Progress in Lattice QCD

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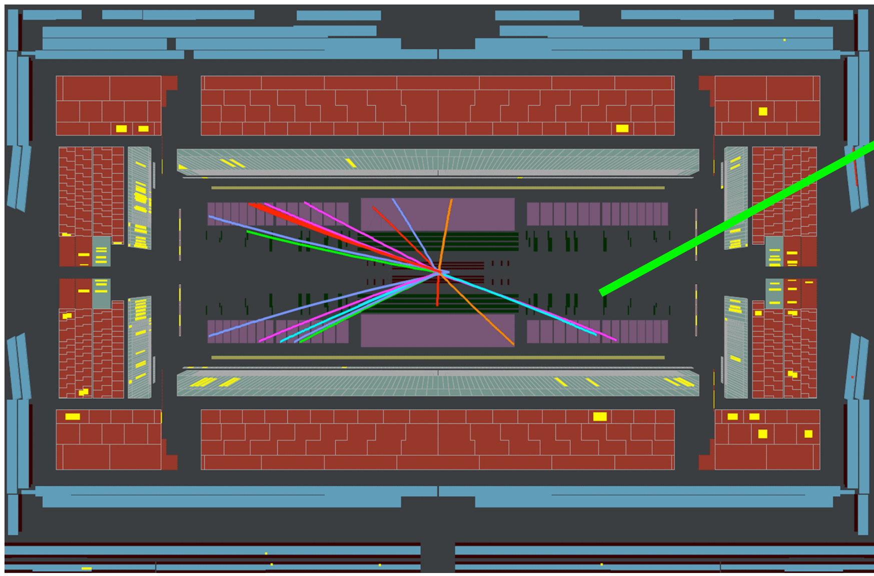
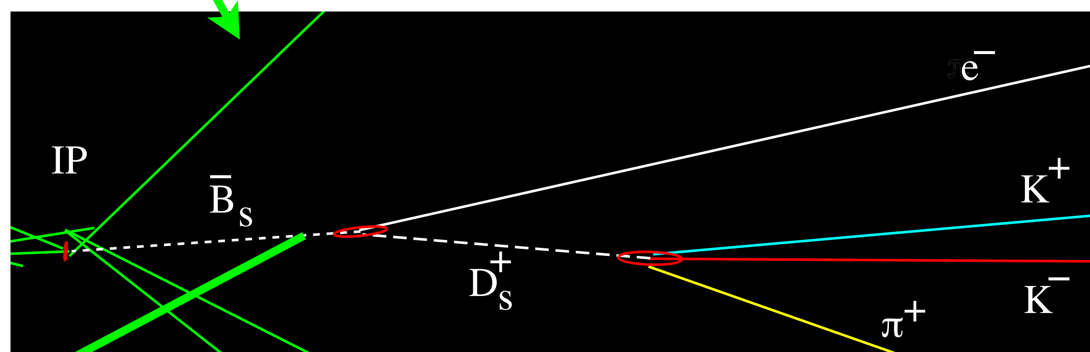
University of Glasgow
HPQCD collaboration

CKM2016
Mumbai, Nov.2016

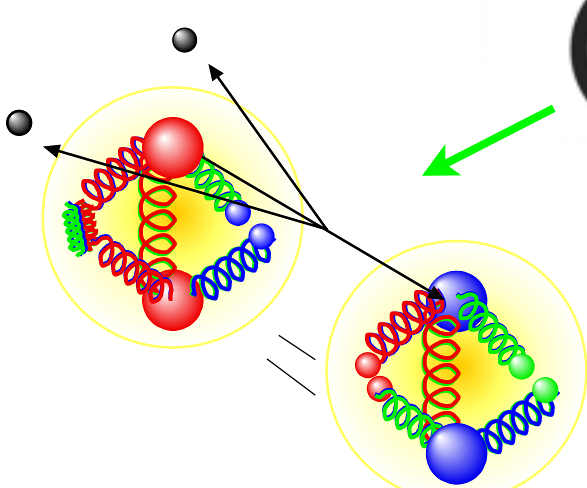
Quark confinement by the strong interaction is a major complication in testing the Standard Model



ATLAS@LHC



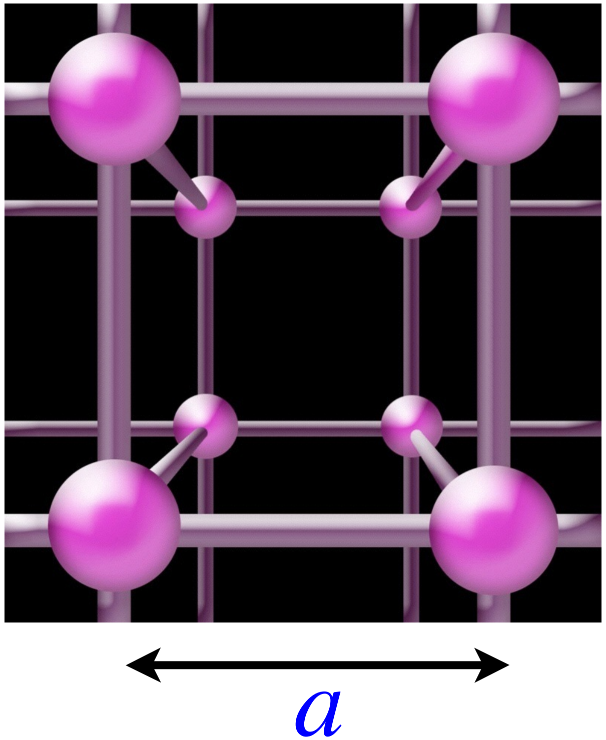
ALEPH@LEP



A fully nonperturbative treatment of Quantum Chromodynamics is necessary to connect observed hadron properties with those of quarks - vital for flavour physics.

Overview of progress in Lattice QCD since 2014 - key messages

- Lattice QCD methods - now working with physical $m_{u/d}$.
- Update of kaon physics - very accurate calculations possible for hadronic quantities needed for Kl2, Kl3. QED/m_u.ne.m_d being added.
- Update of charm physics - use of relativistic actions means accuracy approaching that of kaon for decay constants. Semileptonic form factors being improved.
- Update of bottom physics - lots of work going on, and errors going down, but a lot still to do. Will move to relativistic actions eventually …
- Conclusions

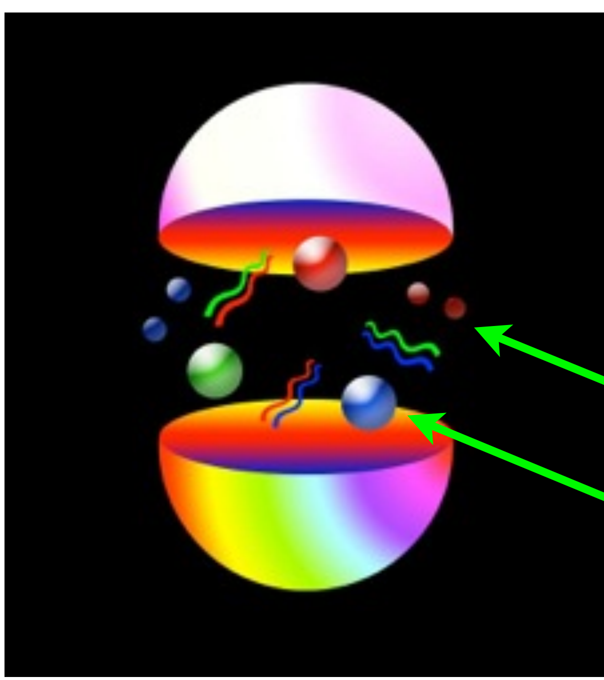


Lattice QCD = fully nonperturbative, based on Path Integral formalism

basic integral $\int \mathcal{D}U\mathcal{D}\psi\mathcal{D}\overline{\psi}\, \exp(-\int \mathcal{L}_{QCD} d^4x)$

discretise quark and gluon fields in a 4-d space-(Euclidean)time volume

a=0.1fm, N = 50^3x100, gives multi-million dimensional integral



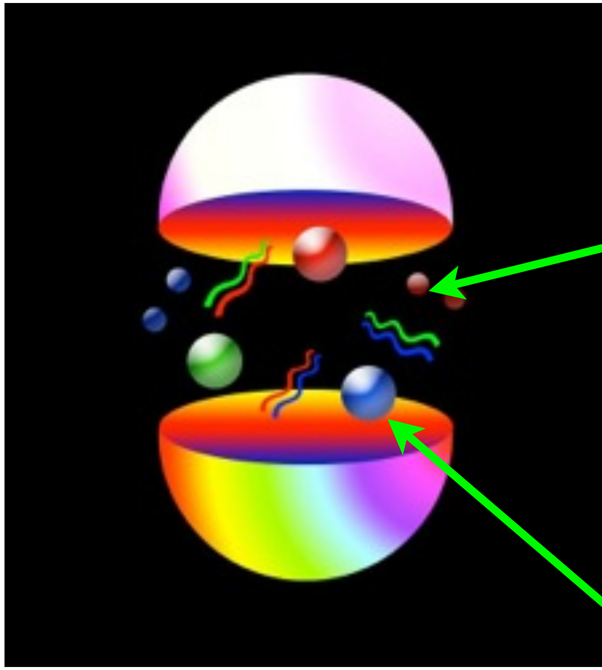
Integrating over quark fields leaves gluon field integral.

Sea quarks appear through $\det M$

Valence quarks through M^{-1}

$\mathcal{L}_{QCD,q} = \overline{\psi} M \psi$ $\quad M = \gamma \cdot D + m_q$

Lattice QCD = two-step procedure



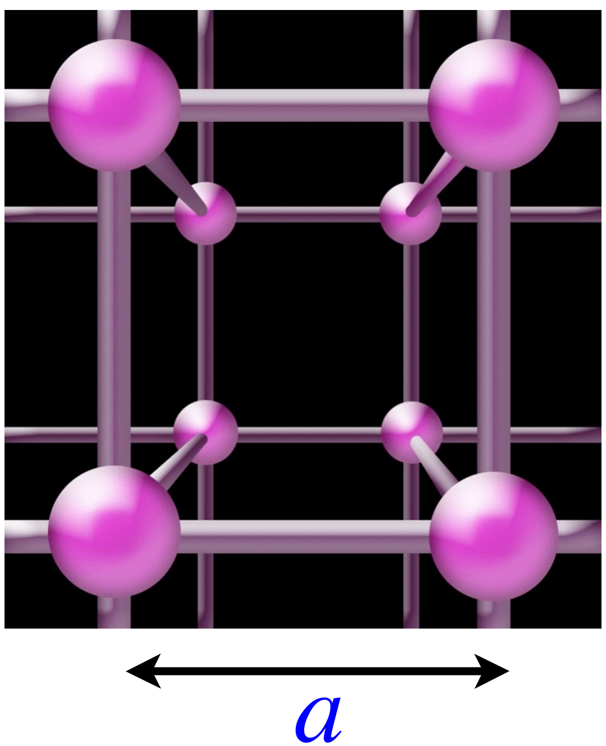
1) Generate sets of gluon fields for Monte Carlo integrn of Path Integral (inc effect of u, d, s, (c) sea quarks)

numerically extremely challenging

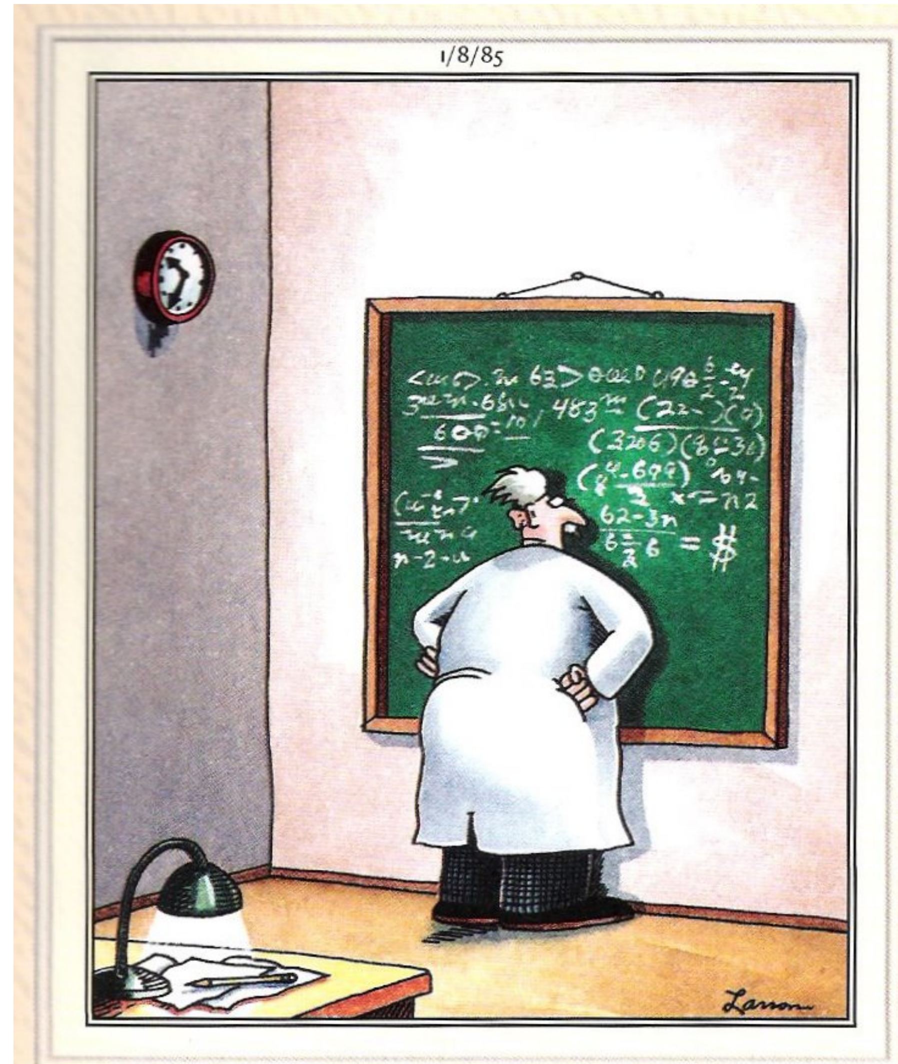
2) Calculate valence quark propagators and combine for “hadron correlators”

numerically costly, data intensive

- Fit for masses and matrix elements
- Determine a and fix m_q to get results in physical units.
- cost increases as $a \to 0, m_l \to phys$ and with statistics, volume.



Darwin@Cambridge - part of UK's DiRAC facility



Einstein discovers that time is actually money.

Calculations need many millions of core-hours of high performance computing time around the world. Costs money!

Lattice 2016 conference:
www.southampton.ac.uk/lattice2016/

Quark formalisms

Many ways to discretise Dirac Lagrangian onto lattice. All should give same answers at physical point. Results now from multiple approaches - comparison tests systematic error analysis.

