QCD critical point, universality and small quark mass

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with Mikhail Stephanov, Phys.Rev. D100 (2019) no.5, 056003

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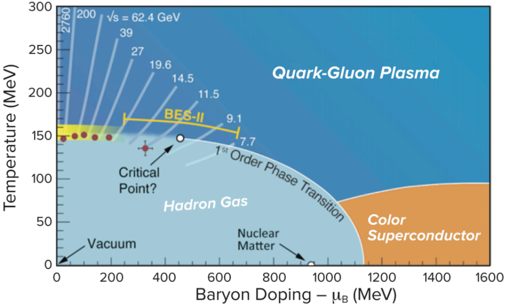
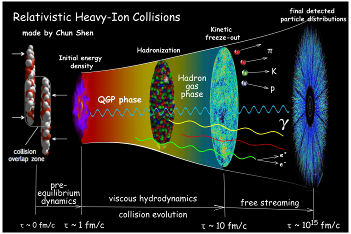
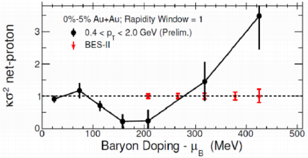
Beam energy scan program at RHIC and BEST topical collaboration





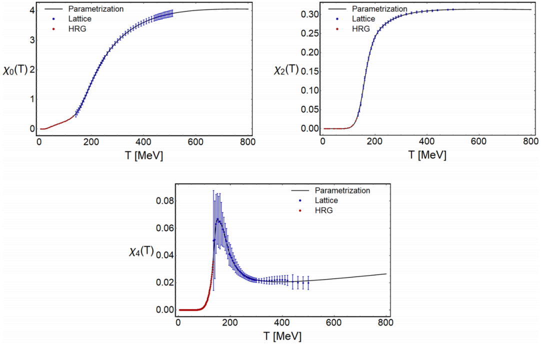
- BES program : Collide heavy-ions at varying $\sqrt{s_{NN}}$ and discover the critical point
- BEST collaboration : Model HICs and make predictions for experiment

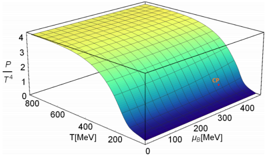


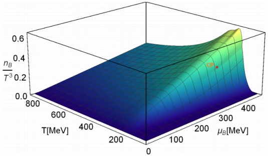
2015 LRPNS

QCD Equation of state at non-zero temperature and chemical potential

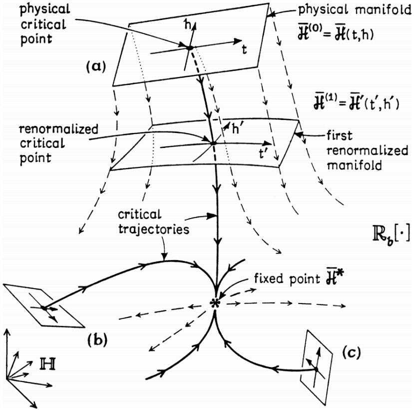
- Taylor expansion upto $O(\mu^6)$ from lattice QCD ***Hot QCD, 20 and Datta, Gavai, Gupta, 18***
- Smooth extrapolation of Lattice EoS to Hadron Resonance gas models ***Bellweid et al., 15***
- CP limits the validity of Taylor expansions
- A class of Hybrid EoSs combining the universal features near CP, Lattice EoS at low μ and Hadron Gas EoS at low T ***Parotto et al, 18***







Universality near an Ising-like critical point



Fisher, 1998

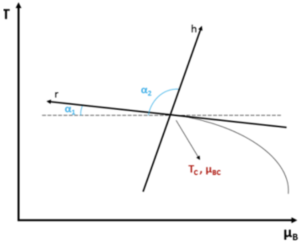
- Universal critical exponents
- Power law scaling
- Critical EoS has the same mathematical form

Static universality class of QCD: 3D Ising model

- Infrared limit is the ϕ^4 theory at the Wilson Fisher fixed point
- Universality class : Symmetry of order parameter and spatial dimensionality of the system

Equation of State near QCD CP from universality

$$P_{QCD}(\mu, T) = -A \overbrace{G(r,h)}^{\text{universal}} + \text{less singular terms}$$



