Search for supersymmetric partners of 3rd generation quarks at CMS

Tom Cornelis

for the CMS Collaboration

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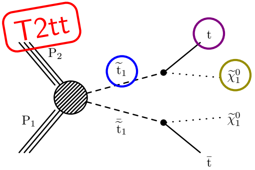
Introduction: supersymmetric partners of 3rd generation quarks

This talk will focus on

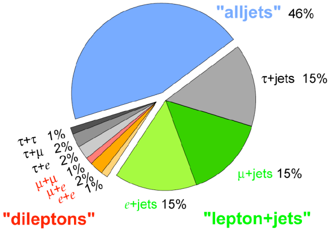
- pair production of **top squarks**, mostly manifesting as $t\bar{t} + p_{\mathrm{T}}^{\mathrm{miss}}$
  - 0ℓ: **SUS-16-049, arXiv:1707.03316, JHEP10 (2017) 005**
  - 1ℓ: **SUS-16-051, arXiv:1706.04402, JHEP10 (2017) 019**
  - 2ℓ: **SUS-17-001, arXiv:1711.00752, Submitted to Phys. Rev. D**
- pair production of **bottom squarks**
  - **SUS-16-032, arXiv:1707.07274, Submitted to Phys. Lett. B**
  - Analysis considers also stop pair production with $\widetilde{t}_1 \rightarrow c\widetilde{\chi}_1^0$ decays

- Use of simplified model spectra
  - Limited set of hypothetical particles and decay chains, describing a specific search channel
  - Minimal set of parameters (usually 2-4 mass parameters)
  - Generic description, results could be interpreted into other scenarios

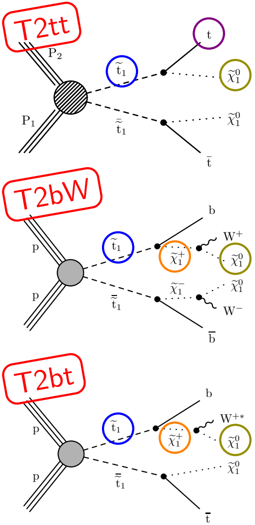
Top squark pair production



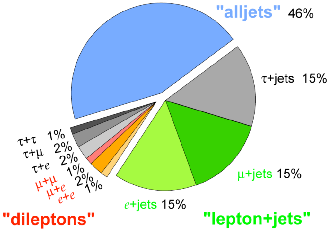
- The simplified model **T2tt** describes top squark pair production
- The top squark ($\tilde{t}_1$) decays into a top (t) and the lightest neutralino ($\tilde{\chi}_1^0$)



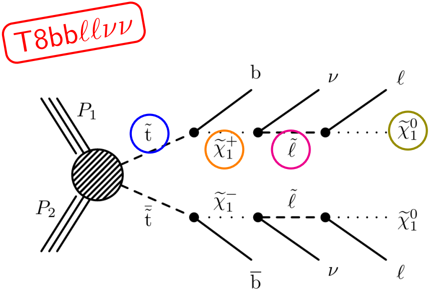
Top squark pair production

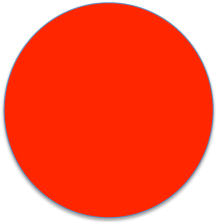


- The simplified model **T2tt** describes top squark pair production
- The top squark ($\tilde{t}_1$) decays into a top (t) and the lightest neutralino ($\tilde{\chi}_1^0$)
- Same final state also in **T2bW** and **T2bt**, where one or both $\tilde{t}_1$ decays to a b quark and a chargino ($\tilde{\chi}_1^+$)



Top squark pair production

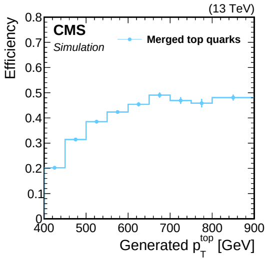


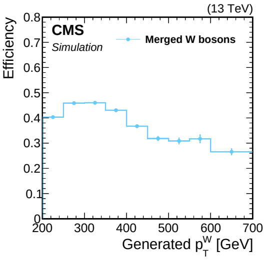


- In the **T8bb**$\ell\ell\nu\nu$ model, the $\tilde{\chi}_1^+$ decays through a slepton ($\tilde{\ell}$) to the $\tilde{\chi}_1^0$
- In this model, we have 100% branching ratio into dileptons, we can therefore only probe it in the 2ℓ analysis

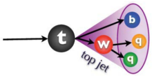
$\widetilde{t}_1\bar{\widetilde{t}}_1$ pair production (0ℓ): search strategy

- Veto leptons, require two jets and $p_{\mathrm{T}}^{\mathrm{miss}} > 250$ GeV
- Top and W-boson reconstruction using multivariate methods

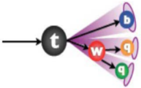




- Highly boosted top/W: merged into one jet, using jet substructure methonds on $R = 0.8$ jets



- Low boost: “resolved top”, using standard $R = 0.4$ jets



$\tilde{t}_1\bar{\tilde{t}}_1$ pair production (0ℓ): event categorization

- Events with low and high $\Delta m(\tilde{t}_1, \tilde{\chi}_1^0)$ are very different
- Introduce two categories, mainly based on