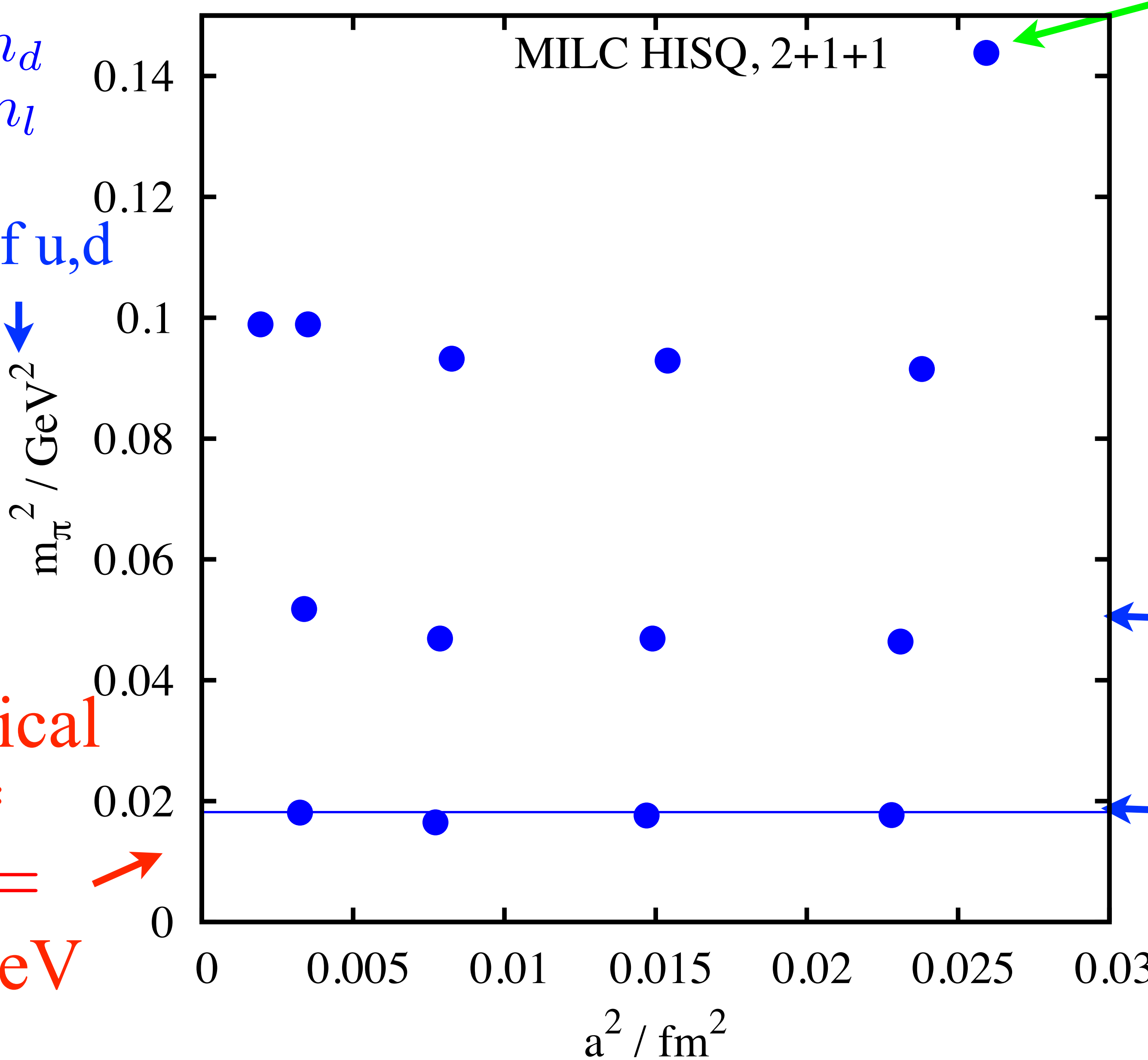
Issues are: Discretisation errors at power a^n
Numerical speed of Dirac matrix inversion
Chiral symmetry/Quark doubling
Normalisation of current operators

relativistic : asqtad, clover, domain-wall, highly-improved staggered quarks (HISQ), twisted mass ….

non-relativistic : HQET, NRQCD

mixed : clover (Fermilab formalism), RHQ

Example parameters for '2nd generation' calculations now being done with staggered quarks.



$m_u = m_d$
$= m_l$

mass of u,d quarks ↓

*physical $m_{u/d}$ *

$m_{\pi^0} =$ 135 MeV

"2nd generation" lattices inc. c quarks in sea

HISQ = Highly improved staggered quarks - very accurate discretisation

E.Follana, et al, HPQCD, hep-lat/0610092.

$m_{u,d} \approx m_s/10$

$m_{u,d} \approx m_s/27$

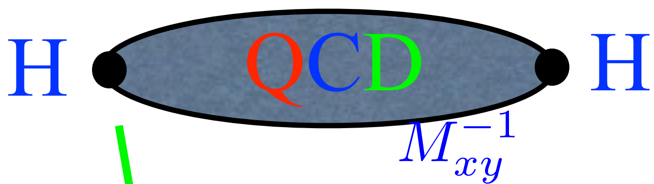
Volume:

$m_\pi L > 3$

$n_f = 1+1+1+1$ with QED now possible (BMW 1406.4088)

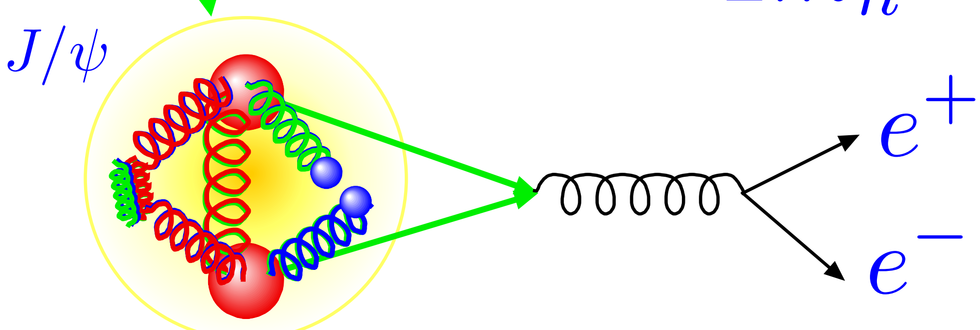
Easiest calculation : meson ‘2point’ functions - give masses and decay constants.

$$\langle 0|H^\dagger(T)H(0)|0\rangle = \sum_n A_n e^{-m_n T} \overset{T}{\underset{\text{large}}{\rightarrow}} A_0 e^{-m_0 T}$$



masses of all hadrons with quantum numbers of H

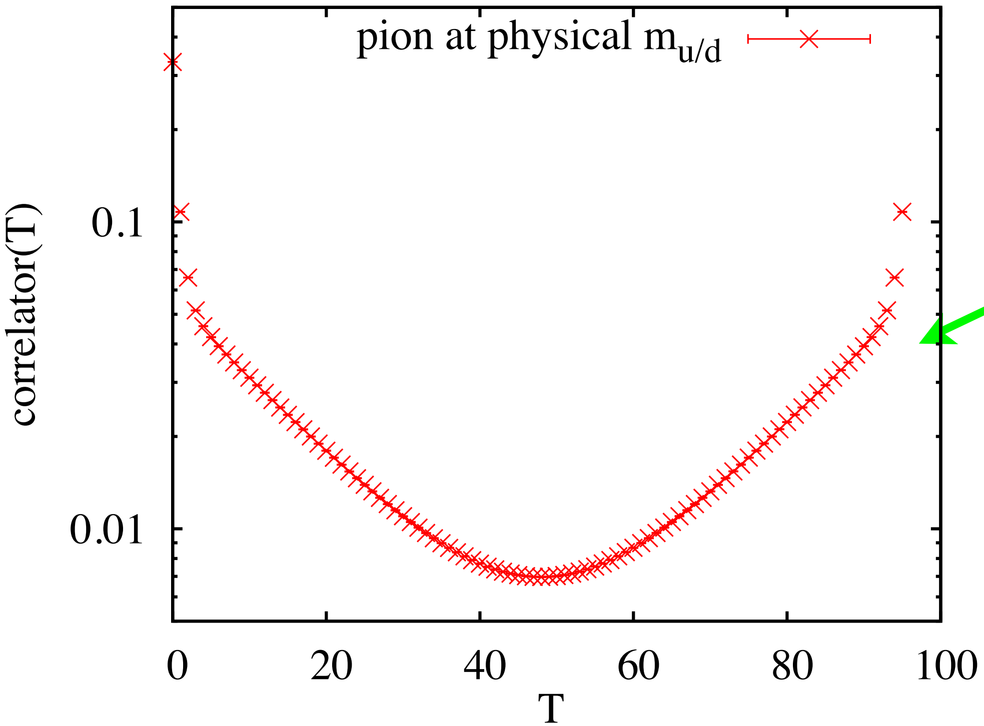
$$A_n = \frac{|\langle 0|H|n\rangle|^2}{2m_n} = \frac{f_n^2 m_n}{2}$$



decay constant parameterises amplitude to annihilate via current H. If H couples to W/photon can compare to exptl decay rate to leptons.

Key issue is normalisation of H. Absolute normalisation comes from symmetry in some cases. Otherwise must ‘match’ to continuum renormln scheme.

Example (state-of-the-art) calculation



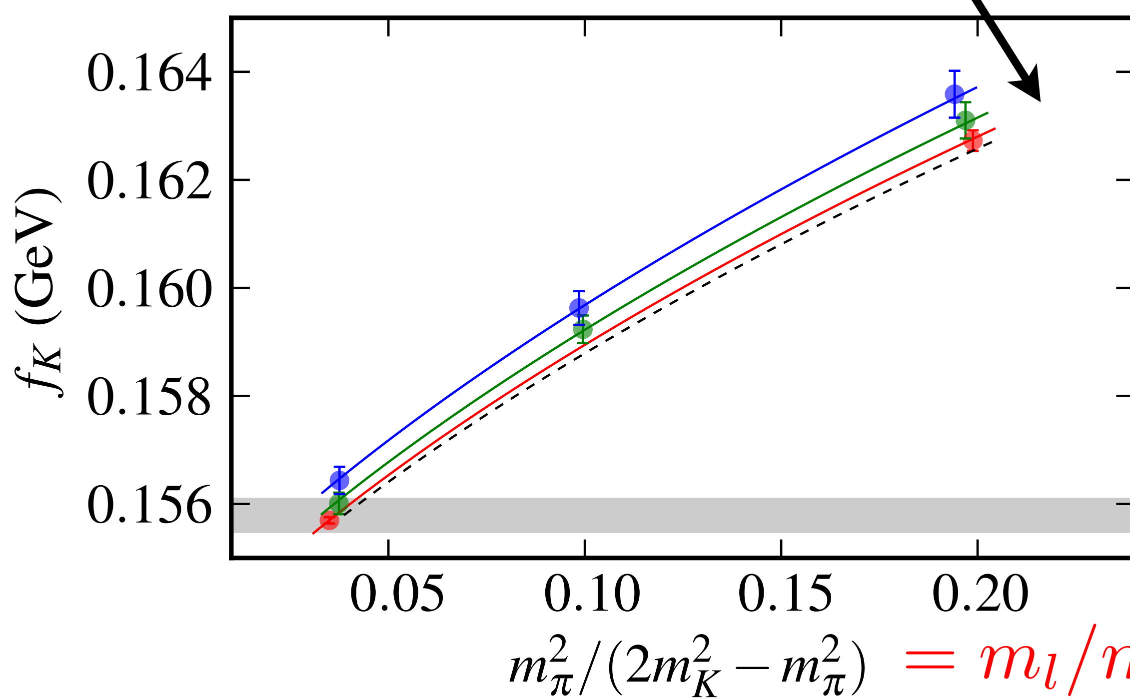
Extract meson mass and amplitude=decay constant (absolutely normalised since PCAC exact) from correlator for multiple lattice spacings and $m_{u/d}$.

Very high statistics

Extrapolation in a

Convert decay constant to GeV units using f_π to fix lattice spacing.

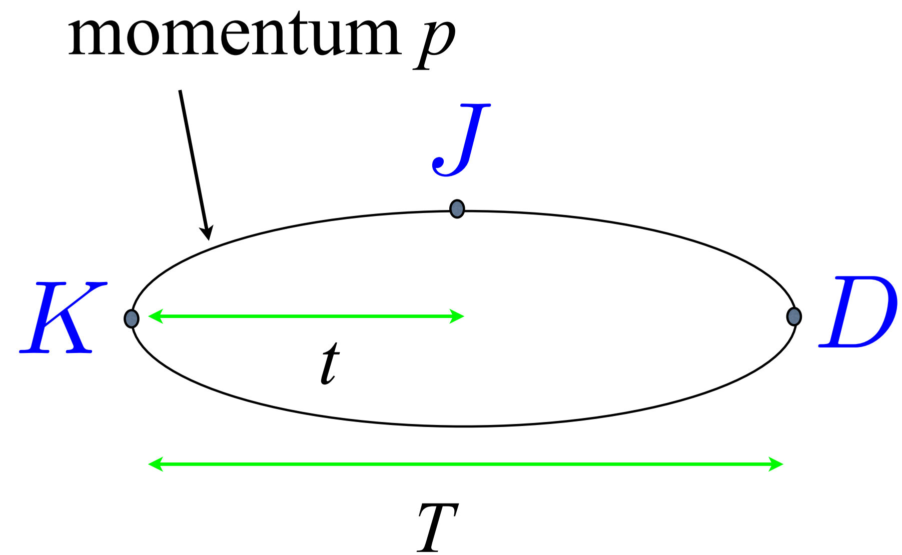
Note: small but visible discretisation errors.



$= m_l/m_s$

R. Dowdall et al, HPQCD, 1303.1670.

3pt functions



Join 3 propagators. Multi-exponential fit as a function of t, T along with 2pt allows extraction of

$\langle K|J|D\rangle$ for a range of q^2 values from q^2_{max} (zero recoil) to $q^2=0$ (max K momentum)

Convert matrix element to form factors e.g

$$< K|V^\mu|D >= f_+(q^2)\left[p_D^\mu + p_K^\mu - \frac{M_D^2 - M_K^2}{q^2}q^\mu\right]$$

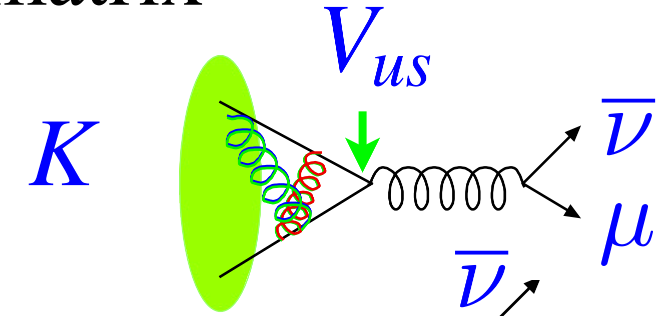
$$+f_0(q^2)\frac{M_D^2 - M_K^2}{q^2}q^\mu \qquad f_0(0) = f_+(0)$$

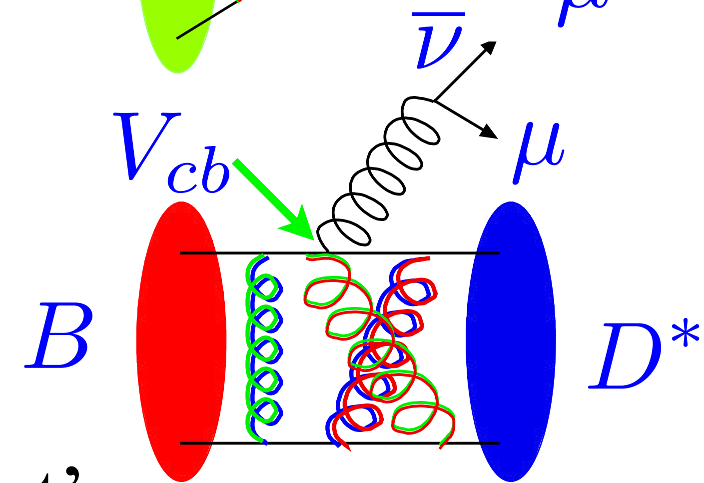
exptl rate depends on f+; f0 appears for τ in final state

Issues for lattice are normln of ops, disc. errors away from zero recoil, fitting form factor shape (now all use z-expansion). Focus on 'gold-plated hadrons', approaches to resonances being developed.