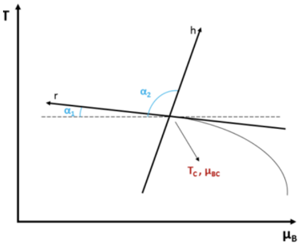
$$\boxed{G(r,h) = r^{\beta(1+\delta)} g(hr^{-\beta\delta})}$$

$$h = \frac{\sin\alpha_1 \; (\mu - \mu_c) + \cos\alpha_1 \; (T - T_c)}{w\, T_c \,\sin(\alpha_2 - \alpha_1)}$$

$$r = \frac{\sin\alpha_2 \; (\mu - \mu_c) + \cos\alpha_2 \, (T - T_c)}{\rho w T_c \sin(\alpha_1 - \alpha_2)}$$

Non-universal mapping between QCD and Ising variables ***[Parotto et.al.(2018)]***

r and h axis on the $T\mu$ plane



Non-universal mapping between QCD and Ising variables [*Parotto et.al.(2018)*]

- r axis is the cross-over line.
  - Lattice QCD: $\alpha_1 \approx 3.85^\circ$ if $\mu_c = 350$MeV
- h axis is more subtle.
  - Close to the CP, physics is symmetric under $h \to -h$
  - Need to account for the lowest order asymmetric corrections to scaling EoS

Landau-Ginsburg potential near a critical point in mean-field theory

$$P(\mu, T) = -\min_{\phi} \Omega(\phi; \mu, T)$$

$$\boldsymbol{\Omega(\phi; \mu, T) = \Omega_0(\mu, T) - \bar{h}(\mu, T)\phi + \frac{\bar{r}(\mu, T)\phi^2}{2} + \frac{u\phi^4}{4} + \dots}$$

Z_2 Symmetry

$$\bar{h} \to -\bar{h}\,, \bar{r} \to \bar{r}\,, \phi \to -\phi$$

Scaling

$$\phi \sim \bar{r}^{1/2}\,, \quad \bar{h} \sim \bar{r}^{3/2}\,, \quad \Omega - \Omega_0 \sim \bar{r}^2$$

Re-parametrization invariance

$$\phi \to f(\phi), \hat{\Omega}(\phi) = \Omega(f(\phi))$$

Effect of the lowest order asymmetric correction to scaling

(h, r) to (T, μ) must be independent of the parametrization of ϕ

$$P(\mu, T) = -\min_{\phi} \Omega(\phi; \mu, T)$$

$$\Omega(\phi; \mu, T) = \Omega_0(\mu, T) - \bar{h}\phi + \frac{\bar{r}\phi^2}{2} + \frac{u\phi^4}{4} + vu\phi^5 + O(\phi^6)$$

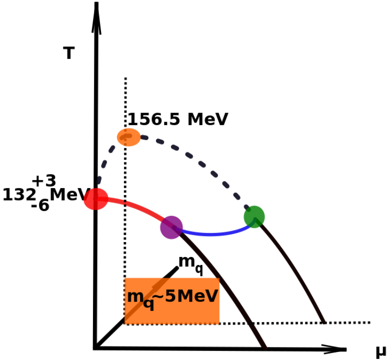
ϕ^5 can be absorbed by a reparameterization:

$$\phi^5 \sim \bar{r}^{5/2} \sim \bar{r}^{1/2}\,(\Omega - \Omega_0) \text{ and } \bar{h} \sim \bar{r}^{1/2}\bar{r}$$

$$\phi \to \phi + v\left(\frac{\bar{r}}{u} - \phi^2\right)$$

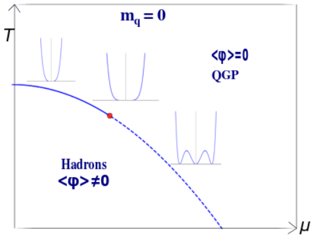
$$\Omega = \left(\Omega_0 - \frac{v\bar{h}\bar{r}}{u}\right) - \overbrace{\left(\mathbf{\bar{h}} - \frac{\mathbf{v\bar{r}^2}}{\mathbf{u}}\right)}^{\mathbf{h}(\mu,\mathbf{T})}\phi + \overbrace{\left(\mathbf{\bar{r}} + \mathbf{2v\bar{h}}\right)}^{\mathbf{r}(\mu,\mathbf{T})}\frac{\phi^2}{2} + \frac{u}{4}\phi^4 + \mathcal{O}(\phi^6, r^3)\,,$$

The world of light quarks

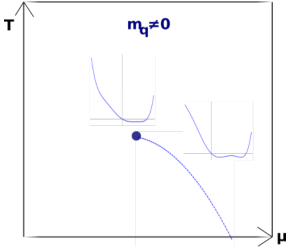


- Two quarks with $m_q << \Lambda_{QCD}$ + a heavy quark
- CP: ϕ^4 theory, fluctuations important in $d = 3$
- TCP: ϕ^6 theory , mean-field in $d = 3$
- Mean-field valid in a region not far from CP