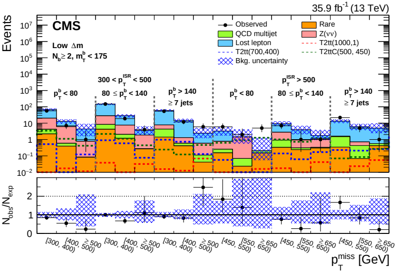
$$M_{\mathrm{T}}(\mathrm{b}_{1,2}, \not{E}_{\mathrm{T}}) \equiv \begin{cases} 0, & N_{\mathrm{b}} = 0, \\ m_{\mathrm{T}}(\mathrm{b}, \not{E}_{\mathrm{T}}), & N_{\mathrm{b}} = 1, \\ \mathrm{Min}[m_{\mathrm{T}}(\mathrm{b}_1, \not{E}_{\mathrm{T}}), m_{\mathrm{T}}(\mathrm{b}_2, \not{E}_{\mathrm{T}})], & N_{\mathrm{b}} \geq 2, \end{cases}$$

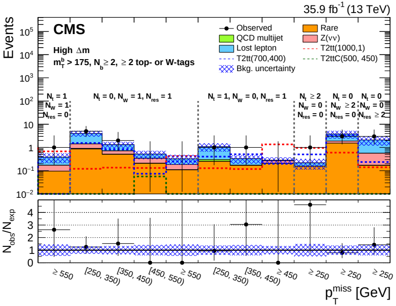
- Low $\Delta m(\tilde{t}_1, \tilde{\chi}_1^0)$ category targets signals with **compressed spectra**
  - low $m_T(b_{1,2}, \not{E})$
  - High $\not{E}$ is caused by stop pair recoiling against ISR jet
  - ISR jet: large-R jet with $p_{\mathrm{T}} > 200$ GeV, fails b-tagging
- High $\Delta m(\tilde{t}_1, \tilde{\chi}_1^0)$ category targets signals with **high mass splittings**
  - high $m_T(b_{1,2}, \not{E})$
  - Extensive use of top and W-tagging algorithms

$\tilde{t}_1\bar{\tilde{t}}_1$ pair production (0ℓ): signal regions

Main backgrounds:

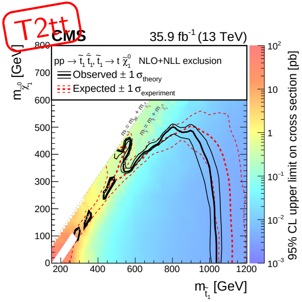
- $t\bar{t}$ and W+jets with lost lepton
- Z+jets and $t\bar{t}Z$ with $Z \to \nu\nu$

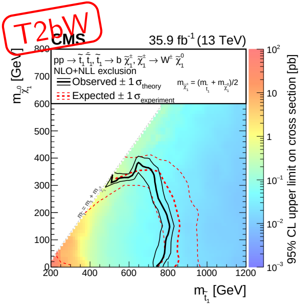


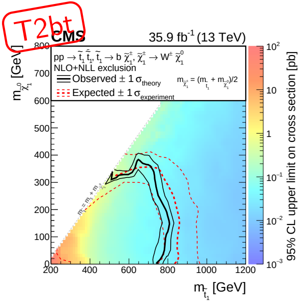


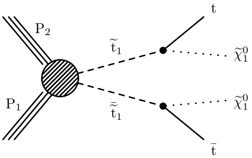
+ more SR for 0 and 1 b-jet categories

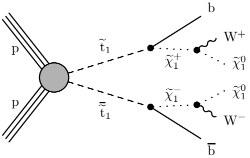
$\tilde{t}_1\bar{\tilde{t}}_1$ pair production (0ℓ): results

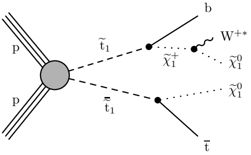




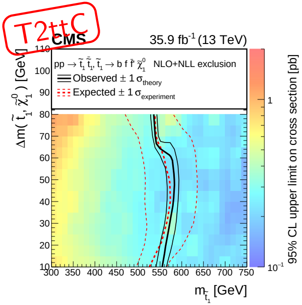


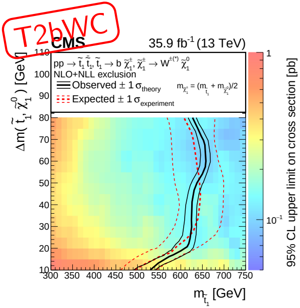


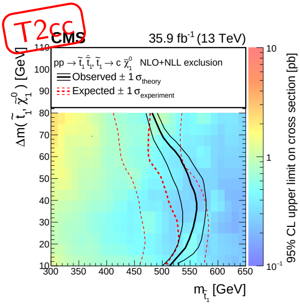


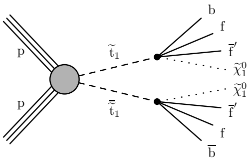


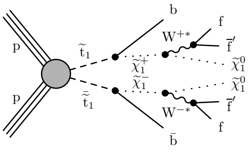
$\tilde{t}_1\bar{\tilde{t}}_1$ pair production (0ℓ): results