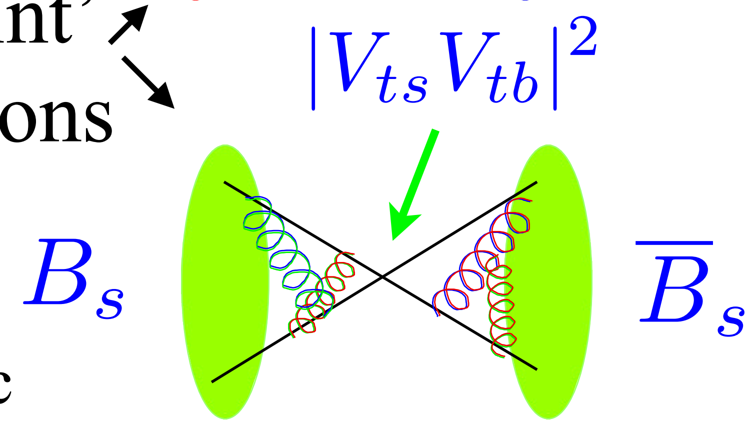
Weak decays give access to CKM matrix

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ \pi \rightarrow l\nu & K \rightarrow l\nu & B \rightarrow \pi l\nu \\ & K \rightarrow \pi l\nu & \\ V_{cd} & V_{cs} & V_{cb} \\ D \rightarrow l\nu & D_s \rightarrow l\nu & B \rightarrow Dl\nu \\ D \rightarrow \pi l\nu & D \rightarrow Kl\nu & \\ V_{td} & V_{ts} & V_{tb} \\ \langle B_d|\overline{B}_d\rangle & \langle B_s|\overline{B}_s\rangle & \end{pmatrix}$$





'3-point' functions



$$Br(M \rightarrow \mu\nu) \propto V_{ab}^2 f_M^2$$

or other hadronic parameter

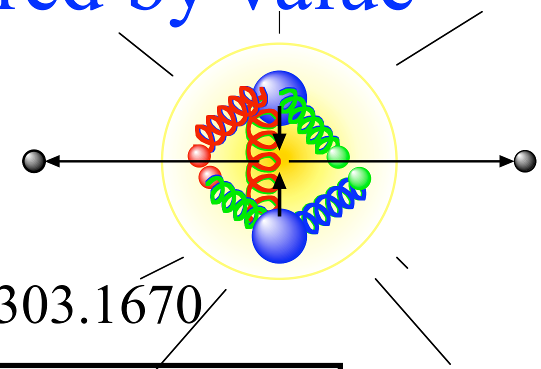
Expt = CKM x theory(QCD)

Accurate CKM element determination requires accurate experiment AND accurate lattice QCD.

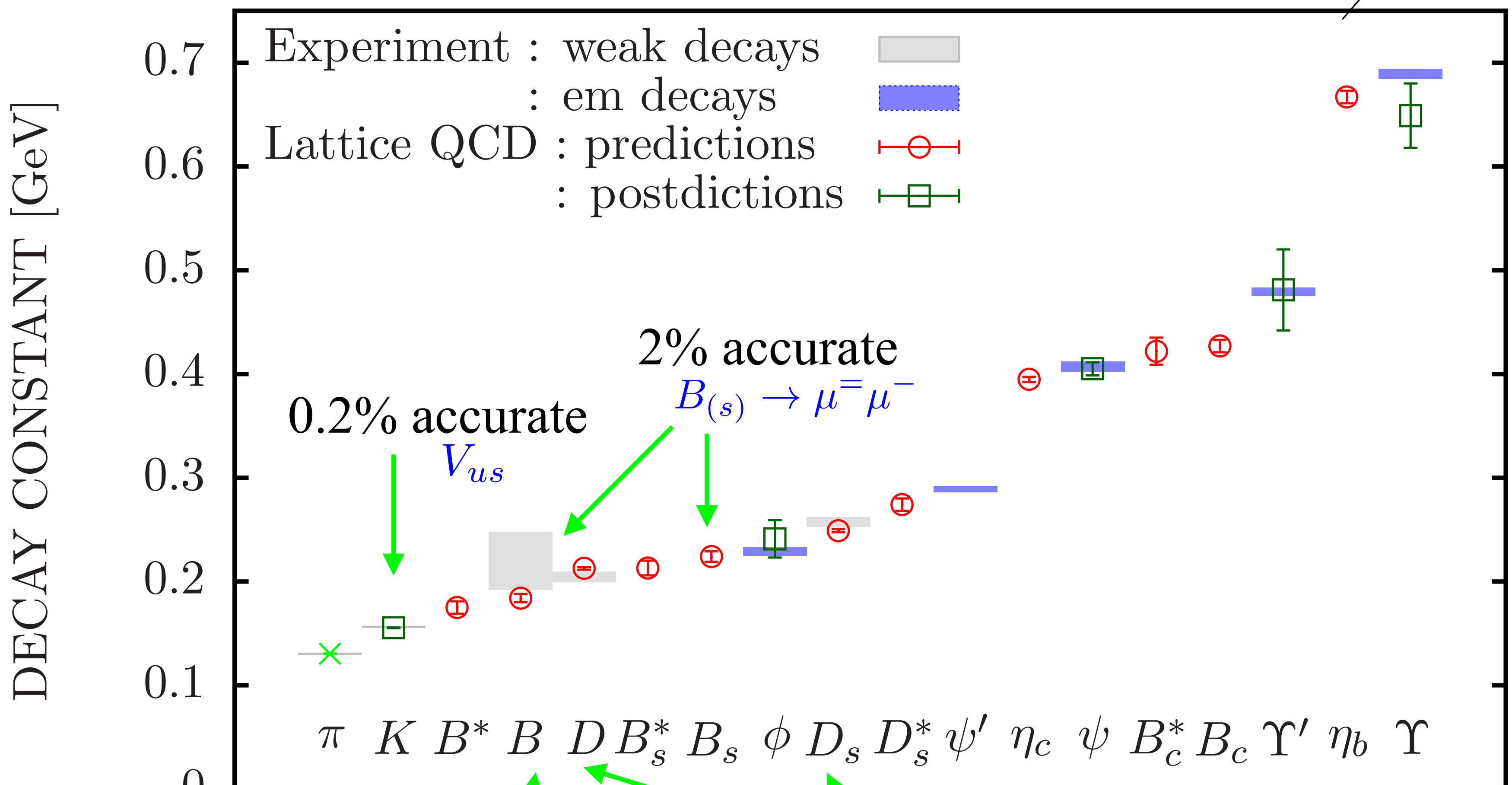
Meson decay constants - summary plot ordered by value

Parameterises hadronic information needed for annihilation rate to W or photon:

$$\Gamma \propto f^2$$



1503.05762, 1408.5768,1302.2644, 1303.1670



2012

$B \to \tau\nu$

V_{ub}

decay constants of vector mesons now being pinned down

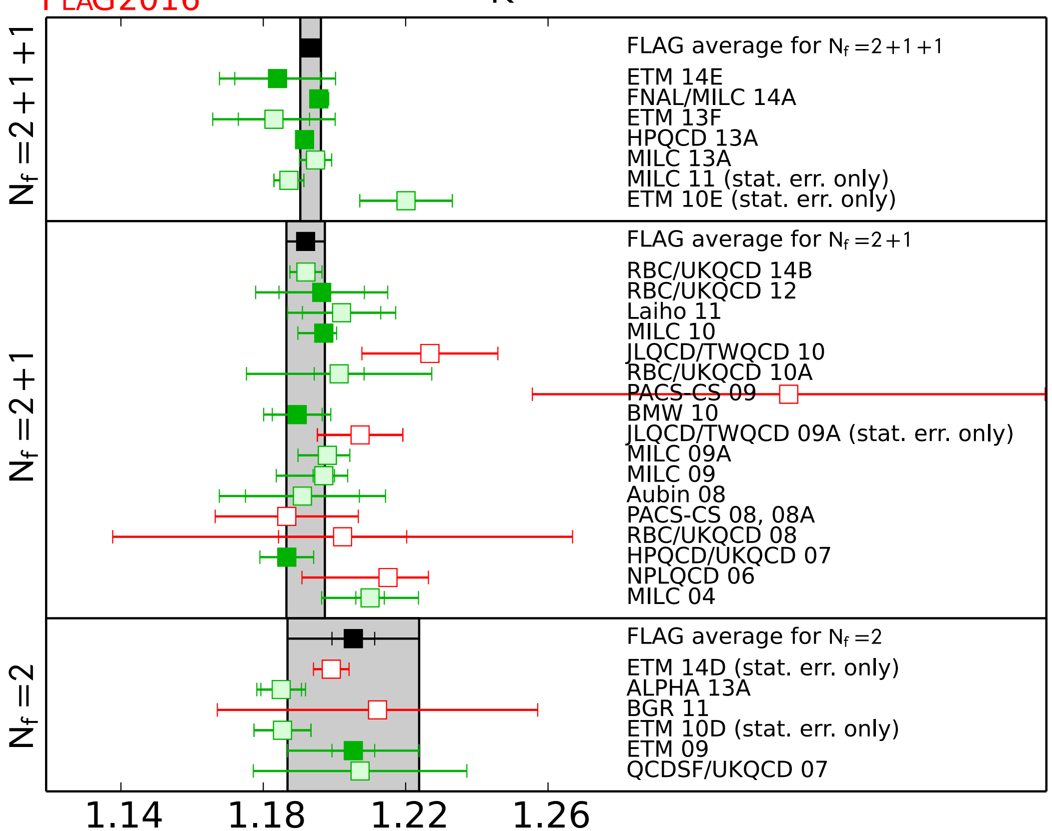
0.5% accuracy from lattice QCD now : **FNAL/MILC** 1407.3772 $V_{cd} V_{cs}$

Kaon physics

High accuracy from several groups for hadronic parameters needed to obtain V_{us} from $K \to \ell\nu, K \to \pi\ell\nu$

FLAG: 1607.00299

0.25%



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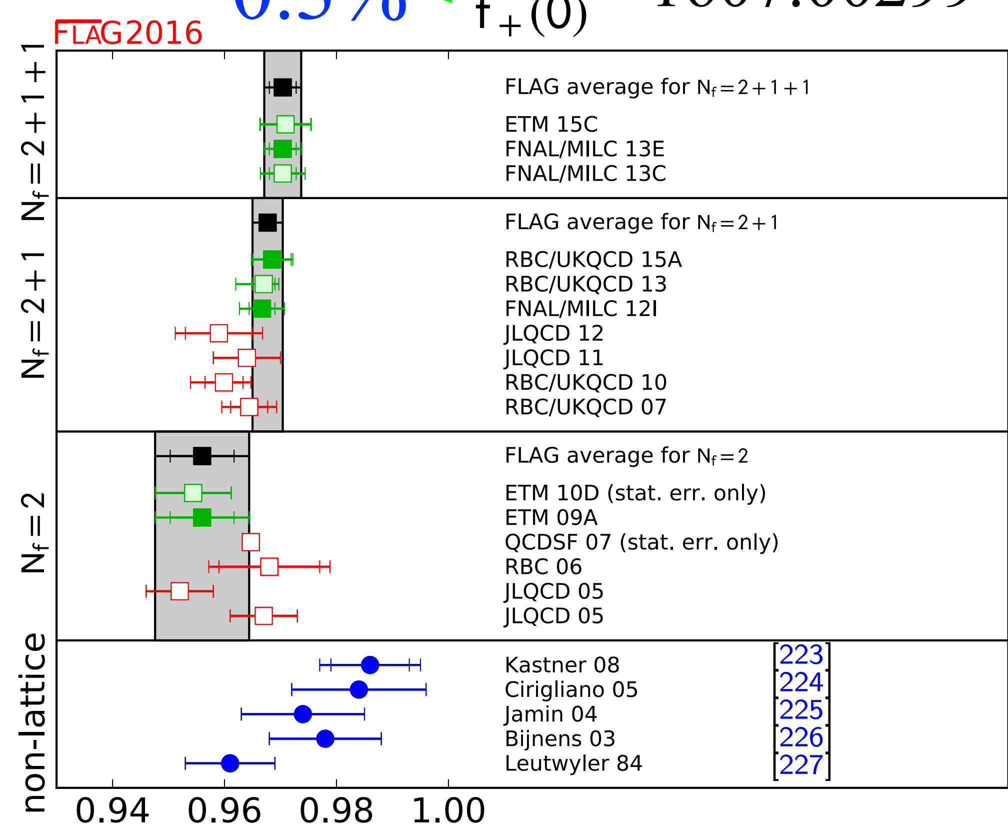


error budget example: HPQCD, 1303.1670

	f_{K^+}	f_{K^+}/f_{π^+}
statistics + *svd* cut	0.13%	0.13%
chiral extrapolation	0.03	0.03
$a^2 \to 0$ extrapolation	0.10	0.10
finite volume correction	0.01	0.01
w_0/a uncertainty	0.02	0.02
f_{π^+} experiment	0.13	0.03
m_u/m_d uncertainty	0.07	0.07
Total	0.22%	0.18%

Lattice uncertainties under good control, reduce further by including QED on lattice

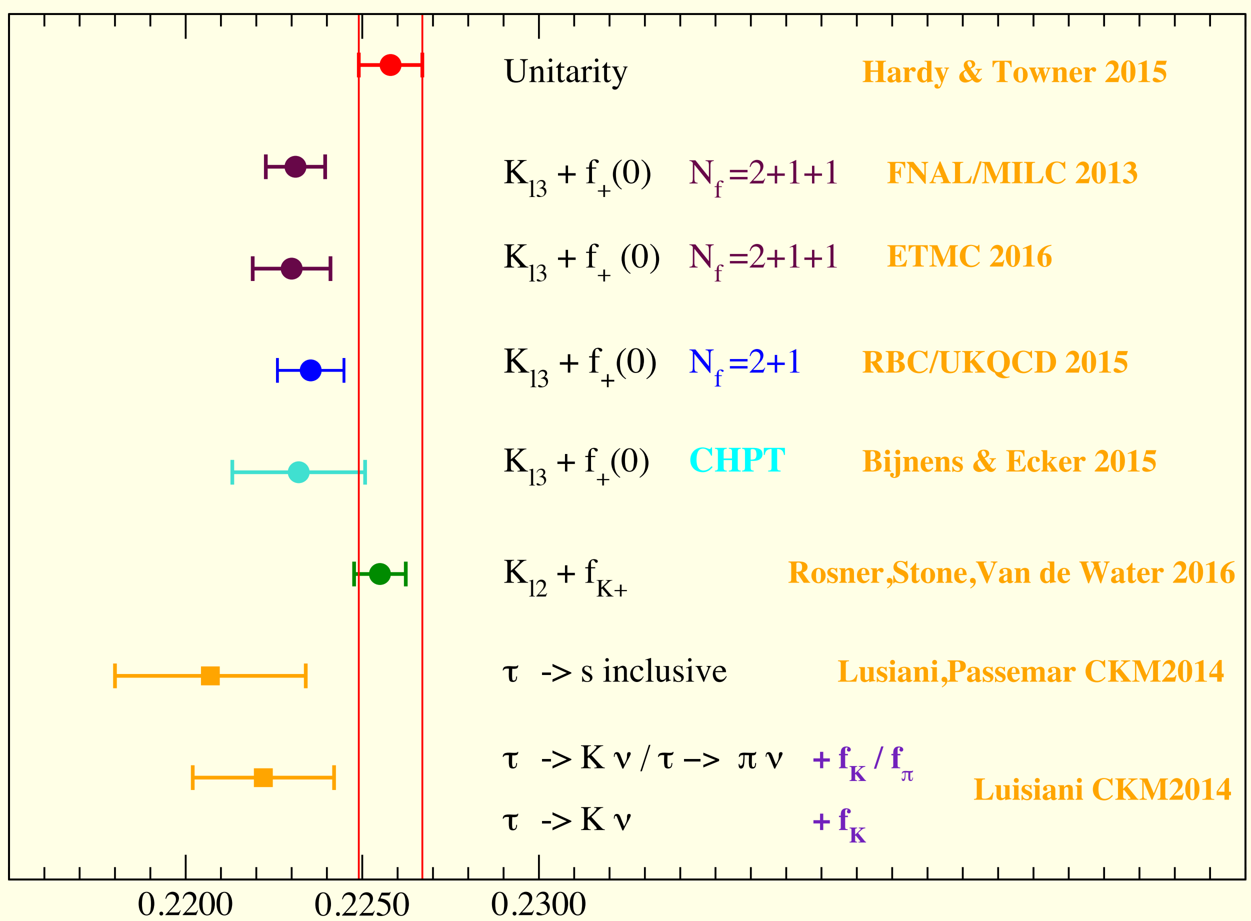
Simula: Thurs 2:30pm

Some tension (2σ) in V_{us} determinations

W coupling to $\overline{u}s$:

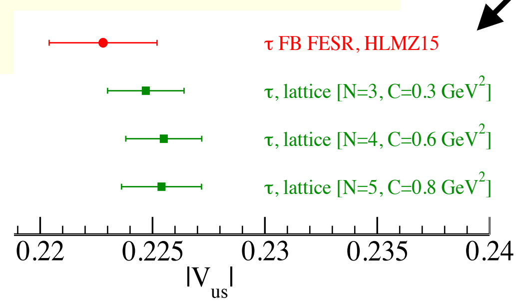
$K \to \ell\nu$ (Kl2) uses axial current,

$K \to \pi\ell\nu$ (Kl3) uses vector current



Gamiz:Lattice2016

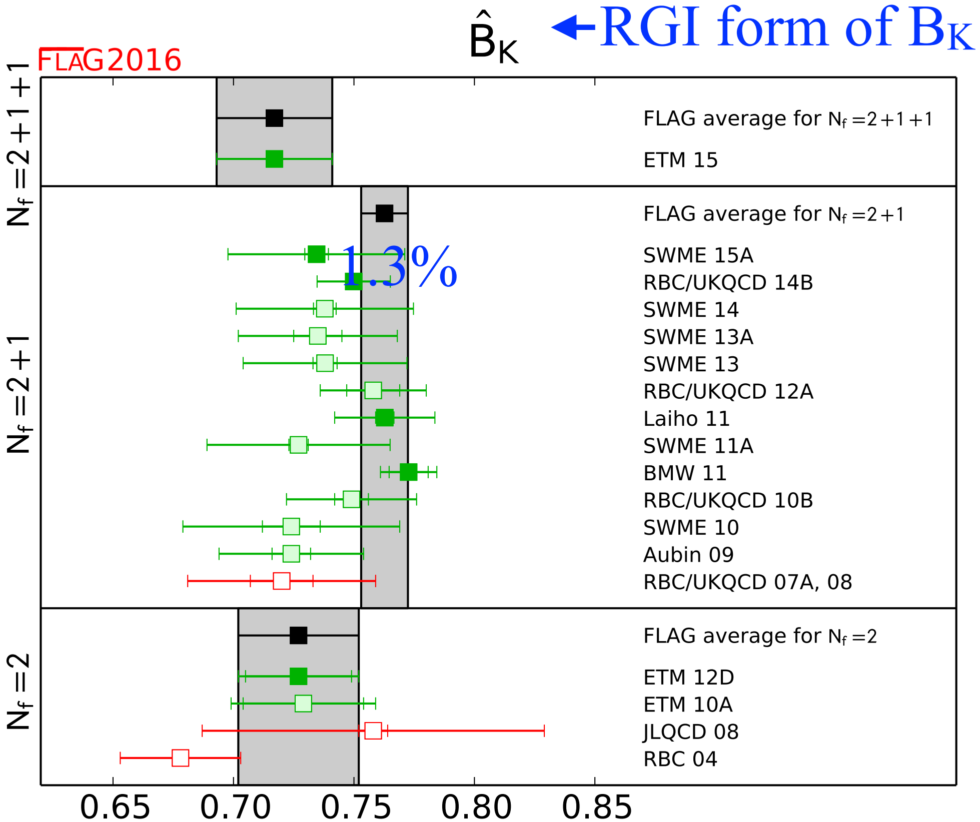
Lattice QCD input from HVP can improve result here

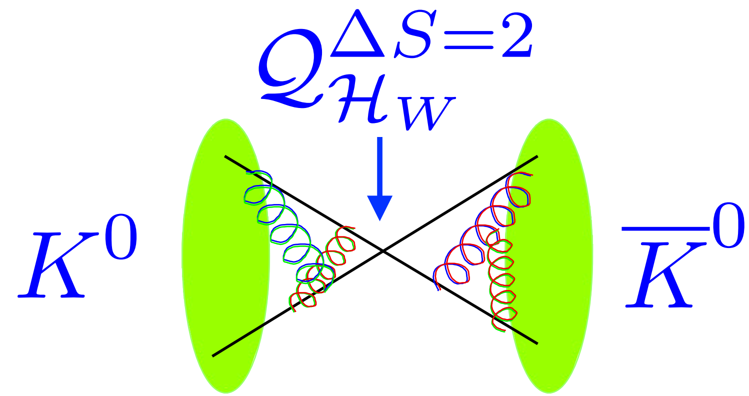


Maltman: Wed,12:30

%-level calcln of 4-quark op. matrix elements for $K^0 - \overline{K}^0$

← RGI form of B_K





SM op. is (V-A)x(V-A) and

$$B_K(\mu) = \frac{\langle \overline{K}^0 | Q | K^0 \rangle}{8 f_K^2 M_K^2 / 3}$$

CKM constraint from ε_K
but main uncty there from V_{cb}

SUSY basis

	ETM 15	SWME 15	RBC/UKQCD:1609.03334	
n_f	$2+1+1$	$2+1$	$2+1$	$2+1$
match	RI − MOM	1 − loop	RI − SMOM	RI − MOM
B_2	0.46(3)(1)	0.525(1)(23)	0.488(7)(17)	0.417(6)(2)
B_3	0.79(5)(1)	0.772(5)(35)	0.743(14)(65)	0.655(12)(44)
B_4	0.78(4)(3)	0.981(3)(61)	0.920(12)(16)	0.745(9)(28)
B_5	0.49(4)(1)	0.751(8)(68)	0.707(8)(44)	0.555(6)(53)

BSM (S/P) op. bag params.

- VSA (B=1) violated
- matching matters

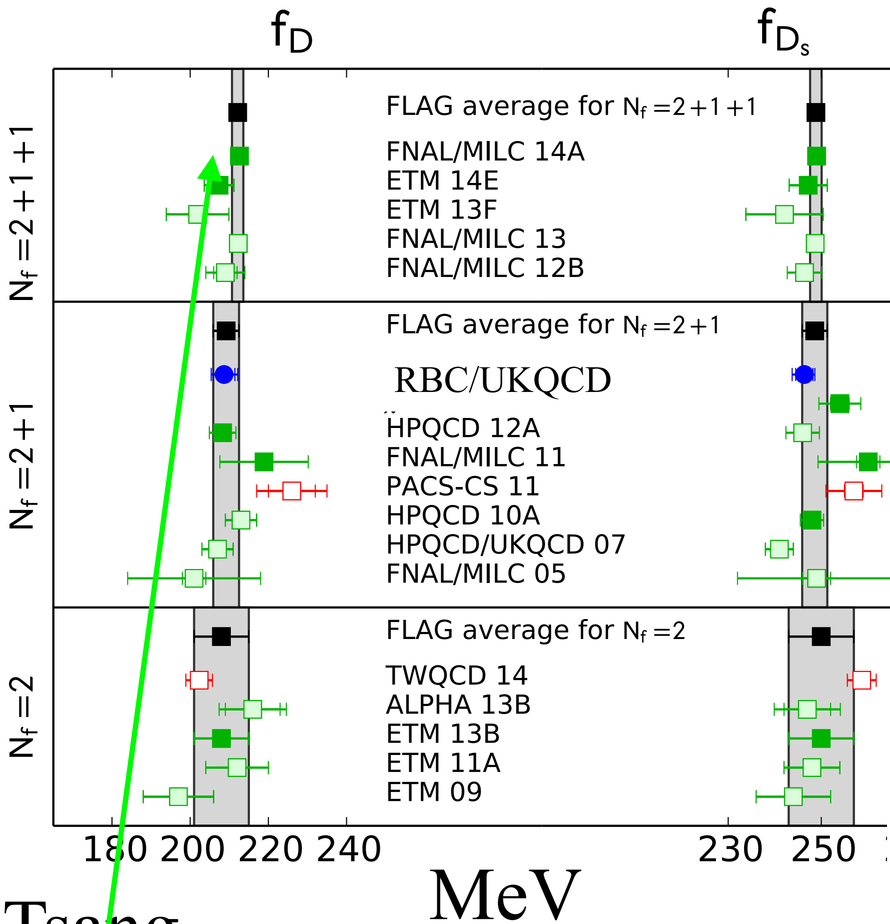
ALSO: first calc of $\text{Re}(\varepsilon'_K/\varepsilon_K) = 1.4(6.9) \times 10^{-4}$

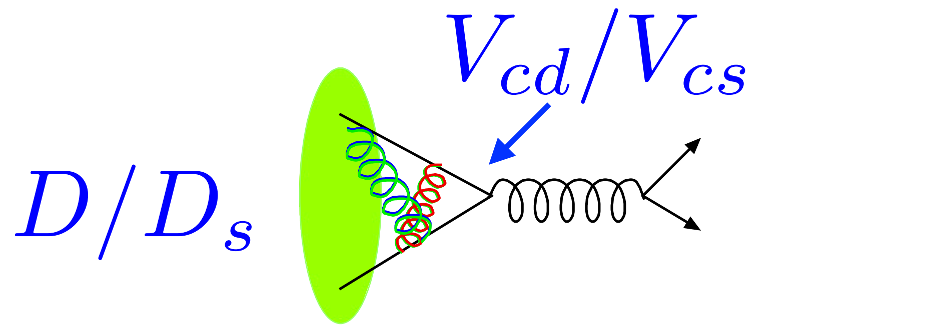
RBC/UKQCD: 1505.07863

Charm physics

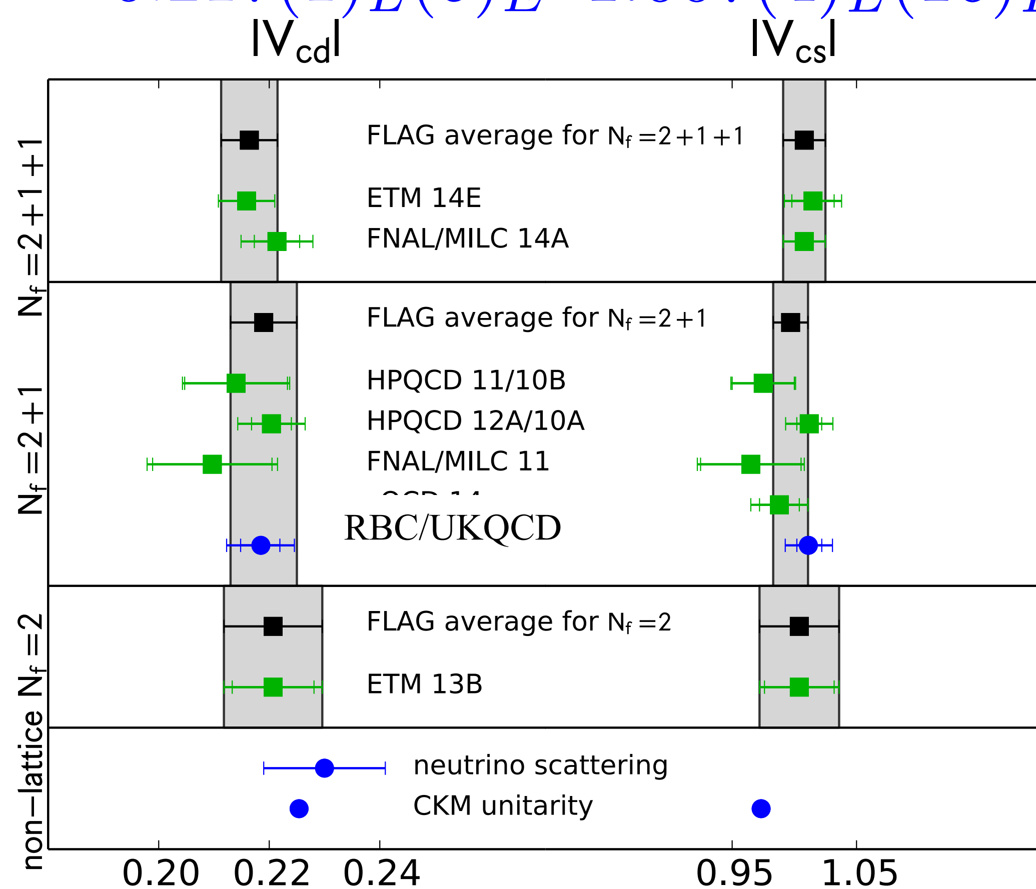
D/D_s decay constants

Tsang, Tues 12:00





$0.217(1)_L(5)_E$ $1.007(4)_L(16)_E$



Tsang

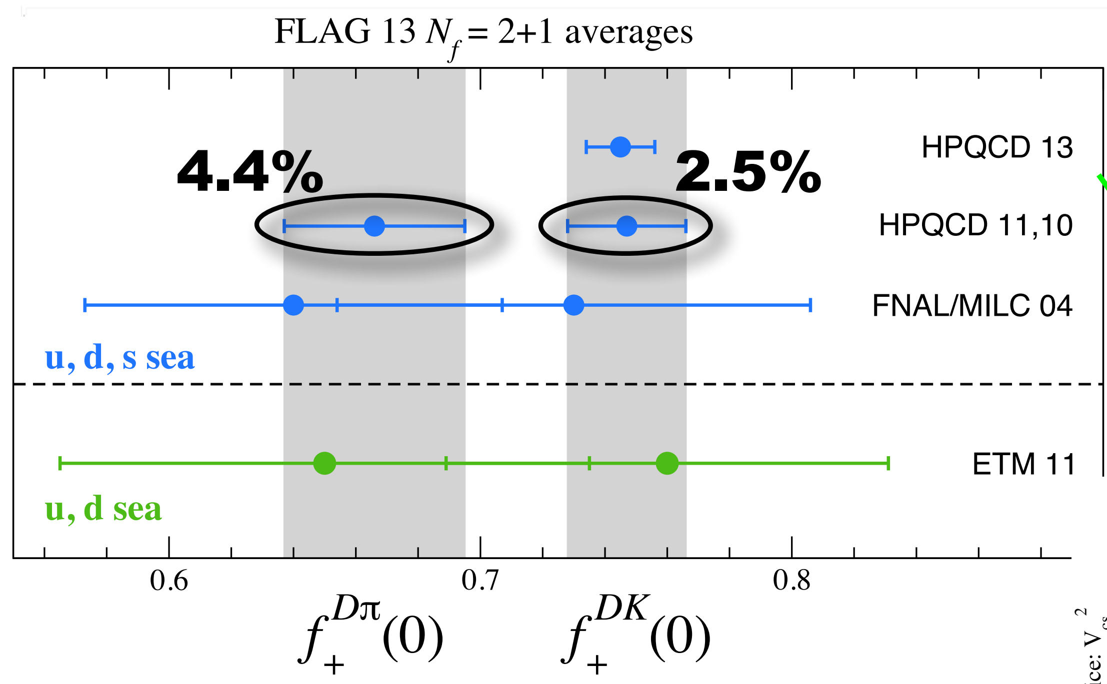
0.5% lattice QCD uncty possible with 2nd generation HISQ calcn (Fermilab/MILC1407.3772)

Limitation on CKM uncty is from exptl branching fraction

D semileptonic decay form factors   Gamiz, Tues 4:10pm

$D \to \pi \quad D \to K$ see also HPQCD, 1311.6987 $D_s \to \phi$

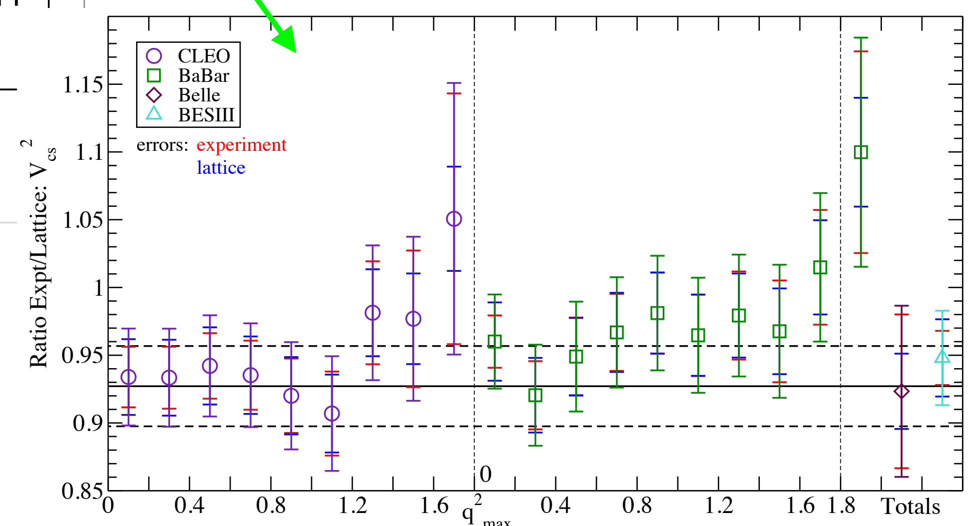
Exptl and lattice results often quoted at q^2=0 for CKM determination - however the whole q^2 range is available !



van der Water Beauty2016

FNAL/MILC (q^2=0, HISQ), HPQCD (full q^2, HISQ) and ETM (TM, full q^2) updating calculations currently

See also D 4-q matrix elements, ETM, 1505.06639, FNAL/MILC, Kronfeld, Lattice 2016.