Effective Landau Ginsburg potential for the chiral condensate



Schematic diagram for $m_q = 0$



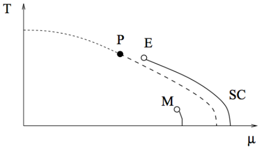
Schematic diagram for $m_q \neq 0$

$$P(\mu, T) = -\min_{\phi} V(\phi; \mu, T) - m_q\phi \text{ where } \frac{\partial P}{\partial m_q} = \phi$$

$$V(\phi; \mu, T) = \frac{a(\mu, T)}{2}\phi^2 + \frac{b(\mu, T)}{4}\phi^4 + \frac{c}{6}\phi^6 \dots$$

TCP at $m_q = 0 : a = b = \phi = 0$, CP at $m_q \neq 0$: $\phi_c = \left(\frac{3m_q}{8c}\right)^{1/5}$, $a_c = 5c\phi_c^4$, $b_c = -\frac{10c}{3}\phi_c^2$

Effective Landau Ginsburg potential near a critical point which is close to a tri-critical point



Phase diagram of QCD[*Stephanov-Rajagopal-Shuryak(1998)*]

$$\boxed{\phi_c \propto m_q^{1/5},\ a_c \propto \phi_c^4,\ b_c \propto \phi_c^2}$$

$$\Omega(\phi,\mu,T) = -m_q\phi + \frac{a(\mu,T)}{2}\phi^2 + \frac{b(\mu,T)}{4}\phi^4 + \frac{c}{6}\phi^6 \ldots$$

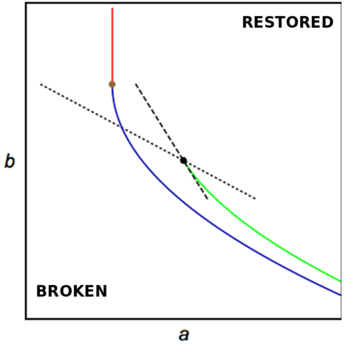
Expand about CP:

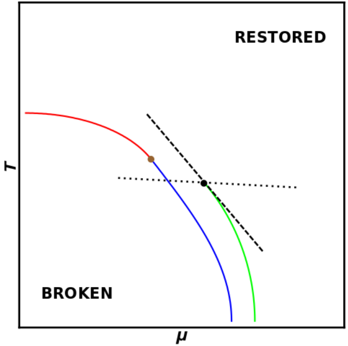
$$\Omega(\phi,\mu,T) = -\bar{h}\tilde{\phi} + \frac{1}{2}\bar{r}\tilde{\phi}^2 + \frac{u}{4}\tilde{\phi}^4 + vu\tilde{\phi}^5 + \tilde{\phi}^6, \boxed{\phi_c \sim m_q^{1/5},\ u \sim m_q^{2/5},\ vu \sim m_q^{1/5}}$$

$$\boxed{h \sim \left(\Delta a + \phi_c^2 \Delta b\right)\phi_c\,,\ r \sim \Delta a + \frac{27}{7}\phi_c^2\Delta b}$$

Scaling in the limit of small quark mass

$$\boxed{\tan\alpha_1 - \tan\alpha_2 = \frac{20}{7a_T^2}\frac{\partial(a,b)}{\partial(\mu,T)}\phi_c^2 \sim m_q^{2/5}}$$

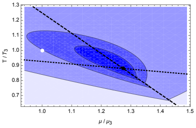




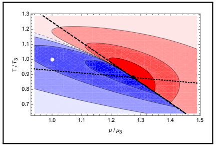
$$\alpha_1 > \alpha_2$$

An example: Random matrix model of Halasz et.al,1998 , $m_q = 5$ MeV

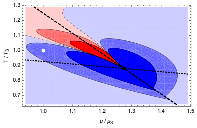
$\alpha_1 \approx 13°, \alpha_2 \approx 1°, \rho \approx 0.5, w \approx 1.4$



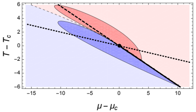
Contours of χ_2

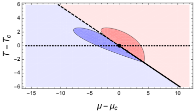


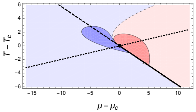
Contours of χ_3



Contours of χ_4

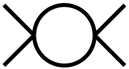






- If the value of skewness along the freeze-out curve is negative, it is indicative that the slope of h axis is negative.