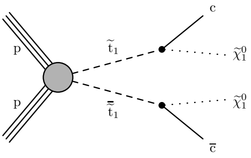




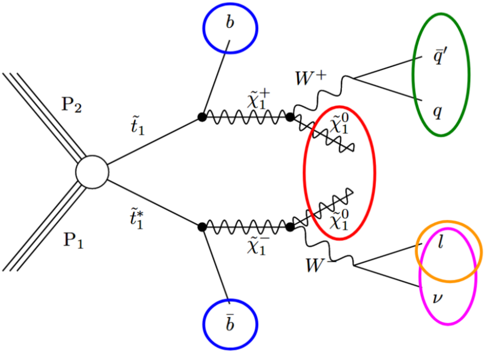








$\tilde{t}_1\bar{\tilde{t}}_1$ pair production (1ℓ): search strategy



- Exactly one charged lepton
- ≥ 2 jets
- ≥ 1 b-tag
- $p_{\mathrm{T}}^{\mathrm{miss}} > 250$ GeV
- $M_T > 150$ GeV, for $t\bar{t}(1\ell)$ and W+jets, only $p_{\mathrm{T}}^{\mathrm{miss}}$ source is neutrino so those backgrounds are bound by W mass
- Minimum $\Delta\phi(j, p_{\mathrm{T}}^{\mathrm{miss}}) > 0.8$

For **compressed** region (requiring 1 ISR jet to provide $p_{\mathrm{T}}^{\mathrm{miss}}$):

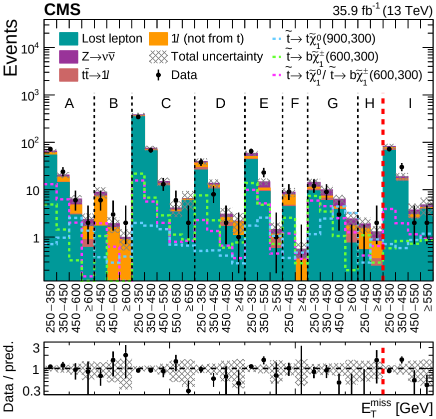
- ≥ 5 jets
- leading jet not b-tagged
- Minimum $\Delta\phi(j, p_{\mathrm{T}}^{\mathrm{miss}}) > 0.5$
- $\Delta\phi(j, \ell) < 2$
- Lepton $p_{\mathrm{T}} < 150$ GeV

$\tilde{t}_1\bar{\tilde{t}}_1$ pair production (1ℓ): signal regions

Categorization in

- $p_{\mathrm{T}}^{\mathrm{miss}}$ and number of jets
- $M_{\ell b}$ (invariant mass of the lepton and its closest b-jet in ΔR)
- Modified topness (using $\vec{p}_{\mathrm{T}}^{\mathrm{miss}} = \vec{p}_{\mathrm{T,W}} + \vec{p}_{\mathrm{T},\nu}$)

$$t_{\mathrm{mod}} = \ln(\min S), \text{ with } S(\vec{p}_{\mathrm{W}}, p_z, \nu) = \frac{(m_{\mathrm{W}}^2 - (p_\nu + p_\ell)^2)^2}{a_{\mathrm{W}}^4} + \frac{(m_{\mathrm{t}}^2 - (p_{\mathrm{b}} + p_{\mathrm{W}})^2)^2}{a_{\mathrm{t}}^4}$$



A: $N_J \leq 3$, $t_{mod} > 10$, $M_{lb} \leq 175$ GeV

B: $N_J \leq 3$, $t_{mod} > 10$, $M_{lb} > 175$ GeV

C: $N_J \geq 4$, $t_{mod} \leq 0$, $M_{lb} \leq 175$ GeV

D: $N_J \geq 4$, $t_{mod} \leq 0$, $M_{lb} > 175$ GeV

E: $N_J \geq 4$, $0 < t_{mod} \leq 10$, $M_{lb} \leq 175$ GeV

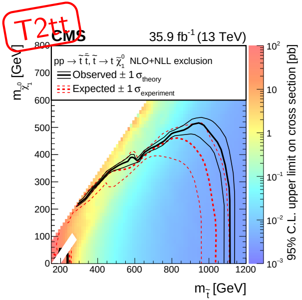
F: $N_J \geq 4$, $0 < t_{mod} \leq 10$, $M_{lb} > 175$ GeV

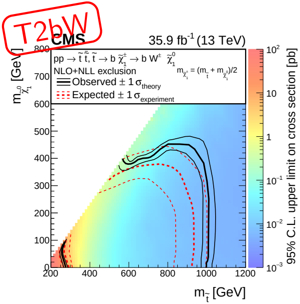
G: $N_J \geq 4$, $t_{mod} > 10$, $M_{lb} \leq 175$ GeV

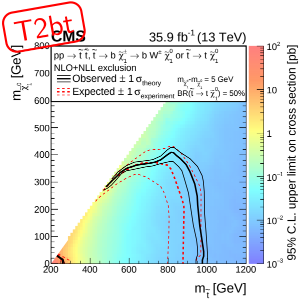
H: $N_J \geq 4$, $t_{mod} > 10$, $M_{lb} > 175$ GeV

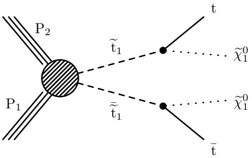
I: Compressed region

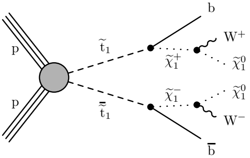
$\widetilde{t}_1\bar{\widetilde{t}}_1$ pair production (1ℓ): results

