School of Mathematics
Tata Institute of Fundamental Research

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Zoom Colloquium

Speaker	:	***Kingshook Biswas***
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Title	:	***Quasi-metric antipodal spaces and maximal Gromov hyperbolic spaces.***
Date & Time	:	***Thursday, 28 October 2021, 5:00 P.M. (Via Zoom)***

Abstract

A quasi-metric antipodal space (Z, ρ_0) is a compact space Z with a continuous quasi-metric ρ_0 which is of diameter one, and which is antipodal, i.e. for any $\xi \in Z$ there exists $\eta \in Z$ such that $\rho_0(\xi, \eta) = 1$. The quasi-metric ρ_0 defines a positive cross-ratio function on the space of quadruples of distinct points in Z, and a homeomorphism between quasi-metric antipodal spaces is said to be Moebius if it preserves cross-ratios.

A proper, geodesically complete Gromov hyperbolic space X is said to be boundary continuous if the Gromov inner product extends continuously to the boundary. Then the boundary ∂X equipped with a visual quasi-metric is a quasi-metric antipodal space. The space X is said to be maximal if for any proper, geodesically complete, boundary continuous Gromov hyperbolic space Y, if there is a Moebius homeomorphism $f : \partial Y \to \partial X$, then f extends to an isometric embedding $F : Y \to X$. We call such spaces maximal Gromov hyperbolic spaces.

We give an explicit description of all maximal Gromov hyperbolic spaces, in particular showing that they are contractible. We show that any proper, geodesically complete, boundary continuous Gromov hyperbolic space embeds isometrically into a maximal Gromov hyperbolic space which is unique up to isometry, and the image of the embedding is cobounded. We prove an equivalence of categories between quasi-metric antipodal spaces and maximal Gromov hyperbolic spaces, namely: any quasi-metric antipodal space is Moebius homeomorphic to the boundary of a maximal Gromov hyperbolic space, and any Moebius homeomorphism $f : \partial X \to \partial Y$ between boundaries of maximal Gromov hyperbolic spaces X, Y extends to a surjective isometry $F : X \to Y$. This is part of a more general equivalence between certain compact spaces called antipodal spaces and their associated metric spaces called Moebius spaces.

Milind Pilankar