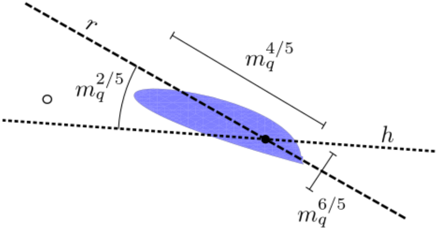
Mean field theory breaks down when

 $\sim$ 

$$u\xi^{\epsilon} \sim 1$$

$$\xi \sim m_q^{-2/5\epsilon}$$

Mean-field theory has a significant range of validity around the critical point and breaks down in a parametrically small region around it.



$$r \sim \xi^{-2}$$

$$h \sim \xi^{-3}$$

Affect of fluctuations on the mixing between r and h

In $d = 4$, two Z_2 odd perturbations of dimension 5

$$\phi^5 \text{ and } \phi^2 \nabla^2 \phi$$

Correction due to V_3 modifies the mapping

$$V_3 = u\phi^5 - \phi^2 \nabla^2 \phi$$

$$\Delta_3 = \beta\delta - 1 = 1/2 + O(\epsilon^2) \sim \log(h/r)$$

Correction due to V_5 doesn't

$$V_5 = u\phi^5 - (10S_5/3)\phi^2 \nabla^2 \phi \, , \; S_5 = O(\epsilon)$$

$$\Delta_5 = 1/2 + \epsilon + O(\epsilon^2)$$

In $d = 3$, $\Delta_5 \sim 1.3 - 1.6 > \Delta_3 \sim 0.56$

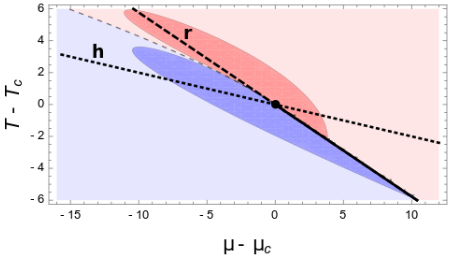
Fluctuations donot modify the scaling of slope difference with quark mass

$$\boxed{\tan\alpha_1 - \tan\alpha_2 = \frac{7}{10} C_{MF} \left(1 + S_5(\epsilon) + O(\epsilon^2)\right) m_q^{2/5}}$$

$$P_{\text{sing}} = -A\, r^{\beta(1+\delta)} \, \left(g(hr^{-\beta\delta}) + v_5\, r^{\Delta_5}\, g_5(hr^{-\beta\delta})\right) + \ldots$$

Phenomenological consequences

$$\boxed{\alpha_1 - \alpha_2 \sim m_q^{2/5} > 0}$$



- Sign of χ_3 along the cross-over line indicates the sign of the slope of h axis
- Enhanced baryon cumulants relative to the prediction based on the often favored assumption $h \perp \mu$, $\frac{\partial^2 P}{\partial \mu^2} \sim \frac{\partial^2 G}{\partial h^2} \sim r^{-\gamma} >> \frac{\partial^2 G}{\partial r^2} \sim r^{-\alpha}$
- Recently, ***M. Martinez, T. Schafer and V. Skokov, 2019*** showed that the smallness of α_2 also results in enhanced critical bulk viscosity.

Locating the critical point from Taylor expansion

- Radius of convergence ($\mu_R^2(T)$) is the distance to the closest singularity from $\mu = 0$
- Taylor expansion around $\mu = 0$,

$$P(\mu, T) = c_0(T) + c_2(T)\mu^2 + c_4(T)\mu^4 \ldots$$

- Convergence radius can be estimated from Taylor coefficients:

$$\boxed{\mu_R(T) = \lim_{n\to\infty} \left(\frac{c_{2n}(T)}{c_{2n+2}(T)}\right)^{1/2}}$$

- If C.P is the closest singularity, we expect to see $\boldsymbol{\mu_{\mathbf{R}}(\mathbf{T_c}) = \mu_{\mathbf{c}}}$.

In the next four slides, I'll show that the behavior of $\frac{c_{2n}(T)}{\mu_c^2 c_{2n+2}(T)}$ at sufficiently large n (need not be too large!) can be predicted universally.