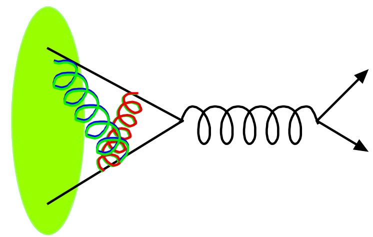
Koponen, HPQCD, 1305.1462

b physics

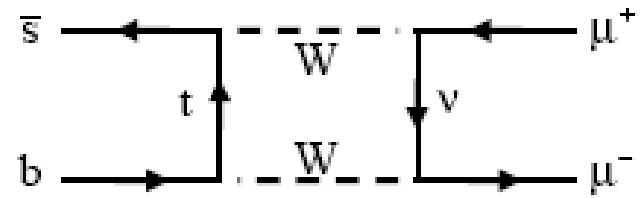
Large b-quark mass ($am_b > 1$ except on very finest lattices) mean non-relativistic effective theory approaches are still important, esp. when b coupled to u/d. Matching the non relativistic current to relativistic continuum QCD in powers of v and α_s then the key issue.

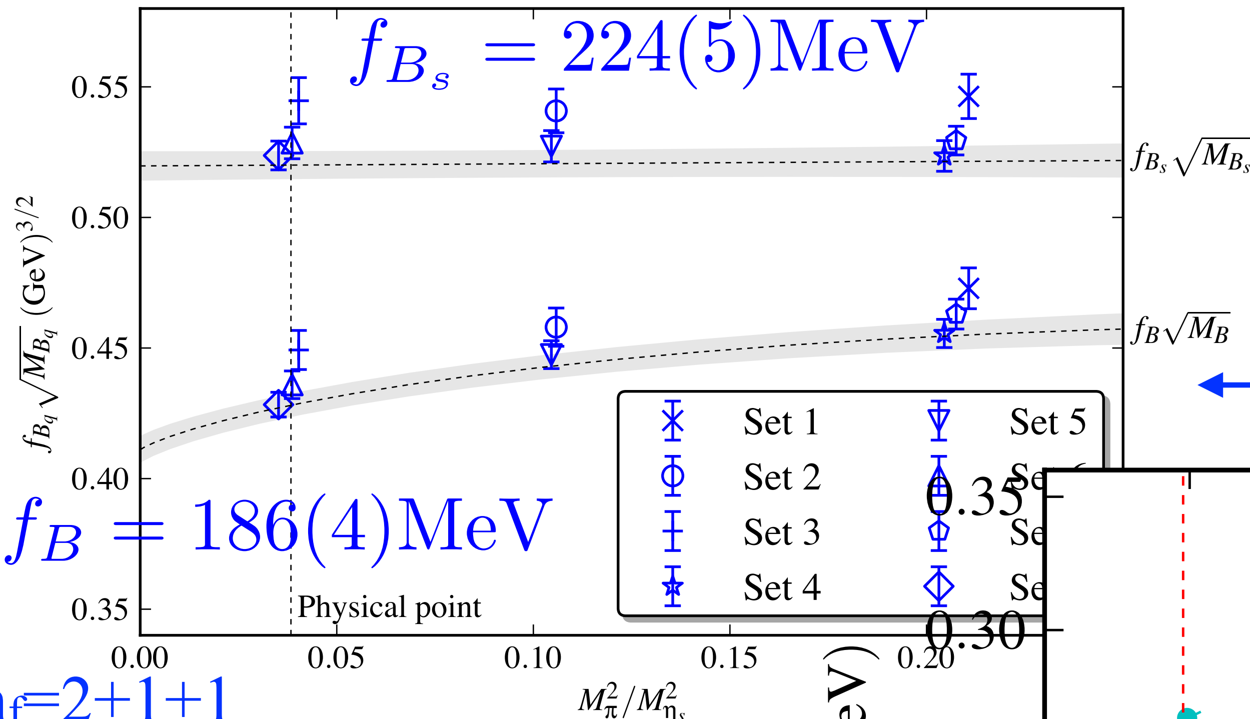
B/Bs decay constants

$B \to \tau\nu$



$B_{s/d} \to \mu^+\mu^-$

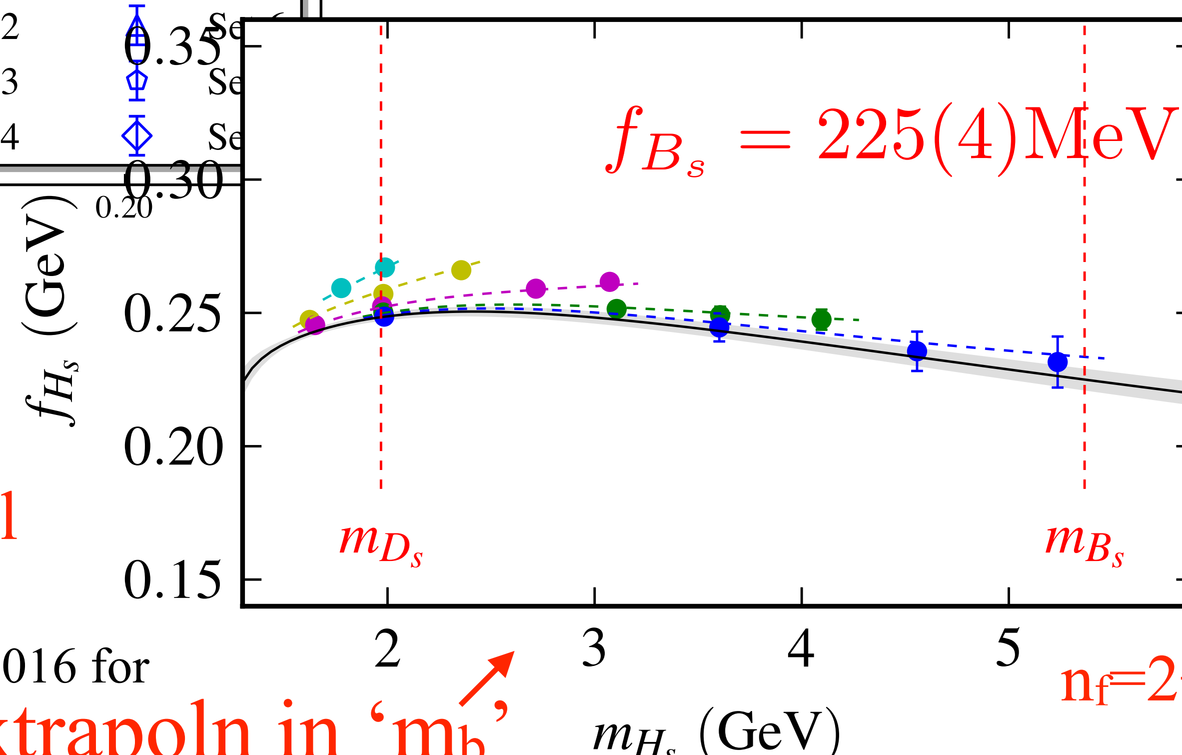




n_f=2+1+1

HPQCD, 1302.2644, NRQCD b and HISQ light. Perturbative current normln.

← extrapoln in $m_{u/d}$



HPQCD, 1110.4510 using HISQ quarks and absolutely normalised axial current.

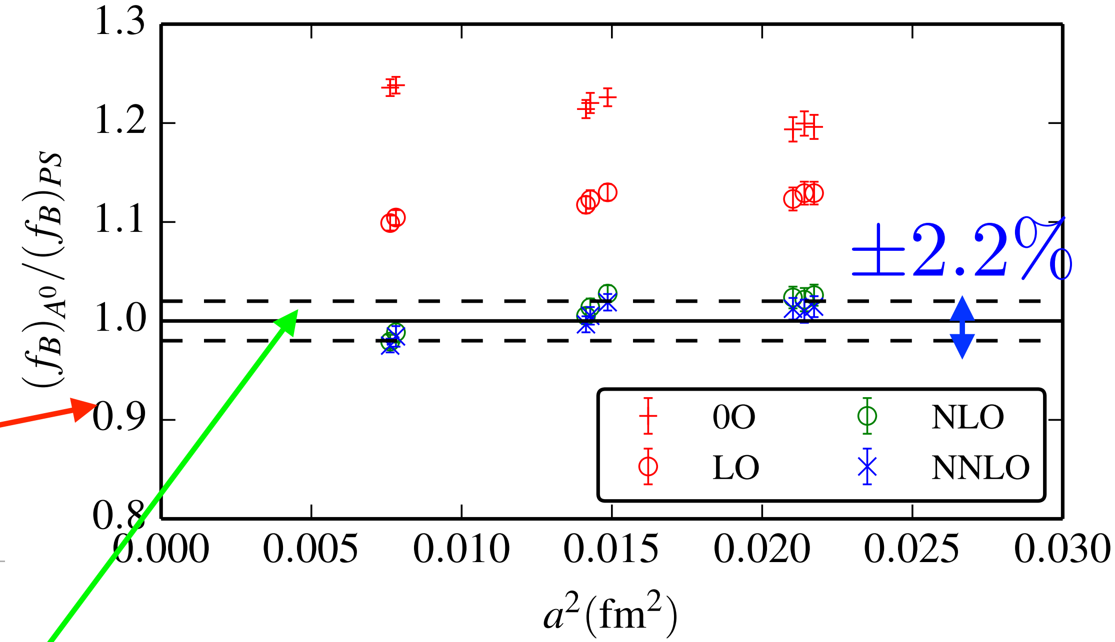
see Komijani, FNAL/MILC,Lattice2016 for update.

extrapoln in 'm$_b$'

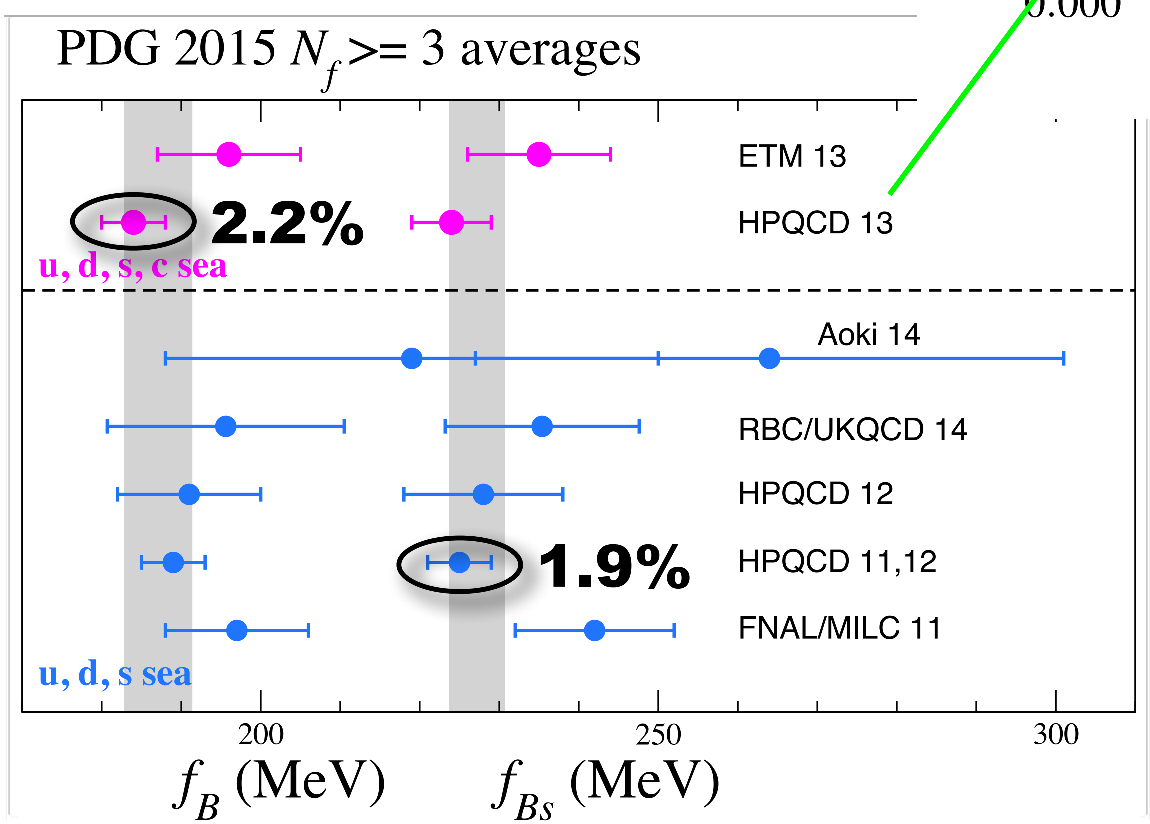
n_f=2+1

Different approaches give good agreement but accuracy worse than K/D

HPQCD, C. Hughes, CD, in prep



new - test accuracy of NRQCD by comparing results from pseudo scalar and temporal axial currents