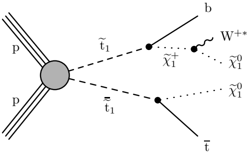




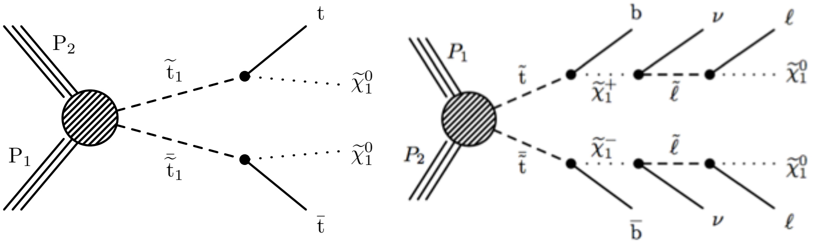








$\widetilde{t}_1\overline{\widetilde{t}}_1$ pair production (2ℓ): Search strategy



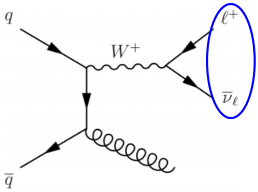
Pre-selections:

- Two **oppositely-charged leptons**
- Veto on additional loose leptons
- **Two jets** or more, with a least one b-tagged
- Veto on $|m_{ll} - m_Z| < 15$ GeV for same-flavour events
- Require $p_{\mathrm{T}}^{\mathrm{miss}} > 80$ GeV
- Additional requirements on $p_{\mathrm{T}}^{\mathrm{miss}}/\sqrt{H_T}$ and $\Delta\phi(j, p_{\mathrm{T}}^{\mathrm{miss}})$ in order to reject further DY events with jet mismeasurements

$\widetilde{t}_1\overline{\widetilde{t}}_1$ pair production (2ℓ): Search strategy

How to get rid of $t\bar{t}$ background in 2ℓ analysis?

- In $W \to \ell\nu$ events, the missing momentum coincidences with the neutrino $\Rightarrow$ from the transverse missing momentum $\vec{p}_{\mathrm{T}}^{\mathrm{miss}}$ we can derive the transverse mass m_T which is bound by the W mass: $m_T < m_W$



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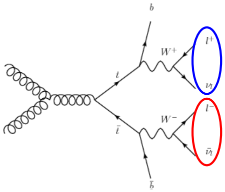
- In $t\bar{t}$ events, we have **two** W-bosons, and if they both decay leptonically, **two** neutrinos



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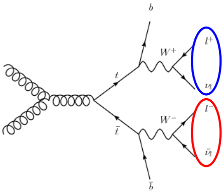
- In $t\bar{t}$ events, we have **two** W-bosons, and if they both decay leptonically, **two** neutrinos
- Try out all possible splittings of $\vec{p}_{\mathrm{T}}^{\mathrm{miss}}$ into $\vec{p}_{\mathrm{T1}}^{\mathrm{miss}}$ and $\vec{p}_{\mathrm{T2}}^{\mathrm{miss}}$



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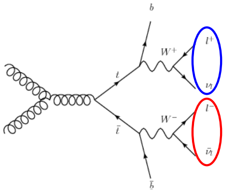
- Try out all possible splittings of $\vec{p}_{\mathrm{T}}^{\mathrm{miss}}$ into $\vec{p}_{\mathrm{T1}}^{\mathrm{miss}}$ and $\vec{p}_{\mathrm{T2}}^{\mathrm{miss}}$
- When both $\vec{p}_{\mathrm{T1}}^{\mathrm{miss}}$ and $\vec{p}_{\mathrm{T2}}^{\mathrm{miss}}$ coincidence with the neutrinos, then:
$\max\left[M_T(\vec{p}_{\mathrm{T}}^{\ell 1}, \vec{p}_{\mathrm{T1}}^{\mathrm{miss}}), M_T(\vec{p}_{\mathrm{T}}^{\ell 2}, \vec{p}_{\mathrm{T2}}^{\mathrm{miss}})\right] < m_W$



$\widetilde{t}_1\bar{\widetilde{t}}_1$ pair production (2ℓ): Search strategy

How to get rid of $t\bar{t}$ background in 2ℓ analysis?

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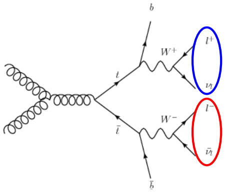
- So the minimization over all possible values, $M_{T2}(\ell\ell)$, is also bound by the W mass



$$M_{T2}(\ell\ell) = \min_{\vec{p}_{\mathrm{T1}}^{\mathrm{miss}} + \vec{p}_{\mathrm{T2}}^{\mathrm{miss}} = \vec{p}_{\mathrm{T}}^{\mathrm{miss}}} \left(\max\left[M_T(\vec{p}_{\mathrm{T}}^{\ell 1}, \vec{p}_{\mathrm{T1}}^{\mathrm{miss}}), M_T(\vec{p}_{\mathrm{T}}^{\ell 2}, \vec{p}_{\mathrm{T2}}^{\mathrm{miss}})\right]\right)$$

How to get rid of $t\bar{t}$ background in 2ℓ analysis?