Convergence radius when an Ising critical point is the closest singularity-I

- Very close to the C.P, along any line other than the first order phase transition curve, the leading behavior of critical part of pressure goes as:

$$P(\mu^2, T_c) \propto h^{\frac{1+\delta}{\delta}}$$

- One can invoke the map from (h, r) to (μ^2, T) to obtain:

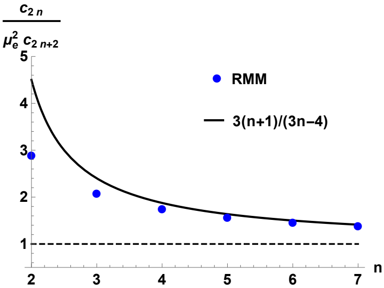
$$P(\mu^2, T_c) \propto \left(1 - \frac{\mu^2}{\mu_c^2}\right)^{a_p}$$

where $a_p = \frac{1+\delta}{\delta}$. At sufficiently large n,

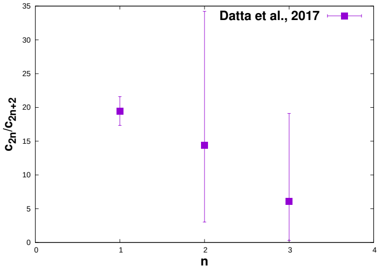
$$\boxed{\frac{c_{2n}(T_c)}{\mu_c^2 c_{2n+2}(T_c)} = 1 + \frac{a_p + 1}{n - a_p} + \dots}$$

If $a_p > -1$, the ratio overestimates μ_c^2 for finite n. In QCD, $a_p \approx 1.2$.

Testing the prediction and implications

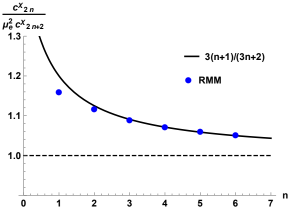


- $\frac{c_{2n}(T_c)}{\mu_c^2 c_{2n+2}(T_c)} = 1 + \frac{a_p+1}{n-a_p}$,
- $a_p = 4/3$
- Eg. $\frac{c_8}{c_{10}} \approx 2\mu_c^2$



- Above plot for $T_c = 0.94T_{\text{crossover}}$
- $a_p \approx 1.2 > -1$
- Can be used to extrapolate to the $n \to \infty$ limit

Convergence radius from Taylor coefficients of susceptibility



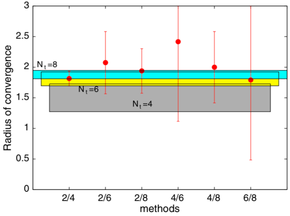


Figure: ***Datta et al., 2017***

- Close to C.P, $\chi \sim \left(1 - \frac{\mu^2}{\mu_c^2}\right)^{a_\chi}$ where $a_\chi = a_p - 2$

$$\boxed{\frac{c^{\chi}_{2n}(T_c)}{\mu_c^2 c^{\chi}_{2}(T_c)} = 1 + \frac{a_\chi + 1}{n - a_\chi}\,,\ a_\chi = a_p - 2}$$

$a_\chi = -2/3 > -1$ for RMM, $a_\chi \approx -0.8 > -1$ for QCD

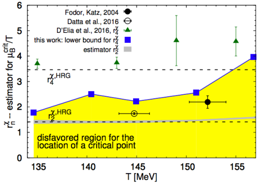
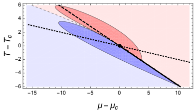


Figure: ***Bazavov et al., 2017***

Summary

$$\boxed{\alpha_1 - \alpha_2 = c\, m_q^{2/5},\ c > 0}$$

- Skewness could possibly be negative along the freeze-out curve
- Enhanced baryon number cumulants
- Enhanced transport coefficients

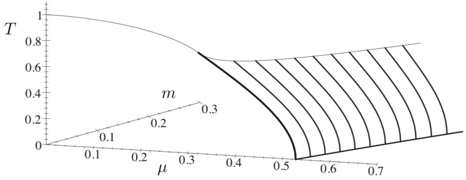


$$\boxed{\lim_{n\to\infty} \frac{c_{2n}(T_c)}{\mu_c^2 c_{2n+2}(T_c)} = 1 + \frac{a_p + 1}{n - a_p}}$$

- $a_p \approx 1.2 > -1$ and $a_\chi \approx -0.8 > -1$

Thank you!

Random Matrix Model



Phase diagram of RMM[*Halasz et al., 1998*]

- Chiral symmetry restoring phase transition at small m_q
- $(\mu_e(m_q), T_e(m_q))$ of Z_2 universality class
- Advantage: Analytically solvable

$$P(\mu, T, m_q) = -\min_{\phi} \Omega(\phi; \mu, T, m_q)$$

$$\Omega(\phi; \mu, T, m_q) = \phi^2 - \frac{1}{2} \ln \left[(\phi + m_q)^4 - 2(\mu^2 - T^2)\,(\phi + m_q)^2 + \left(\mu^2 + T^2\right)^2 \right]$$