van der Water Beauty2016

twisted mass
NRQCD

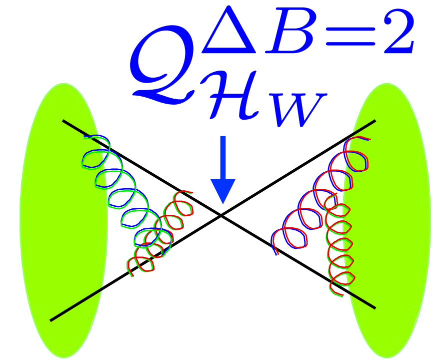
RHQ
NRQCD
HISQ
Fermilab clover

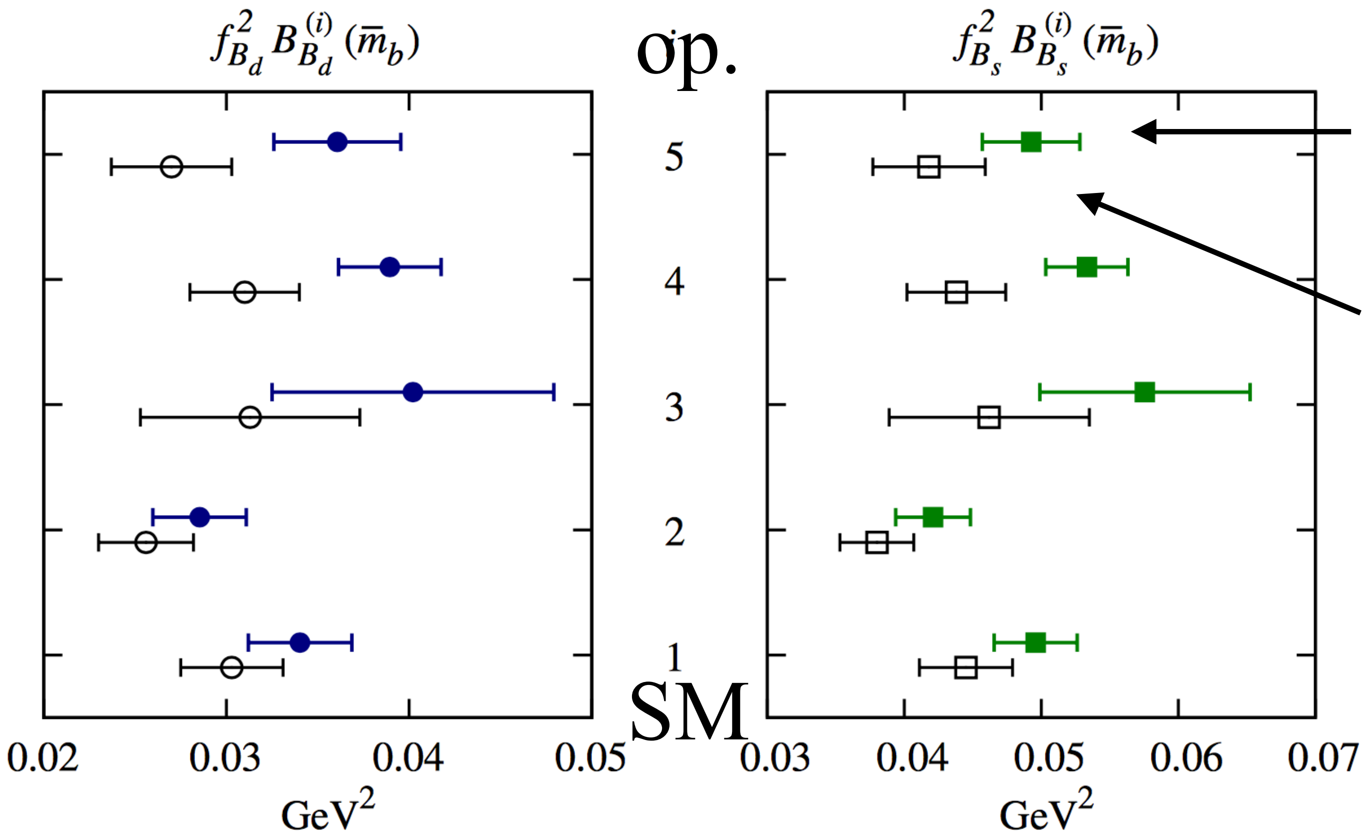
b quark formalism

4-quark op. matrix elements for SM and BSM

Gamiz: Wed. 2:40pm

$B^0_{s,d} - \overline{B}^0_{s,d}$





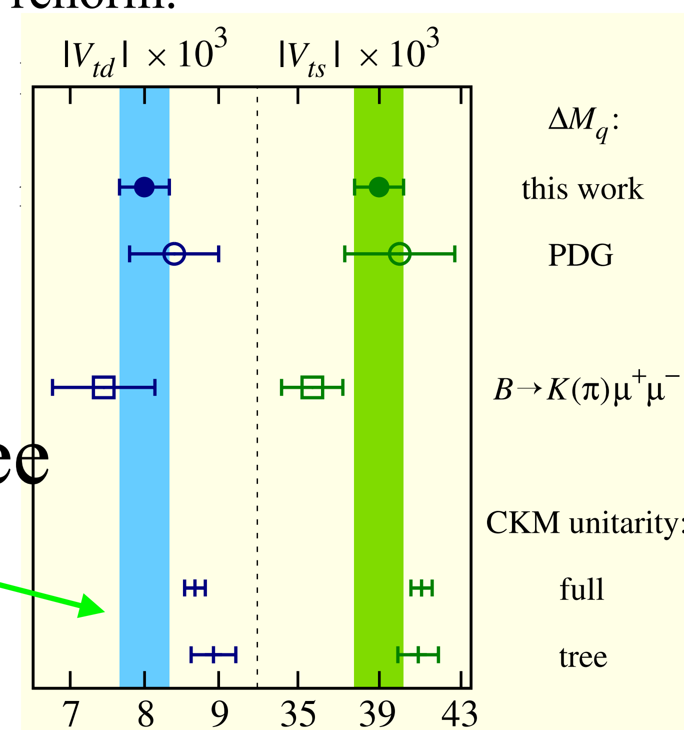
FNAL/MILC:n_f=2+1, 1602.03560, Fermilab b, pert. renorm.

ETM:n_f=2, 1308.1851, TM heavy, extrap to b, RI-MOM renorm.

HPQCD:1411.6989 in progress, n_f=2+1+1, NRQCD/HISQ physical $m_{u/d}$. Direct calc. of bag params reduces unity.

+ Wingate, Lattice2016, ops for $\Delta\Gamma_s$

2σ tension w. tree input unitarity

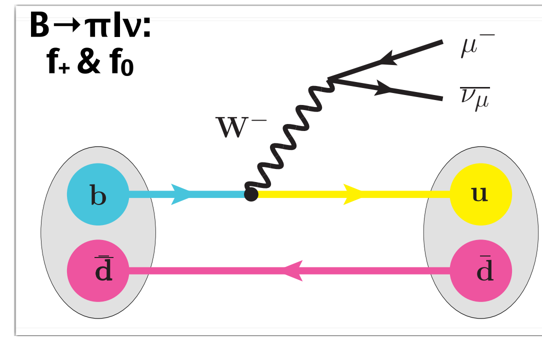


Semileptonic decay processes

T. Kawanai, V_{ub},Mon. 4:20pm
M. Wingate, V_{cb}, Thus. 9:00am
R. van der Water, Beauty2016

1) b to light

• SM tree-level decays, calc. form factor (f_+) needed to extract V_{ub} from exclusive exptl rate



$B \to \pi \ell \nu$ FNAL/MILC:1503.07839; RBC/UKQCD:1501.05373

$B_s \to K \ell \nu$ HPQCD:1406.2279; RBC/UKQCD:1501.05373

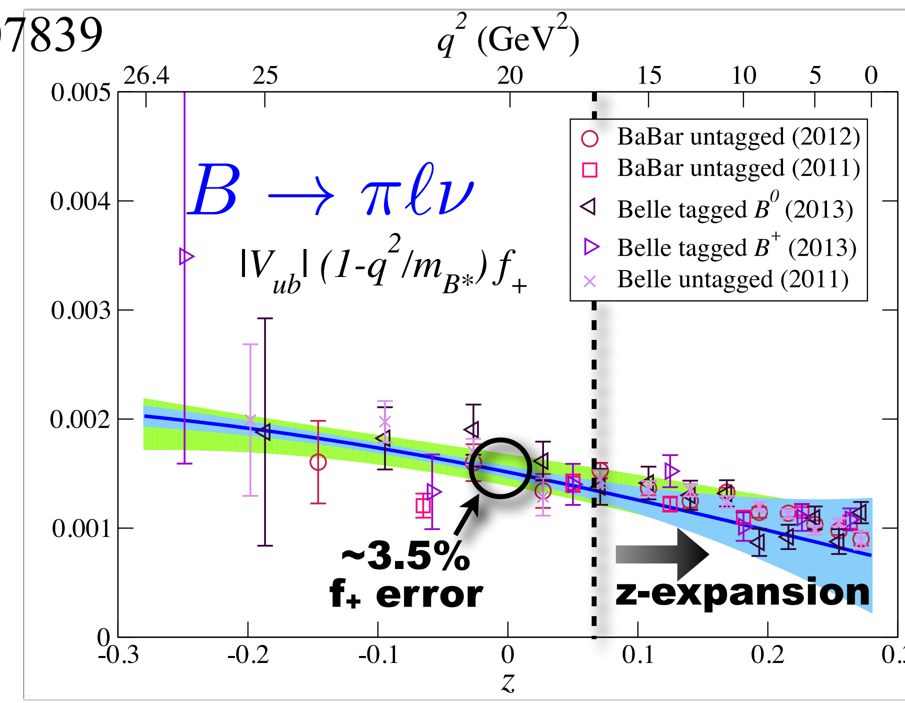
$\Lambda_b \to p \ell \nu$ Detmold, Lehner, Meinel:1503.07839

baryon form factors

Extract form factors at high q^2, close to zero recoil, work with z-expansion in comparison to expt.

$$V_{ub} = 3.72(16) \times 10^{-3}$$

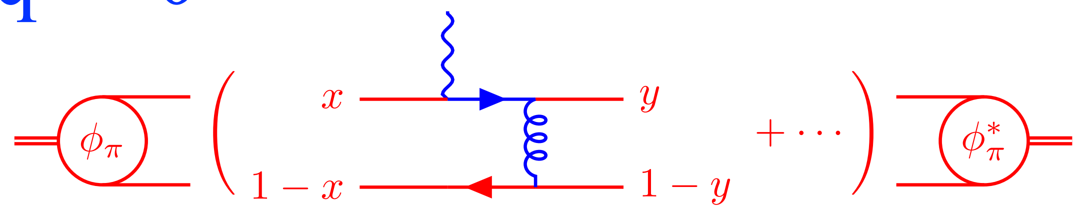
2-3σ lower than inclusive value.

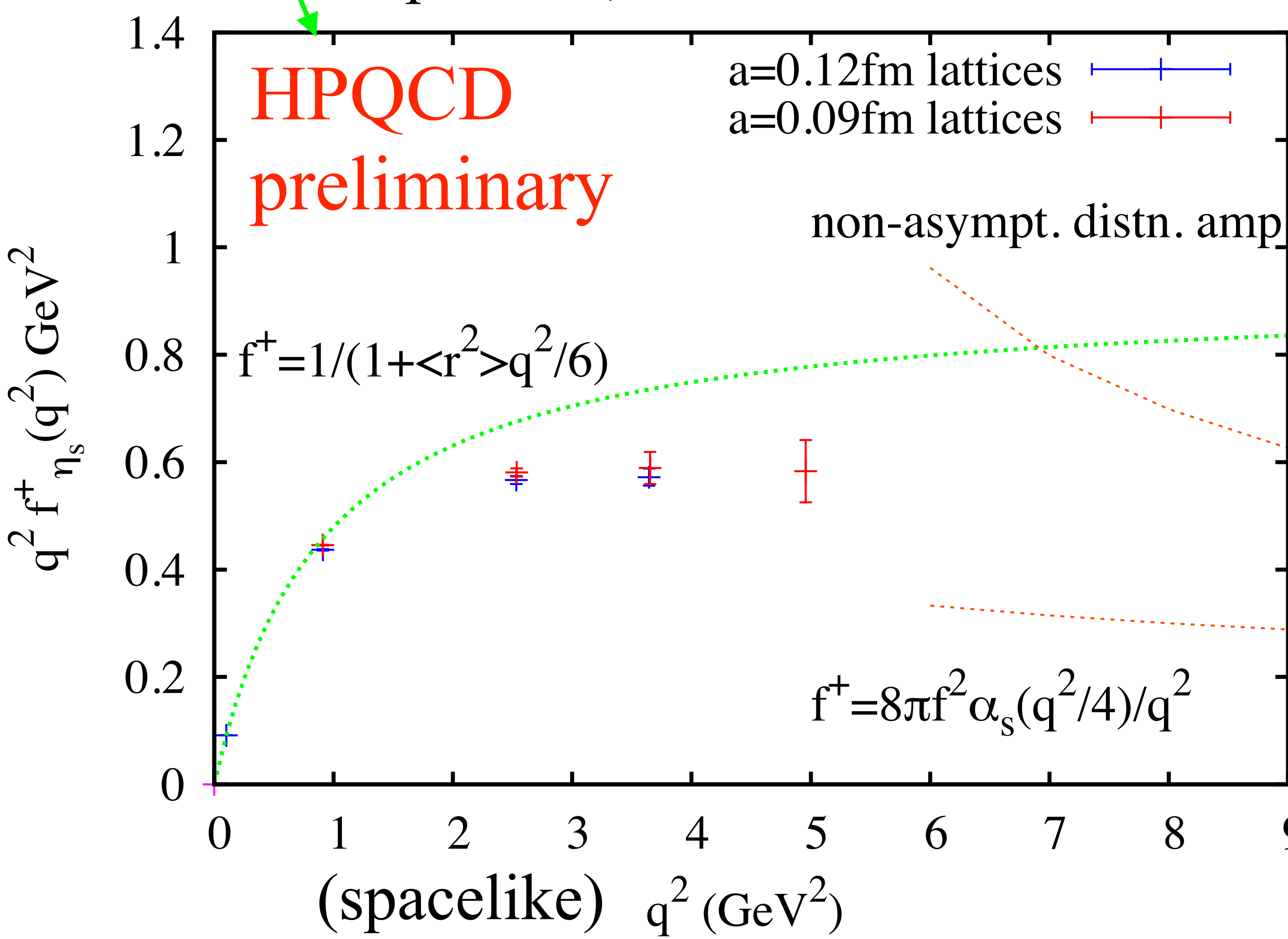


FNAL/MILC:1503.07839

Lattice QCD impact on light-cone-sum-rule calcns of $B \to \pi \ell \nu$ form factor at $q^2 = 0$

Lattice QCD gives meson vector ($s\bar{s}$) form factor at high q^2. Pert. QCD gives in terms of distribution amplitudes, used for $B \to \pi \ell \nu$





Future: can reach 8-10 GeV^2 with finer lattices. JLAB expt will reach 6 GeV^2 for π in 2018-19

HPQCD, J. Koponen, A. Santos et al, in prep. see also: moments of DA calc: Braun et al, 1503.03656

• Flavour-changing neutral currents: f_+, f_0, f_T form factors parameterise SM, BSM processes.