- In $W \to \ell\nu$ events, the missing momentum coincidences with the neutrino $\Rightarrow$ from the transverse missing momentum $\vec{p}_{\mathrm{T}}^{\mathrm{miss}}$ we can derive the transverse mass m_T which is bound by the W mass: $m_T < m_W$
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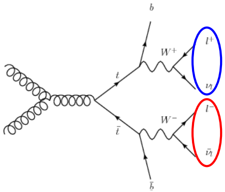
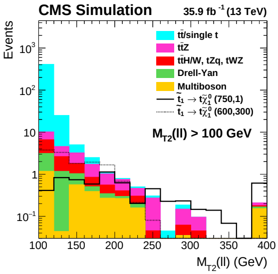
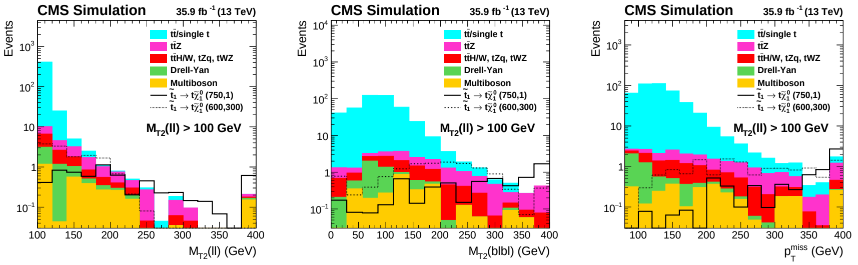
- Try out all possible splittings of $\vec{p}_{\mathrm{T}}^{\mathrm{miss}}$ into $\vec{p}_{\mathrm{T1}}^{\mathrm{miss}}$ and $\vec{p}_{\mathrm{T2}}^{\mathrm{miss}}$
- When both $\vec{p}_{\mathrm{T1}}^{\mathrm{miss}}$ and $\vec{p}_{\mathrm{T2}}^{\mathrm{miss}}$ coincidence with the neutrinos, then:
  $\max\left[M_T(\vec{p}_{\mathrm{T}}^{\,\ell 1}, \vec{p}_{\mathrm{T1}}^{\mathrm{miss}}), M_T(\vec{p}_{\mathrm{T}}^{\,\ell 2}, \vec{p}_{\mathrm{T2}}^{\mathrm{miss}})\right] < m_W$
- So the minimization over all possible values, $M_{T2}(\ell\ell)$, is also bound by the W mass
- Invisible particles, like dark matter would add additional $\vec{p}_{\mathrm{T}}^{\mathrm{miss}}$ to the event, and breaks the bound





$\widetilde{t}_1\bar{\widetilde{t}}_1$ pair production (2ℓ): Search strategy

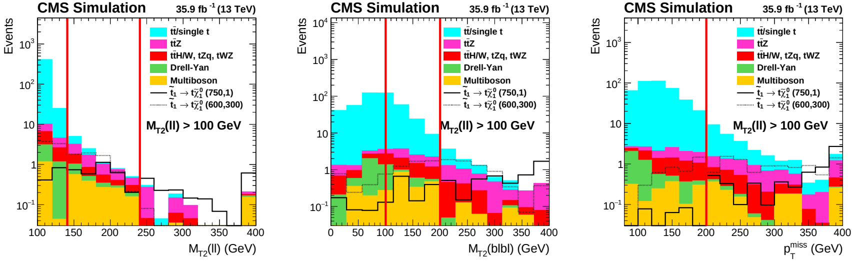
- For both same-flavor and opposite-flavor dilepton events, we construct signal regions in categories of $M_{T2}(\ell\ell)$, $M_{T2}(b\ell b\ell)$ and $p_{\mathrm{T}}^{\mathrm{miss}}$ variables



$M_{\mathrm{T2}}(\mathrm{b}\ell\mathrm{b}\ell)$ (GeV)	$p_{\mathrm{T}}^{\mathrm{miss}}$ (GeV)	$100 < M_{\mathrm{T2}}(\ell\ell) < 140$ GeV	$140 < M_{\mathrm{T2}}(\ell\ell) < 240$ GeV	$M_{\mathrm{T2}}(\ell\ell) > 240$ GeV
0–100	80–200	SR0	SR6	
	>200	SR1	SR7	
100–200	80–200	SR2	SR8	
	>200	SR3	SR9	SR12
>200	80–200	SR4	SR10	
	>200	SR5	SR11	

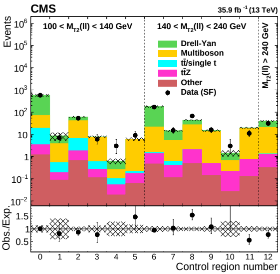
$\widetilde{t}_1\overline{\widetilde{t}}_1$ pair production (2ℓ): Search strategy

- For both same-flavor and opposite-flavor dilepton events, we construct signal regions in categories of $M_{T2}(\ell\ell)$, $M_{T2}(b\ell b\ell)$ and $p_{\mathrm{T}}^{\mathrm{miss}}$ variables

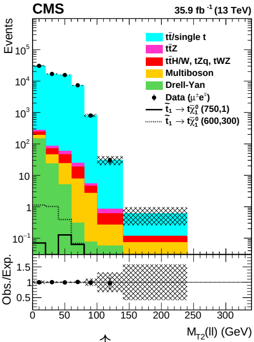


$M_{\mathrm{T2}}(\mathrm{b}\ell\mathrm{b}\ell)$ (GeV)	$p_{\mathrm{T}}^{\mathrm{miss}}$ (GeV)	$100 < M_{\mathrm{T2}}(\ell\ell) < 140$ GeV	$140 < M_{\mathrm{T2}}(\ell\ell) < 240$ GeV	$M_{\mathrm{T2}}(\ell\ell) > 240$ GeV
0–100	80–200	SR0	SR6	SR12
	>200	SR1	SR7	
100–200	80–200	SR2	SR8	
	>200	SR3	SR9	
>200	80–200	SR4	SR10	
	>200	SR5	SR11	

