral formulation of long distance finite temperature physics entirely in terms of hydrodynamical variables on a Schwinger Keldysh contour (hydrodynamicalfluctuations)? Connection to quantum gravity in AdS? All under current investigation.

Conclusions

- Asymptotically AdS_{d+1} gravity reduces, in the long wavelength limit, to the equations $d+1$ dimensional Navier Stokes equations with gravitationally determined dissipative parameters.
- The results from the implementation of this programme has corrected errors in widely accepted results on the general structure of the equations of charged relativistic hydrodynamics and superfluid hydrodynamics.
- Interesting connections between the constraints on constitutive relations from the existence and stability of equilibrium and the local form of the second law of thermodynamics. Possible application to Wald entropy increase. Structural question about the general constraints on constitutive relations.
- Unexplored connections between quantum fluctuations in gravity and the equations of hydrodynamics with fluctuations.