$B \to \pi \ell \bar{\ell}$

$B \to K^{(*)} \ell \bar{\ell}$

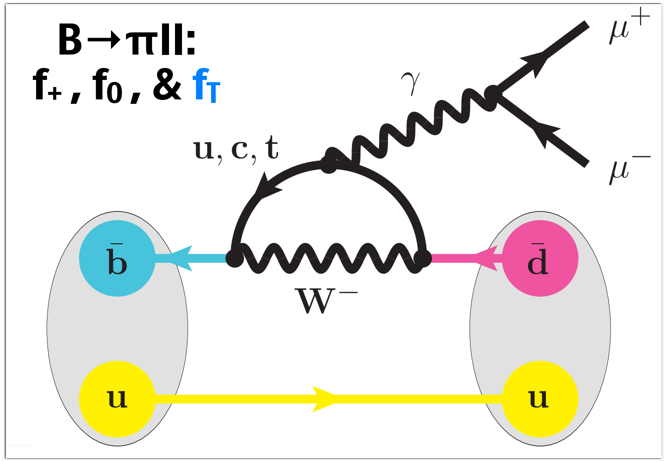
$B_s \to \phi \ell \bar{\ell}$

$\Lambda_b \to \Lambda \ell \bar{\ell}$

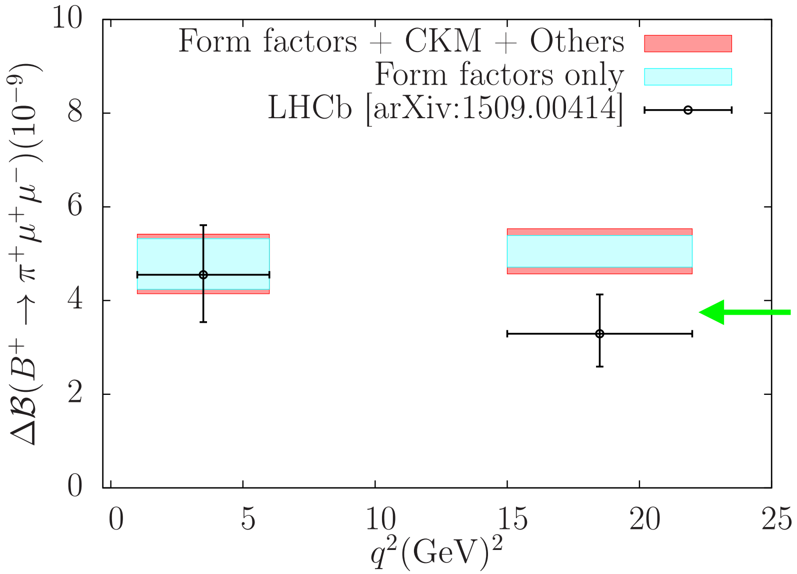
FNAL/MILC:1510.02349
HPQCD:1306.0434

HPQCD:1310.3887

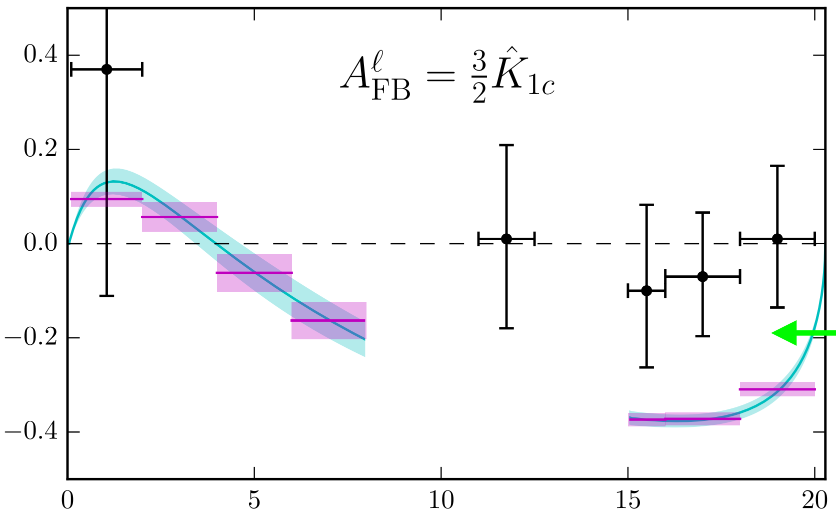
Detmold, Lehner, Meinel:1602.01399



constraints on new physics …



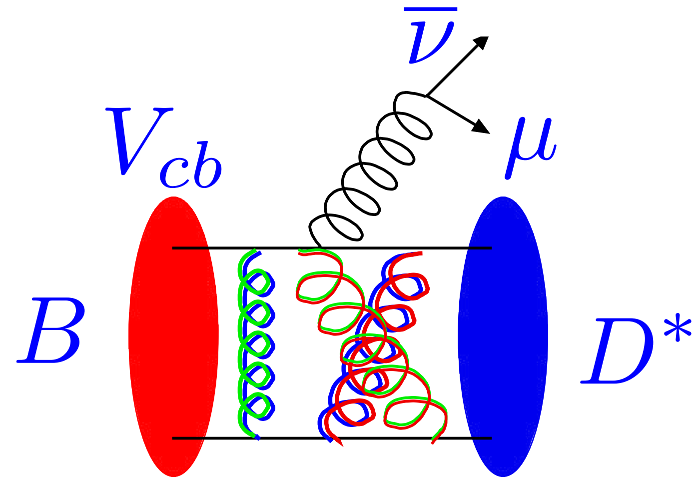
2σ tension



3σ tension

1) b to charm

SM tree-level decays, calc. form factor (f_+) needed to extract V_{cb} from exclusive exptl rate



$B \to D\ell\nu$

near zero recoil

FNAL/MILC: 1503.07237
HPQCD:1505.03925
RBC/UKQCD: Witzel Lattice2016

$\Lambda_b \to \Lambda_c\ell\nu$

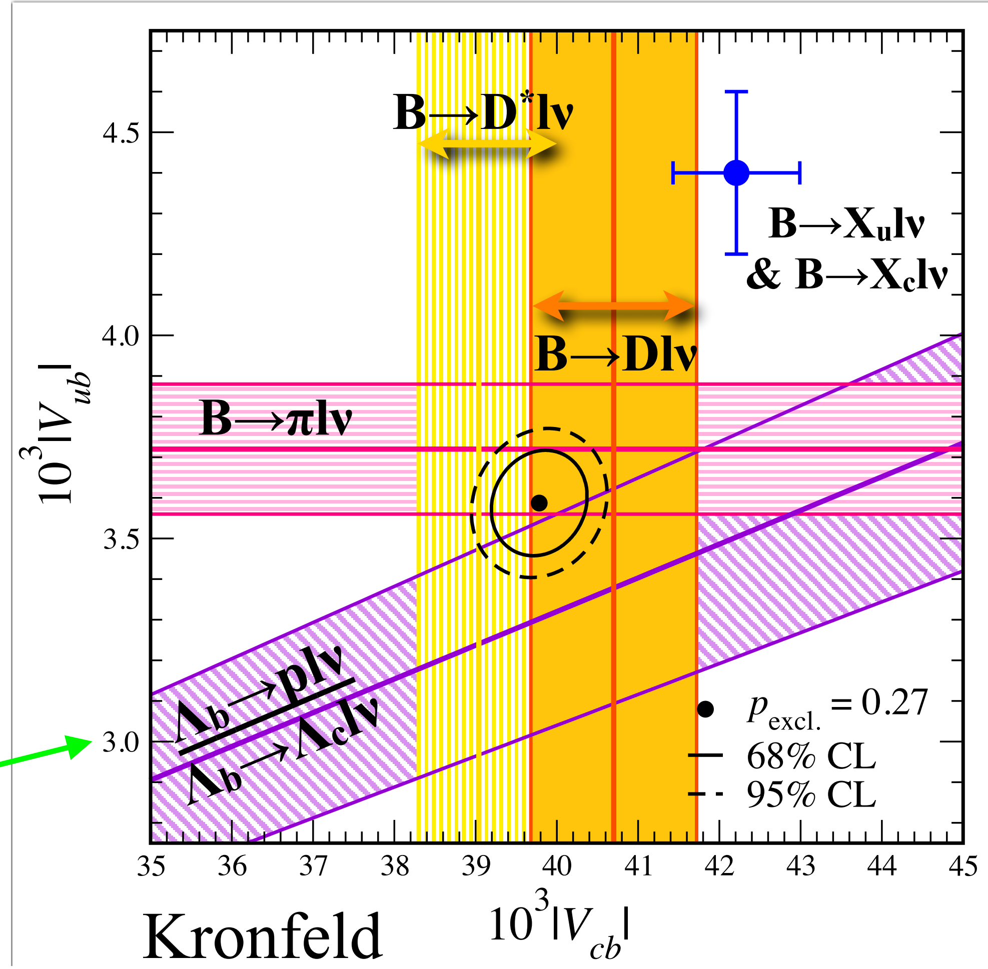
Detmold, Lehner, Meinel: 1503.01421

$B \to D^*\ell\nu$

at zero recoil

FNAL/MILC:1403.0635
HPQCD: Harrison, Lattice2016

Adds to inclusive/exclusive tension for CKM elements

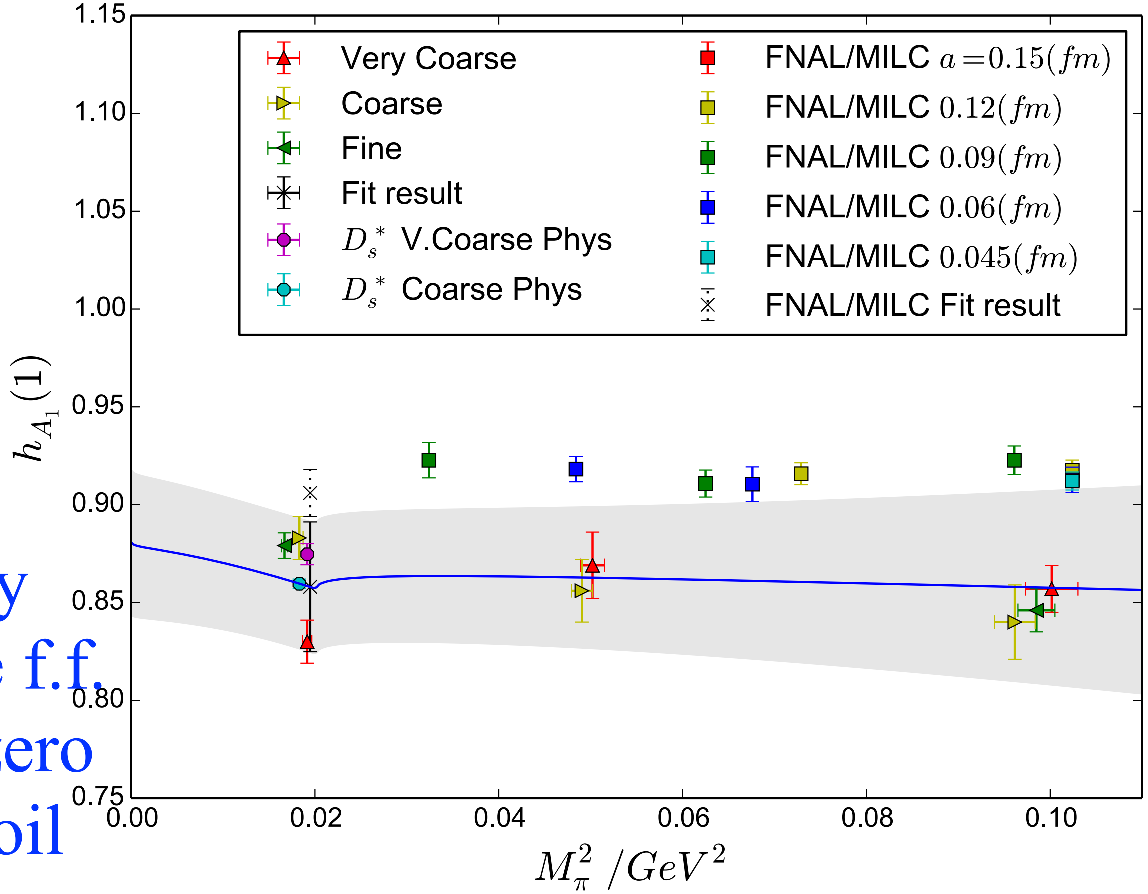


New results for $B \to D^* \ell \nu$ at zero recoil

HPQCD, Harrison et al, Lattice2016; Wingate, Thurs 9am

PREVIOUS: n_f=2+1, Fermilab b + c , asqtad light. Double ratios improve stats/systs. O(α_s) renormln of current.

NEW n_f=2+1+1, using NRQCD b, HISQ c , HISQ light with physical $m_{u/d}$. O(α_s) renormln of current.



only one f.f. at zero recoil

PRELIMINARY

$$V_{cb} = 41.5(1.7) \times 10^{-3}$$

consistent with both

$$V_{cb}^{FNAL/MILC} = 39.5(9) \times 10^{-3}$$

and

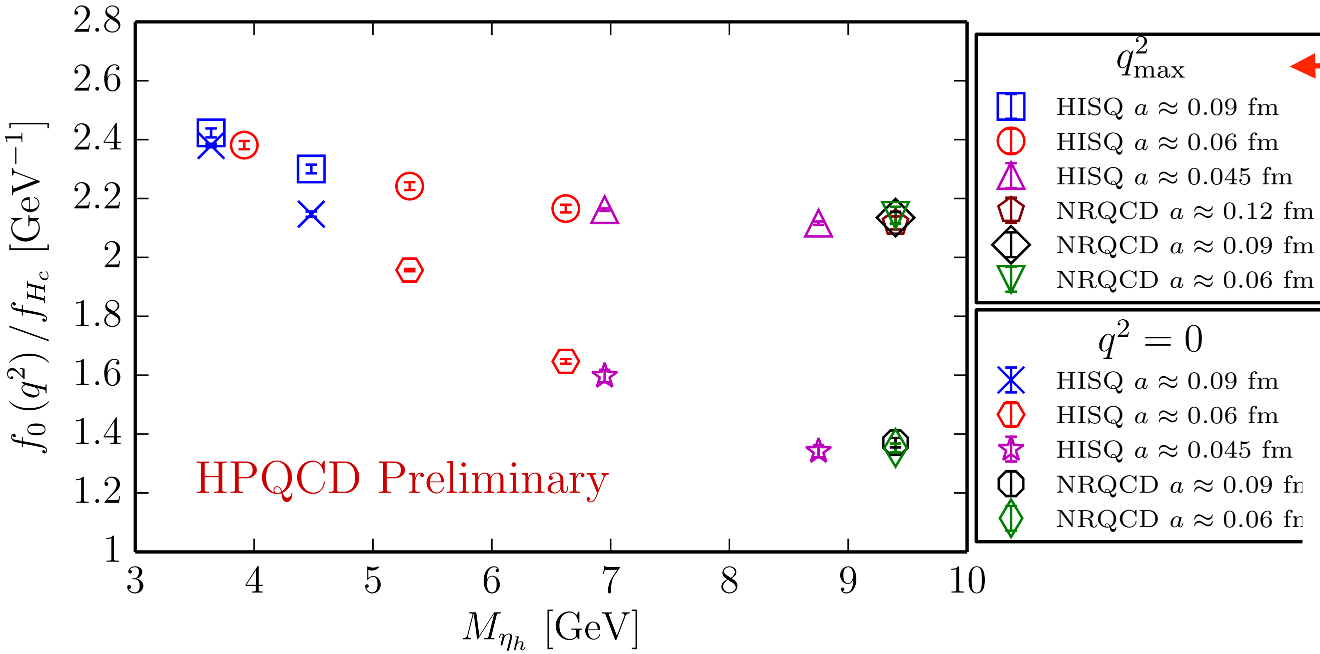
$$V_{cb}^{inclusive} = 42.2(7) \times 10^{-3}$$

need to improve renormln + work at non-zero recoil …

New methods using relativistic formalisms

A. Lytle,
Wed. 10:10

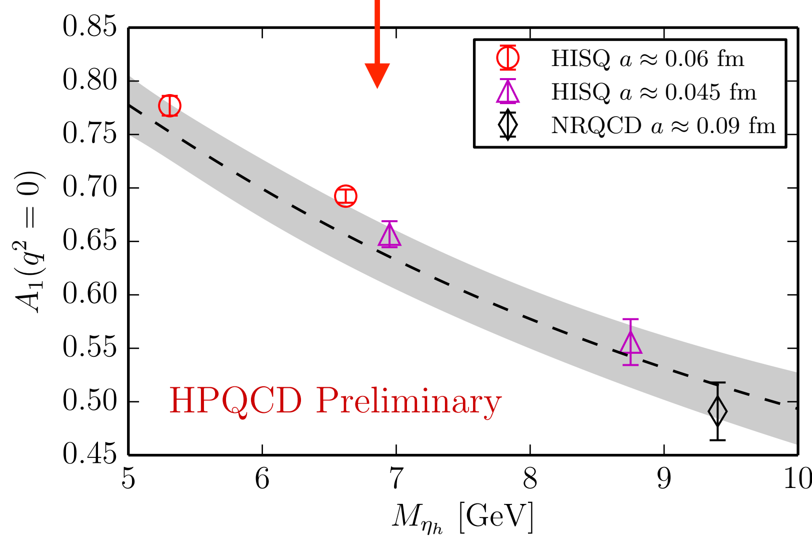
For B_c decays with HISQ b and HISQ c we can
- use currents that are absolutely normalised
- cover the fill q^2 range of the decay
- use this to normalise NRQCD-HISQ current for $B \to D^{(*)}$