$\tilde{t}_1\bar{\tilde{t}}_1$ pair production (2ℓ): Use of control regions

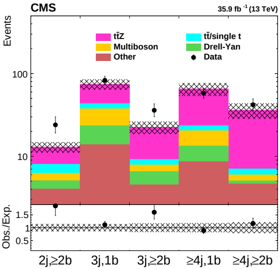


Control regions for Drell-Yan and multibosons: same-flavor events with $|m_{ll} - m_Z| < 15$ GeV and 0 b-jets $\leftarrow$



$\uparrow$

Control region for $t\bar{t}$: low p_T^{miss}

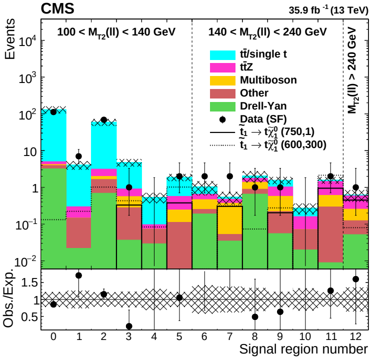


$\nwarrow$

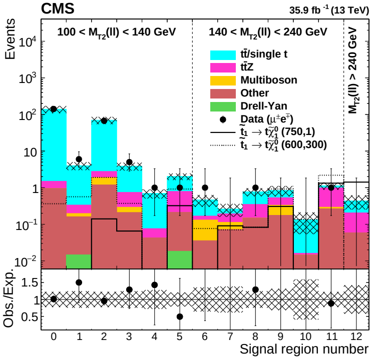
Control region for $t\bar{t}Z$: 3ℓ events

$\widetilde{t}_1\bar{\widetilde{t}}_1$ pair production (2ℓ): Results: signal region yields

- Signal region are split up in same-flavor and opposite-flavor events
- Very good agreement in each of the 26 signal regions
- No significant excess in any of the bins

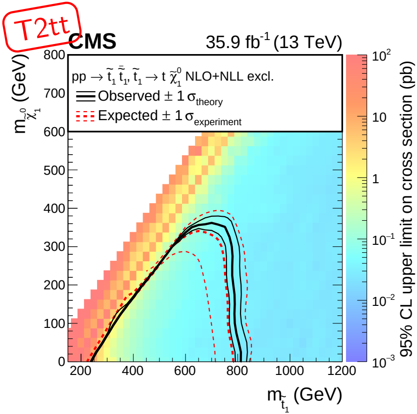


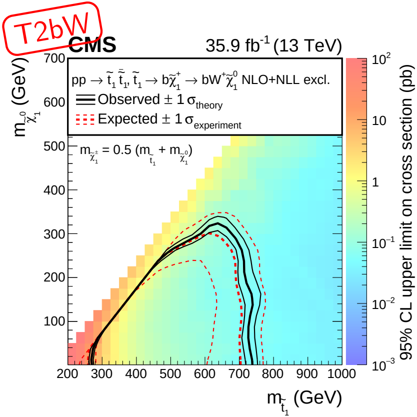
$ee/\mu\mu$ events

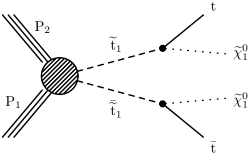


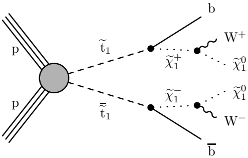
$e\mu$ events

$\tilde{t}_1\bar{\tilde{t}}_1$ pair production (2ℓ): Results



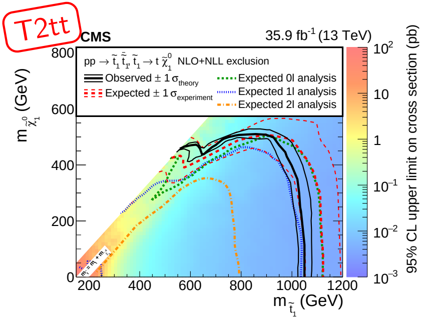


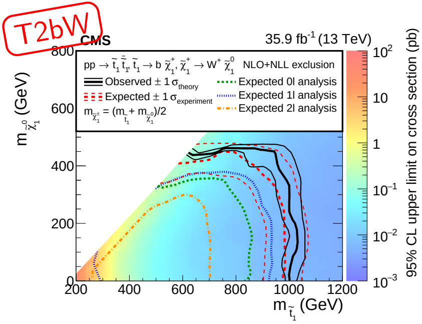




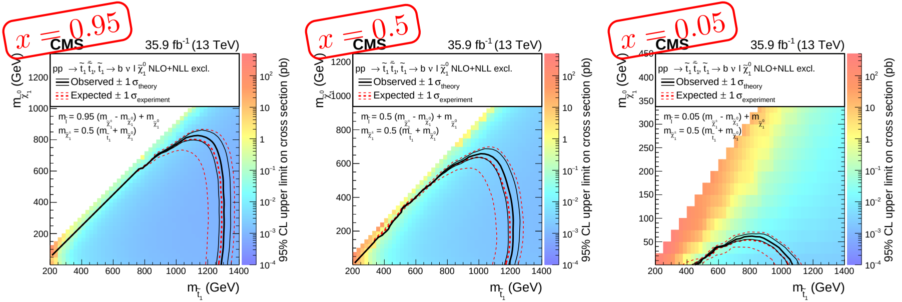
Combining 0ℓ+1ℓ+2ℓ

- The 2ℓ search can be statistically combined with the previous discussed all-jet and semileptonic searches