$B_c \to \eta_c \ell \nu$

extrapolate in heavy quark mass to b

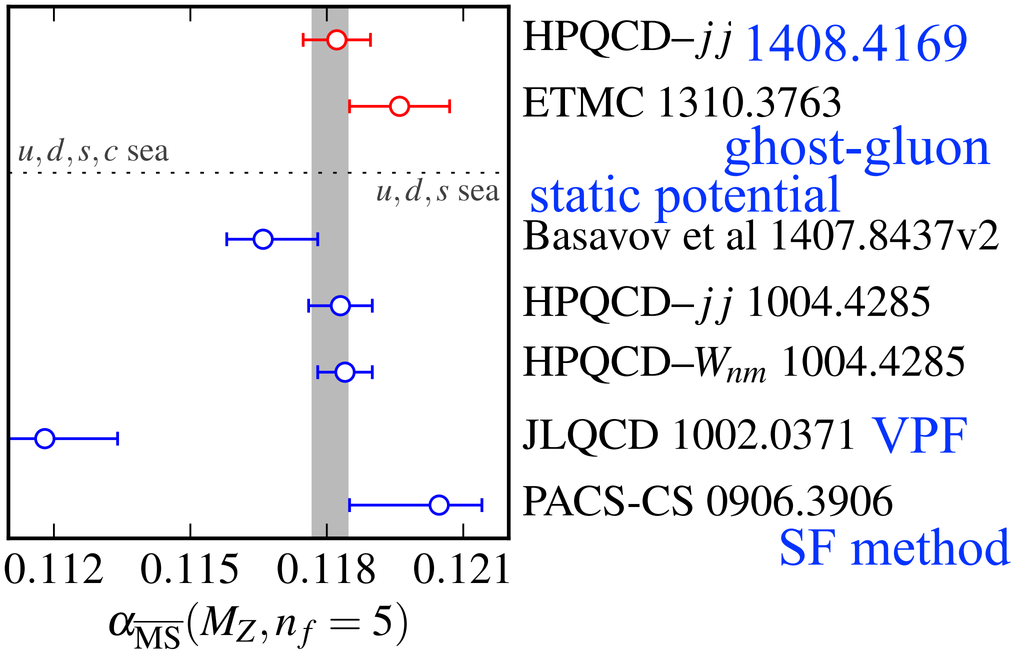
$B_c \to J/\psi \ell \nu$



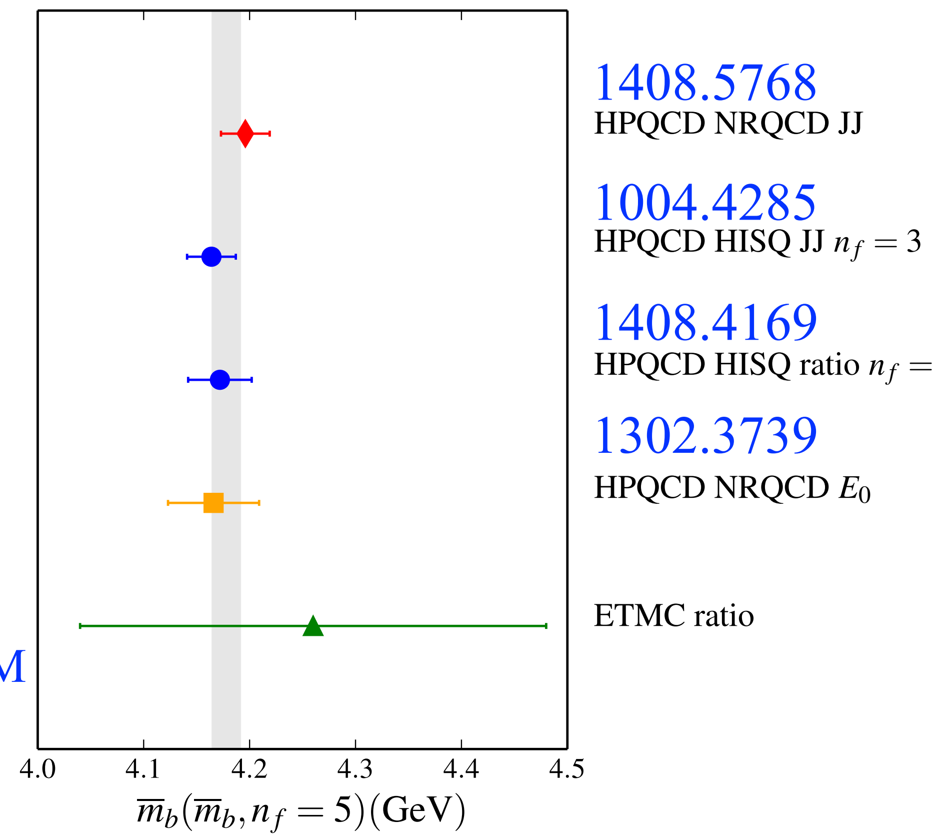
Good agreement at q^2_{max} (zero recoil) and $q^2=0$ shows no large missing renormln for NRQCD-HISQ.

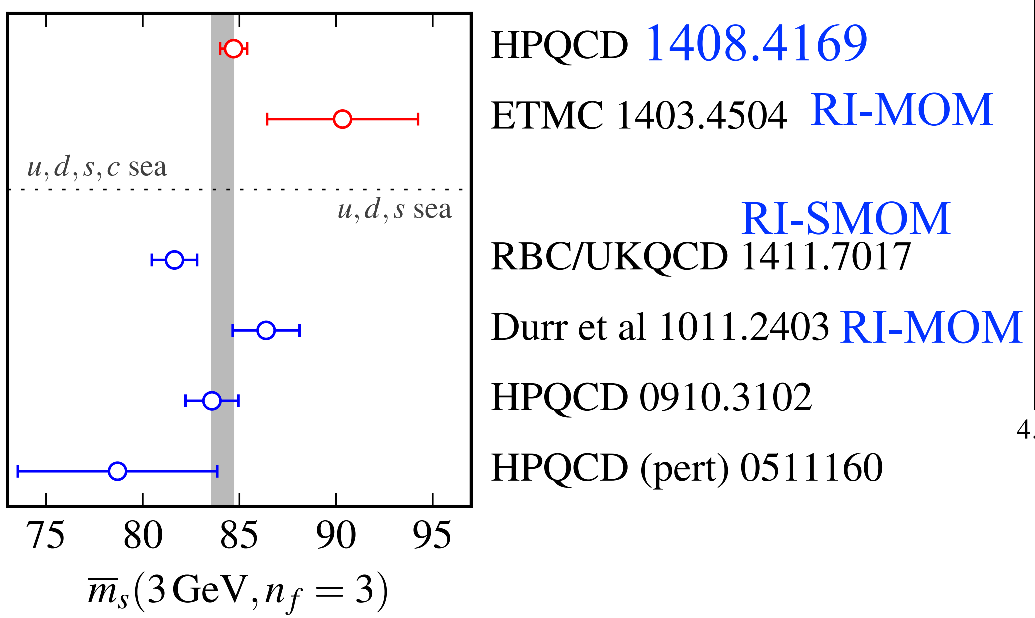
QCD parameters

Quark masses and strong coupling constant



Multiple lattice QCD methods now that agree to high accuracy

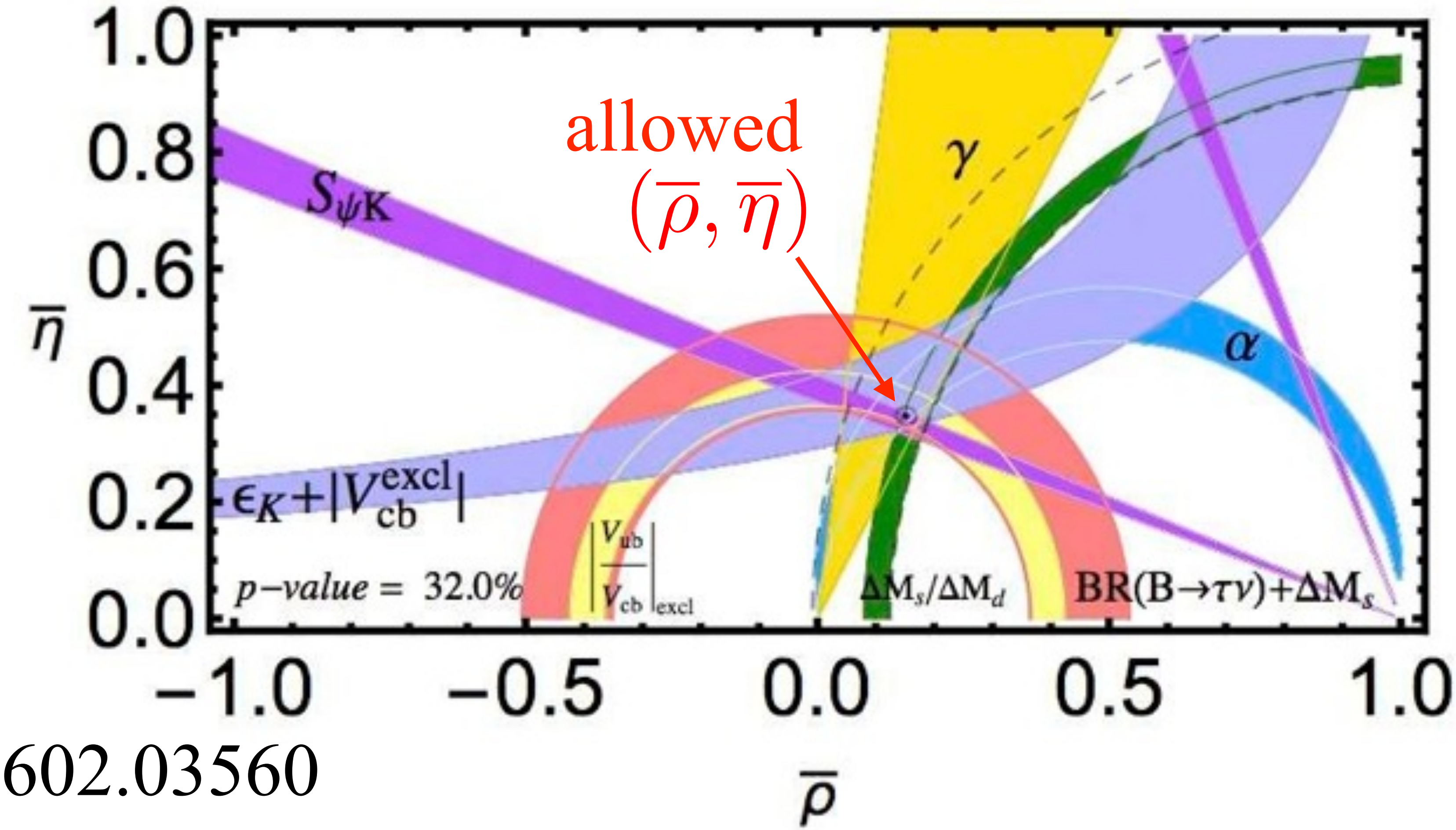




Conclusion

Lattice QCD continues to make progress - reducing errors and testing them using multiple methods.

The impact is seen in the unitarity triangle:



Lunghi:1602.03560

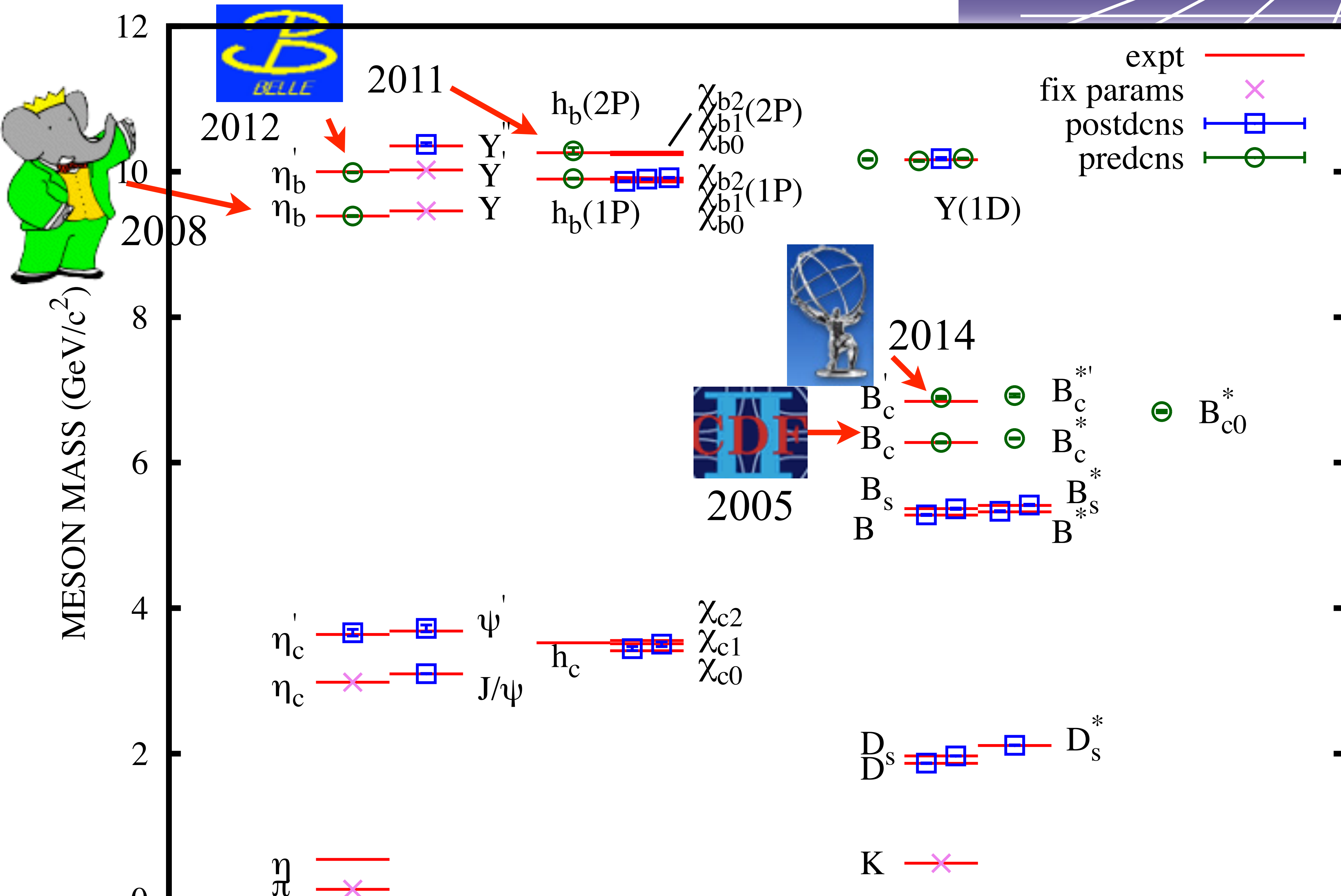
Overview of progress in Lattice QCD since 2014 - key messages

- Lattice QCD methods - now working with physical $m_{u/d}$.
- Update of kaon physics - very accurate calculations possible for hadronic quantities needed for Kl2, Kl3. QED/m_u.ne.m_d being added.
- Update of charm physics - use of relativistic actions means accuracy approaching that of kaon for decay constants. Semileptonic form factors being improved.
- Update of bottom physics - lots of work going on, and errors going down, but a lot still to do. Will move to relativistic actions eventually …
- Conclusions

Spares

The gold-plated meson spectrum

HPQCD



Uncties at few MeV level : Future: inc QED and $m_u \neq m_d$

Error budget for HISQ current-current method

TABLE IV. Error budget [31] for the c mass, QCD coupling, and the ratios of quark masses m_c/m_s and m_b/m_c from the $n_f = 4$ simulations described in this paper. Each uncertainty is given as a percentage of the final value. The different uncertainties are added in quadrature to give the total uncertainty. Only sources of uncertainty larger than 0.05% have been listed.

	$m_c(3)$	$\alpha_{\overline{\mathrm{MS}}}(M_Z)$	m_c/m_s	m_b/m_c
Perturbation theory	0.3	0.5	0.0	0.0
Statistical errors	0.2	0.2	0.3	0.3
$a^2 \to 0$	0.3	0.3	0.0	1.0
$\delta m_{uds}^{\mathrm{sea}} \to 0$	0.2	0.1	0.0	0.0
$\delta m_c^{\mathrm{sea}} \to 0$	0.3	0.1	0.0	0.0
$m_h \neq m_c$ (Eq. (15))	0.1	0.1	0.0	0.0
Uncertainty in w_0, w_0/a	0.2	0.0	0.1	0.4
α_0 prior	0.0	0.1	0.0	0.0
Uncertainty in m_{η_s}	0.0	0.0	0.4	0.0
$m_h/m_c \to m_b/m_c$	0.0	0.0	0.0	0.4
δm_{η_c}: electromag., annih.	0.1	0.0	0.1	0.1
δm_{η_b}: electromag., annih.	0.0	0.0	0.0	0.1
Total:	0.64%	0.63%	0.55%	1.20%

HPQCD, 1408.4169