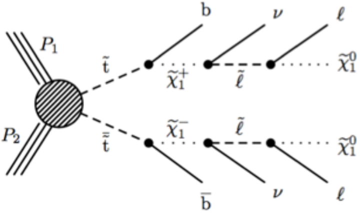


$\tilde{t}_1\bar{\tilde{t}}_1$ pair production (2ℓ): Results for $T8bb\ell\ell\nu\nu$



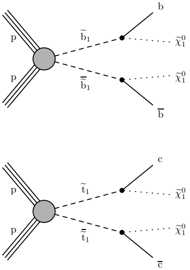
Intermediate mass parameters according to $m_{\tilde{\ell}} = x \cdot (m_{\tilde{\chi}_1^+} - m_{\tilde{\chi}_1^0}) + m_{\tilde{\chi}_1^0}$

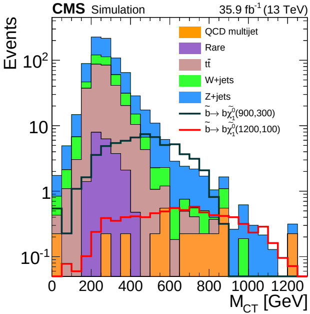
- $x = 0.95$
  - high momentum for the $\tilde{\chi}_1^0$
  - high $p_{\mathrm{T}}^{\mathrm{miss}}$ and $M_{T2}(\ell\ell)$ in the event
  - therefore easy to exclude stop masses up to 1.35 TeV
- $x = 0.5$
  - excludes stop masses up to 1.2 TeV
- $x = 0.05$
  - low momentum for the $\tilde{\chi}_1^0$
  - relatively low $M_{T2}(\ell\ell)$
  - excludes stop masses up to 400 GeV



Bottom and charm decays: search strategy

- Search for direct bottom squark pair or top squark pair production in final states with b and c jets
- Analysis divided in two categories
  - Non-compressed spectra for $\tilde{b}\bar{\tilde{b}}$ pair production with $\Delta m(\tilde{b}, \tilde{\chi}_1^0) > 100$ GeV
  - Compressed spectra for $\Delta m(\tilde{b}/\tilde{t}, \tilde{\chi}_1^0) < 100$ GeV





Non-compressed

- Two leading jets are b-tagged
- Search regions binned in H_T and boosted-corrected contransverse mass M_{CT} defined as

$$M_{CT}^2(j_1, j_2) = 2p_T(j_1)p_T(j_2)\,(1 + \cos\Delta\phi(j_1, j_2))$$

which has endpoint at $(m^2_{\tilde{b}_1} - m^2_{\tilde{\chi}_1^0})/m_{\tilde{b}_1}$

Bottom and charm decays: search strategy

- Search for direct bottom squark pair or top squark pair production in final states with b and c jets
- Analysis divided in two categories
  - Non-compressed spectra for $\widetilde{b}\overline{\widetilde{b}}$ pair production with $\Delta m(\widetilde{b}, \widetilde{\chi}_1^0) > 100$ GeV
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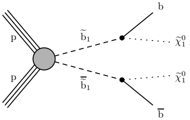
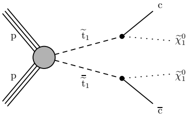
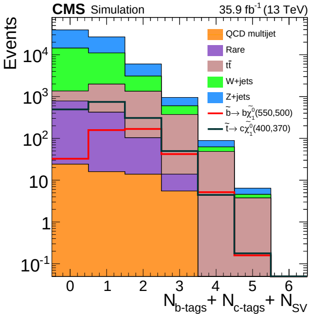


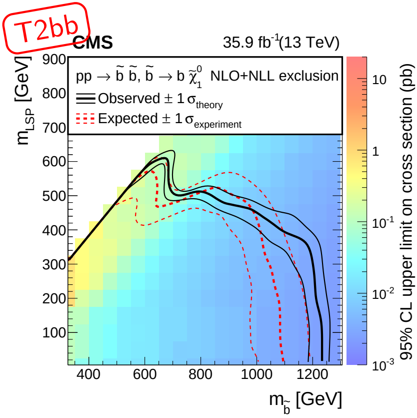


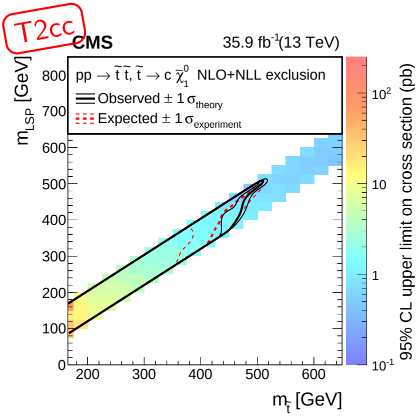


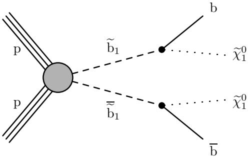
Compressed

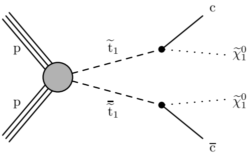
- Require ISR jet
- Binning in H_T, N_b, N_c, N_{SV}

Bottom and charm decays: result









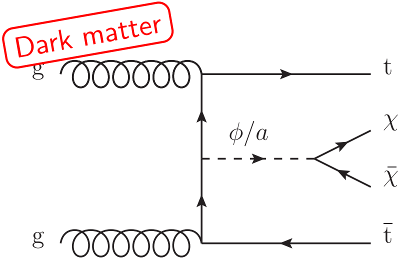
Summary

- Several third generation analyses performed using 2016 LHC data, amounting to 36 fb^{-1}
- Analysis techniques and selections improved (i.e. boosted objects, better use of transeverse mass variables $M_{T2}(\ell\ell)$,…), resulting in a greater sensitivity
- Putting stronger limits on **top squark pair** and **bottom squark pair** production, reaching beyond 1 TeV for some models

Backup slides

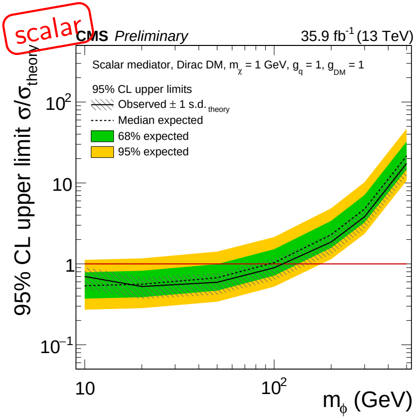
Dark matter reinterpretation

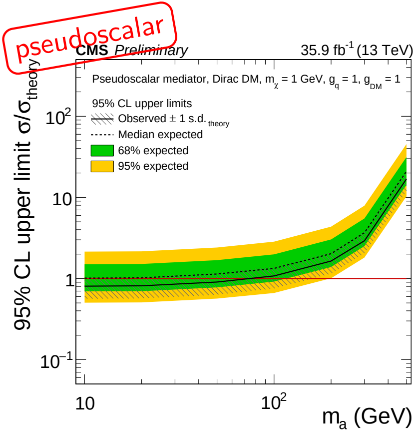
- **Dark matter (DM)** model where the DM candidate χ interacts with Standard Model particles through a scalar mediator a or pseudoscalar mediator ϕ
- Mediator coupling to quark $\propto m_q$, therefore favors coupling to top quark
- Same final state as in the top squark 2ℓ analysis



Dark matter reinterpretation

When interpreting the stop 2ℓ search in a dark matter model, we are able to put the **first limits** on dark matter production in the dileptonic $t\bar{t}$ channel:



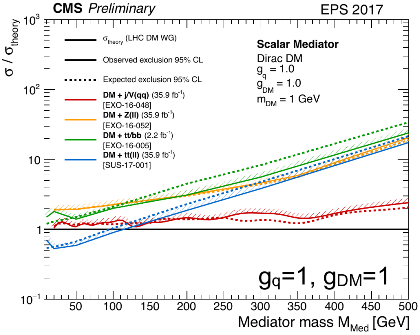


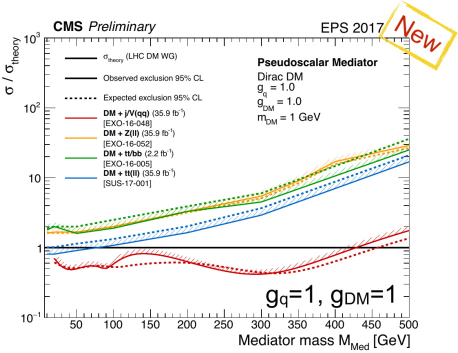
For $m_\chi = 1$ GeV we exclude

- $m_\phi < 100$ GeV for scalar mediators

Dark matter reinterpretation

For a **scalar** mediator, **first** analusis to exclude scaler mediator masses for $m_\chi = 1$ GeV





Dark matter reinterpretation

Limits are also calculated for higher values of m_χ

		Scalar		Pseudoscalar	
m_χ (GeV)	$m_{\phi/a}$ (GeV)	Expected	Observed	Expected	Observed
1	10	$0.54^{+0.25}_{-0.16}$	0.70	$1.01^{+0.49}_{-0.32}$	0.81
1	20	$0.56^{+0.26}_{-0.17}$	0.53	$1.02^{+0.49}_{-0.32}$	0.81
1	50	$0.67^{+0.32}_{-0.21}$	0.59	$1.14^{+0.55}_{-0.36}$	0.91
1	100	$1.04^{+0.48}_{-0.32}$	0.90	$1.33^{+0.65}_{-0.42}$	1.08
1	200	$2.30^{+1.11}_{-0.72}$	1.87	$2.02^{+1.01}_{-0.64}$	1.64
1	300	$4.8^{+2.3}_{-1.5}$	3.8	$3.7^{+1.8}_{-1.2}$	2.9
1	500	$21.6^{+10.9}_{-6.9}$	17.4	$21.0^{+10.4}_{-6.7}$	16.9
10	10	$18.8^{+8.8}_{-5.8}$	16.6	$19.3^{+9.3}_{-6.1}$	15.3
10	15	$17.0^{+8.0}_{-5.2}$	13.8	$15.8^{+7.6}_{-5.0}$	12.7
10	50	$0.72^{+0.33}_{-0.22}$	0.69	$1.08^{+0.52}_{-0.34}$	0.86
10	100	$1.03^{+0.48}_{-0.32}$	0.84	$1.25^{+0.61}_{-0.39}$	0.98
50	10	125^{+61}_{-39}	102	72^{+36}_{-23}	58
50	50	104^{+50}_{-33}	84	62^{+30}_{-19}	49
50	95	52^{+25}_{-16}	43	$20.3^{+10.0}_{-6.4}$	16.2
50	200	$2.32^{+1.14}_{-0.73}$	1.86	$2.05^{+1.02}_{-0.64}$	1.64
50	300	$4.7^{+2.3}_{-1.5}$	3.8	$3.7^{+1.9}_{-1.2}$	3.0